262 Englert Prop up Problem Solving with Proportional Reasoning

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For a set of handouts, and the power point, send an email to me, please

Scheduled for: Thursday, April 18, 2013: 02:45 PM - 04:00 PM, Hyatt Regency , Centennial Ballroom E (6–8) Gallery Workshop Proportional reasoning is one of the most important middle school topics we teach. Create and use easy-to-manage, inexpensive to prepare, hands-on activities to help your students understand proportionality and its connections to many prealgebra topics.

Background info:

According to Lamon (1999) proportional thinkers understand relationships in which two quantities vary together and are able to see how the change in one coincides with the change in another. They recognize real-world contexts for proportional relationships. They develop informal strategies rather than prescribed algorithms for solving proportions and comparing ratios. Proportional reasoning is the ability to recognize, to explain, to think about, to make conjectures about, to graph, to transform, to compare, to make judgments about, to represent, or to symbolize relationships of two simple types (direct proportions – when one quantity gets larger (or smaller) so does the other), and inverse proportions (one quantity increases and the other decreases in such a way that their products remain the same).

According to Van de Walle *"proportional reasoning is difficult to define in a simple sentence or two. It is not something that you either can or cannot do. It is both a qualitative and quantitative process."*

In his book on elementary and middle school mathematics, Van de Walle (2007) describes the big ideas of proportional reasoning:

1. A ratio is a multiplicative comparison of two quantities or measures. A key developmental milestone is the ability of a student to begin to think of a ratio as a distinct entity, different from the two measures that made it up.
2. Ratios and proportions involve multiplicative rather than additive comparisons. Equal ratios result from multiplication or division, not from addition or subtraction
3. Proportional thinking is developed through activities involving comparing and determining the equivalence of ratios and solving proportions in a wide variety of situations and contexts (Van de Walle, 2007, p. 353).

Proportional Reasoning is usually taught in grades 6 to 9. When students are taught this concept at earlier grades, they may not be ready. This encourages students to apply rules without thinking. When this happens, the ability to reason proportionally often does not develop. (Van de Walle, 2007, p. 355)

Instructional Prerequisites to Proportional Reasoning

(1) Students must know the difference between absolute change and relative change (additive thinking vs. and multiplicative thinking).
(2) Students must be able to recognize whether a ratio is an appropriate comparison.
(3) Recognize that quantities in a ratio covary in such a way that the relationship between them is invariant (unchanged).
(4) Students must build increasingly complex ways of unitizing (group as a unit to build up).

Five categories of informal activities to develop proportional reasoning -- Van de Walle suggests five categories of informal activities to develop proportional reasoning. There is no definitive sequencing for these activities, but it is recommended that students have experiences with the first before moving on to other four.

- Identification of multiplicative relationships
- Selection of equivalent ratios
- Comparison of ratios
- Scaling with ratio tables
- Construction and measurement activities

Workshop Activities:

- Sense or nonsense sort – helping students consider what makes sense
- Color tile ratios (materials from ETA)
- Pattern block proportions – Gulliver's Shapes (from ETA)
- Index Card Ratios and proportions
- Proportion matching game – played like dominos
- Proportion dice games (materials from ETA) -- Use standard dice, or dice with more than 6 sides, or use stickers to change the numbers on the faces of some standard dice (to include amounts like 8, 10 and 12) for more variation.
- **Rolling ratios dice game** (2 to 4 players) -- The first player rolls 2 dice and creates a ratio. Players roll their dice to try to create an equivalent ratio. Each matching ratio earns the players involved a point. Play continues clockwise.
- **Proportion Solitaire 1 --** Roll three dice and make a proportion with a missing amount. Determine the missing amount to win the hand.
- **Proportion Solitaire 2 --** Roll five dice and make a proportion using any 4 dice to win the hand.
- Proportion meter (elastic band) – stretch it out to find half of something… What does half look like?
- Percent slider – index card and brad (or paper clip) – evidence that as one amount changes, so does the other; can be used to set up proportions and solve for the missing amount
- Two color counter Ratios (materials from ETA) – model ratios and proportions

- Daily food intake – activity adapted from NCTM's World's Largest Math Event 2000 – Can the cards be arranged to show how much food the organisms eat? Will the order change if we consider consumption relative to body mass?
- Rubber Band Stretchers – another use for rubber bands, this time they are a similar figure generator
- Comparing Elephants – which ones are proportional? How can you tell?
- Nonstandard measurement and ratios – what's the connection?
- Paper folding to create similar figures
- Similar figure concentration game
- Similar figure Yahtzee
- The Similar Mouse Family – a graphing activity
- Similar figures in story problems – find the hidden triangles

Proportions and ratios in literature: *If the World Were a Village* – David Smith
One inch tall – Shel Silverstein *Fifty feet tall (inspired by Shel Silverstein's 'One inch tall')* -- Allen Steble
Counting on Frank – Rod Clement *If you Hopped Like a Frog* – David Schwartz

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