



Assessment for Learning: Uncovering Student Misconceptions through Formative Assessment Lessons

November 8th, 2013

National Council of Teachers Mathematics

Regional Meeting - Louisville, KY

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Who are we?

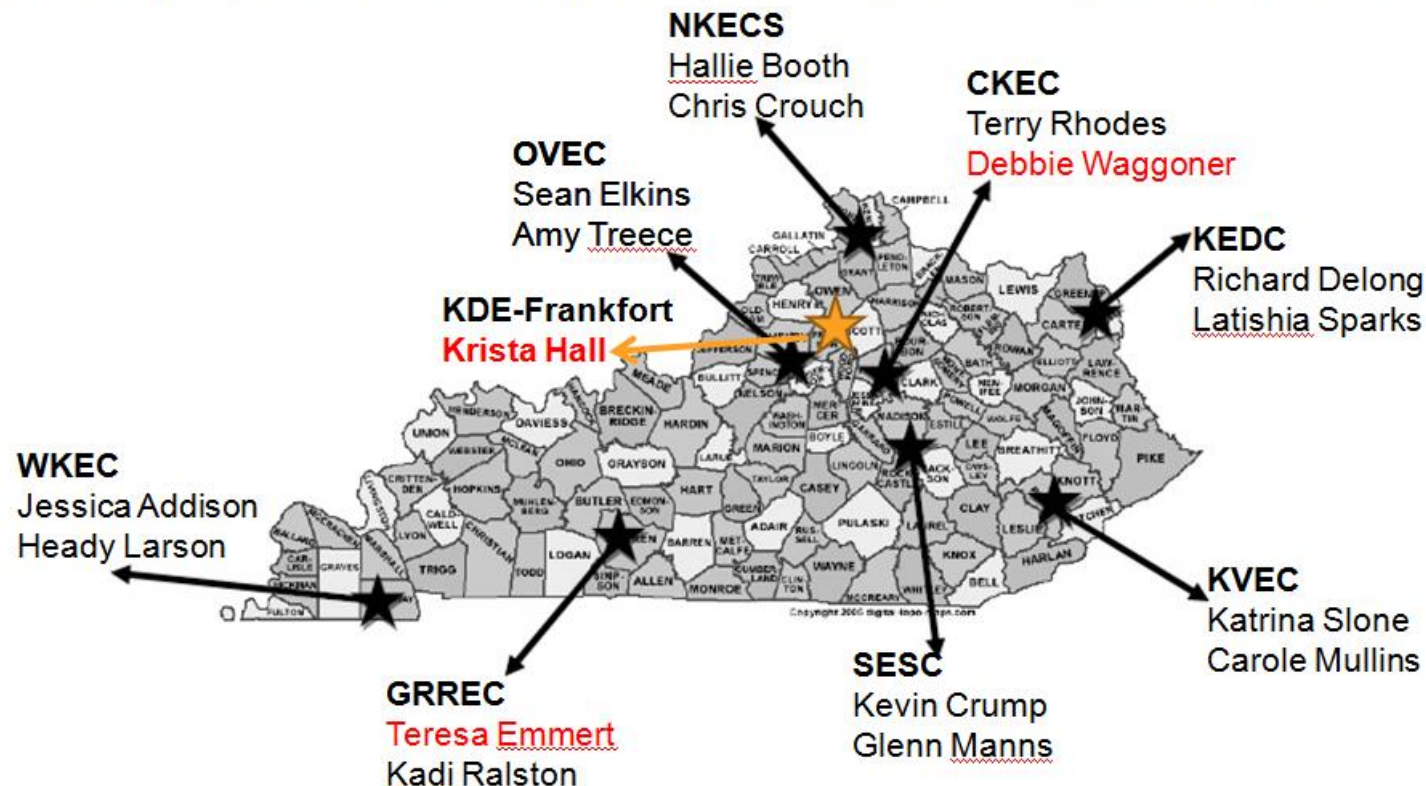
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Kentucky Department of Education Regional Instructional Specialists 2013-2014



Today's Learning Targets



- o I can use student misconceptions to inform my instruction .
- o I can explain the purpose of Formative Assessment Lessons (FALs).
- o I can describe the process of implementing a FAL.
- o I can access and use the materials to plan and perform a FAL.

Formative Assessment???

