Implementing the Mathematical Practices through Interesting Tasks

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Common Core State Standards

- Define what students should understand and be able to do in their study of mathematics. (p. 12)
  - Standards for Mathematical Practices
  - Standards for Mathematical Content

Common Core State Standards

Standards for Mathematical Practices

describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.

Common Core State Standards

If just adopt the content standards, little change will happen.

The Mathematical Practices are the heart of the matter.

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Standards for Mathematical Practices

1. Make sense of problems and persevere in solving them.

- explain to themselves the meaning of a problem and look for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- monitor and evaluate ….
- **… explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.**
- **ask themselves, "Does this make sense?"**
- understand the approaches of others … and identify correspondences between different approaches.

Standards for Mathematical Practices

2. Reason abstractly and quantitatively.

- make sense of quantities and their relationships …
- bring two complementary abilities …: to **decontextualiz**e and **contextualize**,
- Quantitative reasoning entails habits of
  1. **creating a coherent representation …**;
  2. considering the units involved;
  3. attending to the meaning of quantities,…;
  4. and **knowing and flexibly using** different properties of operation and objects.

Standards for Mathematical Practices

3. Construct viable arguments and critique the reasoning of others.

- **understand and use stated assumptions, definitions, and previously established results ….**
- **make conjectures and build a logical progression of statements to explore the truth of their conjectures.**
- **recognize and use counterexamples.**
- **justify their conclusions, communicate them to others, and respond to the arguments of others.**

Standards for Mathematical Practices

4. Model with mathematics.

- apply the mathematics they know to solve problems
- identify important quantities … and **map their relationships using such tools** as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- **interpret their mathematical results in the context of the situation and reflect on whether the results make sense**

Standards for Mathematical Practices

5. Use appropriate tools strategically.

- consider the available tools when solving a mathematical problem. … pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.
- **are sufficiently familiar with tools … to make sound decisions about when**
- detect possible errors … using estimation …
- know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
- able to identify relevant external mathematical resources, … and use them to pose or solve problems.
- are able to use technological tools to …understanding of concepts.

Standards for Mathematical Practices

6. Attend to precision.

- **try to communicate precisely to others.**
- try to use clear definitions ….
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure, and label axes …
- calculate accurately and efficiently, with a degree of precision appropriate for the problem context.

Standards for Mathematical Practices

7. Look for and make use of structure

- **look closely to discern a pattern or structure.**
- recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- **can step back for an overview and shift perspective.**

Standards for Mathematical Practices

8. Look for and express regularity in repeated reasoning.

- notice if calculations are repeated, and **look for general methods shortcuts.**
- **maintain oversight of the process, while attending to the details.**
- **evaluate the reasonableness of their intermediate results.**

Implementing the Mathematical Practices through Interesting Tasks

- Consecutive Number Problem
- The Pool Border Problem
- The 8's Problem

Consecutive Number Problem

What Numbers Can Be Written as the Sum of Consecutive Natural Numbers?

Some numbers, such as 21 can be written as the sum of 2 natural numbers:

21 = 10 + 11.

Some numbers can be written as the sum of consecutive natural numbers in more than one way:

39 = 19 + 20

39 = 12 + 13 + 14

Consecutive Number Problem

Some numbers cannot be written as the sum of 2 consecutive natural numbers.

Determine which numbers less than 30 can be written as the sum of consecutive natural numbers. As you make this determination, keep a record of all your work. We will need to refer to that as we discuss the problem.

After exploring the possibilities, **make some conjectures related to this task.**

What Generalizations?

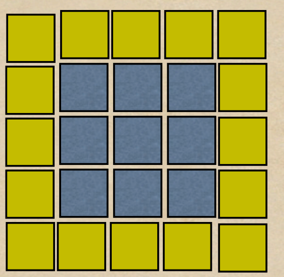
Text

The Pool Border Problem

- Tat Ming is designing square swimming pools. Each pool has a square center that is the area of the water. Around each pool there is a border of square tiles. How can you determine the number of tiles in the border for any size square pool?

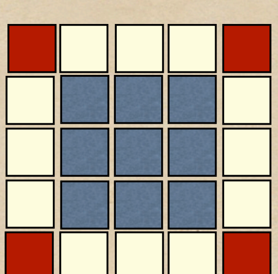
- Ferrini-Mundy, Joan; Lappan, Glenda; and Phillips, Elizabeth. “Experiences with Patterning” *Teaching Children Mathematics* 3 (February 1997): 282–89.

