

**TEACHER BELIEFS  
AFFECT PRACTICE:  
PROPORTIONAL REASONING  
AND LINEAR EQUATIONS**

**Todd Abel**

Appalachian State  
University (Boone,  
NC)

**Ashley Lamar**

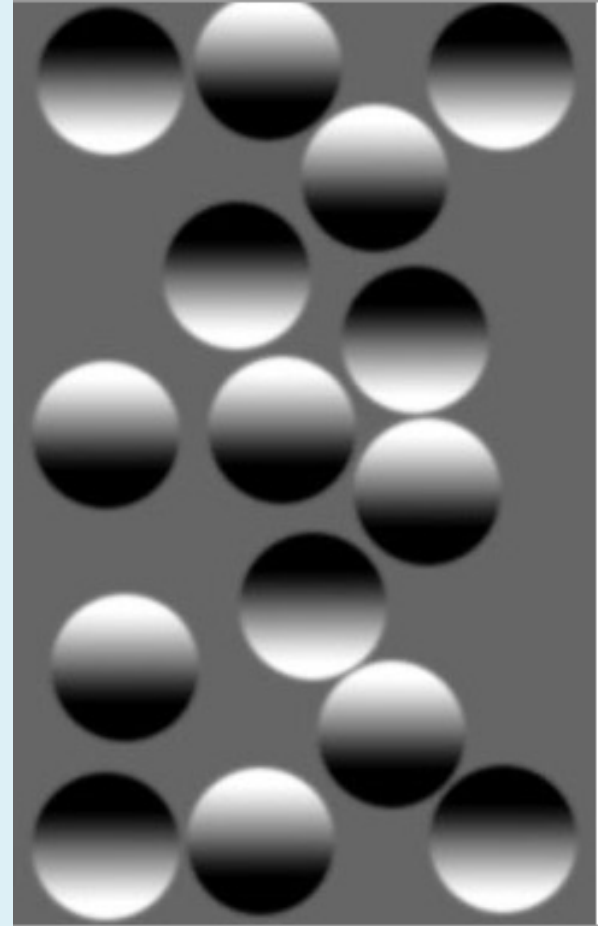
The Ben Franklin  
Academy (Atlanta,  
GA)

**Adam Abel**

E.B. Stanley Middle  
School (Abingdon,  
VA)

# BELIEFS AFFECT PRACTICE

- What do you see?
- If we have expectations about how the world behaves, we attempt to interpret our experiences within the framework of our expectations



# BELIEFS AFFECT PRACTICE

“The brain interprets something different than it actually is, but it doesn’t mean it’s made a mistake. It took the information it had and did its best job with it.”



# BELIEFS AFFECT PRACTICE

- Psychological research supports preconceptions shaping experience
- Teachers tend to teach the way they were taught (e.g. Raymond, 1997)
- Learners accommodate new experiences into their existing frameworks (Erlwanger, 1973; Abel, 2010)
- As teachers, this means that the way we tend to think about mathematics, or even just the way we tend to think about people learning mathematics, guides the way we approach teaching mathematics.

