Using Quadratic Equations to Build Calder Mobiles

by Danielle Passno
The Spence School
New York, NY
dpassno@spenceschool.org

Alexander Calder

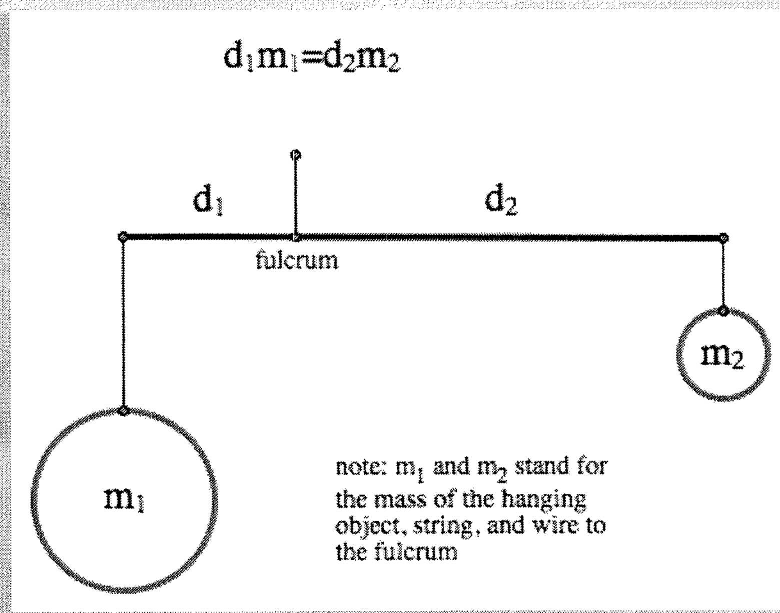
- American sculptor (1898 – 1976)
- Family of artists yet Calder also studied as an engineer
- Created sculptures using wire, thus creating line drawings in three dimensions
- Most famous works: mobiles and Cirque Calder

Resources

- www.calder.org - The Calder Foundation website that offers biographies of Calder, images of his art, his journal entries about his art, and essays on his art
- *The Way Things Work* by David Macaulay – a book I loved as a child; it offers an easy-to-understand description of levers as well as interesting drawings to illustrate the ideas
- I can also be a resource to you – dpassno@spenceschool.org

How Levers Work

- The product of the mass and distance on one side of the fulcrum must equal the product of the mass and distance on the other side of the fulcrum.

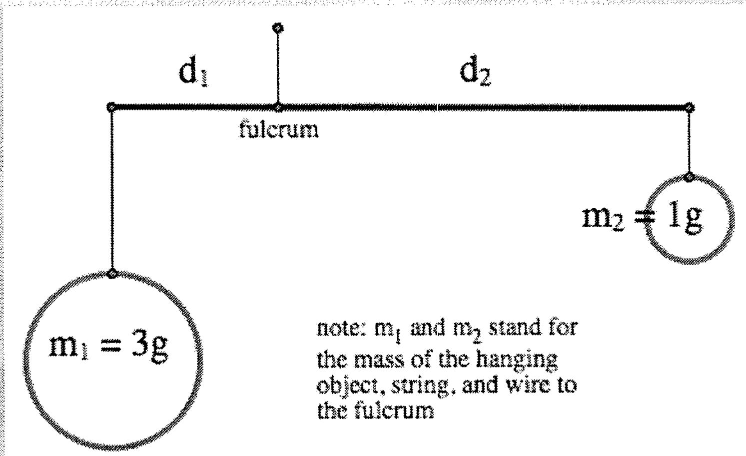


WARNING!

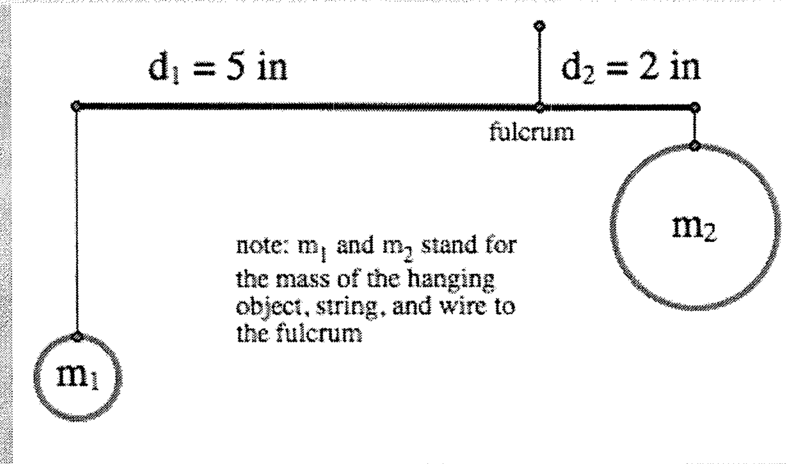
- To simplify this project, I consistently ignore the distribution of the mass along the wire on each side of the fulcrum. How the mass is distributed along the wire is negligible when building the mobiles.

