

# Using Student-Response Systems in Entry-Level College Mathematics Courses

NCTM Regional Conference, Louisville,  
KY  
November 7, 2013

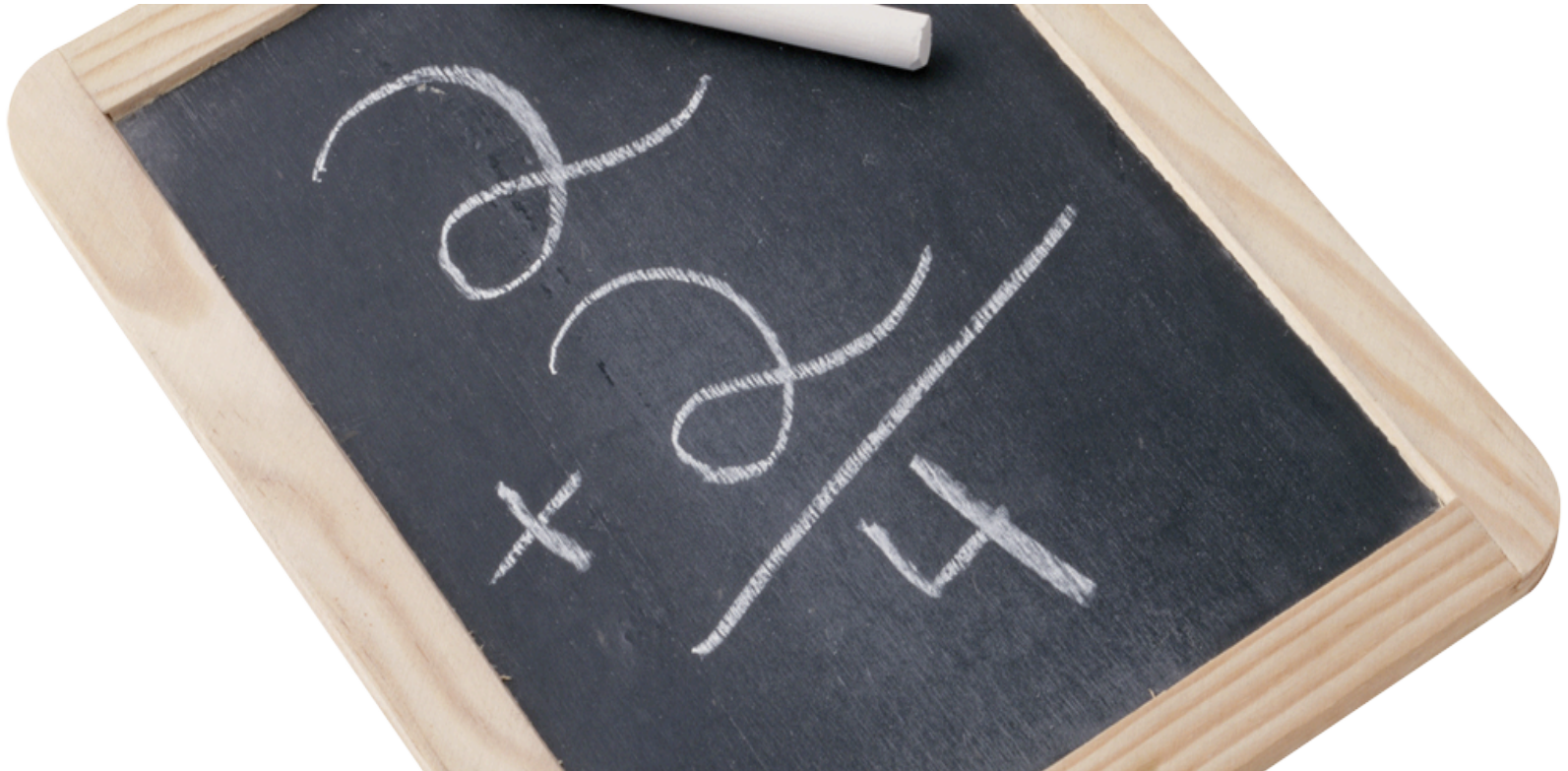
Jonathan Engelman, MA  
Kettering College  
[jonathan.engelman@kc.edu](mailto:jonathan.engelman@kc.edu)



# Roadmap

- ◆ Formative Assessment
- ◆ What are clickers?
- ◆ One clicker sequence, with results
- ◆ Debrief
- ◆ Plethora of example questions

# What is the goal of teaching?



# The Three Questions:

# The Three Questions:

- 1. Where is the learner right now?