# A COUPLE EXAMPLES OF TEACHER BELIEFS

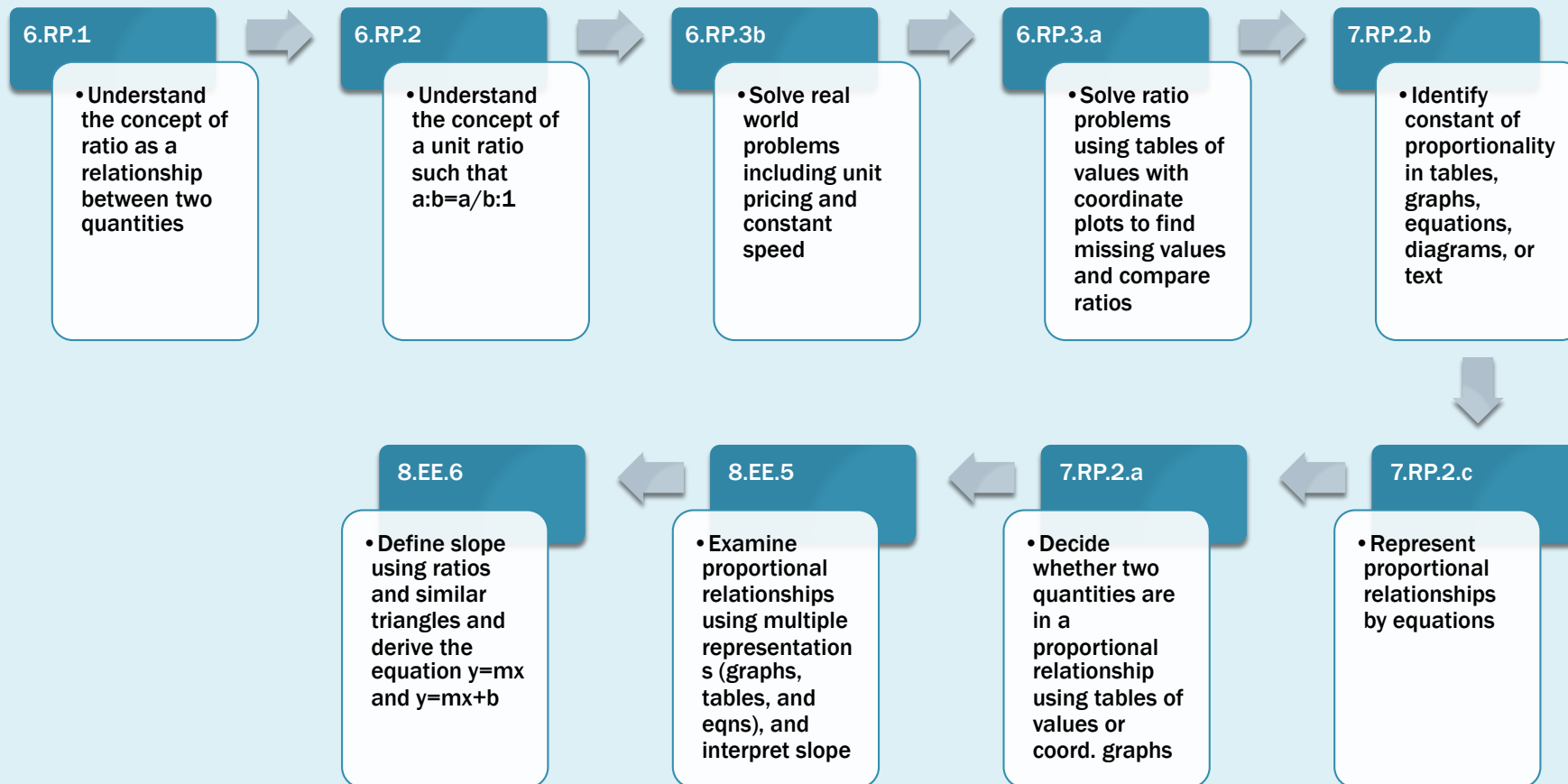
- **Where I come from:**
  - A strange and wonderful assortment of experiences
- **When I was a kid:**
  - Narrative-based strategies for problem solving
- **Look at me now:**
  - Wrong calculation creatively derived is better than robotic repetition

# A COUPLE EXAMPLES OF TEACHER BELIEFS

- **Where I come from:**
  - Sleazy used car salesman
- **When I was a kid:**
  - Tangible, concrete examples
- **Look at me now:**
  - More comfortable with teaching as I was taught – tried to make things tangible

# PROPORTIONAL REASONING AND LINEAR EQUATIONS

How ratio and proportional reasoning progress in the Common Core (Please see [TurnonCCMath.net](http://TurnonCCMath.net))



# PROPORTIONAL REASONING AND LINEAR EQUATIONS – A LESSON TRAJECTORY

The Snowy Tree cricket is sort of a natural thermometer. Its chirps are slow enough to be counted, they synchronize their chirps, and how fast they chirp is related to temperature. If we measure the temperature change from the moment they begin chirping and measure the corresponding chirp rate, we see:

Chirps/Min	Temperature Change
60	15° F
100	25° F
128	32° F

Can you predict the temperature change if you count:

- 80 chirps/min
- 40 chirps/min
- 125 chirps/min
- 117 chirps/min



Photo from [songsofinsects.com](http://songsofinsects.com)