Pool Border Problem



Pool Border Problem

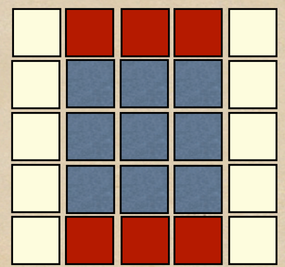
4(3) + 4



4s + 4

Pool Border Problem

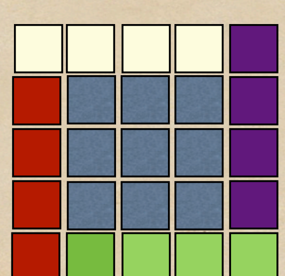
$2(5) + 2(3)$



$2(s + 2) + 2s$

Pool Border Problem

$4\ (4)$



$4(s + 1)$

The 8’s Problem

How many different ways can you make 1000 using numbers with the numeral 8 and the operation of addition?

The 8’s Problem

What numbers can we use?

8? 88? 888? 8888?

ETA Hands2Mind

Common Core State Standards in Mathematics |ETA/Cuisenaire

http://www.etacuisenaire.com/ccss/visual-supersource.jsp

The Super Source

Color Tiles Grades 5-6

1. Make sense of problems and persevere in solving them.
Questions which encourage students to think about all possible solutions encourage perseverance by having students keep looking for solutions even when it becomes a challenge.

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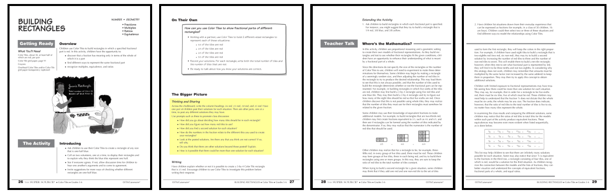
BUILDING RECTANGLES

BUILDING RECTANGLES

NUMBER • GEOMETRY

- Fractions
- Multiples
- Ratios
- Equivalence

Getting Ready

What You'll Need

Color Tiles, about 30, at least half of which are red, per pair

Color Tile grid paper, page 91

Crayons

Overhead Color Tiles and/or Color Tile grid paper transparency (optional)

Overview

Children use Color Tiles to build rectangles in which a specified fractional part is red. In this activity, children have the opportunity to:

- discover that a fraction has meaning only in terms of the whole of which it is a part
- find different ways to represent the same fractional part
- recognize multiples, equivalence, and ratios

The Activity

Introducing

- Ask children to use their Color Tiles to create a rectangle of any size that is one-half blue.
- Call on two volunteers, one at a time, to display their rectangles and to explain why they think the blue tiles represent one half.
- See if everyone agrees. If not, allow discussion time for children to hear one another's arguments and to come to agreement.
- Invite suggestions for more ways of checking whether different rectangles are one-half blue.

On Their Own

How can you use Color Tiles to show fractional parts of different rectangles?

- Working with a partner, use Color Tiles to build 2 different-sized rectangles to represent each of these situations:
  - 5/6 of the tiles are red
  - 2/3 of the tiles are red
  - 2/8 of the tiles are red
  - 3/5 of the tiles are red
- Record your solutions. For each rectangle, write both the total number of tiles and the number of tiles that are red.
- Be ready to talk about how you know your solutions are correct.

The Bigger Picture

Thinking and Sharing

Across the chalkboard, write the column headings *5/6 red, 2/3 red, 2/8 red,* and *3/5 red.* Have one pair of children post their solutions for each situation. Then ask other pairs, one at a time, to post any different solutions they may have.

Use prompts such as these to promote class discussion:

- How did you go about deciding how many tiles should be in each rectangle?
- How did you figure out how many red tiles to use?
- How did you find a second solution for each situation?
- How do the numbers in the fraction relate to the different tiles you used to create your rectangles?
- Look at the posted solutions. Are there any that you think are not correct? If so, tell why.
- Do you think that there are other solutions beyond those posted? Explain.
- How is it possible that there could be more than one solution for each situation?

Writing

Have children explain whether or not it is possible to create a 3-by-4 Color Tile rectangle that is 3/5 red. Encourage children to use Color Tiles to investigate this problem before writing their response.