# The Three Questions:

- 1. Where is the learner right now?
- 2. Where is the learner going?

# The Three Questions:

- 1. Where is the learner right now?
- 2. Where is the learner going?
- 3. How will the learner get there?

# Formative Assessment Characteristics



# Formative Assessment Characteristics

- ◆ 1. Intended outcomes of learning are clearly stated and shared

# Formative Assessment Characteristics

- ◆ 1. Intended outcomes of learning are clearly stated and shared
- ◆ 2. Designed to collect quality evidence to inform teaching and improve learning

# Formative Assessment Characteristics

- ◆ 1. Intended outcomes of learning are clearly stated and shared
- ◆ 2. Designed to collect quality evidence to inform teaching and improve learning
- ◆ 3. Formative feedback to improve learning is provided to each student

# Formative Assessment Characteristics

- ◆ 1. Intended outcomes of learning are clearly stated and shared
- ◆ 2. Designed to collect quality evidence to inform teaching and improve learning
- ◆ 3. Formative feedback to improve learning is provided to each student
- ◆ 4. Students are engaged in the assessment process and, to the extent possible, in planning their own next steps for learning

# Questioning

An Assessment for Learning Strategy



# 3 Things to Take Away About Questioning

- ◆ Plan questions ahead of time
- ◆ Questions directly connect to lesson objectives
- ◆ Leave enough time for students to think, respond to the prompt, and respond to one another

# Planning Your Questions

- ◆ Previous Material/Warm Up Questions:
- ◆ Pre-Assessment Questions
- ◆ Engagement Questions
- ◆ Checking for Understanding Questions
- ◆ Wrap Up Questions

# Clickers in 30 Seconds



# Clicker Questions

- ◆ Three types:
  - ◆ Multiple Choice
  - ◆ Numeric
  - ◆ Alpha Numeric
- ◆ Can be timed or un-timed
- ◆ Can be scored for correctness or participation

# Example Sequence: Exponents

- ◆ Use the product, quotient, zero exponent, negative exponent, power, and special product rules for exponents in order to simplify them.
- ◆ Use order of operations to simplify real number expressions containing exponents

# Critical Thinking Question

◆ Simplify:  $(-3)^2$

# Critical Thinking Question

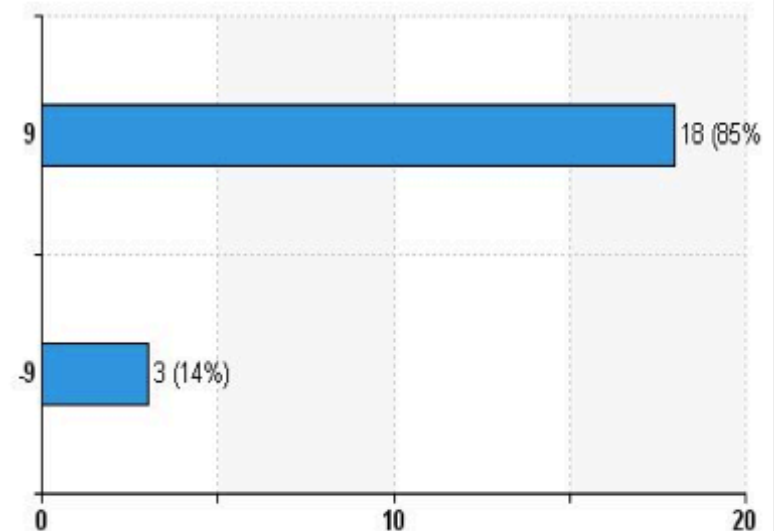
Question Type: Numeric  
Significant Digits: 8  
Time Started: 11:26:40 AM  
Correct Answer(s): 9

Maximum Score: 1.00  
Number of Responses: 21  
Number Missing: 0  
Class Average: 1.00