Exit Slips

entrance slips

bell ringers

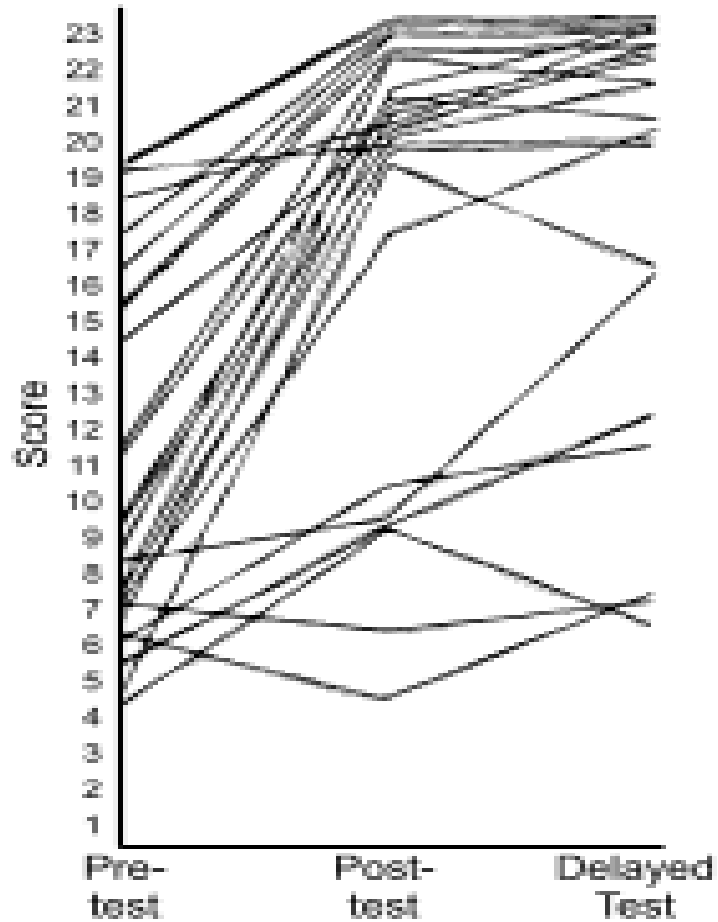
THUMBS UP/DOWN

Clickers

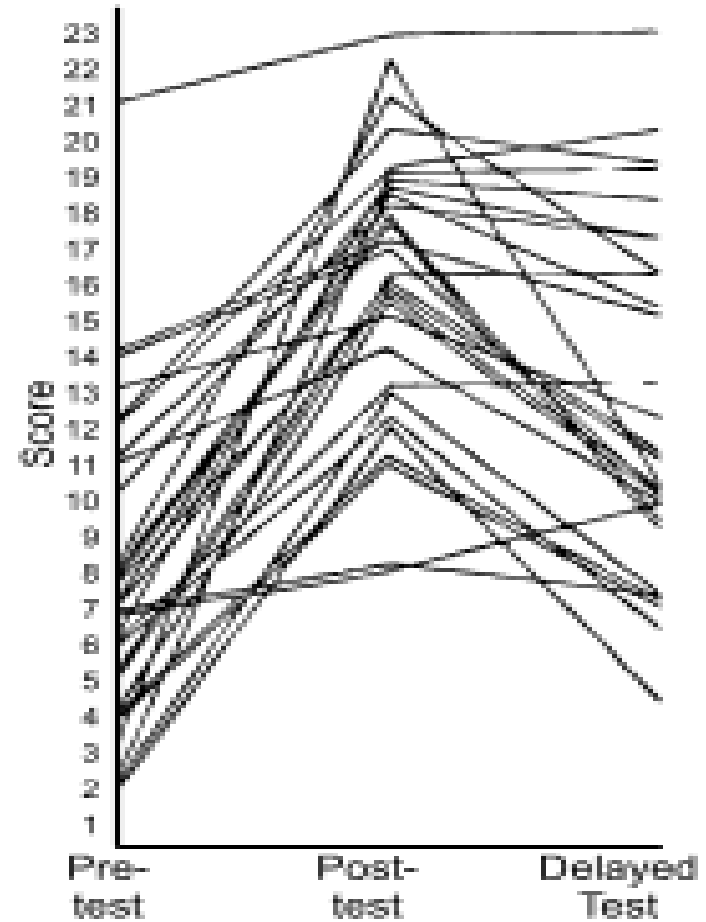
Common Assessments

WHITE BOARDS

Why Now?



Formative assessment techniques



Direct instruction

vs.

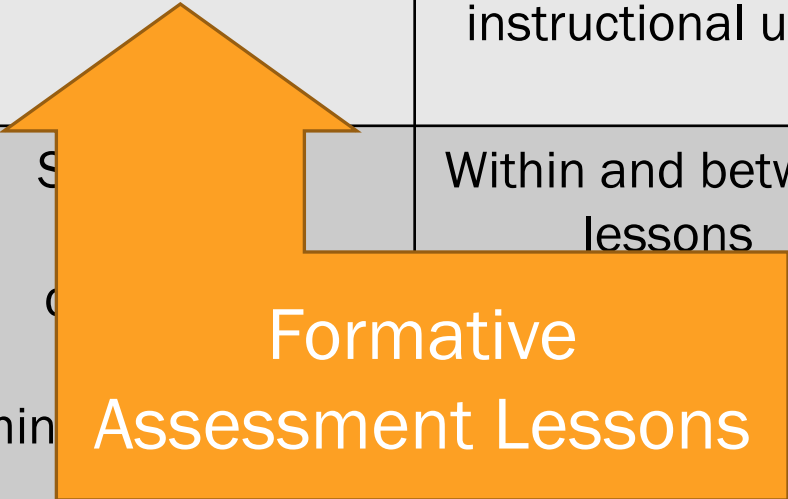


Five “Key Strategies” for Effective Formative Assessment

1. Clarifying, sharing, and understanding goals for learning and criteria for success with learners
2. Engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students’ learning.
3. Providing feedback that moves learning forward.
4. Activating students as owners of their own learning.
5. Activating students as learning resources for one another.

