Interesting Ideas, Manipulatives and Engaging Activities for Teaching Geometry Topics

Participants will use manipulatives and investigations to increase student success in the Geometry classroom -- ***Erin Schneider, Atherton High School, Louisville, KY***

Solving Puzzles in Teams

Welcome! This professional development session will challenge you to use different problem-solving **strategies**. You will also be introduced to different tools and resources that you can use throughout the course as you **investigate** new ideas, solve problems, and share mathematical ideas.

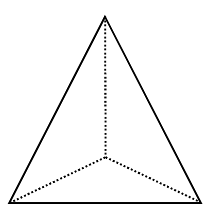
We will then reflect on how each investigation implemented a standard of mathematical practice from the CCSSM.

All problems used today come from CPM Educational Program's Core Connection Series. Go to www.cpm.org for more information or email Erin Schneider at schneider@cpm.org. All investigations are geared toward participants increasing their conceptual understanding as well as using skills in real-world experiences; hence engaging in abstract and quantitative reasoning to enhance student success in Geometry.

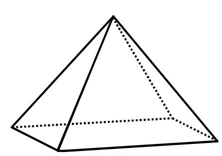
1-1. BUILDING WITH YARN

Work with your team to make each of the shapes you see below out of a single loop of yarn. You may make the shapes in any order you like. Before you start, review the team roles that are described on the next page. Use these roles to help your study team work together today. When you make one of the shapes successfully, call your teacher over to show off your accomplishment.

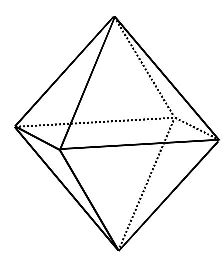
A Tetrahedron



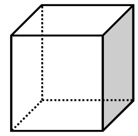
A Square-Based Pyramid



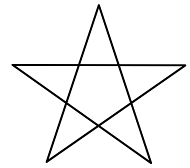
An Octahedron

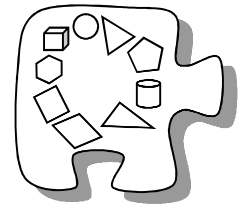


A Cube



A Star





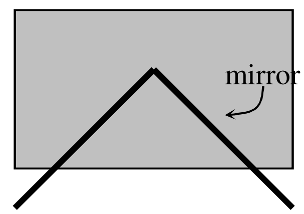
What shapes can you find?
Building a Kaleidoscope

Today you will learn about angles and shapes as you study how a kaleidoscope works.

1-37. BUILDING A KALEIDOSCOPE

How does a kaleidoscope create the complicated, colorful images you see when you look inside? A hinged mirror and a piece of colored paper can demonstrate how a simple kaleidoscope creates its beautiful repeating designs.

Your Task: Place a hinged mirror on a piece of colored, unlined paper so that its sides extend beyond the edge of the paper as shown at right. Explore what shapes you see when you look directly at the mirror, and how those shapes change when you change the angle of the mirror. Discuss the questions below with your team. Be ready to share your responses with the rest of the class.



Discussion Points

What is this problem about? What is it asking you to do?

What happens when you change the angle (opening) formed by the sides of the mirror?

How can you describe the shapes you see in the mirror?

1-38. To complete your exploration, answer these questions together as a team.

a. What happens to the shape you see as the angle formed by the mirror gets bigger (wider)? What happens as the angle gets smaller?

b. What is the smallest number of sides the shape you see in the mirror can have? What is the largest?

c. With your team, find a way to form a **regular hexagon** (a shape with six equal sides and equal angles).

d. How might you describe to another team how you set the mirrors to form a hexagon? What types of information would be useful to have?

THE REFLECTION OF LIGHT

2-28. You know enough about angle relationships now to start analyzing how light bounces off mirrors. **Examine** the two diagrams below. Diagram A shows a beam of light emitted from a light source at A. In Diagram B, someone has placed a mirror across the light beam. The light beam hits the mirror and is reflected from its original path.