Answer	#	%	Performance Points
-9	3	14%	1
9	18	86%	1

CRITICAL THINKING QUESTION  
#5

SIMPLIFY:  $(-3)^2$



# Note

- ◆ The following question was added after section 1 and before sections 3 and 4

# Critical Thinking Question

💧 Simplify:  $-3^2$

# Critical Thinking Question

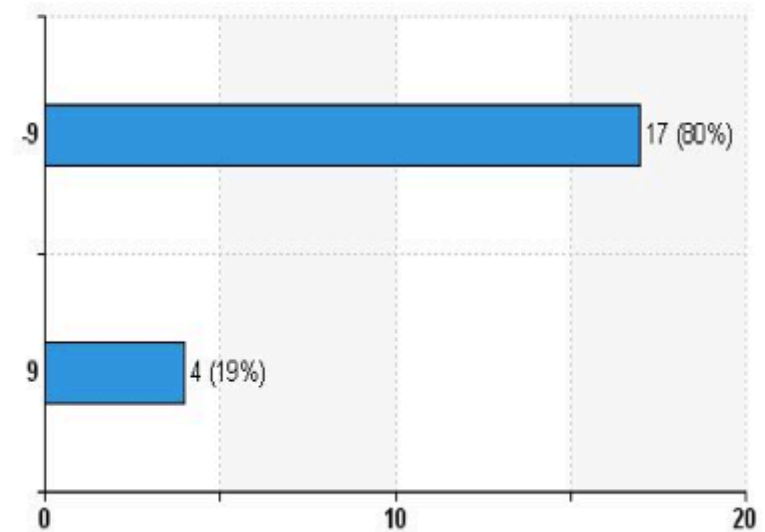
Question Type: Numeric  
Significant Digits: 8  
Time Started: 11:27:42 AM  
Correct Answer(s): -9

Maximum Score: 1.00  
Number of Responses: 21  
Number Missing: 0  
Class Average: 1.00

Answer	#	%	Performance Points
-9	17	81%	1
9	4	19%	1

CRITICAL THINKING QUESTION  
#6

SIMPLIFY:  $-3^2$



# Critical Thinking Question

• Simplify:  $\frac{x^7}{x^{-4}}$



# Critical Thinking Question

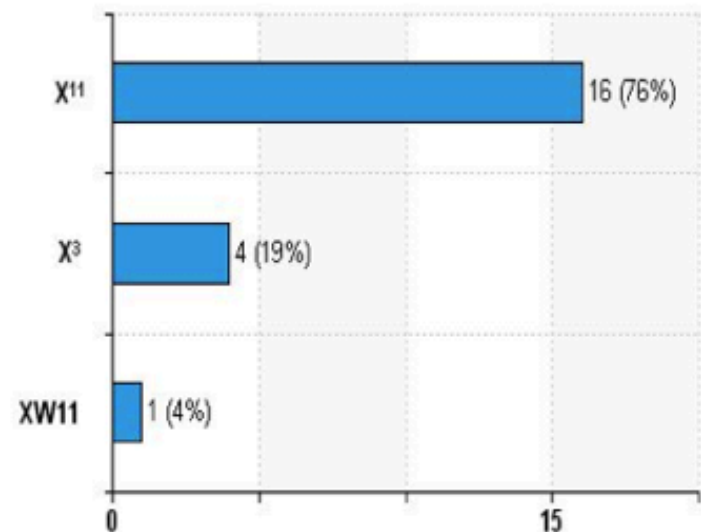
Question Type: Alphanumeric  
Significant Characters: 16  
Time Started: 11:51:12 AM  
Correct Answer(s): X^11

Maximum Score: 1.00  
Number of Responses: 21  
Number Missing: 0  
Class Average: 1.00

Answer	#	%	Performance Points
XW11	1	5%	1
X^11	16	76%	1
X^3	4	19%	1

CRITICAL THINKING QUESTION  
#7

USE THE RULES OF EXPONENTS TO SIMPLIFY:

$$\frac{x^7}{x^{-4}}$$


# Critical Thinking Question

- $-2^4$  vs  $(-2)^4$
- A) Both -16
- B) Both 16
- C) 16, -16
- D) -16, 16
- E) None of the above