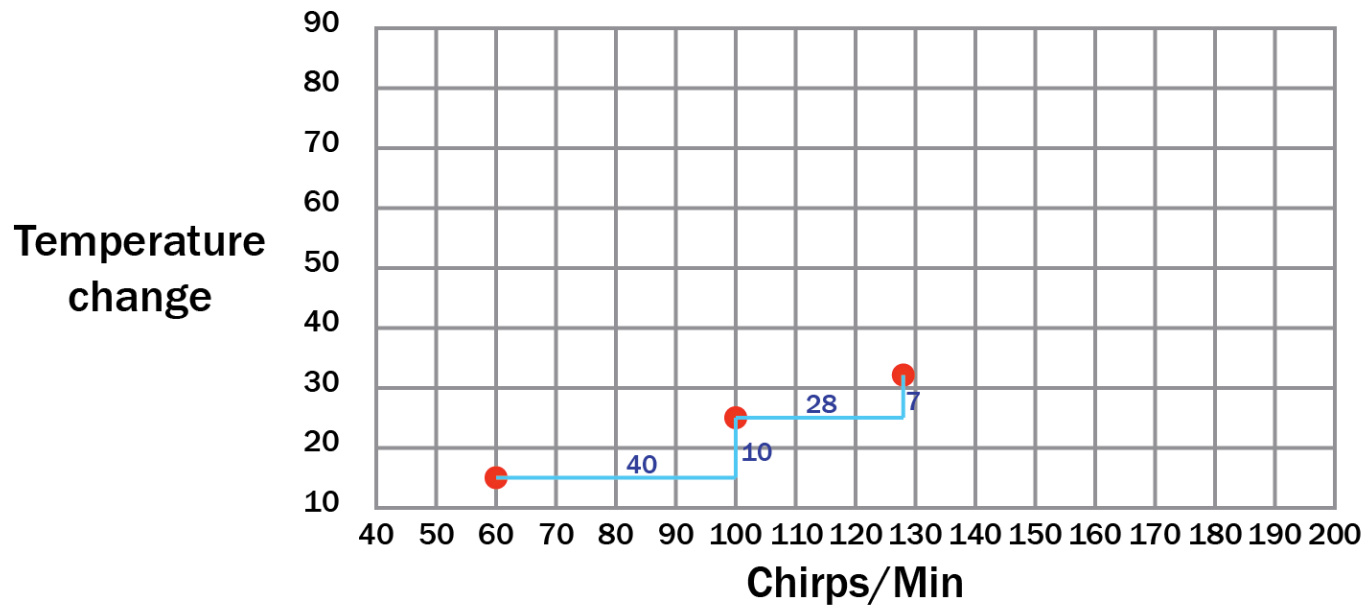


# PROPORTIONALITY AND LINEAR EQUATIONS – A LESSON TRAJECTORY

- Students might notice that an increase of 40 chirps/min corresponds to a  $10^\circ$  temperature change.
  - OR, that they increase by 4 chirps/min for each  $1^\circ$  change in temperature.
  - Looking at the data, we always see that  $\frac{\text{Temperature change}}{\text{Chirps / Min}} = \frac{1}{4}$
  - That tells us that  $\text{Temperature Change} = \frac{1}{4}(\text{Chirps/Min})$ 
    - Let's just make that easier to write and say  $T = \frac{1}{4} \cdot C$  where T is temperature change and C is chirps per minute.