Practice Lever Problems #1

Given the following scenarios, use inequalities to relate m_1 and m_2 and to relate d_1 and d_2.



$m_1 > m_2$
$d_1 < d_2$



$d_1 > d_2$
$m_1 < m_2$

Practice Lever Problems #2

If $d_1 = 2$ inches and $d_2 = 4$ inches and if the mass of the total tier is 10 g, what should be the mass of m_1 and m_2?

$m_1 + m_2 = 10$
$2m_1 = 4m_2$

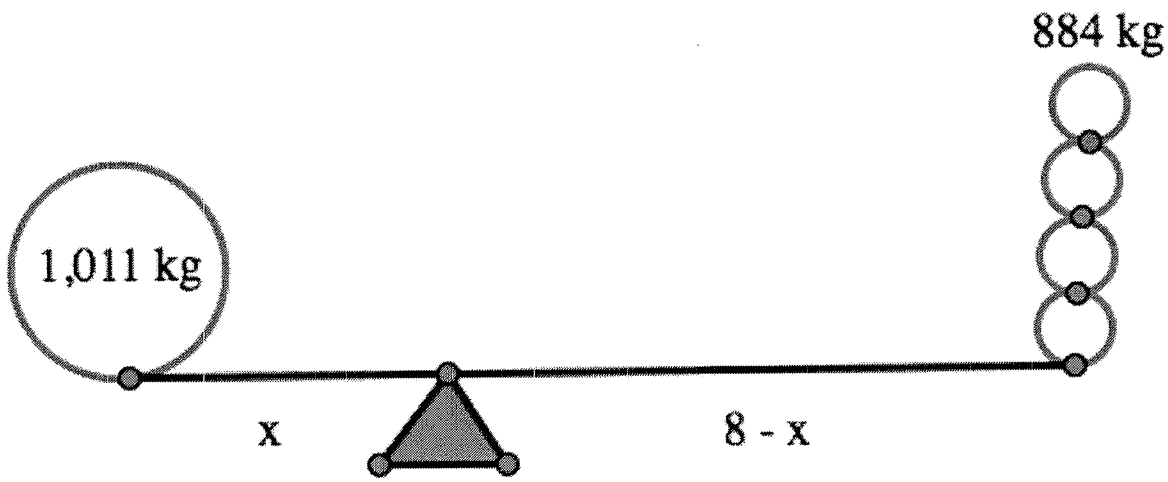
Using substitution...
$2m_2 + m_2 = 10$

$$m_2 = \frac{10}{3}\,g \text{ and } m_1 = \frac{20}{3}\,g$$

Lever Homework #2

A log of length 8 meters is resting asymmetrically on top of a rock. Each meter of the log has a mass of 25 kg. A single parent woolly mammoth, with a mass of 1,011 kg, sits on one end of the log, and four of his woolly mammoth babies, each with a mass of 221 kg, sit on the other end of the log. Where is the fulcrum placed if the daddy woolly mammoth and the baby woolly mammoths are perfectly balanced on the log?

Lever Homework #2



$m_1 = 1{,}011 + 25x$
$d_1 = x$
$m_2 = 884 + 25(8 - x) = 1{,}084 - 25x$
$d_2 = 8 - x$

$x(1{,}011 + 25x) = (8 - x)(1{,}084 - 25x)$
$25x^2 + 1{,}011x = 25x^2 - 1{,}284x + 8{,}672$
Notice that we can cancel the x^2 terms.
$x = 3.78$ m

Lever Homework #4

A mobile has a 5 g mass and a 9 g mass hanging from each side. The distance from the fulcrum to the 9g mass is 4 in. The distance from the fulcrum to the 5g mass is x in.

- Find the total mass on each side of the fulcrum if the mass of the wire is 0.5 g per 1 in.
- Write an equation that represents the mobile in balance.
- Solve the equation for x using the quadratic formula.

Lever Homework #4

a)
$m_1 = 9 + 4(0.5) = 11$
$m_2 = 5 + 0.5x$

b)
$m_1 = 11$
$d_1 = 4$
$m_2 = 5 + 0.5x$
$d_2 = x$

$4(11) = x(5 + 0.5x)$
$0 = 0.5x^2 + 5x - 44$

c) $x = 5.63$ in

- Notice that the only way to create a quadratic equation is to solve for one of the distances when the total length of the wire is unknown.