Typology of Kinds of Formative Assessment

Type	Focus	Length
Long-cycle	Across marking periods, quarters, semesters, years	4 weeks to 1 year
Medium-cycle	Within and between instructional units	1 to 4 weeks
Short-cycle	Within and between lessons	24-48 hours
Minute		5 seconds to 2 hours



Formative Assessment Lessons

How many beads are hidden under the cloud?



NOTE - the cloud is not drawn to scale, and

HINT - remember planes often fly through clouds....

Sample Responses to Discuss

Here is a sample response from a student in your class.

For the first part of the problem, I was thinking about how many beads were in the jar. I counted them and found there were 100 beads in total. Then I saw that there were 20 beads under the cloud. So I subtracted 20 from 100 and got 80. That means there were 80 beads in the jar.

What's the teacher doing now?

Moving around to groups, taking notes, asking questions and answering questions with questions!

_____ 's Solution

Timothy's Solution

W	1	2	3	4	5	6	7	11	W
B		2	4	8	16	32		14	

$14 = 2$
 $5 = 27$

$$\begin{array}{r} 14 \\ + 27 \\ \hline 41 \end{array} \text{ B}$$

$11 + 41 = 52 \text{ total}$

Hannah's Solution

115 beads because if you follow the pattern...

$$16 - 2 = 14 + 5 = 19,$$

$$19 + 32 = 51, 51 + 6 = 57$$

$$64 - 5 = 58 + 57 = 115 \text{ beads}$$

Curtis' Solution

B	1	2	3	4	$\boxed{5}$	$\boxed{6}$	7
W	2	4	8	32	$\boxed{256}$	8192	2697152
				$\boxed{30}$		$\boxed{8187}$	
						8484	

Tony's Solution

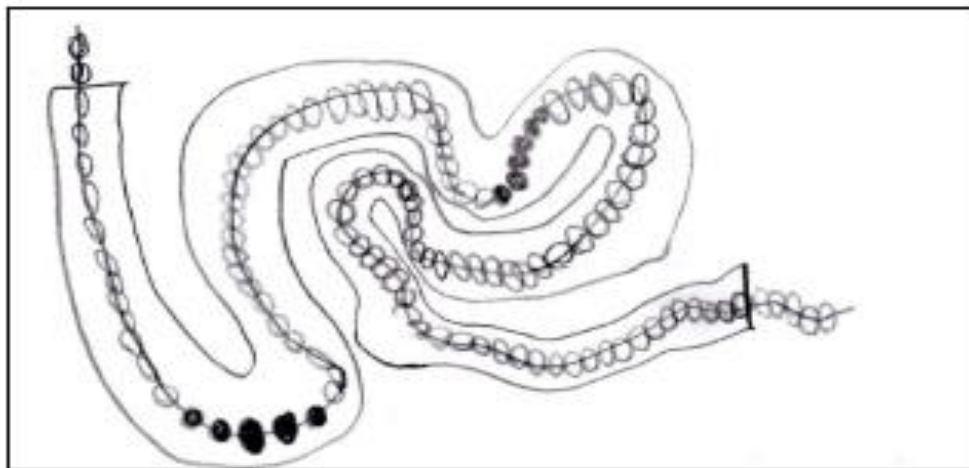
Black	White
1	2
2	4
3	8
4	
?	16
.	$\frac{2}{5}$
7	
	16
576 = 11	$\frac{7}{9}$
	<u>11 + 9 = 20</u>

Claire's Solution

P	B	W
1	1	2
2	2	4
3	3	8
4	4	16
5	5	32
6	6	64
7	7	128

$14 + 5 + 32 + 6 + 60 = \underline{117}$

Victoria's Solution



Sam's Solution

$$\begin{array}{r} 14 \\ + 5 \\ \hline 19 \end{array}$$

$$\begin{array}{r} 19 \\ + 32 \\ \hline 51 \end{array}$$

$$\begin{array}{r} 51 \\ + 6 \\ \hline 57 \end{array}$$

$$\begin{array}{r} 57 \\ + 59 \\ \hline 116 \end{array}$$

$$\begin{array}{r} 64 \\ + 5 \\ \hline 69 \end{array}$$

$$\begin{array}{r} 16 \\ + 2 \\ \hline 18 \end{array}$$

$$\begin{array}{r} 18 \\ + 14 \\ \hline 32 \end{array}$$

the pattern was black + 1 then white + 2

so 1 black, 2 white, 2 black, 4 white, 3 black, 8 white, 4 black, 16 white, 5 black, 32 white, and so on.

116 beads

Rick's Solution

$\rightarrow 5 + 6 = 11$
 $\rightarrow 16 + 32 + 64 = 112$
 $\frac{112}{105} = 1.066$ showing

Black Set	beads	$y = x$
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	

White Set	beads	$y = 2x$
1	2	
2	4	
3	6	
4	8	
5	10	
6	12	
7	14	

Number of Beads Pattern

Key
 ● = black $y = x$
 ○ = white $y = 2x$

Collaborative work

- (1) Share your method with your partner(s) and your ideas for improving your individual solution.
- (2) Together in your group, agree on the best method for completing the problem.
- (3) Produce a poster, showing a joint solution to the problem.
- (4) Make sure that everyone in the group can explain the reasons for your chosen method, and describe any assumptions you have made.
- (5) Check your work.