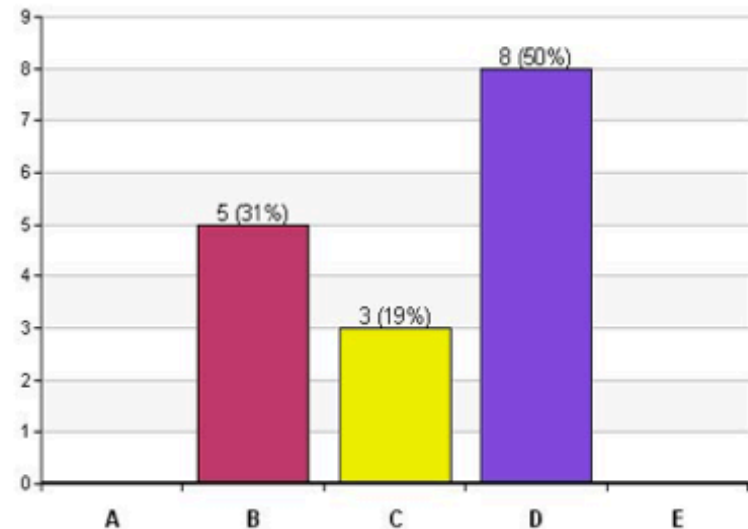
# Critical Thinking Question

Answer	#	%	Performance Points
A	0	0%	1
B	5	31%	1
C	3	19%	1
D	8	50%	1
E	0	0%	1

CRITICAL THINKING QUESTION # 2

■  $-2^4$  vs  $(-2)^4$

- A) Both -16
- B) Both 16
- C) 16, -16
- D) -16, 16
- E) None of the Above



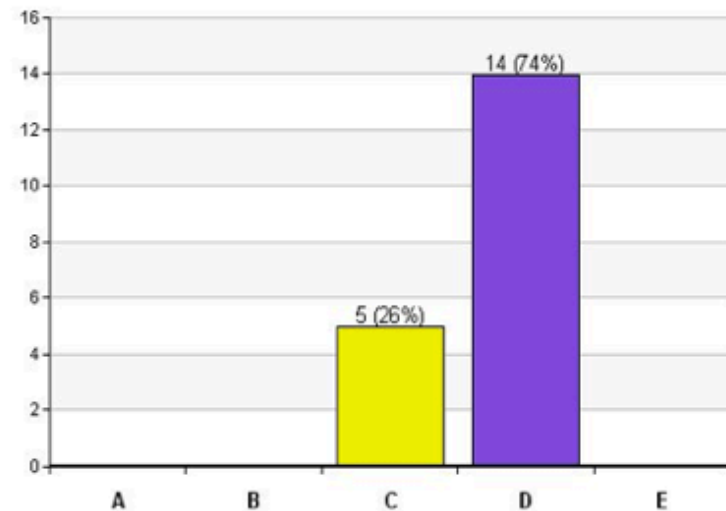
# Critical Thinking Question

Answer	#	%	Performance Points
A	0	0%	1
B	0	0%	1
C	5	26%	1
D	14	74%	1
E	0	0%	1

CRITICAL THINKING QUESTION # 2

■  $-2^4$  vs  $(-2)^4$

- A) Both -16
- B) Both 16
- C) 16, -16
- D) -16, 16
- E) None of the Above



# Debrief

- ◆ What did you notice about this questioning strategy?
- ◆ What worked about the strategy?
- ◆ When could this strategy be useful?
- ◆ How might you implement the strategy in your class?
- ◆ <http://mathquest.carroll.edu/>

# Debrief: Formative Assessment

- ◆ Asking questions that point toward the goal/s for the day
- ◆ Change instruction based on results
- ◆ Class discussion: “Why is that the right answer?”

# Questioning “on the Fly”

- ◆ Wait time – first and second
- ◆ How can the prompts we developed in the first strategy help you increase your wait time?
  - ◆ “what do others think?”
  - ◆ “how do you know if that is true or not?”
  - ◆ “who can add on to what Lindsay just said?”