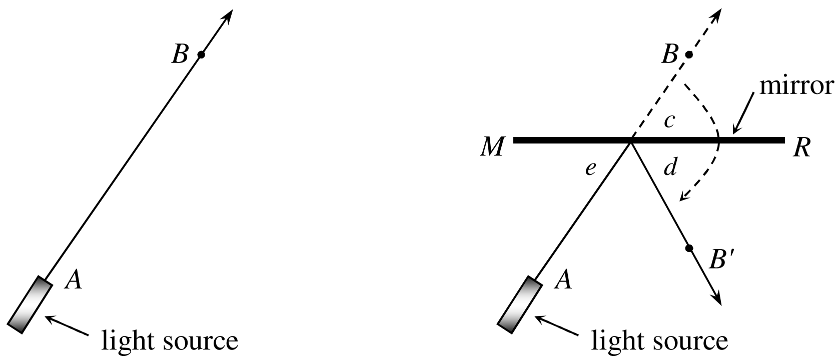


Diagram A **Diagram B**

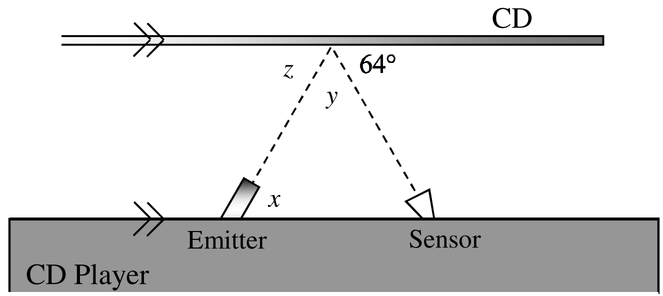
a. What is the relationship between angles *c* and *d*? Why?

b. What is the relationship between angles *c* and *e*? How do you know?

c. What is the relationship between angles *e* and *d*? How do you know?

d. Write a conjecture about the angle at which light hits a mirror and the angle at which it bounces off the mirror.

2-29. A CD player works by bouncing a laser off the surface of the CD, which acts like a mirror. An emitter sends out the light, which bounces off the CD and then is detected by a sensor. The diagram below shows a CD held parallel to the surface of the CD player, on which an emitter and a sensor are mounted.



a. The laser is supposed to bounce off the CD at a 64° angle as shown in the diagram above. For the laser to go directly into the sensor, at what angle does the emitter need to send the laser beam? In other words, what does the measure of angle x have to be? **Justify** your conclusion.

b. The diagram above shows two parts of the laser beam: the one coming out of the emitter and the one that has bounced off the CD. What is the angle ($\measuredangle y$) between these beams? How do you know?

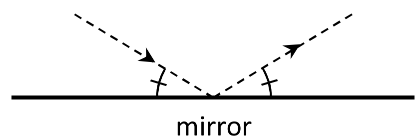
2-30. ANGLE RELATIONSHIPS TOOLKIT

Obtain a Lesson 2.1.3 Resource Page ("Angle Relationships Toolkit") from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the new angle relationships you have studied so far in Chapter 2. Then, in the space provided, add a diagram and a description of the relationship for each special angle relationship you know. Be sure to specify any relationship between the measures of the angle (such as whether or not they are always congruent). In later lessons, you will continue to add relationships to this toolkit, so be sure to keep this resource page in a safe place. At this point, your toolkit should include:

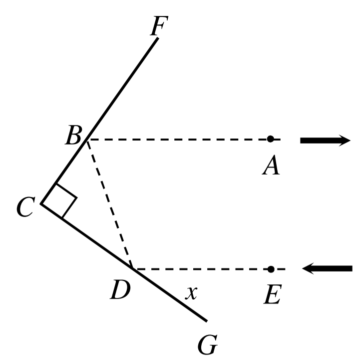
- Vertical angles
- Straight angles
- Corresponding angles
- Alternate interior angles
- Same-side interior angles

2-50 SOMEBODY'S WATCHING ME, Part Two

Remember Mr. Douglas' trick from problem 2-1? You now know enough about angle and line relationships to analyze why a hinged mirror set so the angle between the mirrors is 90° will reflect your image back to you from any angle. Since your reflection is actually light that travels from your face to the mirror, you will need to study the path of the light. Remember that a mirror reflects light, and that the angle the light hits the mirror will equal the angle it bounces off the mirror.