Positive Points of Project

- Creative application of solving quadratic equations
- Easy interdisciplinary connections (physics and art)
- Inexpensive materials
- Mobiles can be displayed in school with explanations of calculations

Some drawbacks…

- Some important physical concepts are ignored in the project
- Construction can be challenging for students (and teachers!) with poor motor skills and/or little patience

Alexander Calder Mobile Project

Day 1 – The Life of Alexander Calder
HW: Reading & Journal
Day 2 – Exploring Levers
HW: Lever WS 1
Day 3 – Building Mobiles – Part I
HW: Finish Construction of Hanging Forms
Day 4 – Using Quadratic Equations
HW: Lever WS 2
Day 5 – Mobile Calculations
HW: Finish Calculations and Diagrams of Mobile
Day 6 – Building Mobiles – Part II
HW: Reflection on Project

Calculations: Drawing accurate diagrams and completing accurate calculations is essential to the success of your mobile. As you build your mobile, you will need to complete calculations to determine distances. I will collect all of these drawings and calculations at the end of the project.

Mobile: After we explore Calder's mobiles, you will have the opportunity to build your own. You will be responsible for bringing in some of the materials for the project (such as wire clothes hangers).

Materials: - wire, pen, mechanical pencil, poster board (or other hangable object), ruler, string, scissors, scale

Grading for Mobile Project

-Each homework assignment will be collected and graded on a 5-point scale.
-Your mobile and calculations are worth half a test grade (20 points) and will be graded according to the following rubric.

Mobile	Did you build a 3-tier mobile? Is it balanced?	8
Calculations	Are your calculations correct? Did you turn in a legible copy of your calculations? Did you turn in a diagram of your mobile with labeled distances and masses?	8
Aesthetics	Is your mobile well crafted and intriguing? Is it unique?	4
Comments		Final Grade (20)

Algebra I
Calder Mobile Project
Homework

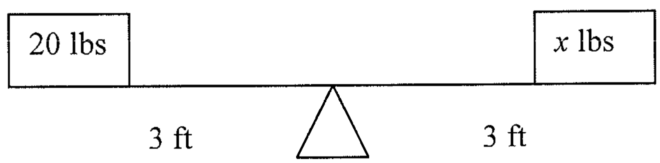
Name____

Levers

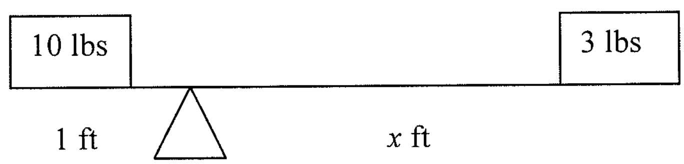
(You may use a calculator to help you with the arithmetic in these problems.)

1) Find a value for x that will keep the seesaw balanced. (The measurement in lbs includes the mass of the seesaw from fulcrum to end.

a.



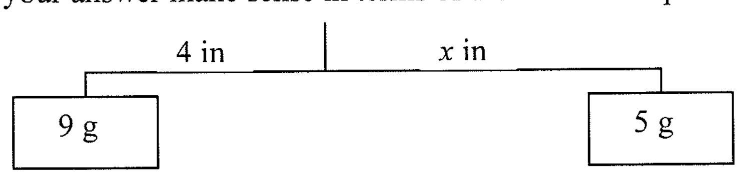
b.



2) A log of length 8 meters is resting asymmetrically on top of a rock. Each meter of the log has a mass of 25 kg. A single parent woolly mammoth, with a mass of 1,011 kg, sits on one end of the log, and four of his woolly mammoth babies, each with a mass of 221 kg, sit on the other end of the log. Where is the fulcrum placed if the daddy woolly mammoth and the baby woolly mammoths are perfectly balanced on the log?

3) You are building a mobile in your 8^{th} grade algebra class. As you design your mobile, you find that the mass on one side of your fulcrum is 12 g. The fulcrum is placed 2 inches to the right of the 12 g mass, which is a distance of 1/3 of the entire tier. How much mass needs to be on the other side of the fulcrum?

4) The following mobile has a 5 g mass and a 9 g mass hanging from each side.
   a. Find the total mass on each side of the fulcrum if the mass of the wire is 0.5 g per 1 in.
   b. Write an equation that represents the mobile in balance.
   c. Solve the equation for x using the quadratic formula.
   d. Does your answer make sense in terms of the mobile? Explain how you know.