How Did You Work?

Post-Task Reflection: Beads Under the Cloud

Tick the boxes and complete the sentences that apply to your work.

1.) Check one, then complete the sentence below:

Our group work was better than my own individual work.

OR

My own individual work is better than our group work.

I prefer (*circle one*) **our method** / **my method** because:

2.) Check one, then complete the sentence below:

Our method is similar to: _____
(*add name of sample response*)

OR

Our method is different from **all** of the sample responses.

I prefer (*circle one*) **our method** / **the sample response** method because:

3.) Check one and complete the sentence:

We checked our method by: _____

OR

We could check our method by: _____

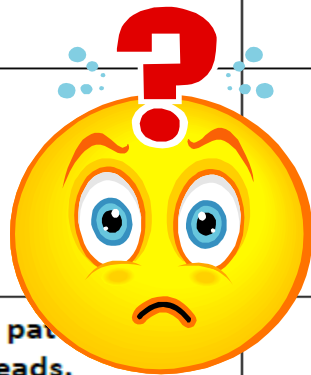
Misconceptions & Feedback Questions



- What *misconceptions* might your students have with this FAL or an activity like this one?
- What possible *feedback questions* could you ask to *move their learning forward*?

Common issues - Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student forgets to look at one of the sets of beads (only looking at white or black, and not both).	<ul style="list-style-type: none">• <i>How could you simplify this into an easier task?</i>• <i>What sort of diagram might be helpful?</i>
Student work is unsystematic. Student sees the patterns as two separate entities and does not see the relationship between the white and black patterns or how they alternate on the string.	<ul style="list-style-type: none">• <i>How do the black beads grow?</i>• <i>What patterns do you notice?</i>• <i>What is the same and what is different about the patterns of the black & white beads?</i>
Student assumes the picture of the cloud is to scale and that not very many beads can fit under the cloud.	<ul style="list-style-type: none">• <i>What assumptions can you make about the size of the cloud?</i>• <i>Are all math diagrams always drawn to scale?</i>
Student writes answers without explanation.	<ul style="list-style-type: none">• <i>How could you explain/show how you reached your conclusions so that someone in another class understands?</i>• <i>How can you use words and/or variables to describe the patterns?</i>
Student does not generalize.	<ul style="list-style-type: none">• <i>Can you describe a visual pattern in the black beads and the white beads? How could I find out the number of white beads that follow the set of ten black beads? Or the set of 20 black beads?</i>
Student correctly identifies the pattern but does not consider both the black and the white beads.	<ul style="list-style-type: none">• <i>Think of another way of solving the problem. Is this method better or worse than your original one? Explain your answer. Can you extend your solution to include exponents?</i>



Provide feedback that moves students forward.

Beads Under the Cloud
I can generate and analyze patterns.

How many beads are hidden under the cloud?

NOTE - the cloud is not drawn to scale, and
HINT - remember planes often fly through clouds...

Name _____

0	
1	● ● ●
2	●
3	● ● ● ● ●
4	● ● ● ● ●
5	● ● ● ● ●
6	● ● ● ● ●
7	● ● ● ● ●
8	● ● ● ● ●
9	● ● ● ● ●

I can identify numbers 0-9 by counting and matching.

Students	Misconceptions	What will you do next to help clear up misconceptions?
All	The size of the cloud - they thought the size of the cloud related to the # of beads under it.	Explain, more in depth, what "drawn to scale" means.
Majority	All sample responses are correct answers.	Discussion about why you give sample responses - to see strategies. Their job is to analyze the responses.
AD KF CB	Some of the beads from the pattern "under" the cloud are "out" of the cloud.	Look at ways to subtract those beads AFTER solving problem to avoid confusion

Students	Misconceptions	What will you do next to help clear up misconceptions?
Number ID	Don't know ⁶⁹ the numeral	• number structure with fingers • flashing numeral cards
Random dots	Mis-counting	Modeling organized counting
Ten Frame	Filling the ten frame	Model counting aloud when dotting

This problem solving Formative Assessment Lesson is designed to be part of an instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of the unit.

Mathematical goals

This problem solving lesson is intended to help you assess how well students are able to identify patterns (both linear and exponential) in a realistic context: the number of beads of different colors that are hidden behind the cloud. In particular, this problem solving lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns
- Describing and explaining findings clearly and effectively.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

4-OA: Generate and analyze patterns.

5-OA: Analyze patterns and relationships

6-EE: Represent and analyze quantitative relationships between dependent and independent variables.

8-F: Use functions to model relationships between quantities.

