Pairing Mathematics & Figure Skating

Grades 6-8 Session

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Figure skating is the winter Olympic sport that the most viewers watch. We will explore proportional relationships describing aspects of pairs figure skating. Topics include the international judging system, conservation of momentum, and skating paths on the ice. We aim to raise mathematical literacy by giving students motivating contexts.

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CONSERVATION OF MOMENTUM

A lady and a man skate together as part of a pairs figure skating team.

M_1 and M_2 are the skaters' masses and V_1 and V_2, respectively, are the skaters' individual velocities. Their momentums are expressed as $M_1 * V_1$ and $M_2 * V_2$.

When the two skaters have the same momentum, the equation representing their momentums is the following:

$$M_1 * V_1 = M_2 * V_2$$

Answer the following questions using the above momentum equation:

The lady weighs less than the man. If the lady and man want to achieve the same momentum, will the lady travel faster or will the man travel faster? Explain.

If the lady's mass is 60 kg and the man's mass is 120 kg, and if the man is traveling at 2 meters per second, what does the lady's velocity need to be in order for her to have the same momentum as the man? Show your work.

Find the man's mass given the following information: the lady and the man have the same momentum; the lady's mass is 75 kg and her speed is 5 meters per second, and the man is traveling at 2.5 meters per second. Show two different ways to solve this problem. Which way do you think is more efficient? Explain.

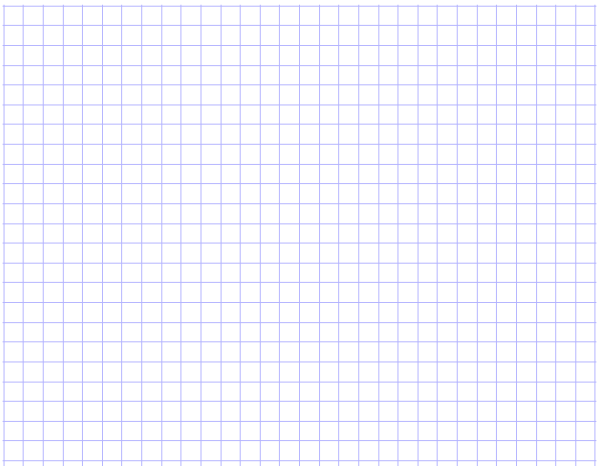
Use the momentum equation from the prior page, $M_1 * V_1 = M_2 * V_2$. Assume the lady's mass is 75 kg and the man's mass is 100 kg. For the purposes of this activity, assume that the lady's velocity is the independent variable and that the man's velocity is the dependent variable. Complete the following table:

Lady's Velocity	Man's Velocity
0	
	4
5	
9	
	28

Will the relationship between the lady's velocity and the man's velocity be linear, quadratic, exponential, or another type of function? Explain.

Write an equation that helps you find the man's velocity given the lady's velocity.

On the coordinate grid below, produce a graph representing the lady's velocity vs the man's velocity if they have the same momentum.



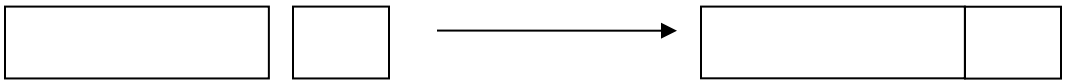
What is the slope of your graph?

What does the slope of your graph represent?

CONSERVATION OF MOMENTUM:

INELASTIC COLLISION

In a pairs skating program, there are many instances when the lady and man are skating separately and then come together to skate as a unit. For example, the lady may jump towards the man and he catches her, and they then skate as a unit. A simple diagram showing what happens in this situation is the following:



Each box represents one skater's mass, and when the skaters join together, their masses are added. If M_1 and M_2 are the skaters' masses and the V_1 and V_2, respectively, are the skaters' individual velocities initially, then V_3 is the combined velocity of the pairs team when the man is carrying the woman. By conservation of linear momentum, the momentum generated by the individual skaters separately must equal the momentum of the skaters skating as a team.

The equation for conservation of momentum in this situation is the following:

$$M_1 * V_1 + M_2 * V_2 = (M_1 + M_2) * V_3$$

We will assume that during the course of a skating program, skaters' masses remain the same at the beginning and at the end; we also assume that no additional forces are acting on the system.

In this activity, we would like to examine how the combined velocity changes as the skaters' individual velocities change.