# PROPORTIONALITY AND LINEAR EQUATIONS – A LESSON TRAJECTORY

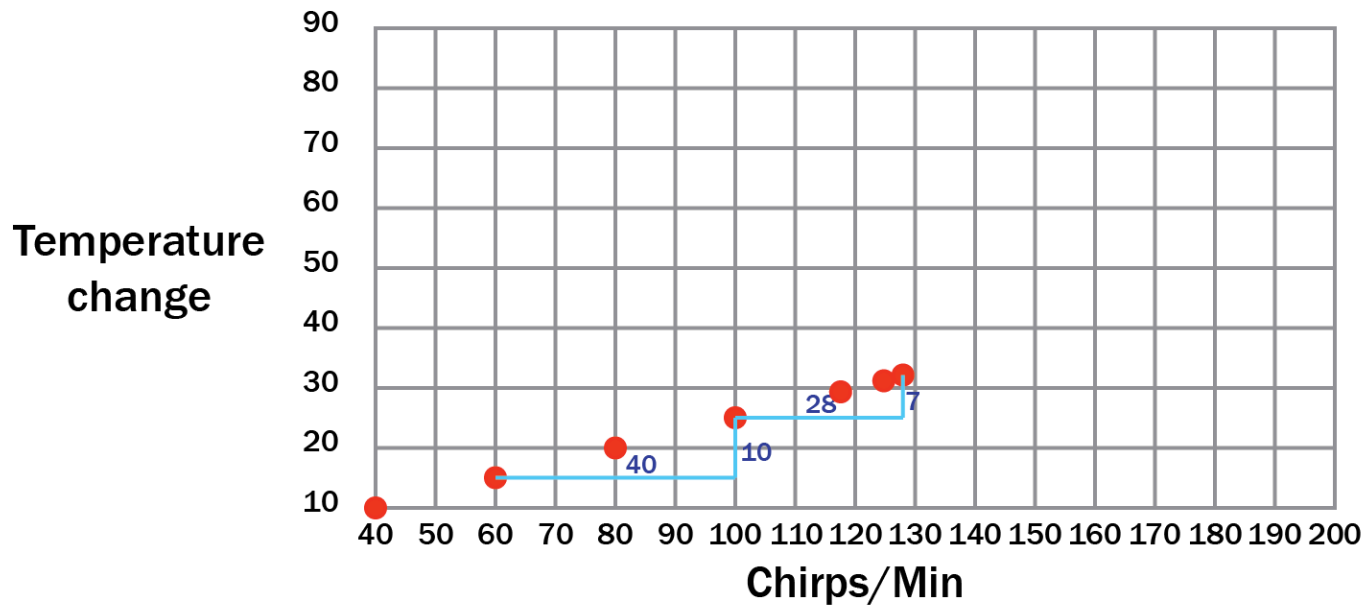
- So if we plot the points we started with:



Note that the ratio of the base and height of the “triangles” between points is always  $\frac{1}{4}$

# PROPORTIONALITY AND LINEAR EQUATIONS – A LESSON TRAJECTORY

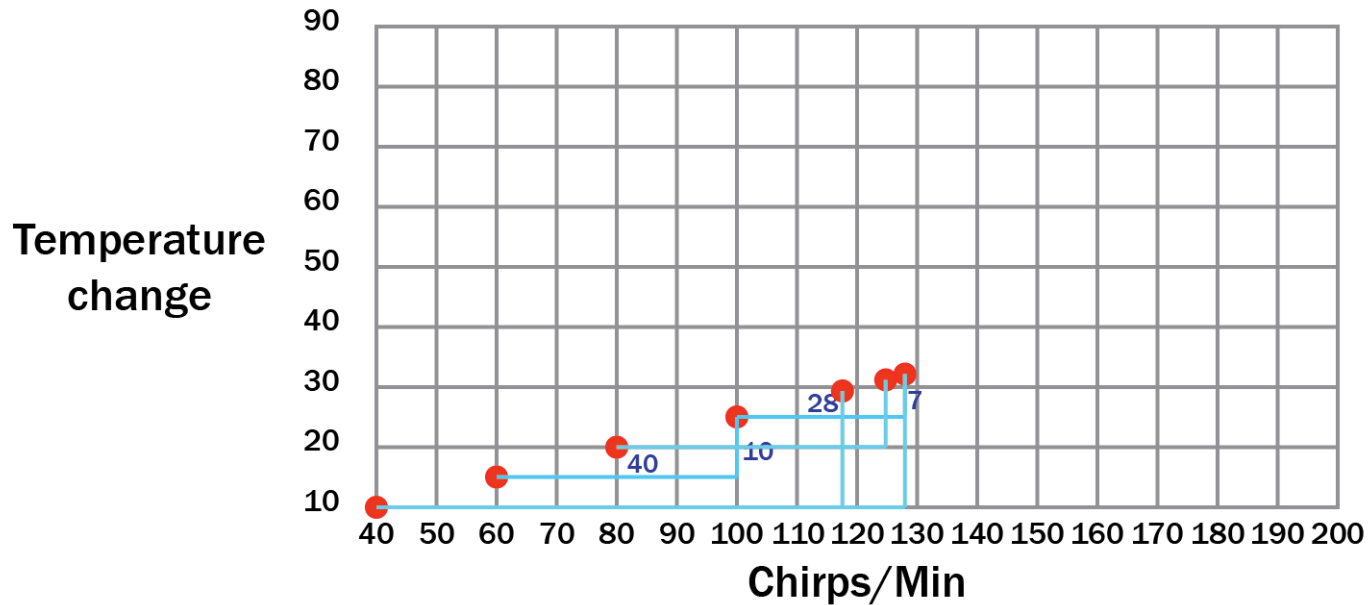
- Now add the other points we found:



Do you notice anything interesting?