E-LE: Linear, Quadratic, and Exponential Models

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

FAL Outline:

1. Pre-Assessment – Individual student work
← Determine FEEDBACK QUESTIONS
2. Intro Lesson
3. Collaborative Activity
← Oral FEEDBACK QUESTIONS
4. Whole Class Discussion
5. Post-Assessment
← Written FEEDBACK QUESTIONS

Formative Assessment Lesson

Mathematics Assessment Project

- o Designed and developed well-engineered assessment tools (FALs) to support US schools in implementing the Common Core State Standards for Mathematics (CCSS).
- o Funding is provided by the Bill and Melinda Gates Foundation through the University of California, Berkeley.
- o <http://map.mathshell.org/materials/lessons.php>
- o KDE mathematics specialists are developing FALs for grades K-5.
- o www.debbiewaggoner.com
- o www.teresaemmert.weebly.com

Re-teaching vs. Re-engagement



Re-teaching

Teach unit again

Address missing basic skills

On the same or similar problems

Practice more

Cognition lower

Re-engagement

Revisit student thinking

Address conceptual understanding

Examine task from different perspective

Critique student approaches

Cognition higher

Two Kinds of FALs

Concept Focused

- Specific content is central to the activity
- Generally one correct answer, but may be a variety of ways to get that answer
- Usually includes a small group/pairs activity that requires manipulation of mathematical information (often in the form of card sorts, etc.)

Problem Solving Focused

- Activity draws on knowledge about a variety of content
- Sometimes a number of answers are plausible but must be defended
- Usually includes a small group/pairs activity that requires analyzing sample student work in order to look at different strategies for solving the problem at hand.

Both types include both concepts and problem solving, but each puts more emphasis on one than the other.

NUMBER PUZZLES

1.) A restaurant is open 24 hours a day. The manager wants to divide the day into work shifts of equal length. Show the different ways this can be done. The shifts should not overlap, and all shifts should be a whole number of hours long.

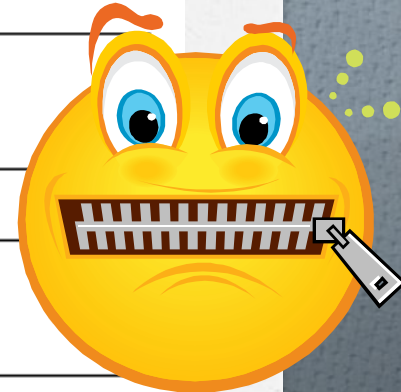
2.) Sammie's Game Store wants to rent a space of 32 square units. Find all the possible ways Sammie can arrange the squares. How are the rectangles you found related to the factors of 32?

Pre-Assessment

3.) Lewis has chosen a mystery number. His number is larger than 12 and smaller than 40 and it has exactly three factors. What could his number be?

4.) How many rectangles can you build with a prime number of square tiles?

5.) Which group of numbers – evens or odds- has more prime numbers? Why?

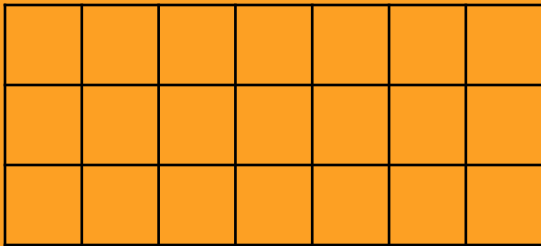


What is the Mystery Number?

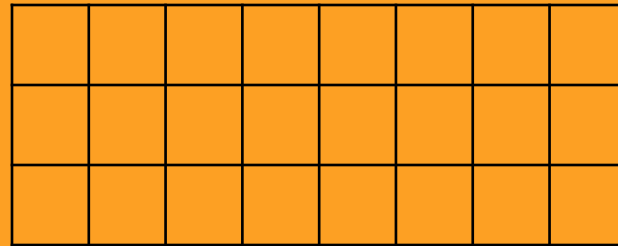
Puzzle 1

- 1-This number of tiles will make a rectangle 3 tiles wide.
- 2-This number of tiles will make a rectangle 4 tiles wide.
- 3- This number is greater than 20.
- 4- This number is less than 30.