Extending the Activity

1. Ask children to build rectangles in which each fractional part is specified. For instance, you might suggest that they try to build a rectangle that is 1/4 red, 3/8 blue, and 3/8 yellow.

Teacher Talk

Where's the Mathematics?

In this activity, children use proportional reasoning and a geometric setting to create their own models of fractional representations. As they build rectangles and test to see whether their rectangles fit the given conditions, children have an opportunity to enhance their understanding of what is meant by a fractional part of a whole.

Since the directions do not specify the size of the rectangles or the number of Color Tiles to use, children will need to experiment to make these determinations for themselves. Some children may begin by making a rectangle of a seemingly random size, and then adjusting the number of red tiles in the rectangle to try to produce the desired relationship. This may lead them to see that this is not always possible, and that the number of tiles used to build the rectangle determines whether or not the fractional part can be represented. For example, in building rectangles in which five sixths of the tiles are red, children may first build a 2-by-3 rectangle using five red tiles and one blue tile. They may then build a 2-by-4 rectangle and try to figure out how many of the eight tiles should be red so that five sixths are red. Once children discover that this is not possible using whole tiles, they may realize that the number of tiles they must use for their rectangles must somehow be related to the given fraction.

Some children may use their knowledge of equivalent fractions to make additional models. For example, to build rectangles that are two-thirds red, children may first create fractions equivalent to 2/3, such as 4/6 and 6/9, and then see if rectangles can be formed using the number of tiles indicated by the denominator. If so, they may realize that the numerator is the number of red tiles that should be used.

Other children may realize that for a rectangle to be, for example, three-fifths red, in every group of five tiles used, three must be red. These children may form groups of five tiles, three in each being red, and try to build their rectangles using one or more groups. In this way, they are sure to keep the ratio of red tiles to the total number of tiles constant.

When trying to build a second rectangle for a given situation, some children may think that if they add one red and one non-red tile to the set of tiles

2. Have children list situations drawn from their everyday experience that can be expressed as fractions (for example, in a class of 30 children, 16 are boys). Children could then select two or three of these situations and find different ways to model the relationships using Color Tiles.

used to form the first rectangle, they will keep the colors in the right proportion. For example, if children have used eight tiles to build a rectangle that is two-eighths red (two red, six non-red), they may try to build a second solution by increasing the number of red tiles to three and the number of non-red tiles to seven. This will enable them to build a ten-tile rectangle; however, when they test to see what fractional part is represented by red, they will find it to be three tenths and not two eighths. In considering why this strategy does not work, children may remember that amounts must be multiplied by the same factor (not increased by the same addend) to keep them in proportion. They may then try to apply this concept to obtain additional solutions.

Children with limited exposure to fractional representations may have trouble seeing how there could be more than one solution for each situation. They may say, for example, that in order for a rectangle to be five-sixths red, there must be six tiles, five of which must be red. These children may need help to understand that the fraction $5/6$ does not dictate that the whole must be six units; the whole may be any size. The fraction does indicate, however, that the ratio of red tiles to the total number of tiles is five to six, no matter how many tiles represent the whole.

In examining the class results and comparing the different solutions, some children may notice that the ratios of red tiles to total tiles for the models within each part of the activity produce equivalent fractions. These equivalences may become even more evident when listed sequentially, as is done below.

$5/6$	=	$10/12$	=	$15/18$	=	$20/24$	=	$25/30$	=	...
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$3/5$	=	$6/10$	=	$9/15$	=	$12/20$	=	$15/25$	=	...

This list may help children to see that there are infinitely many solutions possible for each situation. Some may also notice that since $1/4$ is equivalent to the fractions in the third row, a rectangle consisting of four tiles, one of which is red, would be a solution for the third situation. As children recognize the connection between the models and the lists of fractions, they can better visualize and understand the concepts of equivalent fractions, fractional parts of a whole, and equal ratios.

The Super Source

Color Tiles Grades 5-6

2. Reason abstractly and quantitatively.
This description of student thinking is an example of abstract and quantitative reasoning. We can see a student using what they understand about proportional relationships to model equivalent fractions.

The Super Source

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3. Construct viable arguments and critique the reasoning of others.
Students are prompted not just to find solutions but also to be prepared to explain their reasoning.

