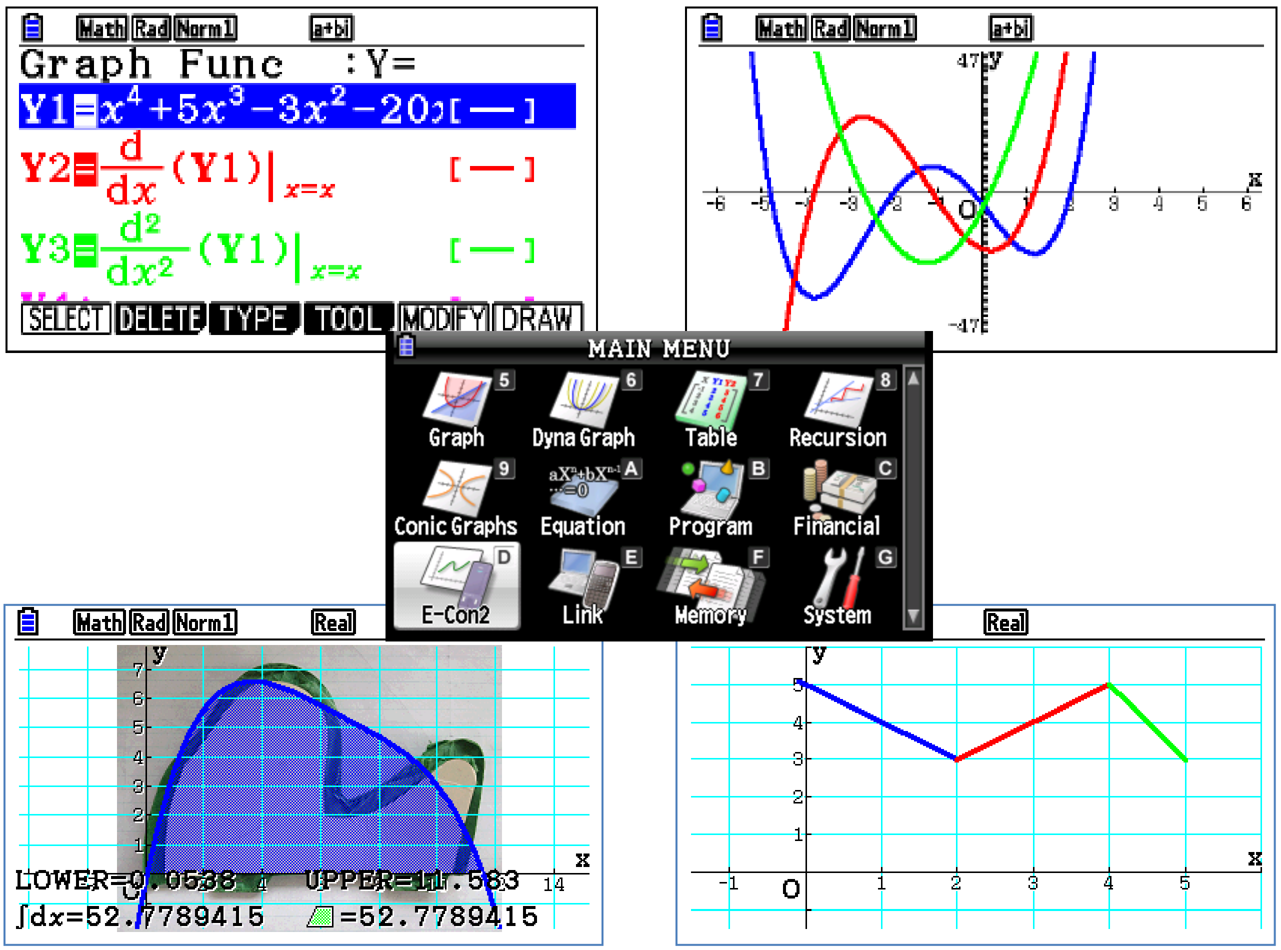
Calculus Investigations

Deedee Stanfield Henderson

deedeehenderson@aol.com

Oxford, Alabama



- Investigate Limit Rules
- Discover Derivative Rules and Graphs
- Compare Characteristics of the Graphs of f, f′, & f
- Calculate and Display are under a curve

Introduction to Limits

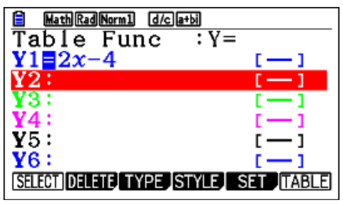
What is a limit? ____

<u>Formal Definition</u>: If *f*(*x*) becomes arbitrarily close to a unique number *L* as *x* approaches *c* from the left and right sides, then the **limit** of *f*(*x*) as *x* approaches *c* is *L*. This is written as $\lim_{x \to c} f(x) = L$.