# Pitfalls

- ◆ How much time to spend on each question?
- ◆ Cost of technology
- ◆ Multiple choice questions



# Resource



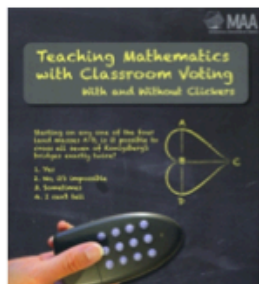
## MathQUEST/MathVote



### Resources for Clickers and Classroom Voting in Collegiate Mathematics

This page contains resources for classroom voting in mathematics, also known as ConcepTests for Peer Instruction or "Clicker Questions". These materials were developed through two NSF-funded projects: "MathQUEST: Math Questions to Engage Students" (2006-2009) and "MathVote: Teaching Mathematics with Classroom Voting" (2010-2013). These projects involve not only creating and testing libraries of classroom voting questions, but also involve the study of these teaching methods. For more details see our Project Summaries below. These projects have been run here at [Carroll College's, Department of Mathematics, Engineering, and Computer Science](#) by [Holly Zullo](#) and [Kelly Cline](#).

- [Question Libraries, Links, and Resources for Classroom Voting and Clickers in Mathematics](#): Compilations of classroom voting questions, published papers, web sites, and other links that may be helpful if you want to use classroom voting in mathematics.
- [Frequently Asked Questions](#)
- [Math QUEST: Project Summary](#)
- [MathVote: Project Summary](#)



*New book on classroom voting in mathematics:*

#### [Teaching Mathematics with Classroom Voting: With and Without Clickers](#)

Kelly Cline and Holly Zullo eds.

Published by the Mathematical Associate of America

Now available as an e-book or print-on-demand.

# Resource

## Question Libraries for Specific Courses

(These are the libraries that we use here at Carroll College. For libraries from people at other institutions see our [Resources](#) page, which has links to libraries for Statistics, College Algebra, Group Theory, and more.)

- [Differential Equations](#)
- [Linear Algebra](#)
- [Series, Sequences, and Difference Equations](#)
- [Multivariable Calculus](#)
- [Integral Calculus](#)
- [Differential Calculus](#)
- [Precalculus](#)
- [Algebra](#)
- [Statistics](#)

This material is based upon work supported by the National Science Foundation under Grants DUE 536077 and 0836775. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

[The Carroll College Department of Mathematics, Engineering, and Computer Science](#)

[kcline@carroll.edu](mailto:kcline@carroll.edu)

# Resource

## Classroom Voting Questions: Algebra

This library of questions was developed by Project MathVote starting in 2010. Please e-mail us for a teacher's edition containing solutions and past voting results. If you use these questions in class, we would appreciate it if you could send us your voting results, the percentages of your class voting for each of the options.

[Click Here](#), for the complete set of algebra questions, ordered following the topic headings below (as in Hall and Mercer's "Beginning and Intermediate Algebra" 3rd edition).

[Click Here](#), for these questions in a larger font, which may be more useful if you plan to cut and paste these questions from the pdf file into PowerPoint.

- [The Real Number Line](#)
- [Graphs of Linear Equations in Two Variables](#)
- [Solving Linear Equations in One Variable Using the Addition-Subtraction Principle](#)
- [Solving Linear Equations in One Variable Using the Multiplication-Division Principle](#)
- [Slope of a Line and Applications of Slope](#)
- [Solving Systems of Linear Equations in Two Variables Graphically and Numerically](#)
- [Solving Linear Inequalities Using the Multiplication-Division Principle](#)
- [Solving Compound Inequalities](#)
- [Negative Exponents and Scientific Notation](#)
- [Adding and Subtracting Polynomials](#)
- [Multiplying Polynomials](#)
- [Special Products of Binomials](#)
- [An Introduction to Factoring Polynomials](#)
- [Factoring Trinomials of the Form  \$x^2+bx+c\$](#)
- [Factoring by Grouping and a General Strategy for Factoring Polynomials](#)
- [Solving Equations by Factoring](#)
- [Extraction of Roots and Properties of Square Roots](#)
- [Complex Numbers and Solving Quadratic Equations with Complex Solutions](#)
- [Functions and Representations of Functions](#)

[mathquest.carroll.edu/libraries/ALG.student.02.04.pdf](http://mathquest.carroll.edu/libraries/ALG.student.02.04.pdf)

# Resource

## Classroom Voting Questions: Precalculus

This library contains questions from the [Cornell GoodQuestions Project](#), ConcepTests to Accompany Calculus, by Hughes-Hallett et al, and others we have written ourselves. We have been developing this collection of questions here at Carroll College since fall 2004. Please e-mail us for a teacher's edition containing solutions and past voting statistics.

[Click Here](#), for the complete set of precalculus questions, ordered following the topic headings below.