Your task: Explain why the mirror bounces your image back to you from any angle. Include in your analysis:



- Use angle relationships to find the measures of all the angles in the figure. (Each team member should choose a different x value and calculate all of the other angle measures using his or her selected value of x.)
- What do you know about the paths the light takes as it leaves you and as it returns to you? That is, what is the relationship between $\overline{BA}$ and $\overline{DE}$?
- Does Mr. Douglas' trick work if the angle between the mirrors is not 90°?

2-54. EXTENSION

Hold a 90° hinged mirror at arm's length and find your own image. Now close your right eye. Which eye closes in the mirror? Look back at the diagram from problem 2-46. Can you explain why this eye is the one that closes?

2-91. HEIGHT LAB

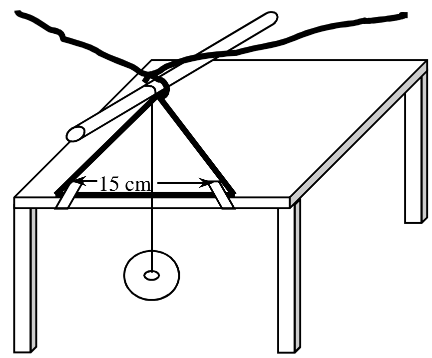
What is the height of a triangle? Is it like walking up any side of the triangle? Or is it like standing at the highest point and looking straight down? Today your team will build triangles with string and consider different ways height can be seen for triangles of various shapes.

a. Use the materials given to you by your teacher to make a triangle like the one in the diagram below.

(1) Tie one end of the short string to the weight and the other to the end of a pencil (or pen).

(2) Tape a 15 cm section of the long string along the edge of a desk or table. Be sure to leave long ends of string hanging off each side.

(3) Bring the loose ends of string up from the table and cross them as shown in the diagram. Then put the pencil with the weight over the crossing of the string. Cross the strings again on top of the pencil.



b. Now, with your team, build and sketch triangles that meet the three conditions below. To organize your work, assign each team member one of the jobs described at right.

(1) The height of the triangle is inside the triangle.

(2) The height of the triangle is a side of the triangle.

(3) The height of the triangle is outside of the triangle.

Student jobs:

- hold the pencil (or pen) with the weight.
- make sure that the weight hangs freely
- draw accurate sketches.
- hold the strings that make the sides of the triangles.

c. Now make sure that everyone in your team has sketches of the triangles that you made.

2-92. How can I find the height of a triangle if it is not a right triangle?

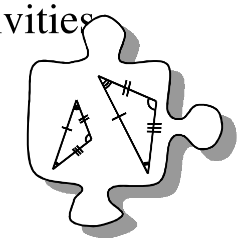
a. On the Lesson 2.2.4 Resource Page there are four triangles labeled (1) through (4). Assume you know the length of the side labeled "base." For each triangle, draw in the height that would enable you to find the area of the triangle.
Note: You do not need to find the area.

b. Find the triangle for part (b) at the bottom of the same resource page. For this triangle, draw all three possible heights. First **choose** one side to be the base and draw in the corresponding height. Then repeat the process of drawing in the height for the other two sides, one at a time.

c. You drew in three pairs of bases and heights for the triangle in part (b). Using centimeters, measure the length of all three sides and all three heights. Find the area three times using all three pairs of bases and heights. Since the triangle remains the same size, your answers should match.

3-1. WARM-UP STRETCH

Before computers and copy machines existed, it sometimes took hours to enlarge documents or to shrink text on items such as jewelry. A pantograph device (like the one below) was often used to duplicate written documents and artistic drawings. You will now employ the same geometric principles by using rubber bands to draw enlarged copies of a design. Your teacher will show you how to do this.



What can I do with similar triangles?