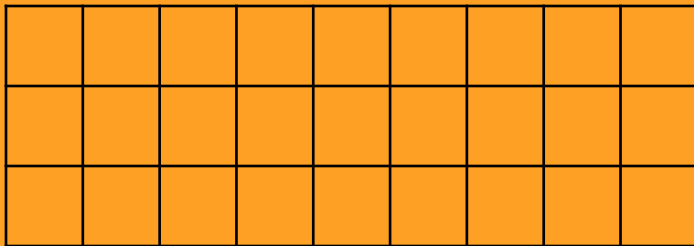
Arrays 3 Squares Wide



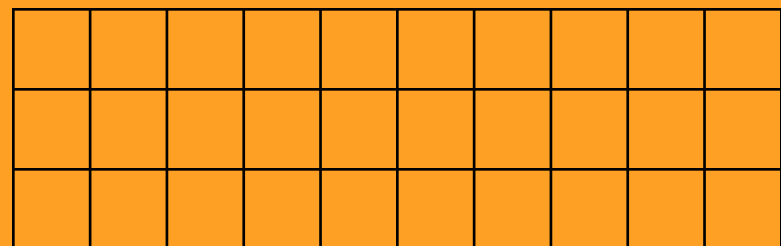
3 tiles wide
7 tiles long **21**



3 tiles wide
8 tiles long **24**

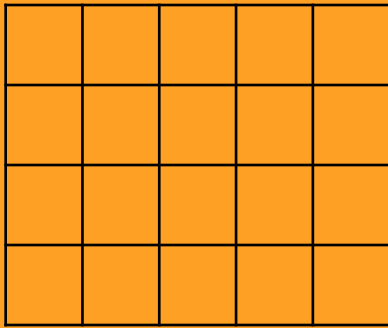


3 tiles wide
9 tiles long **27**

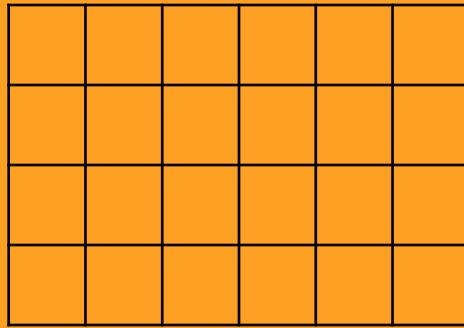


3 tiles wide
10 tiles long **30**

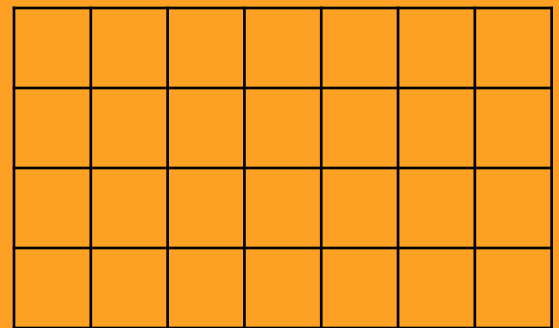
Arrays 4 Squares Wide



4 tiles wide
5 tiles long **20**

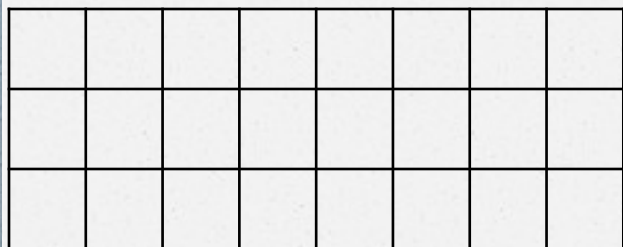


4 tiles wide
6 tiles long **24**

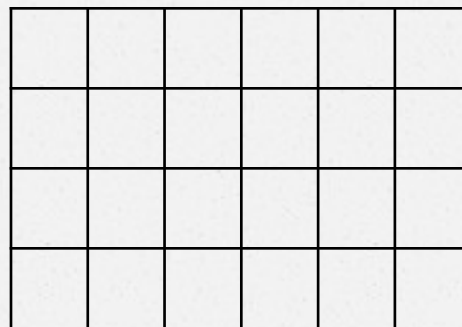


4 tiles wide
7 tiles long **28**

Are Any Arrays the Same?



3 tiles wide
8 tiles long **24**



4 tiles wide
6 tiles long **24**

Could 24 be our mystery
number?

Hundreds Chart

Lets look at the numbers between 20 and 30.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Which of these numbers is a multiple of 3?

Which of these numbers is a multiple of 4?

Which number between 20 and 30 is a multiple of 3 AND 4?

CARD SET A: CLUES

A – This number will make a rectangle 3 tiles wide.	F – This number is a multiple of 5.
B – This number has exactly 8 factors.	G – This number has an odd number of factors.
C – 1 factor of this number is 4.	H – This number is a square number.
D – This number is not a multiple of 5 or 7.	I – This number is not even and is less than 50.
E – Add the digits of this number and the sum is odd.	J – The product of the digits of this number is greater than 20.
Blank Clue Card	

CARD SET B: TASK CARDS

<p>NUMBER PUZZLE 1:</p> <p>Clue 1 – A Clue 3 – C Clue 2 – B Clue 4 – D</p>	<p>NUMBER PUZZLE 5:</p> <p>Clue 1 – E Clue 3 – G Clue 2 – F Clue 4 – H</p>
<p>NUMBER PUZZLE 2:</p> <p>Clue 1 – A Clue 3 – I Clue 2 – E Clue 4 – F</p>	<p>NUMBER PUZZLE 6:</p> <p>Clue 1 – E Clue 3 – J Clue 2 – I Clue 4 – D</p>
<p>NUMBER PUZZLE 3:</p> <p>Clue 1 – A Clue 3 – C Clue 2 – H Clue 4 – G</p>	<p>NUMBER PUZZLE 7:</p> <p>Clue 1 – A Clue 3 – B Clue 2 – E Clue 4 – F</p>
<p>NUMBER PUZZLE 4:</p> <p>Clue 1 – A Clue 3 – F Clue 2 – E Clue 4 – I</p>	<p>NUMBER PUZZLE 8:</p> <p>Clue 1 – D Clue 3 – E Clue 2 – I Clue 4 – Blank (write a fourth clue)</p>