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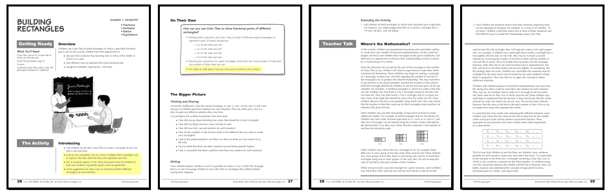

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Where's the Mathematics?

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The Super Source

Color Tiles Grades 5-6

4. Model with mathematics.
This question asks students to use mathematics in descriptive modeling - connect the mathematical model of color tiles & fractions to a variety of real-world situations. They can then take this knowledge and apply it to using mathematics to solve real world problems.

The Super Source

Color Tiles Grades 5-6

5. Use appropriate tools strategically.
The stage is set for students to use manipulatives (color tiles) as part of their strategy for completing this task. The teacher models for students how the tool supports successful completion of the task and the use of tools in explaining mathematical reasoning.

Educational Manipulatives &
Supplemental Materials for PreK-Grade 12.

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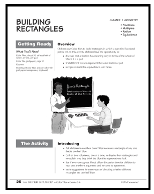

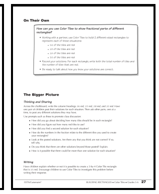
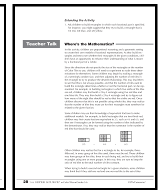
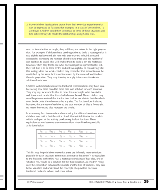

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Since the directions do not specify the size of the rectangles or the number of Color Tiles to use, children will need to experiment to make these determinations for themselves. Some children may begin by making a rectangle of a seemingly random size, and then adjusting the number of red tiles in the rectangle to try to produce the desired relationship. This may lead them to see that this is not always possible, and that the number of tiles used to build the rectangle determines whether or not the fractional part can be represented. For example, in building rectangles in which five sixths of the tiles are red, children may first build a 2-by-3 rectangle using five red tiles and one blue tile. They may then build a 2-by-4 rectangle and try to figure out how many of the eight tiles should be red so that five sixths are red. Once children discover that this is not possible using whole tiles, they may realize that the number of tiles they must use for their rectangles must somehow be related to the given fraction.

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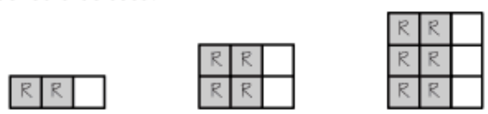

2/3 = 4/6 = 6/9

Other children may realize that for a rectangle to be, for example, three-fifths red, in every group of five tiles used, three must be red. These children may form groups of five tiles, three in each being red, and try to build their rectangles using one or more groups. In this way, they are sure to keep the ratio of red tiles to the total number of tiles constant.

When trying to build a second rectangle for a given situation, some children may think that if they add one red and one non-red tile to the set of tiles

2. Have children list situations drawn from their everyday experience that can be expressed as fractions (for example, in a class of 30 children, 16 are boys). Children could then select two or three of these situations and find different ways to model the relationships using Color Tiles.

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This list may help children to see that there are infinitely many solutions possible for each situation. Some may also notice that since $1/4$ is equivalent to the fractions in the third row, a rectangle consisting of four tiles, one of which is red, would be a solution for the third situation. As children recognize the connection between the models and the lists of fractions, they can better visualize and understand the concepts of equivalent fractions, fractional parts of a whole, and equal ratios.

The Super Source

Color Tiles Grades 5-6

6. Attend to precision.
By asking students to write about a challenging task, they are pushed to be precise in their description of the situation and its (non-) solution. They must use pictures, numbers, and words to describe why the problem posed is impossible.

The Super Source

Color Tiles Grades 5-6

7. Look for and make use of structure.
Students will express what they've noticed about the various situations (choosing rectangle size, finding a second solution) and in doing this will share the structure they're finding in the situations.