Applying Similarity

In previous lessons, you have learned methods for finding similar triangles. Once you find triangles are similar, how can that help you? Today you will apply similar triangles to analyze situations and solve new applications. As you work on today's problems, ask the following questions in your team:

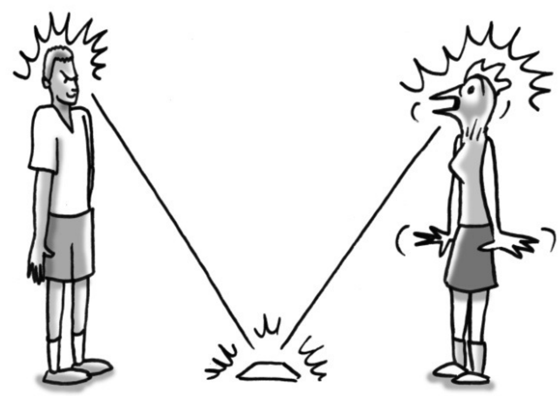
What is the relationship?

Are any triangles similar? What similarity conjecture can we use?

3-105. YOU ARE GETTING SLEEPY…

Legend has it that if you stare into a person's eyes in a special way, you can hypnotize them into squawking like a chicken. Here's how it works.

Place a mirror on the floor. Your victim has to stand exactly 200 cm away from the mirror and stare into it. The only tricky part is that you need to figure out where you have to stand so that when you stare into the mirror, you are also staring into your victim's eyes.



If your calculations are correct and you stand at the *exact* distance, your victim will squawk like a chicken!

a. Choose a member of your team to hypnotize. Before heading to the mirror, first analyze this situation. Draw a diagram showing you and your victim standing on opposite sides of a mirror. Measure the heights of both yourself and your victim (heights to the eyes, of course), and label all the lengths you can on the diagram. (Remember: your victim will need to stand 200 cm from the mirror.)

b. Are there similar triangles in your diagram? **Justify** your conclusion. (Hint: Remember what you know about how light reflects off mirrors.) Then calculate how far you will need to stand from the mirror to hypnotize your victim.

c. Now for the moment of truth! Have your teammate stand 200 cm away from the mirror, while you stand at your calculated distance from the mirror. Do you make eye contact? If not, check your measurements and calculations and try again.

7-12. IS THERE MORE TO THIS CIRCLE?

Circles can be folded to create many different shapes. Today, you will work with a circle and use properties of other shapes to develop a three-dimensional shape. Be sure to have **reasons** for each conclusion you make as you work. Each person in your team should start by obtaining a copy of a circle from your teacher and cutting it out.

a. Fold the circle in half to create a crease that lies on a line of symmetry of the circle. Unfold the circle and then fold it in half again to create a new crease that is perpendicular to the first crease. Unfold your paper back to the full circle. How could you convince someone else that your creases are perpendicular? What is another name for the line segment represented by each crease?

b. On the circle, label the endpoints of one diameter *A* and *B*. Fold the circle so that point *A* touches the center of the circle and create a new crease. Then label the endpoints of this crease *C* and *D*. What appears to be the relationship between $\overline{AB}$ and $\overline{CD}$? Discuss and **justify** with your team. Be ready to share your **reasons** with the class.

c. Now fold the circle twice to form creases $\overline{BC}$ and $\overline{BD}$ and use scissors to cut out $\triangle BCD$. What type of triangle is $\triangle BCD$? How can you be sure? Be ready to convince the class.

7-13. ADDING DEPTH

Your equilateral triangle should now be flat (also called two-dimensional). **Two-dimensional** shapes have length and width, but not depth (or "thickness").

a. Label the vertices of $\triangle BCD$ if the labels were cut off. Then, with the unmarked side of the triangle facedown, fold and crease the triangle so that *B* touches the midpoint of $\overline{CD}$. Keep it in the folded position.

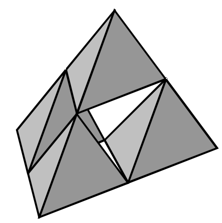
What does the resulting shape appear to be? What smaller shapes do you see inside the larger shape? **Justify** that your ideas are correct (for example, if you think that lines are parallel, you must provide evidence).

b. Open your shape again so that you have the large equilateral triangle in front of you. How does the length of a side of the large triangle compare to the length of the side of the small triangle formed by the crease? How many of the small triangles would fit inside the large triangle? In what ways are the small and large triangles related?

c. Repeat the fold in part (a) so that C touches the midpoint of $\overline{BD}$. Unfold the triangle and fold again so that D touches the midpoint of $\overline{BC}$. Create a three-dimensional shape by bringing points B, C, and D together. (A **three-dimensional** shape has length, width, and depth.) Use tape to hold your shape together.

d. Three-dimensional shapes formed with polygons have **faces** and **edges**, as well as **vertices**. Faces are the flat surfaces of the shape, while edges are the line segments formed when two faces meet. Vertices are the points where edges intersect. Discuss with your team how to use these words to describe your new shape. Then write a complete description. If you think you know the name of this shape, include it in your description.