CARD SET C: MYSTERY NUMBERS

36

45

24

30

72

47

100

29

HUNDREDS BOARDS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Instructions for Working Together

- *Place the Clue Cards and Mystery Numbers face up so everyone in the group can see the clues and numbers. Solve one Number Puzzle at a time.*
- *Find the four clues that match the Number Puzzle your group is solving. Divide the Clue Cards among the group.*
- *Solve for the Mystery Number by using a strategy your group is comfortable with. Every group member should help solve for the mystery.*
- *After your group has reached a consensus, match the Number Puzzle Card to the Mystery Number Card.*
- *Set these two cards aside and place the Clue Cards face up with the other clues.*
- *Choose another Number Puzzle to solve.*

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This lesson is intended to help you assess how well students are able to use “clues” about numbers including: factors, multiples, prime, composite, square, even, odd, etc. In particular, this unit aims to identify and help students who have difficulties with:

- understanding the difference between primes and composites.
- understanding the difference between factors and multiples.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grade, with emphasis on:

Operations and Algebraic Thinking 4.OA

- Gain familiarity with factors and multiples

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:


1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
6. Attend to precision.
7. Look for and make use of structure.

Misconceptions & Feedback Questions



- What *misconceptions* might your students have with this FAL or an activity like this one?
- What possible *feedback questions* could you ask to *move their learning forward*?

Common issues - Suggested questions and prompts:

Common Issues	Suggested questions and prompts
<p>Student doesn't find all factor pairs for a given number.</p> 	<ul style="list-style-type: none">• <i>How can you make sure you haven't left out any factor pairs for a number?</i>• <i>How can you use rectangular arrays to decide if you have all the factor pairs? How can you use a factor tree to decide if you have all the factor pairs? How could you make a chart to decide if you have all the factor pairs?</i>
<p>Student doesn't have a strategy to find the factors of a number.</p>	<ul style="list-style-type: none">• <i>Can you think of a way that we can model this problem?</i>• <i>If you were given square tiles, could you model a rectangle with 10 tiles? How does this rectangle's width and length compare to the factors of 10?</i>
<p>Conceptual understanding of prime numbers. Student doesn't realize that a prime number will have only one factor pair, only one rectangular array, and it will not have a factor tree if it is prime. Student should also recognize the only EVEN prime number is 2.</p>	<ul style="list-style-type: none">• <i>How many other rectangular arrays can you find for each number? What do you notice about the number of rectangular arrays possible for each number? Which number(s) do not have a factor tree? Why not?</i>• <i>Why are more of the prime numbers odd than even?</i>
<p>Not understanding that if there are an odd number of factors for a number, the number is a square number.</p>	<ul style="list-style-type: none">• <i>Which rectangular arrays are actually squares? How many total factors do each of your numbers have? How can you tell if a number will have a square as one of its arrays?</i>

Provide feedback that moves students forward.

NUMBER PUZZLES

1.) A restaurant is open 24 hours a day. The manager wants to divide the day into work shifts of equal length. Show the different ways this can be done. The shifts should not overlap, and all shifts should be a whole number of hours long.

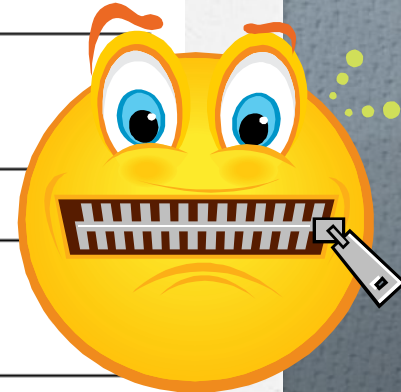
2.) Sammie's Game Store wants to rent a space of 32 square units. Find all the possible ways Sammie can arrange the squares. How are the rectangles you found related to the factors of 32?

Post-Assessment

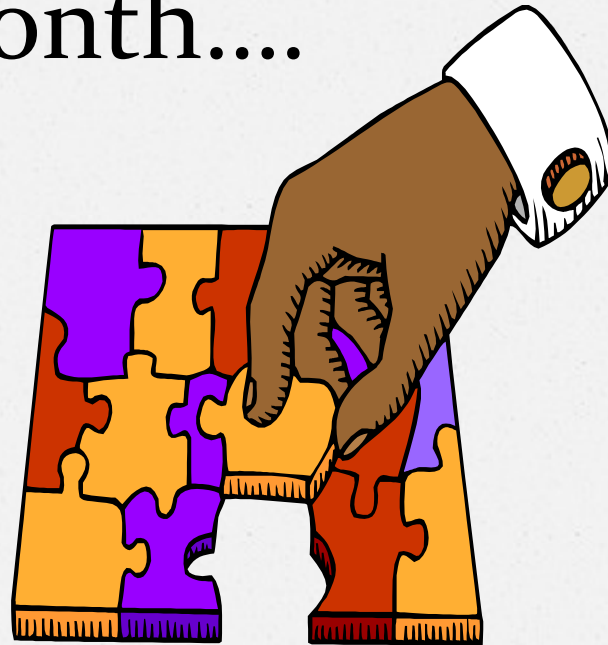
3.) Lewis has chosen a mystery number. His number is larger than 12 and smaller than 40 and it has exactly three factors. What could his number be?