[Click Here](#), for these questions in a larger font, which may be more useful if you plan to cut and paste these questions from the pdf file into PowerPoint.

- [The Coordinate Plane](#)
- [Polar Coordinates](#)
- [Parametric Equations](#)
- [Lines and Parabolas](#)
- [Functions and Change](#)
- [Exponential Functions](#)
- [New Functions From Old: Compositions, Inverses, and Transforms](#)
- [Logarithmic Functions](#)
- [Angles](#)
- [Evaluating Trigonometric Functions](#)
- [Trigonometric Functions: Amplitudes, Periods, and Graphs](#)
- [Inverse Trigonometric Functions](#)
- [The Sum, Difference, Double, and Half Angle Formulas](#)
- [The Law of Sines and the Law of Cosines](#)
- [Solving Trigonometric Equations](#)
- [The Trigonometric Form of Complex Numbers](#)
- [Conic Sections](#)
- [Powers, Polynomials, and Rational Functions](#)
- [Polynomials, Synthetic Division, and Rational Functions](#)

# Questions?

◆ [jonathan.engelman@kc.edu](mailto:jonathan.engelman@kc.edu)

# Bonus Questions

# Critical Thinking Question

- ◆ Convert 8 m to mm:
  - ◆ A) 0.08 mm
  - ◆ B) 0.8 mm
  - ◆ C) 80 mm
  - ◆ D) 800 mm
  - ◆ E) 8000 mm

# Critical Thinking Question

- ◆ Convert 7.68 kg to pounds. Round to 2 decimal places
  - ◆ A) 3.48 lb
  - ◆ B) 3.49 lb
  - ◆ C) 16.90 lb
  - ◆ D) 16.91 lb
  - ◆ E) None of the above



# Critical Thinking Question

- ◆ True or False: 1 is a prime number.
- ◆ A) True
- ◆ B) False

# Critical Thinking Question

◆ Find the GCF of 360 and 1350.

# Critical Thinking Question

💧 Fill in the blank:  $-4$  \_\_\_\_\_  $-(-5)$

💧 A)  $>$

💧 B)  $<$

💧 C)  $\geq$

💧 D)  $\leq$

💧 E)  $=$

# Critical Thinking Question

- ◆ Simplify without using a calculator:

$$\frac{(-10 + 4) \cdot (-3)}{-7 - 2}$$

# Critical Thinking Question

- ◆ Simplify without using a calculator:

$$\frac{21}{5} \div \frac{7}{15}$$

# Critical Thinking Question

◆ Simplify  $\sqrt{32}$

◆ A)  $\sqrt{32}$

◆ B)  $2\sqrt{16}$

◆ C)  $16\sqrt{2}$

◆ D)  $4\sqrt{2}$

◆ E)  $2\sqrt{4}$

# Critical Thinking Question

◆ Solve:  $-(9-3x)-(4+2x)-4=-(-2-5x)-x$

# Critical Thinking Question

- ◆ In a chemistry class, 12 liters of a 12% alcohol solution must be mixed with a 20% solution to get a 14% solution. How many liters of the 20% solution are needed?



# Critical Thinking Question

- ◆ The weight of an object above Earth varies inversely as the square of its distance from the center of Earth. If an astronaut in a space vehicle weights 57 pounds when 6700 miles from the center of Earth, what does the astronaut weight when 4090 miles from the center?

# Critical Thinking Question

Factor:  $2x^2 + x - 36$

A)  $(2x - 4)(x + 9)$

B)  $(x + 4)(2x - 9)$

C)  $(2x + 4)(x - 9)$

D)  $(x - 4)(2x + 9)$

E) None of <sup>43</sup>the Above

# Critical Thinking Question

- ◆ For any value of  $x$ , the point  $(x,0)$  lies on the \_\_\_\_\_-axis.

# Critical Thinking Question

- ◆ For any value of  $y$ , the point  $(0,y)$  lies on the \_\_\_\_\_-axis.

# Critical Thinking Question

- ◆ Calculate the slope between the points  $(-4,3)$  and  $(-3,4)$ .

# Critical Thinking Question

- Write the slope-intercept form of the equation of the line passing through  $(5,8)$  with a slope of  $-2$ .