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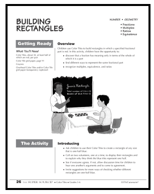
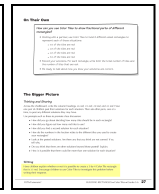
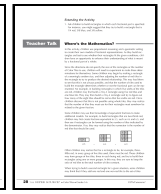
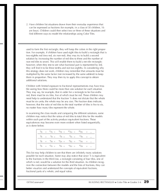

BUILDING RECTANGLES

NUMBER • GEOMETRY

- Fractions
- Multiples
- Ratios
- Equivalence

Getting Ready

What You'll Need

Color Tiles, about 30, at least half of which are red, per pair

Color Tile grid paper, page 91

Crayons

Overhead Color Tiles and/or Color Tile grid paper transparency (optional)

Overview

Children use Color Tiles to build rectangles in which a specified fractional part is red. In this activity, children have the opportunity to:

- discover that a fraction has meaning only in terms of the whole of which it is a part
- find different ways to represent the same fractional part
- recognize multiples, equivalence, and ratios

The Activity

Introducing

- Ask children to use their Color Tiles to create a rectangle of any size that is one-half blue.
- Call on two volunteers, one at a time, to display their rectangles and to explain why they think the blue tiles represent one half.
- See if everyone agrees. If not, allow discussion time for children to hear one another's arguments and to come to agreement.
- Invite suggestions for more ways of checking whether different rectangles are one-half blue.

On Their Own

How can you use Color Tiles to show fractional parts of different rectangles?

- Working with a partner, use Color Tiles to build 2 different-sized rectangles to represent each of these situations:
  - 5/6 of the tiles are red
  - 2/3 of the tiles are red
  - 2/8 of the tiles are red
  - 3/5 of the tiles are red
- Record your solutions. For each rectangle, write both the total number of tiles and the number of tiles that are red.
- Be ready to talk about how you know your solutions are correct.

The Bigger Picture

Thinking and Sharing

Across the chalkboard, write the column headings *5/6 red, 2/3 red, 2/8 red,* and *3/5 red.* Have one pair of children post their solutions for each situation. Then ask other pairs, one at a time, to post any different solutions they may have.

Use prompts such as these to promote class discussion:

- How did you go about deciding how many tiles should be in each rectangle?
- How did you figure out how many red tiles to use?
- How did you find a second solution for each situation?
- How do the numbers in the fraction relate to the different tiles you used to create your rectangles?
- Look at the posted solutions. Are there any that you think are not correct? If so, tell why.
- Do you think that there are other solutions beyond those posted? Explain.
- How is it possible that there could be more than one solution for each situation?

Writing

Have children explain whether or not it is possible to create a 3-by-4 Color Tile rectangle that is 3/5 red. Encourage children to use Color Tiles to investigate this problem before writing their response.

Extending the Activity

1. Ask children to build rectangles in which each fractional part is specified. For instance, you might suggest that they try to build a rectangle that is 1/4 red, 3/8 blue, and 3/8 yellow.

Teacher Talk

Where's the Mathematics?

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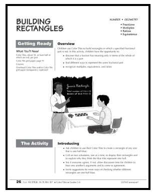

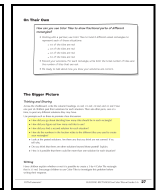

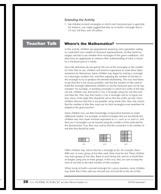

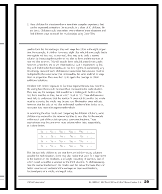

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The Super Source
Color Tiles Grades 5-6

8. Look for and express regularity in repeated reasoning.
The solution relates to equivalent fractions and the fact that there is an infinite variety of fractions which are equivalent to the given fraction. Students are seeing the regularity and repeated reasoning of equivalent fractions when they are able to respond to this question appropriately.

Contact Me

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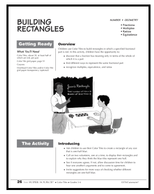
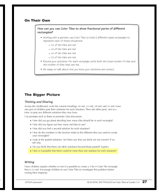
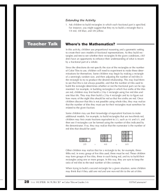
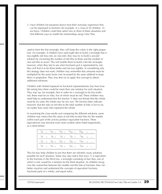

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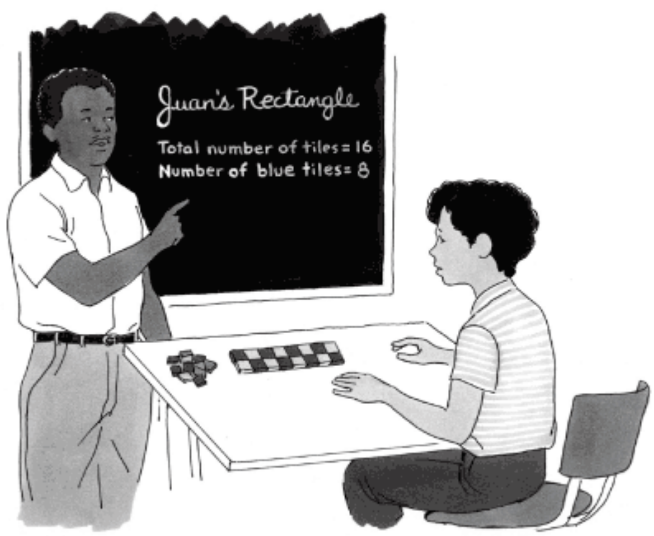