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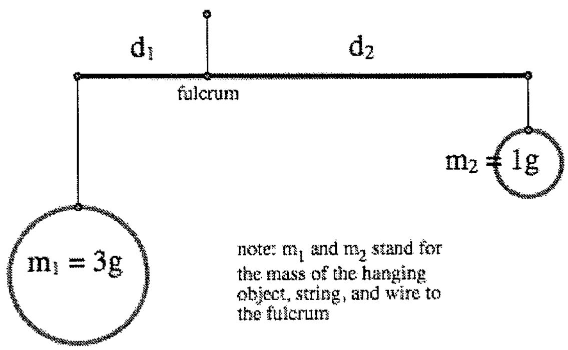
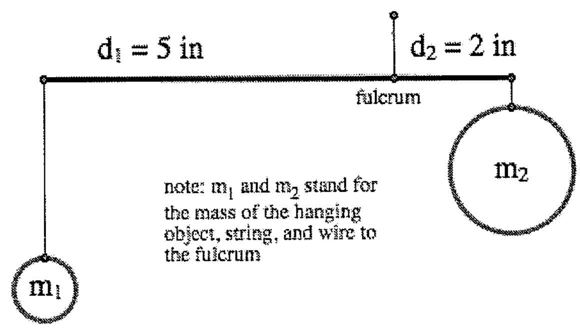
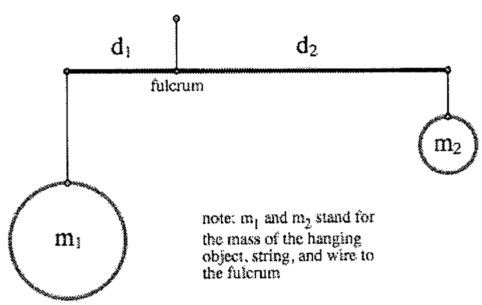
Algebra I
Calder Mobile Project
Practice Lever Problems

1) Given the following scenarios, use inequalities to relate m_1 and m_2 and to relate d_1 and d_2.







2) If $d_1 = 2$ inches and $d_2 = 4$ inches and if the mass of the total tier is 10 g, what should be the mass of m_1 and m_2?

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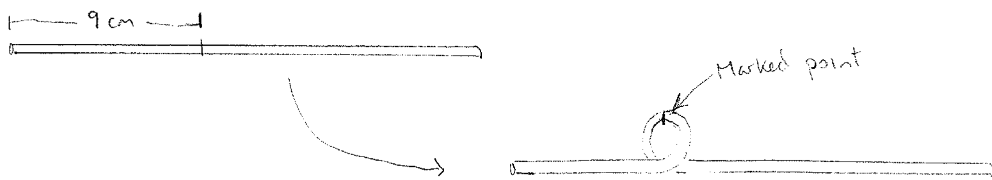
3) When constructing a mobile, you find that one of your construction paper cutouts is 6 grams. You find the other to be 10 grams. The total length of the wire used for the tier is 7 inches. The mass of each inch of wire is 2 grams. Find d_1 and d_2.

4) When constructing a mobile, you find that one of your construction paper cutouts is 5 grams. You find the other to be 6 grams. The distance from the 5-gram mass to the fulcrum is 4 inches. The mass of each inch of wire is 2 grams. Find d_2.

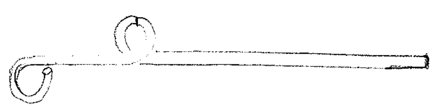
Step-by-step guide to building a mobile

Assumptions:
The mass of one of your hanging objects (including the string attached to the object) is 1.3 g.
The mass of your other hanging object (including the attached string) is 1.04 g.
The mass of 1 cm of wire is .067 g.
You are creating loops in your wire using a standard mechanical pencil.

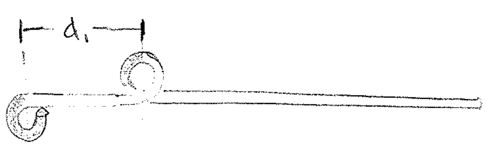
Begin by measuring 9 cm (an arbitrary distance) of wire from tip of wire to 9 cm out on the wire. Mark a point on the wire. This will be the center of the fulcrum. Take your pencil and wrap the wire around the pencil, keeping the marked point equally in the center as you wrap.



Use the clip on your pencil to create a loop at the very end of the wire.



Measure the distance from the fulcrum to point where the object will hang from the loop (basically the middle of the loop).



If $d_1 = 5$ cm, then the shaded wire must be 4 cm.

This distance will be your d_1 value ($d_1 = 5$ cm). Notice that is about 4 cm less than the original marked distance. This means that the loop on the end of the wire and half of the fulcrum loop make up 4 cm of wire.