# Critical Thinking Question

- Find the slope of the line perpendicular to  $2x+3y=6$ .

# Critical Thinking Question

---

- ❖ Does the graph of the function open up or down?

$$f(x) = x^2 + 2x + 6$$

- ❖ A) Up
- ❖ D) Down



# Critical Thinking Question

---

- ❖ What is the equation for the axis of symmetry for the function?

$$f(x) = x^2 + 2x + 6$$

# Critical Thinking Question

---

- ❖ What is the  $y$ -coordinate of the vertex of the function?

$$f(x) = x^2 + 2x + 6$$

# Critical Thinking Question

---

- ❖ What is the  $y$ -coordinate of the  $y$ -intercept of the function?

$$f(x) = x^2 + 2x + 6$$

# Critical Thinking Question

---

❖ What are the  $x$ -intercepts of the function?

$$f(x) = x^2 + 2x + 6$$

- ❖ A)  $x=2,3$
- ❖ B)  $x=-2,-3$
- ❖ C)  $x=-2,3$
- ❖ D)  $x=2,-3$
- ❖ E) No solution

# Critical Thinking Question

---

❖ What is the domain of the function?

$$f(x) = x^2 + 2x + 6$$

# Critical Thinking Question

---

- ❖ What is the range of the function?

$$f(x) = x^2 + 2x + 6$$

# Critical Thinking Question

- ◆ If \$20,000 is invested at 3% annual interest, how much money, to the nearest cent, is in the account after 4 years if compounded continuously?

# Critical Thinking Question

- Which of the following ordered pairs is a solution to the system?
- $-2x+3y=11$  and  $-2x+2y=4$ 
  - A)  $(-4,1)$
  - B)  $(1,3)$
  - C)  $(5,7)$
  - D)  $(-1,3)$
  - E) None of the above



# Critical Thinking Question

- Find the angle of least positive measure that is coterminal with:  $539^\circ$

# Critical Thinking Question

Find  $\csc \theta$  if  $\sin \theta = \frac{2}{3}$ .

# Critical Thinking Question

- Use identities to find  $\sec \theta$  if  $\sin \theta = -\frac{\sqrt{3}}{2}$  with  $\theta$  in quad. IV.

# Critical Thinking Question

- Given a triangle with angles  $A$ ,  $B$ , and  $C$ , and opposite sides  $a$ ,  $b$ , and  $c$ , find the measurements of the remaining angle and sides assuming that  $B=30$  degrees,  $C=100$  degrees, and  $b=20$  ft.
  - A)  $A=50$  degrees,  $c=39.39$  ft,  $a=30.64$  ft
  - B)  $A=50$  degrees,  $a=10.5$  ft,  $c=20.26$  ft
  - C)  $A=50$  degrees,  $a=30.64$  ft,  $c=30.64$  ft
  - D)  $A=230$  degrees,  $a=30.64$  ft,  $c=39.39$  ft
  - E) None of the above

# Critical Thinking Question

- Find the (arithmetic mean, median, mode, range, midrange, standard deviation) of the following set of data:
  - 10, 8, 11, 11, 11, 13, 15

# Critical Thinking Question

- ◆ A coin is flipped three times in a row. What is the probability of getting 3 tails in a row?

# Critical Thinking Question

- ◆ If a single die is rolled, what is the probability of a 2 or an odd face coming up? Round your answer to three decimal places.

# Critical Thinking Question

- ◆ A jar contains 4 red spheres, 3 blue spheres, and 2 yellow spheres. Two spheres are drawn *without* replacement. Find the probability that a yellow sphere is drawn and then a blue ball, in that order. Round your answer to three decimal places.



# Critical Thinking Question

- ◆ Evaluate the expression for  $x=3$  and  $y=-5$ .

$$\frac{x - 3y}{2} + xy$$

# Critical Thinking Question

- Evaluate the expression:  $\frac{x^{-2}y^3z^{-4}}{x^{-3}y^5z^5}$ 
  - A)  $x^{2/3}y^{3/5}z^{-4/5}$
  - B)  $xy^2z^9$
  - C)  $x^{-5}y^8z$
  - D)  $\frac{x}{y^2z^9}$
  - E) None of the above