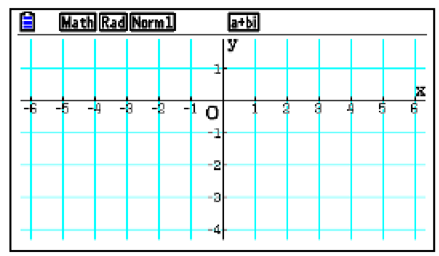
Our goals today are to: 1) Understand the limit definition.
2) Use the definition of a limit to estimate limits.
3) Determine whether limits of functions exist.

I. Estimate a Limit Numerically

1. Estimate $\lim_{x \to 3} (2x-4)$ by creating a table by using the table feature of your graphing calculator.
(Since $x \to 3$, that is x is <u>approaching</u> a value of 3, use values that greater and less than 3)



x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							



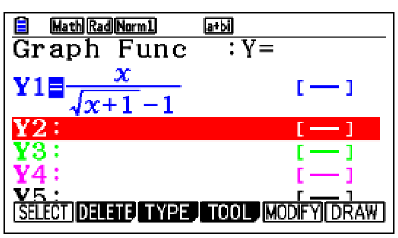
From the table, it appears that the closer x gets to 3, the closer $f(x)$ =_____.

Look at the graph of $f(x) = 2x-4$ to verify the information from the chart.

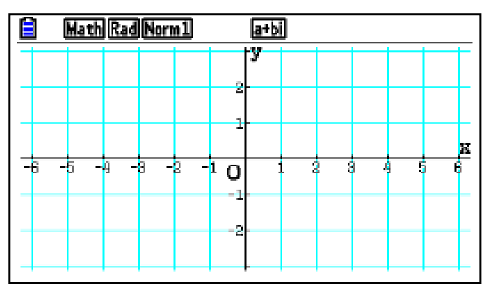
$$\lim_{x \to 3} (2x-4) = \_\_\_\_\_\_\_$$

For all **<u>polynomials</u>**, such as this linear example, limits can be found through **<u>direct substitution</u>** because polynomial functions have unrestricted domains. That is, $\lim_{x \to 3} (2x-4) = [2(3)-4] =$ _____

2. Find the limit of $\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$ by using your graphing calculator to complete the table listed below.



x	-.01	-.001	-.0001	0	.0001	.001	.01
$f(x)$							

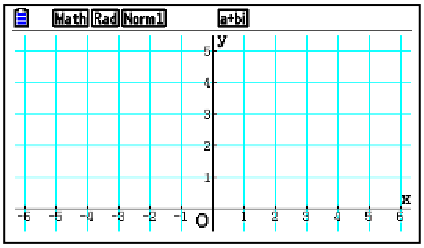


Although $f(0)$ is undefined, as the values approach 0 from the left and right, $f(x)$ approaches _______. Graph and use the trace feature to "look" at $f(0)$. $\lim_{x \to 0} \frac{x}{\sqrt{x+1}-1}$ = _________

This is an example of a function that has a <u>point</u> of discontinuity, yet based upon the limit definition:
If *f*(*x*) becomes __________ close to a unique number *L* as *x* approaches *c* from the _____ and _____ sides, then the **limit** of *f*(*x*) as *x* approaches *c* is *L*.

II. Limits that fail to exist.

3. Unbounded Behavior: Investigate $f(x) = \frac{1}{x^2}$ graphically. For what value is $f(x)$ undefined? x =_____



This tells you where the limit ***MIGHT*** fail to exist.
Investigate the limit using a table as x values approach this value.

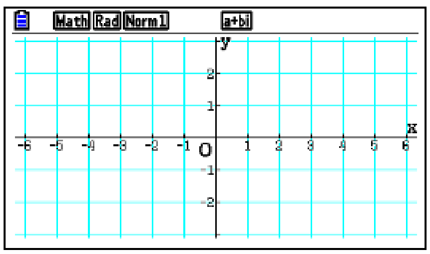
x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$							

From the graph and the table, what does the graph appear to be approaching? ______________

Recall our formal definition: If $f(x)$ becomes arbitrarily close to a unique ________ L as x approaches c from the right and left sides, then the **limit** of $f(x)$ as x approaches c is L.