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Now let's focus on the mass on this side of the fulcrum. Not only will this mass include the mass of one of your hanging objects but also the mass of the amount of wire you have on that side of the fulcrum. Remember that you have 9 inches of wire to consider when creating the mass.

$m_1 =$ mass of hanging object + mass of wire

$m_1 = 1.30 + .067(9)$

$m_1 = 1.903$ grams

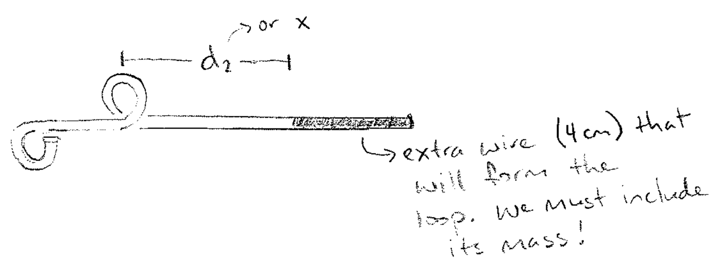
Thus, $d_1 m_1 = 5(1.903) = 9.515$.

Now let's consider the other side of the mobile. We want to solve for what the distance should be for this other side of the mobile. We'll call this unknown distance *x*, or our d_2 value.

What about the mass on this side of the object? We know the mass should be $m_2 =$ mass of hanging object + mass of wire. The mass of the hanging object is given to be 1.04 g. The mass of our wire should be the mass of one inch of wire multiplied by how many cm of wire we have. We've said $d_2 = x$, but *remember that we also need to include the mass of the wire that will make up half of the fulcrum loop and the hanging loop*, which is 4 cm of wire. Thus the mass of the wire becomes: $.067(x+4)$.

$m_2 = 1.04 + .067(x+4)$

$m_2 = .067x + 1.308$



Thus, $d_2 m_2 = x(.067x + 1.308)$.

Because $d_1 m_1 = d_2 m_2$, we know that $9.515 = x(.067x + 1.308)$, a quadratic equation. We can solve the quadratic equation as follows:

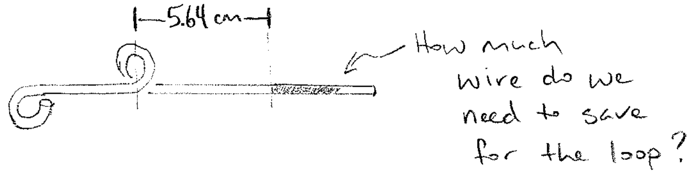
$$9.515 = .067x^2 + 1.308x$$

$$0 = .067x^2 + 1.308x - 9.515$$

Use the zero feature of a graphing calculator or the quadratic formula to solve for the positive value of x.

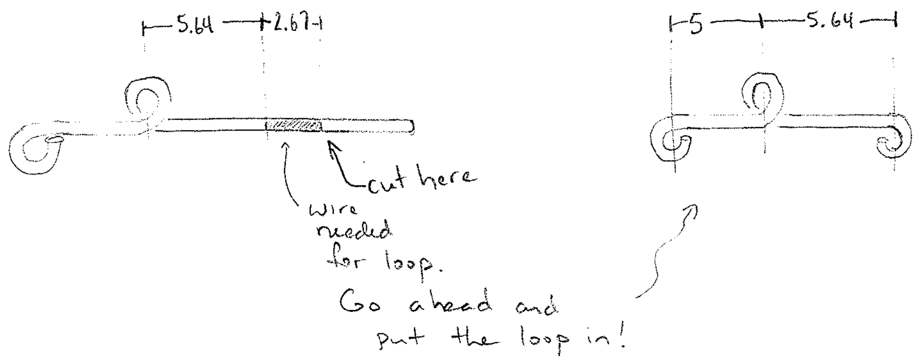
$x = 5.64$ cm

So how do we know how far to measure from the fulcrum to get a distance of 5.64 cm? Remember that we still have to put in a hanging loop at the end of the wire.



We know that half of the fulcrum loop and the hanging loop together account for 4 cm of wire. Because we already have half of the fulcrum loop on the wire, we just need to account for the hanging loop. The fulcrum loop and the hanging loop together make up 1.5 loops. Four cm for every 1.5 loops means the hanging loop accounts for 2.67 cm of wire. Thus, we need to add this extra distance to our solved for distance before cutting the wire.

Distance from fulcrum to where we should cut the wire $= 5.64 + 2.67 = 8.31$ cm.



Now we know where to cut the wire!

Attach the hanging objects, and you should have a balanced (or close to it) mobile!