Statistical Reasoning: Convincing Evidence Versus Proof

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Convincing Evidence Versus Proof

- Key distinction in statistical inference
- Makes drawing conclusions in inferential settings tricky.
- Understanding what conclusions are reasonable and wording conclusions correctly is conceptually difficult for many students.

Activities That Helps Students Understand What Conclusions Make Sense

- Mystery Bags
- Cookie Game
- (If time) Confidence Intervals—Can You Hear Me Now?

Mystery Bag 1

- Mix of milk chocolate and dark chocolate candies
- Sample 10 candies from the bag
- What do we now know about the population of candies in the bag?

p = proportion of dark chocolate candies

Statement: $p \neq 0$
convincing evidence or proof?
Statement: $p = 0$
convincing evidence or proof?
Statement: $p \neq 1$
convincing evidence or proof?
Statement: $p = 1$
convincing evidence or proof?
Statement: $p \neq 0.5$
convincing evidence or proof or ???
Statement: $p = 0.5$
convincing evidence or proof or ??????

p = 0.5

- Have we proven p = 0.5?
  - No
- Are we convinced that p = 0.5?
  - Sample outcome is consistent with what we would expect to see if p = 0.5, but it is also consistent with p = 0.49, p = 0.51, etc.
- Are we convinced that $p \neq 0.5$?
  - No
- What are we convinced of?
  - Nothing!

Mystery Bag 2

- . Have we proven p = 0.5?
  - No
- Have we proven p ≠ 0.5?
  - No
- Are we convinced that p ≠ 0.5?
  - Yes
- Difference between convincing evidence and proof

A hierarchy

- Proof
- Convincing evidence (that a claim is false)
- No convincing evidence that a claim is false.
- The conclusion that is NOT possible based on a sample (except in really rare cases of claims about proportions being 0 or 1): Convincing evidence that a claim is true.

Understanding Convincing Evidence-- The Logic of Hypothesis Testing

- The Cookie Game

Discussion Points

- Cookie Game illustrate all elements of a statistical hypothesis test
  - Competing claims about a population, one of which is initially assumed to be true (the null hypothesis)
  - Observation
  - Assessment of how likely observed outcome would be if the null hypothesis is true
  - A decision based on whether the observed outcome would have been likely or unlikely to occur when the null hypothesis is true

Discussion Points

- Convincing evidence vs. proof
- What if saw 2 of 5? Would this be proof that the true proportion is 0.5? Convincing evidence??
- Relationship between probability assessment and choice of significance level

Important that students understand…

Possible conclusions in a hypothesis test are

1. Convincing evidence against the null hypothesis
2. No convincing evidence against the null hypothesis

AND

No convincing evidence against the null hypothesis
IS NOT THE SAME AS
Convincing evidence that the null hypothesis is true

And While We Are on the Subject of Difficult Interpretations

- Interpreting confidence level…
- "Can you hear me now?" activity.

Normal Population with mean μ

- 10 random samples of size 25. Pick a sample and calculate a 90% confidence interval for μ.

Sample	Sample mean	Sample standard deviation
1	101.67	9.58
2	98.51	9.40
3	96.45	8.59
4	100.14	6.53
5	98.20	11.52
6	102.87	9.39
7	100.83	8.86
8	100.07	9.67
9	102.13	9.01
10	102.31	11.06

Normal Population with mean μ

- 10 samples of size 25

Sample	90% confidence interval for μ
1	(98.396, 104.953)
2	(95.296, 101.728)
3	(93.516, 99.393)
4	(97.906, 102.372)
5	(94.259, 102.146)
6	(99.658, 106.081)
7	(98.486, 103.181)
8	(96.761, 103.382)
9	(99.052, 105.215)
10	(98.523, 106.091)

Meaning of 90% Confidence

- Common student error (maybe even more common than a correct answer!): The probability that the population mean is in my interval is 0.9.
- Ask students what a probability of 0.9 means. The get to the 90% of the time, in the long run, …
- Then play the "Can you hear me now?" game.
  - Actual population mean is 100. Is it in your interval.
    How about now?
    How about now?
    How about now?
  - This interpretation of confidence level doesn't make sense because NOTHING is random here!

THANKS!

- Thanks for attending this session.
- Copies of Powerpoint slides are on the NCTM conference web site, or you can email me for a copy.
- Questions and Comments?
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