# PROPORTIONALITY AND LINEAR EQUATIONS – A LESSON TRAJECTORY

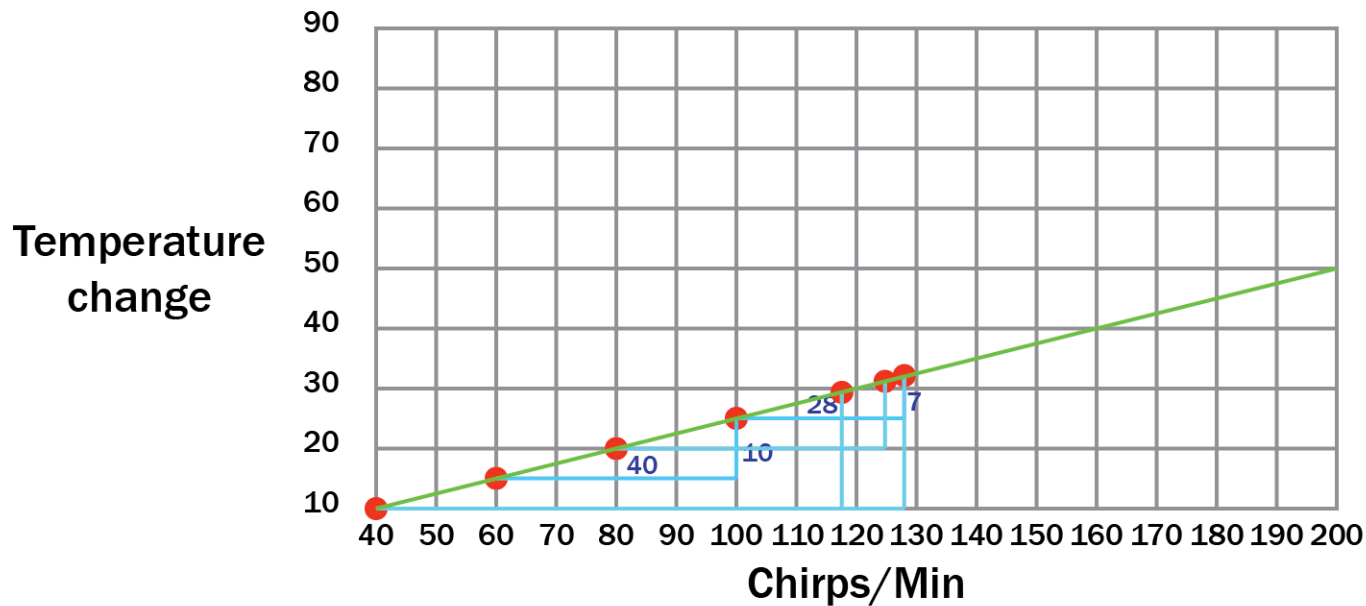
- Look at the other possible “triangles”:



Notice that any “triangle” we use to connect to points has the same ratio!

# PROPORTIONALITY AND LINEAR EQUATIONS – A LESSON TRAJECTORY

- They're also lined up quite nicely:



In fact, every pair of points on this line has the same property. And every point has the relationship  $T = \frac{1}{4} \cdot C$

**AMAZING!**

# PROPORTIONAL REASONING AND LINEAR EQUATIONS – A LESSON TRAJECTORY

Now, instead of looking at the Temperature Change from when they start chirping, let's compare the chirp rate to the actual temperature:

Chirps/Min	Temperature
60	55° F
100	65° F
128	72° F

Can you predict the temperature if you count:

- 80 chirps/min
- 40 chirps/min
- 125 chirps/min
- 117 chirps/min

# PROPORTIONAL REASONING AND LINEAR EQUATIONS – A LESSON TRAJECTORY

At what temperature do they first start chirping?