What does this imply about $\lim_{x \to 0} \frac{1}{x^2}$? _______ Why? ____

4. Left and Right Hand Behavior: Investigate the graph of $f(x) = \frac{|x|}{x}$. For what value is $f(x)$ undefined? x =_____



This tells you where the limit ***MIGHT*** fail to exist.
Investigate the limit using a table as x values approach this value.

x	-2	-1	-0.5	0	0.5	1	2
$f(x)$							

What does the graph appear to be approaching from the left? $x \to 0^-$ =__________.

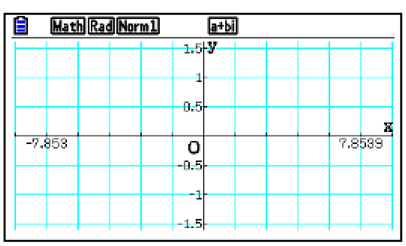
What does the graph appear to be approaching from the right? $x \to 0^+$ =__________.

Recall our formal definition AGAIN!! If $f(x)$ becomes arbitrarily close to a unique number L as x approaches c from the _____ and ______ sides, then the **limit** of $f(x)$ as x approaches c is L.

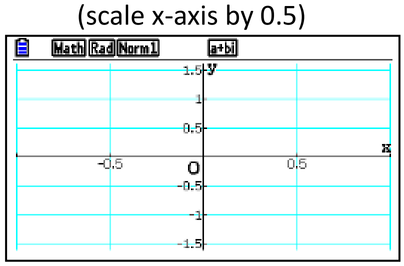
What does this imply about $\lim_{x \to 0} \frac{|x|}{x}$? _______ Why? ____

5. Oscillating Behavior: Investigate the graph of $f(x) = \sin\frac{1}{x}$.

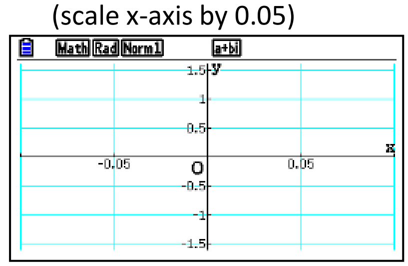
Adjust to a Trig View Window



Adjust view window between -1 & 1 (scale x-axis by 0.5)



Adjust view window between -0.1 & 0.1 (scale x-axis by 0.05)

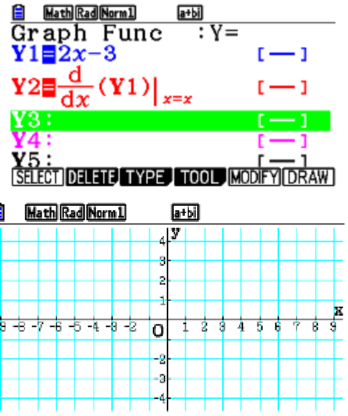


ONE LAST TIME…recall the formal definition: If $f(x)$ becomes _________ close to a ________ number L as x approaches c from the _______and _______ sides, then the **limit** of $f(x)$ as x approaches c is L.

What does this imply about $\lim_{x \to 0} \sin\frac{1}{x}$? _____ Why? ____

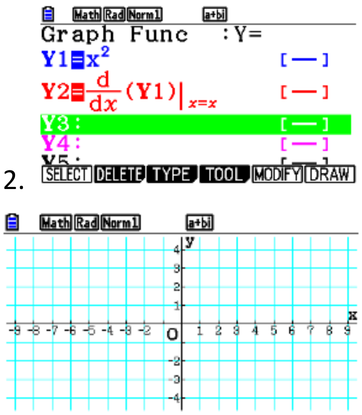
Discovering Derivative Rules

The derivative of a function represents the behavior of the slope of the function at each point along its domain. Graph Y and Y' in order to discover patterns and methods of formulating derivatives analytically. Below each graph, write the equation you *think* would produce the graph of the derivative and then check by graphing it as your Y3 graph. Repeat the process for Y".

1.


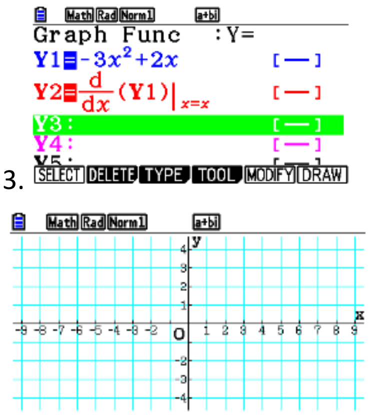
$y'' =$ ________________

$y'' =$ ________________

2.