Program the "Pairs linear momentum.xls" spreadsheet or use the following table to calculate V_3 given several sets of values of M_1, M_2, V_1, and V_2.

M1	V1	M2	V2	M1*V1+M2*V2	M1+M2	V3

Determine the answers to these two questions:

1) Tom and Lily are pairs partners who have the same mass. In one part of their program, Tom starts from a standstill and Lily skates at a nonzero initial velocity and jumps towards him. They would like to describe what their combined velocity will be when he catches her and they join together to skate. Tom thinks that their combined velocity will be the same as Lily's initial velocity. Lily thinks that their combined velocity will be half of her initial velocity. Who is right? Explain how you know.

2) Tom and Lily are pairs partners who have the same mass. In another part of their program, Tom and Lily each complete side by side jumps next to each other. On their exit to the jumps, Tom and Lily join up and skate as a unit. In relation to their individual jump exit velocities, what will Tom and Lily's combined velocity be?

Answer the following questions, assuming that the lady's mass M_1 = 60 kg, and that the man's mass M_2 = 90 kg. Show your work.

3) Allison and Erin are trying to answer the question, "How fast is the man traveling individually if the lady was traveling individually 2 m/s and their combined velocity is 4 m/s?" Allison thinks that the answer is 6 m/s because the average of 2 m/s and 6 m/s is 4 m/s. Erin doesn't think that Allison's answer is correct, but cannot figure out why. How would you help Erin and Allison determine the correct answer?

4) If the lady's and the man's individual velocities are both increased by the same percentage, by what percentage would their combined velocity V_3 be?

5) If the lady's individual velocity increased by 15%, what would the team's combined velocity be?

6) How much faster does the lady need to travel individually in order to increase the team's velocity by 10%?

7) After they have been gliding in the lift for some time, the man lets the lady down and each skater resumes skating on his/her own. How would you model this situation using an equation and your knowledge of conservation of linear momentum?

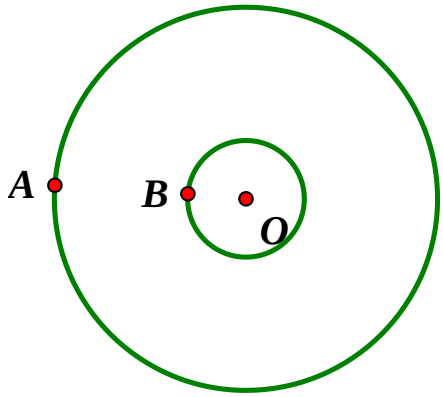
8) If the pair is skating together and the man lets the lady down forward, would each of their individual velocities be faster or slower than their combined velocity? Why? How would the situation change if the man let the lady down behind him instead of in front?

9) Assume the pairs team's initial velocity of 5 m/s and the lady has a final velocity of 7 m/s after she lands. What is the man's final velocity?

Create two additional questions that we can answer using information from the conservation of linear momentum equation & the related table of values. Provide solutions to these questions on a separate sheet of paper.

TRACINGS ON THE ICE

Below are two projections of the skate blade tracings that are created during Keauna MacLaughlin and Rockne Brubaker's death spiral.



The projection on the left, depicting concentric circles, is a bird's eye view of the tracings that Keauna and Rockne form when completing the death spiral. In both projections, Rockne's center of mass is at point O. The circle representing Keauna's blade trajectory has radius, OA. The circle representing Rockne's blade trajectory has radius, OB.

Answer the following questions about the death spiral:

- Does Keauna or Rockne travel faster during the death spiral? Explain.

- If OA = 10 feet, and OB = 2 feet, answer the following questions:
  - How much further does Keauna travel than Rockne? Explain.

- If the death spiral took 8 seconds to complete, what is the difference in speeds of Keauna and Rockne? Explain.

- Keauna and Rockne's friend, Lisa, is trying to answer previous two questions and she first calculates the difference between the lengths of OA and OB. Is this a valid first step to solve the above two questions? If so, how could she use it, and if not, explain why not.

- If Rockne's rate is 1.05 feet per second, then how much time did the death spiral take? What is Keauna's speed during the death spiral? Explain.

- Keauna and Rockne completed a death spiral that lasted 4 seconds. Keauna's rate was 12.56 feet per second. What is the length of OA? Explain.