

### Assessing Student Understanding of Fractions

#### An Interview Protocol for Teachers

Presented by:  
**Trena L. Wilkerson**  
Department of Curriculum & Instruction  
Baylor University  
Trena.Wilkerson@baylor.edu

of the  
National Council of Teachers of Mathematics Regional Conference  
Louisville, Kentucky  
November 2013

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#### During this session

1. Identify **key concepts** in fractions.
2. Examine **tasks for assessment** purposes of fractional understanding in children.
3. Analyze **task for significance** in revealing student understanding.
4. Determine **how to use results** to inform instruction.
5. Perform **select tasks and discuss** how they might be used in their classrooms.
6. Share how tasks have been used in elementary classrooms for the past 7 years and the **results**.

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#### Common Core State Standards: K-2

1. Kindergarten: Counting & Shapes
2. Grade 1: **Partition** circles and rectangles into two and four equal shares, **describe the shares** using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. **Describe the whole** as two of, or four of the shares. Understand for these examples that **decomposing** into more equal shares creates smaller shares.
3. Grade 2: **Partition** circles and rectangles into two, three, or four equal shares, **describe the shares** using the words halves, thirds, half of, a third of, etc., and **describe the whole** as two halves, three thirds, four fourths. Recognize that **equal shares** of identical wholes need not have the same shape.

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#### CCSS-M-Grade 3

1. Develop understanding of fractions as numbers.
2. Understand a fraction  $\frac{1}{b}$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $\frac{a}{b}$  as the quantity formed by  $a$  parts of size  $\frac{1}{b}$ .
3. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
4. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Geometry Domain: Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.  
Note: limited to fractions with denominators 2, 3, 4, 6, 8.

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#### Fraction Research Study: Example #1 (K, 1<sup>st</sup>, 2<sup>nd</sup>, & 3<sup>rd</sup>)

- Question and Procedure
  - Present the child with paper and pencil.
  - **Can you show me what one-half looks like?**
  - If no response or child seems confused or only writes symbolically, ask: Can you draw me a picture or shape and shade in one-half?
- Discuss with a partner the possible student responses.

\*Modified from Larson, S. (2005). Teaching fractions and ratios for understanding. Mahwah, NJ: Lawrence, Erlbaum Associates.  
\*\*2004 3<sup>rd</sup> Grade Math TAKS test

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#### Kindergarten Examples

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#### First Grade Examples

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#### Second Grade Examples

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#### Fraction Research Study: Example #1 Scoring Guide


	4	3	2	1	0
Child creates picture or diagram that shows 2 equal parts or notates it correctly.					
Child shows picture that shows 2 unequal parts.					
Child shows picture that shows parts other than 1/2.					
Child shows picture or drawing that is unrelated to 1/2 or any other fractional part.					

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**Consider For Each Question**


What does this question reveal about student understanding?

How might the results inform instruction?

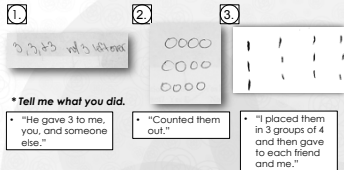
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**Fraction Research Study: Example #2  
(K, 1<sup>st</sup>, 2<sup>nd</sup>, & 3<sup>rd</sup>)**

- Question and Procedure
  - Present with 12 Teddy Bear Counters. (Grade 3 used Pencils.)
  - Show me how you would share these Teddy Bears (or Pencils) fairly between you and 2 friends; that would be 3 people. By fairly I mean each of you would get the same amount.
- Discuss with a partner the possible student responses.


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**Kindergarten Examples**

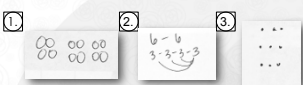


**\*Tell me what you did.**

- "He gave 3 to me, you, and someone else."
- "Counted them out."
- "I placed them in 3 groups of 4 and then gave to each friend and me."


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**First Grade Examples**

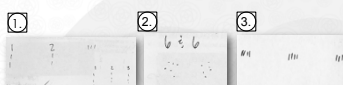


**\*Tell me what you did.**

- "Four each."
- "I just subtracted 4 from 12 and subtracted some more."
- "Split them into 3 groups of 3." (The other 3 bears were put to the side and not included in explanation)


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**Second Grade Examples**

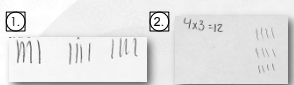


**\*Tell me what you did.**

- "I put one in each group until I ran out. (4 in each group)"
- "12's an even number so I cut them in 1/2 and it = 6."
- "4 + 4 = 8, 8 + 4 = 12. Added by 4's."


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**Third Grade Examples**




**\*Tell me what you did.**

- "Gave each person a pencil. Each group has 4 pencils, there are 3 groups."
- "4x3=12. Each person gets 4 so I put 4 pencils for each person."