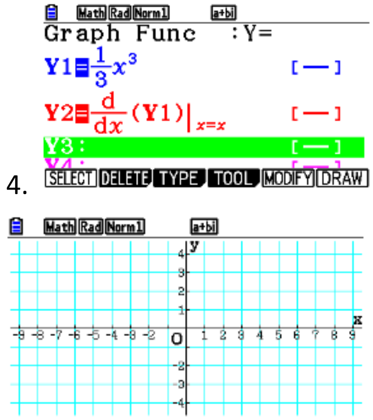
$y' =$ ________________

$y'' =$ ________________

3.


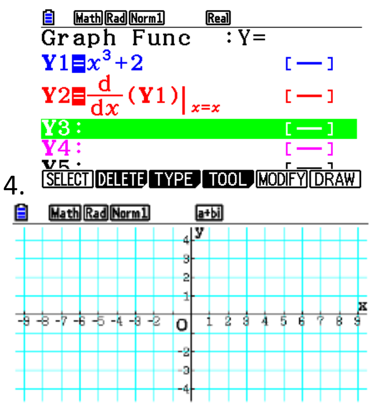
$y' =$ ________________

$y'' =$ ________________

4.


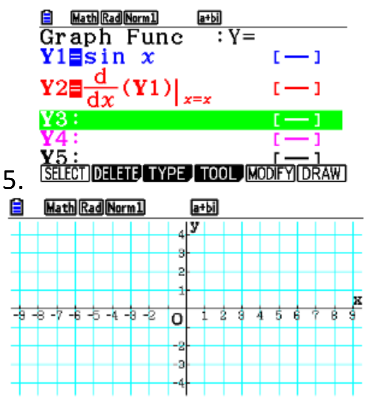
$y' =$ ________________

$y'' =$ ________________

4.


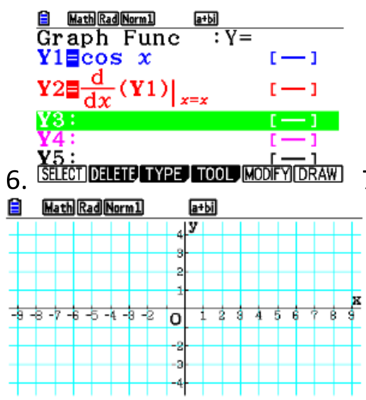
$y' =$ ________________

$y'' =$ ________________

5.


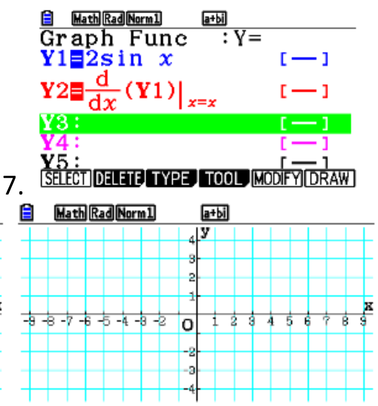
$y' =$ ________________

$y'' =$ ________________

6.


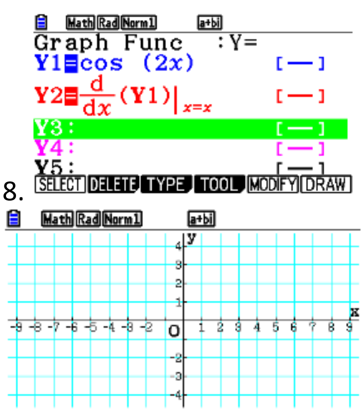
$y' =$ ________________

$y'' =$ ________________

7.


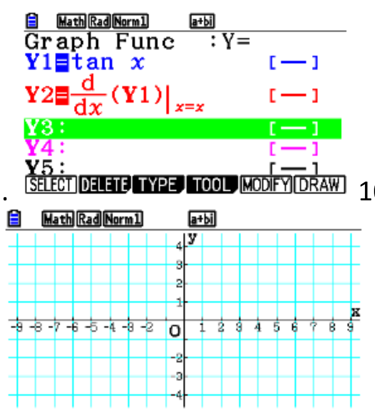
$y' =$ ________________

$y'' =$ ________________

8.


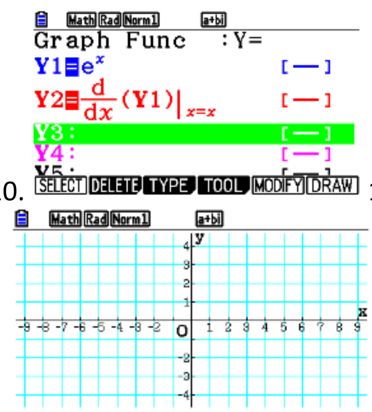
$y' =$ ________________

$y'' =$ ________________

9.