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In this activity, children use proportional reasoning and a geometric setting to create their own models of fractional representations. As they build rectangles and test to see whether their rectangles fit the given conditions, children have an opportunity to enhance their understanding of what is meant by a fractional part of a whole.

Since the directions do not specify the size of the rectangles or the number of Color Tiles to use, children will need to experiment to make these determinations for themselves. Some children may begin by making a rectangle of a seemingly random size, and then adjusting the number of red tiles in the rectangle to try to produce the desired relationship. This may lead them to see that this is not always possible, and that the number of tiles used to build the rectangle determines whether or not the fractional part can be represented. For example, in building rectangles in which five sixths of the tiles are red, children may first build a 2-by-3 rectangle using five red tiles and one blue tile. They may then build a 2-by-4 rectangle and try to figure out how many of the eight tiles should be red so that five sixths are red. Once children discover that this is not possible using whole tiles, they may realize that the number of tiles they must use for their rectangles must somehow be related to the given fraction.

Some children may use their knowledge of equivalent fractions to make additional models. For example, to build rectangles that are two-thirds red, children may first create fractions equivalent to 2/3, such as 4/6 and 6/9, and then see if rectangles can be formed using the number of tiles indicated by the denominator. If so, they may realize that the numerator is the number of red tiles that should be used.

Other children may realize that for a rectangle to be, for example, three-fifths red, in every group of five tiles used, three must be red. These children may form groups of five tiles, three in each being red, and try to build their rectangles using one or more groups. In this way, they are sure to keep the ratio of red tiles to the total number of tiles constant.

When trying to build a second rectangle for a given situation, some children may think that if they add one red and one non-red tile to the set of tiles

2. Have children list situations drawn from their everyday experience that can be expressed as fractions (for example, in a class of 30 children, 16 are boys). Children could then select two or three of these situations and find different ways to model the relationships using Color Tiles.

used to form the first rectangle, they will keep the colors in the right proportion. For example, if children have used eight tiles to build a rectangle that is two-eighths red (two red, six non-red), they may try to build a second solution by increasing the number of red tiles to three and the number of non-red tiles to seven. This will enable them to build a ten-tile rectangle; however, when they test to see what fractional part is represented by red, they will find it to be three tenths and not two eighths. In considering why this strategy does not work, children may remember that amounts must be multiplied by the same factor (not increased by the same addend) to keep them in proportion. They may then try to apply this concept to obtain additional solutions.

Children with limited exposure to fractional representations may have trouble seeing how there could be more than one solution for each situation. They may say, for example, that in order for a rectangle to be five-sixths red, there must be six tiles, five of which must be red. These children may need help to understand that the fraction $5/6$ does not dictate that the whole must be six units; the whole may be any size. The fraction does indicate, however, that the ratio of red tiles to the total number of tiles is five to six, no matter how many tiles represent the whole.

In examining the class results and comparing the different solutions, some children may notice that the ratios of red tiles to total tiles for the models within each part of the activity produce equivalent fractions. These equivalences may become even more evident when listed sequentially, as is done below.

$5/6$	=	$10/12$	=	$15/18$	=	$20/24$	=	$25/30$	=	...
$2/3$	=	$4/6$	=	$6/9$	=	$8/12$	=	$10/15$	=	...
$2/8$	=	$4/16$	=	$6/24$	=	$8/32$	=	$10/40$	=	...
$3/5$	=	$6/10$	=	$9/15$	=	$12/20$	=	$15/25$	=	...

This list may help children to see that there are infinitely many solutions possible for each situation. Some may also notice that since $1/4$ is equivalent to the fractions in the third row, a rectangle consisting of four tiles, one of which is red, would be a solution for the third situation. As children recognize the connection between the models and the lists of fractions, they can better visualize and understand the concepts of equivalent fractions, fractional parts of a whole, and equal ratios.