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**Fraction Research Study: Example #2  
Scoring Guide**

Q	4	3	2	1	0
Show me	Child separates teddy Bears (or Pencils) into 3 groups with 4 in each group.	Child separates teddy Bears (or Pencils) into 3 equal groups with less than 4 in each.	Child separates teddy Bears (or Pencils) into 3 unequal groups or separates into incorrect number of groups.	Child separates teddy Bears (or Pencils) into groups other than 3.	NU
Tell me	Child gives correct fractional name with correct justification.	Child gives incorrect or no fractional name but gives a valid mathematical justification.	Child gives correct name with a partial or incorrect justification.	Child gives incorrect or no fractional name with no or non-mathematical justification.	NU

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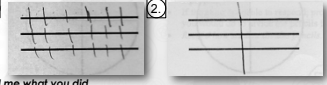
**Fraction Research Study: Example #3  
(K & 1<sup>st</sup>)**

- Question and Procedure
  - Present child with 3 Twizzlers®.
  - Show me how you would share these three pieces fairly between you and a friend; that is 2 people. It's OK to cut some or all of them if you need to do so.
- Discuss with a partner the possible student responses.

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**Kindergarten Examples**

**\*Record how student divides the Twizzlers®.**




**\*Tell me what you did.**

- "Passed them out and cut them. I counted them."
- "Cut in two, cut in half, cut it."

**\*How much does each friend get?**

- "10 pieces"
- "3, 3 pieces"

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### First Grade Examples

\* Record how student divides the Twizzlers®.

\* Tell me what you did.

- "Cut each in half."
- "Spill in half, 2 share it!"

\* How much does each friend get?

- "3, 3 pieces, one-half."
- "2 pieces"

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### Fraction Research Study: Example #3 Scoring Guide

Q	4	3	2	1	0
show me	Child breaks twizzlers into equal pieces and distributes them equally between 2 people.	Child breaks twizzlers into equal pieces but distributes them unequally between 2 people.	Child distributes 1 twizzler® per person and gives 1st piece to himself, friend or no one at all.	Child splits twizzlers unequally.	
tell me	Child gives correct fractional name with correct justification.	Child gives incorrect or no fractional name but gives a valid mathematical justification.	Child gives correct name with a partial or incorrect justification.	Child gives incorrect or no fractional name with no or non-mathematical justification.	

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### Fraction Research Study: Example #4 (2<sup>nd</sup> & 3<sup>rd</sup>)

- Question and Procedure
  - Present child with the Pizza Picture.
  - Say: Let's pretend that these are pizzas. Ted gets the pizza (the shaded part) in the first pan and Mandy gets the pizza (the shaded part) in the second pan.
  - Who gets more pizza or do they get the same amount?
- Discuss with a partner the possible student responses.

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### Second Grade Examples

\* Who gets more pizza or do they get the same amount?

Mandy	The Same
<ul style="list-style-type: none"> <li>"Mandy gets 4 and Ted gets 3. Mandy's pieces are bigger, it's 4/8 and Ted gets 3/6."</li> <li>"Mandy has more pieces than Ted."</li> <li>"because she has 4 and Ted only has 3."</li> </ul>	<ul style="list-style-type: none"> <li>"He gets 1/4 and she gets 1/4."</li> <li>"I think they get the same because this one (Ted's) is bigger than this one (Mandy) because they cover each other up. (uses her fingers to show they cover each other up)"</li> </ul>

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### Third Grade Examples

\* Who gets more pizza or do they get the same amount?

Mandy	The Same	Ted
<ul style="list-style-type: none"> <li>"She has more pieces. She had 4 pieces, Ted gets 3"</li> <li>"She gets 8 pieces and he gets 6."</li> </ul>	<ul style="list-style-type: none"> <li>"Pizzas are the same size."</li> <li>"Both have 1/4 (half) shaded."</li> </ul>	<ul style="list-style-type: none"> <li>"2 of Mandy's make 1 of Ted's and Ted has 3 pieces that are bigger."</li> </ul>

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### Fraction Research Study: Example #4 Scoring Guide

4	3	2	1	0
Responds by saying they both get the same amount, because the shaded part is the same area just cut into different sized pieces	Responds correctly, but mathematical justification is weak or unclear	Responds incorrectly, with valid mathematical justification	Responds incorrectly with no or incorrect mathematical justification	

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### Sample Findings

- Increased understanding of the concept of "one-half"
- Mathematical vocabulary improved
- Students began to reason and communicate more
  - Not always at a highly rigorous level for some students
- Partitioning techniques became somewhat sophisticated
- students demonstrated growth in understanding of fractions with both discrete and continuous models
  - Usually better with discrete—but mixed results across grade levels

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### Sample Findings-Continued

- Understanding of fair share and partitioning improved
- Equivalency concepts-improved but still difficult
  - 3<sup>rd</sup> grade-better with discrete model
- Students comfort level with fractions increased. No longer that scary!!!

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### Sample NCTM Resources

- Developing Essential Understanding of Rational Numbers for Teaching Mathematics in Grades 3-5
- Putting Essential Understanding of Fractions into Practice, Grades 3-5
- Making Sense Fractions, Ratios and Proportions, 64th Yearbook (2002)
- September 2012: An Appetite for Fractions! *Teaching Children Mathematics*

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