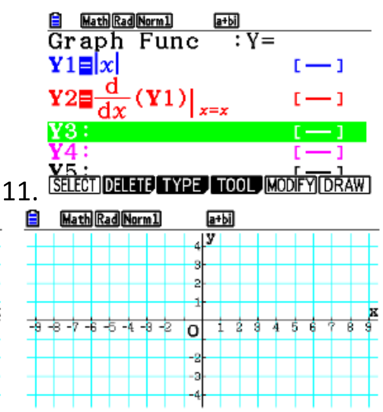
$y' =$ ________________

$y'' =$ ________________

10.


$y' =$ ________________

$y'' =$ ________________

11.


$y' =$ ________________

$y'' =$ ________________

Comparing characteristics of f, f', and f"

From the Main Menu, enter the Graph Mode and enter the function $f(x) = x^4 + 5x^3 - 3x^2 - 20x - 4$. We are going to investigate the graphs of the first and second derivatives.

First graph the function and the first derivative. If you draw vertical lines at the points where the first derivative is zero, what do you notice? ____

____

Now, return to the graph entry screen and graph the function and the second derivative only. Draw vertical lines where the second derivative is zero. Do you notice any relationship between the two graphs?

____

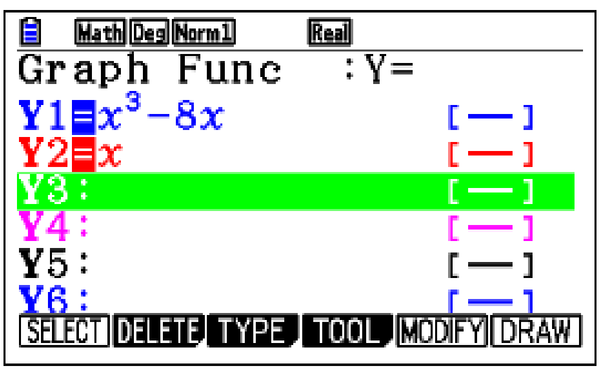
____

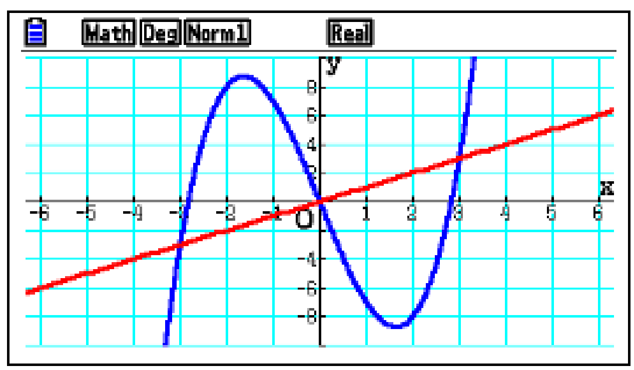
Lastly, graph the first derivative and the second derivative. Sketch vertical lines where the second derivative has a value of zero. What do you notice? _______________________________________

____

Integration and Area Between Curves

Graph $f(x) = x^3 - 8x$ & $f(x) = x$ in the same view window as shown below.