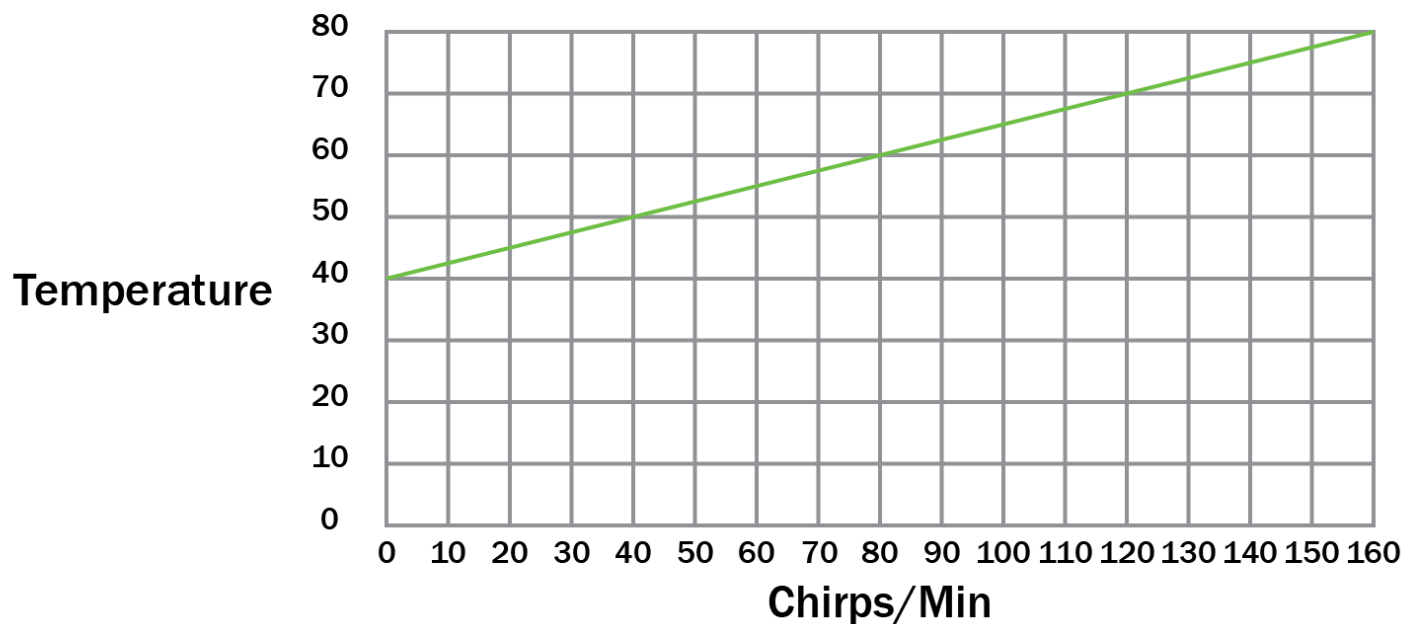
Chirps/Min	Temperature
60	55° F
100	65° F
128	72° F

Can you find a way to predict the temperature if you count the number of chirps/min?

$$T = \frac{1}{4} \cdot C + 40$$

# PROPORTIONAL REASONING AND LINEAR EQUATIONS – A LESSON TRAJECTORY

What if we start to look into the graph of these points?



How does this compare to what we found before?