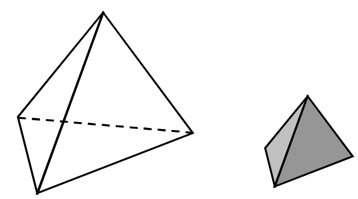
7-14. Your team should now have 4 three-dimensional shapes (called **tetrahedra**). (If you are working in a smaller team, you should quickly fold more shapes so that you have a total of four.



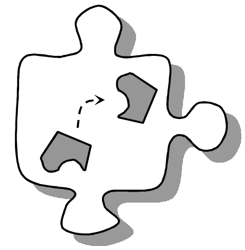
a. Put four tetrahedra together to make an enlarged tetrahedron like the one pictured at right. Is the larger tetrahedron similar to the small tetrahedron? How can you tell?

b. To determine the edges and faces of the new shape, pretend that it is solid. How many edges does a tetrahedron have? Are all of the edges the same length? How does the length of an edge of the team shape compare with the length of an edge of one of the small shapes?

c. How many faces of the small tetrahedral would it take to cover the face of the large tetrahedron? Remember to count gaps as part of a face. Does the area of the tetrahedron change in the same way as the length?



Enlarged tetrahedron Original



What shapes can I create with triangles?

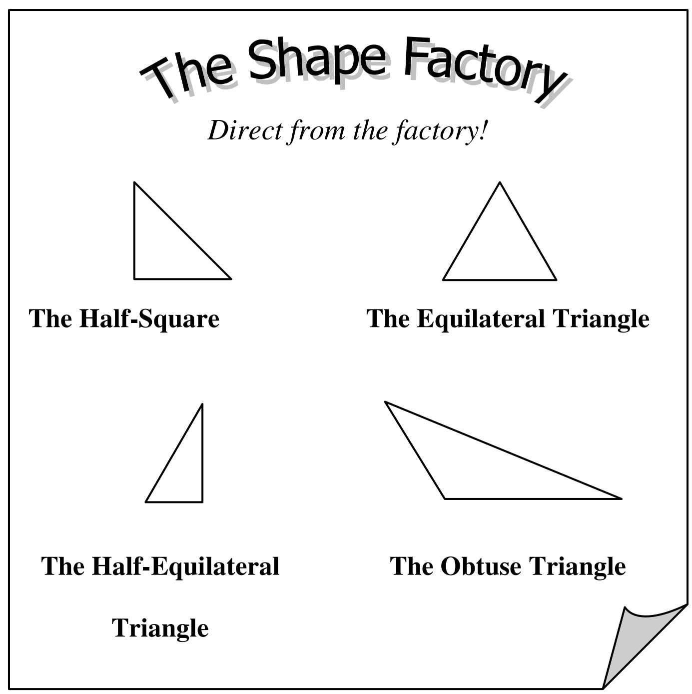
Using Transformations to Create Shapes

You practiced reflecting, rotating and translating figures. Since these are rigid transformations, the image always had the same size and shape as the original. In this lesson, you will combine the image with the original to make new, larger shapes from four basic "building-block" shapes.

As you create new shapes, consider what information the transformation gives you about the resulting new shape. By the end of this lesson, you will have generated most of the shapes that will be the focus of this course.

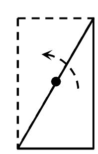
1-90. THE SHAPE FACTORY

The Shape Factory, an innovative new company, has decided to branch out to include new shapes. As Product Developers, your team is responsible for finding exciting new shapes to offer your customers. The current company catalog is shown at right.

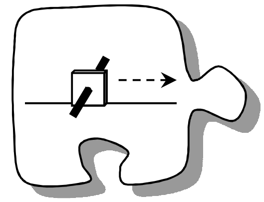


Since your boss is concerned about production costs, you want to avoid buying new machines and instead want to reprogram your current machines.