4.) How many rectangles can you build with a prime number of square tiles?

5.) Which group of numbers – evens or odds- has more prime numbers? Why?

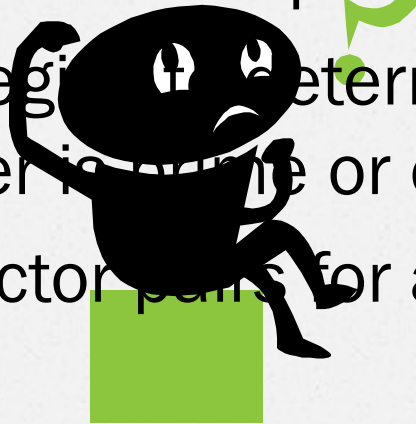


Write your own Number Puzzle
Clues for number concepts your
students will be studying the next
month....



Learning Targets

- Define prime and composite numbers.
- Explain strategies to determine whether a whole number is prime or composite.
- Identify all factor pairs for any given number 1-100.
- Recognize and explain that a whole number is

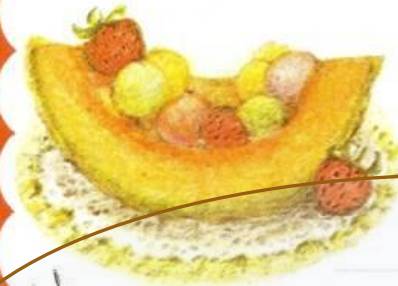


If I gave an exit slip, quiz question, or other short cycle formative assessment for each of these targets, would it tell me if they really got and were able to do the standard?

- a
- D
- n
- G

From the kitchen of...

Susan Wunderlich



Red Velvet Cake with Cream Cheese Frosting

$2\frac{1}{2}$ c. flour
 $1\frac{1}{2}$ c. sugar 1 tsp. soda
2 c. salad oil 1 tsp. salt
2 eggs beaten 2 Tbsp. cocoa
1 tsp. vinegar 1 c. buttermilk
2 oz. red food coloring

Cream sugar & oil in bowl. Add eggs & beat well. Add vinegar & food coloring. Beat well. Sift flour, soda, salt & cocoa together. Add to creamed mixture alternately with buttermilk. Add vanilla & beat well. Pour into 2(3) greased & floured cake pans. Bake at 350° 30-35 minutes.

Frosting - cream 1 stick margarine & 8 oz. cream cheese. Add 1 Tbsp. vanilla, then 1 box confec. sugar. Then 1 c. chopped nuts if desired.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Today's Learning Targets



- o I can use student misconceptions to inform my instruction .
- o I can explain the purpose of Formative Assessment Lessons (FALs).
- o I can describe the process of implementing a FAL.
- o I can access and use the materials to plan and perform a FAL.

Who are we?

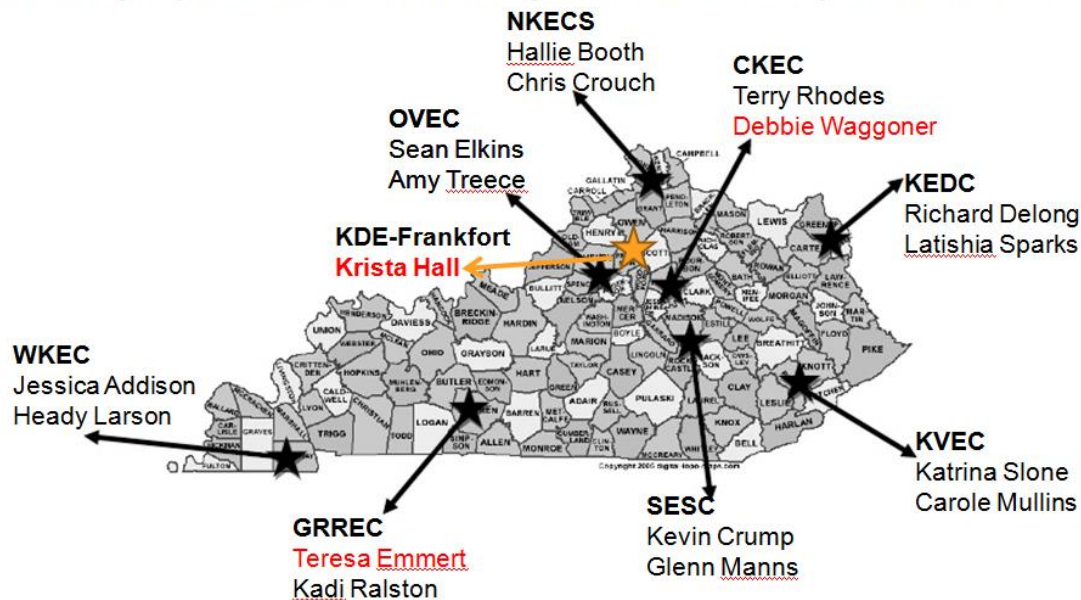
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