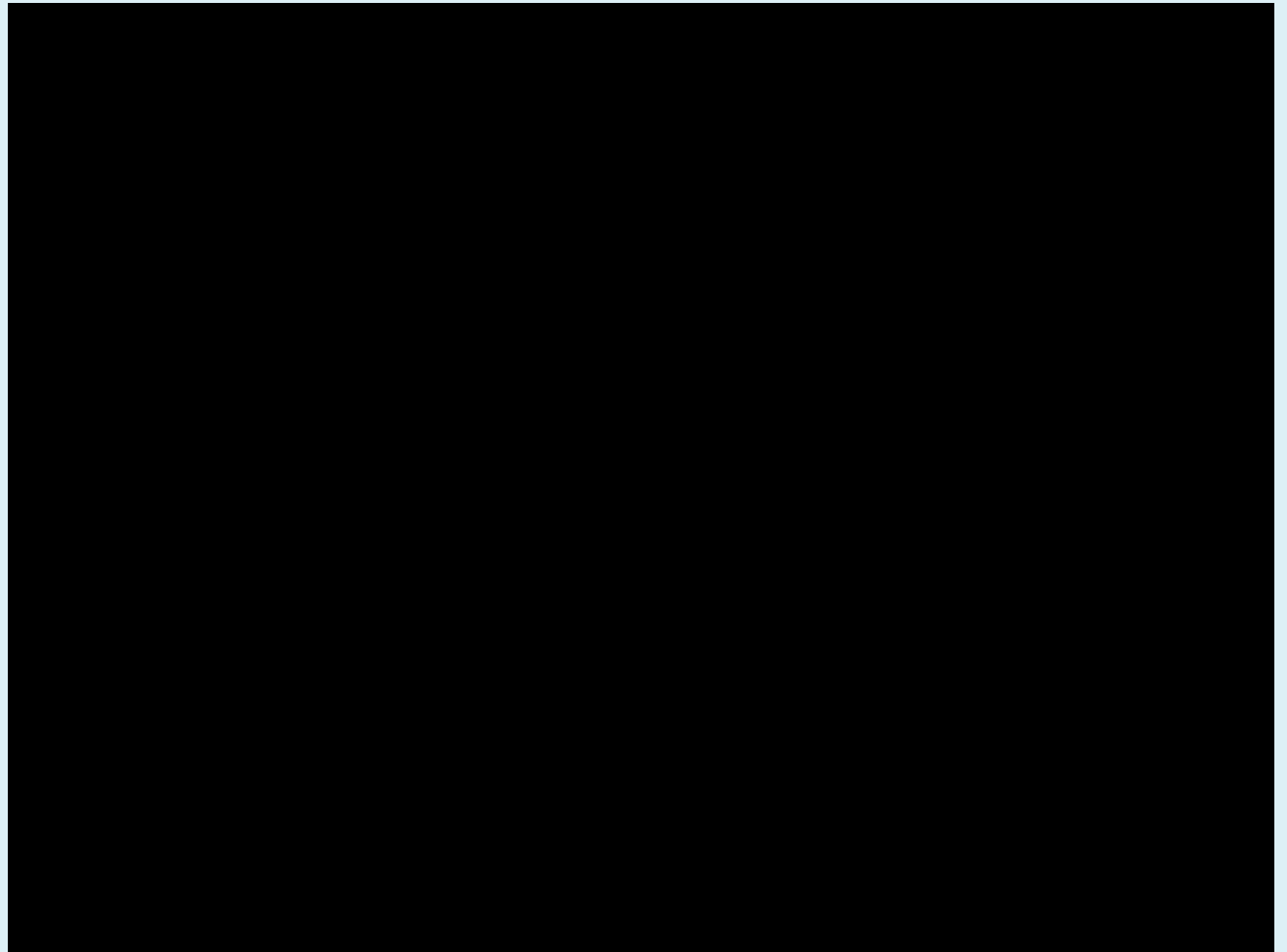
# PROPORTIONAL REASONING AND LINEAR EQUATIONS – AND BELIEFS

- Lessons may be designed to have a certain cognitive demand, but teachers can increase or decrease that demand by the way they enact the lesson (e.g., Stein, Smith, Henningsen, & Silver, 2000).
- Our choices in enacting a lesson are guided by our own beliefs and experiences, but examination of those beliefs and experiences can help us understand the effects, and either mitigate or enhance them.

# BACK TO BELIEFS

Listen and watch carefully (but silently for others' sake) –

We'll quiz you on the answer at the end.



# IF YOU FORGET EVERYTHING WE SAID – THINK ABOUT THIS:

- What are your basketballs? – the things you draw your focus and attention
  
- What are your gorillas? – the things that you miss for focusing so hard on the basketball

# THANK YOU!

**Todd Abel (abelta@appstate.edu)**

**Assistant Professor of Mathematics Education**

**Dept. of Mathematical Sciences**

**Appalachian State University: Boone, NC**

**Ashley Lamar (ealamar@benfranklinacademy.org)**

**Mathematics and Science Teacher**

**The Ben Franklin Academy: Atlanta, GA**

**Adam Abel (adama@wcs.k12.va.us)**

**Science Teacher**

**E.B. Stanley Middle School**

**Washington Co. Schools: Abingdon, VA**