The factory machines not only make all the shapes shown in the catalog, but they also are able to rotate or reflect a shape. For example, if the half-equilateral triangle is rotated 180° about the **midpoint** (the point in the middle) of its longest side, as shown at right, the result is a rectangle.



Your Task: Your boss has given your team until the end of this lesson to find as many new shapes as you can. Your team's reputation, which has suffered recently from a series of blunders, could really benefit by an impressive new line of shapes formed by a triangle and its transformations. For each triangle in the catalog, determine which new shapes are created when it is rotated or reflected so that the image shares a side with the original triangle. Be sure to make as many new shapes as possible. Use tracing paper or any other reflection tool to help.



How can I create it?

Using Symmetry to Study Polygons

In Chapter 1, you used a hinged mirror to study the special angles associated with regular polygons. In particular, you **investigated** what happens as the angle formed by the sides of the mirror is changed. Today, you will use a hinged mirror to determine if there is more than one way to build each regular polygon using the principals of symmetry. What about other types of polygons? What can a hinged mirror help you understand about them?

As your work with your study team, keep these focus questions in mind:

Is there another way?

What types of symmetry can I find?

What does symmetry tell me about the polygon?

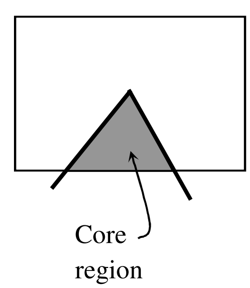
7-40. THE HINGED MIRROR TEAM CHALLENGE

Obtain a hinged mirror, a piece of unlined colored paper, and a protractor from your teacher.

With your team, spend five minutes reviewing how to use the mirror to create regular polygons. (Remember that a **regular polygon** has equal sides and angles). Once everyone remembers how the hinged mirror works, select a team member to read the directions of the task below.

Your Task: Below are four challenges for your team. Each requires you to find a creative way to position the mirror in relation to the colored paper. You can tackle the challenges in any order, but you must work together as a team on each. Whenever you successfully create a shape, do not forget to measure the angle formed by the mirror, as well as draw a diagram on your paper of the core region in front of the mirror. If your team decides that a shape is impossible to create with the hinged mirror, explain why.

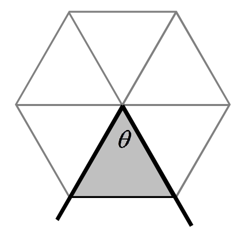
- Create a regular hexagon.
- Create an equilateral triangle at least <u>two</u> different ways.
- Create a rhombus that is <u>not</u> a square.
- Create a circle.



7-41. ANALYSIS

How can symmetry help you to learn more about shapes? Discuss each question below with the class.

a. One way to create a regular hexagon with a hinged mirror is with six triangles, as shown in the diagram at right. (Note: the gray lines represent reflections of the bottom edges of the mirrors and the edge of the paper, while the core region is shaded.)

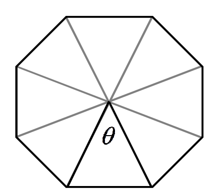


What is special about each of the triangles in the diagram? What is the relationship between the triangles? Support your conclusions. Would it be possible to create a regular hexagon with 12 triangles? Explain.

b. If you have not done so already, create an equilateral triangle so that the core region in front of the mirror is a right triangle. Draw a diagram of the result that shows the different reflected triangles like the one above. What special type of right triangle is the core region? Can all regular polygons be created with a right triangle in a similar fashion?

c. In problem 7-36, your team formed a rhombus that is not a square. On your paper, draw a diagram like the one above that shows how you did it. How can you be sure your resulting shape is a rhombus? Using what you know about the angle of the mirror, explain what must be true about the diagonals of a rhombus.

7-42. Use what you learned today to answer the questions below.



a. **Examine** the regular octagon at right. What is the measure of angle θ? Explain how you know.

b. Quadrilateral $ABCD$ at right is a rhombus. If $BD = 10$ units and $AC = 18$ units, then what is the perimeter of $ABCD$? Show all work.