1. Find the integral value between the curves using the first and last intersection points.

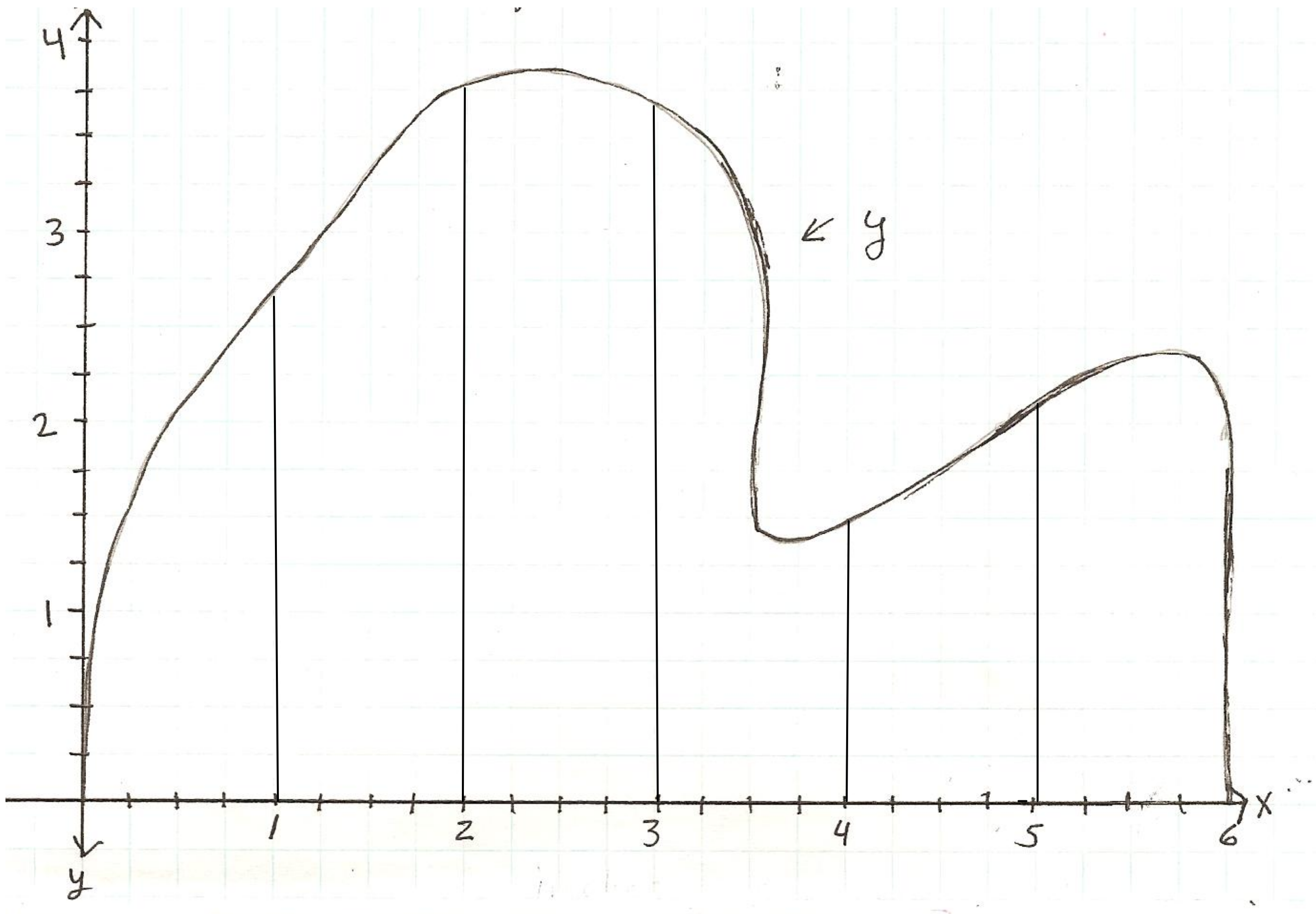
2. Determine the area trapped between the two curves.

3. Why are these different?

Slicing a Mushroom: Finding Area and Volume Activity

1. Estimate the area, in square inches, trapped between the curve and the x-axis by counting squares.

Area = ______________________



2. Section off the area with Wikki Stixs using 6 1-inch sub-intervals by the Midpoint Riemann Sum method.

The symbol below, an integral, is used to represent area under a curve. It represents the **S**um of the rectangles under the curve. Calculate the areas of the rectangles formed by the midpoints and use it to estimate the total area under the curve.

Area = $\int_0^6 Y1dx =$ ________________________________

What is the significance of the 0 and 6 on the integral to this problem?____

How could this *estimate* become more accurate? ____

3. Find the polynomial equation that best fits the mushroom by using the regression feature of your calculator. Use the points that are labeled on the graph. Round the coefficients to the hundredths place.

Y1=____

4. Enter the equation from step 3 using the graph mode and find the area by "integrating" between the roots.

Area = $\int_0^6 Y1dx =$ ________________________________

Now for volume….

5. Imagine the shape being spun about the x-axis in such a way that it "forms" a solid. Find the volume of this solid using the same 6 slices as in step 2 where the height of the graph at each midpoint represents the length of the radius each cylinder.

Volume of a cylinder: V=____________________________

6. Using the equation from step 3, find the volume of this solid if revolved about the x-axis.

Volume of a solid revolved about the x-axis is given by: $V=\pi\int_0^6 Y1^2\,dx =$ ______________________________________

7. Using the Casio Picture Converter, any picture can be imported and then many explorations can be done. Would the following regression equation be a "good" estimate? Explain.