# Critical Thinking Question

Factor:  $25x^2 - 20x + 4$

A)  $(5x - 1)(5x - 4)$

B)  $(5x + 1)(5x + 4)$

C)  $(5x - 2)^2$

D)  $(5x + 2)^2$

E) None of<sup>68</sup> the Above

# Critical Thinking Question

Factor:  $x^4 - 1$

A)  $(x - 1)(x + 1)(x^2 + 1)$

B)  $(x^2 - 1)(x^2 + 1)$

C)  $(x - 1)(x + 1)^3$

D)  $(x - 1)^2(x + 1)^2$

E) None of the Above

# Critical Thinking Question

◆ Multiply and simplify:  $\frac{x - 3}{2x + 4} \cdot \frac{10x + 20}{5x - 15}$

# Critical Thinking Question

• Simplify:  $\left(\frac{x^{-1/4}}{y^{-2/3}}\right)^{-12}$

• A)  $\left(\frac{y}{x}\right)^{11}$

• B)  $\frac{y^{18}}{x^{48}}$

• C)  $\frac{x^{48}}{y^{18}}$

• D)  $\frac{x^3}{y^8}$

• E)  $\frac{y^8}{x^3}$

# Critical Thinking Question

◆ Simplify:  $\frac{2 + \sqrt{-4}}{1 + i}$

# Critical Thinking Question

- ◆ If  $(x-2)(x+1)=10$ , then
  - ◆ A)  $x-2=10$  or  $x+1=10$
  - ◆ B)  $x=4$  or  $x=-3$
  - ◆ C)  $x=2$  or  $x=-1$
  - ◆ D)  $x=5$  or  $x=-2$
  - ◆ E) None of the above



# Critical Thinking Question

💧 Solve and check your answer:

$$\sqrt{6y - 11} = 2y - 7$$

# CRITICAL THINKING QUESTION

- ◆ Solve the inequality:  $4 < 2 - 2x \leq 14$
- ◆ A)  $-1 > x \geq -6$
- ◆ B)  $1 > x \geq -6$
- ◆ C)  $-2 > x \geq -12$
- ◆ D)  $-1 > x \geq -12$
- ◆ E) None of the above

# Critical Thinking Question

---

- ❖ A diameter of a circle has endpoints  $(4,-3)$  and  $(-2,5)$ . What is the equation of this circle?
- ❖ A)  $(x - 1)^2 + (y - 1)^2 = 10$
- ❖ B)  $(x - 1)^2 + (y - 1)^2 = 25$
- ❖ C)  $(x + 1)^2 + (y + 1)^2 = 100$
- ❖ D)  $(x - 3)^2 + (y + 1)^2 = 10$
- ❖ E) None of the above

# Critical Thinking Question

- ◆ Are the given lines parallel, perpendicular, or neither?
- ◆ Line 1:  $2x+4y=12$
- ◆ Line 2:  $2x-y=4$ 
  - ◆ A) Parallel
  - ◆ B) Perpendicular
  - ◆ C) Neither

# Critical Thinking Question

- ◆ True or false: If  $a$  and  $b$  are in the domain of  $f(x)=1/x$ , then  $a+b$  is also in the domain of  $f(x)$ .
- ◆ A) True
- ◆ B) False

# Critical Thinking Question

- True or False: The function  $f(x) = x^3 + x + 1$  is an odd function.
  - D) False
  - B) True

# Critical Thinking Question

- ◆ True or False: The graph of  $y=f(x)$  and  $y=f(-x)$  cannot be the same.
- ◆ A) True
- ◆ B) False

# Critical Thinking Question

- \* True or False: You get the same graph by shifting the graph of  $y = x^2$  two units up, reflecting the shifted graph in the  $x$ -axis or by reflecting the graph of  $y = x^2$  in the  $x$ -axis, and then shifting the reflected graph up two units.
- \* D) True
- \* E) False



# Critical Thinking Question

- True or False: The functions  $f$  and  $g$  are inverses of each other:

$$f(x) = g(x) = \frac{1}{x}$$

- A) True
- B) False

# Critical Thinking Question

◆ True or False:

$$\log_a(u + v) = \log_a u + \log_a v$$

◆ A) True

◆ B) False

# Clicker Question

- *The terminal side of  $\theta$  in standard position contains the given point. Find the exact value of  $\csc(\theta)$ .*
  - $(-3,4)$

# Critical Thinking Question