Exceptional & Free Online Resources for Teaching Probability

2013 NCTM Regional Conference – Louisville

Sarah DeLeeuw & Ann Kong

November 8, 2013

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Introductions

Who are we ?

Who are YOU ?

THE MATH SALUTE

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We are …

ILLUMINATIONS
Resources for Teaching Math

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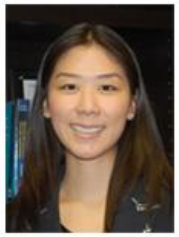
Thinkfinity | verizon Foundation

Lessons Interactives

Contact Us

NCTM Staff dedicated to the Illuminations Project.

Online Projects Manager
Sarah DeLeeuw



Illuminations Coordinator
Ann Kong

NCTM Headquarters Office

search site

Lessons Interactives

NCTM Standards | Common Core Math Standards

Pre-K-2 3-5
6-8 9-12

Number & Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

SEARCH

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Welcome

~~50~~	60	~~43~~	~~45~~
~~36~~	~~46~~	~~29~~	31
47	~~57~~	~~40~~	~~42~~
~~35~~	~~45~~	28	~~30~~

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What'd You Get?

166

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Session 166

Whoa!

How did that work?

Check out NCTM Illuminations

YouTube to find out.

Ready. Set. GO!

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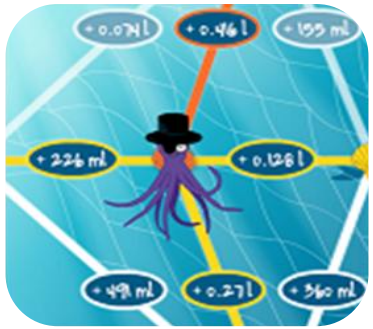
  (coming soon)

http://illuminations.nctm.org

>600 lesson plans



>100 interactives

>120 Brainteasers

Challenge others. Challenge yourself.™ CalculatioN NatioN illuminations.nctm.org

12 games

Bright Ideas

e-newsletter

success stories

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Illuminations

Lessons include:

Activities Include:

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calculationnation.nctm.org

CALCULATION NATION

- **12** interactive math games
- Two players compete from any two computers
  - Ranks based on wins/losses
  - Tracks where in the world competitors live
- Lesson Plans: **http://illuminations.nctm.org/CN**
- More games and features on the way…

The Game of SKUNK

My math teacher might collect homework today; should I do it?

CHOICE vs CHANCE

S	K	U	N	K

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Playing Skunk

Goal: *Accumulate the greatest possible sum the 5 rounds.*

- To accumulate points, a pair of dice is rolled.
- Everyone standing is an active player. Sit to become inactive.
- An *active* player gets the sum of the dice and records it on their score sheet, unless a "one" comes up.
- If a "one" comes up, then….
  1. The round is over
  2. ALL points in the column are wiped out.
- If "double ones", then…
  1. The round is over
  2. ALL points accumulates in *prior* columns gets wiped out too.

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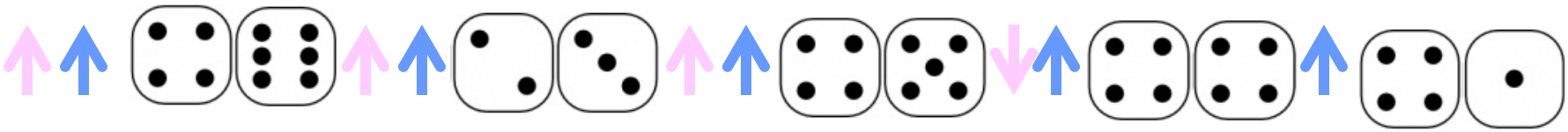
Mock Game : Jane vs. John



S	K	U	N	K
10				
5				
9				
24				

S	K	U	N	K
10				
5				
9				
3				
0				

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Let's Get Movin!!

The Pedagogy

- Should we find out who the winner is?

  winning vs. self-improvement

- Should this game be played more than once?

  playing for fun vs. playing after figuring out the math

- What should the teacher do while students are playing the game? *Think-pair-share!*

SKUNK- The Big Picture

- Which part of this game is **CHOICE**?

  Which games involve mostly choice?

- Which part of this game is **CHANCE**?

  Which games involve mostly chance?

- Create a strategy for the game.

  Write it down first. Then, share it with a neighbor.

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SKUNK- The Math

- Write down all the possible outcomes.
- On average, how many good rolls happen before a 1 or 1-1 occurs?
    - How would this change how you play the game?
- When a “one” does *not* come up, what is the average score on a single roll of a pair of dice?

	1	2	3	4	5	6
1	**XX**	X	X	X	X	X
2	X	4	5	6	7	8
3	X	5	6	7	8	9
4	X	6	7	8	9	10
5	X	7	8	9	10	11
6	X	8	9	10	11	12

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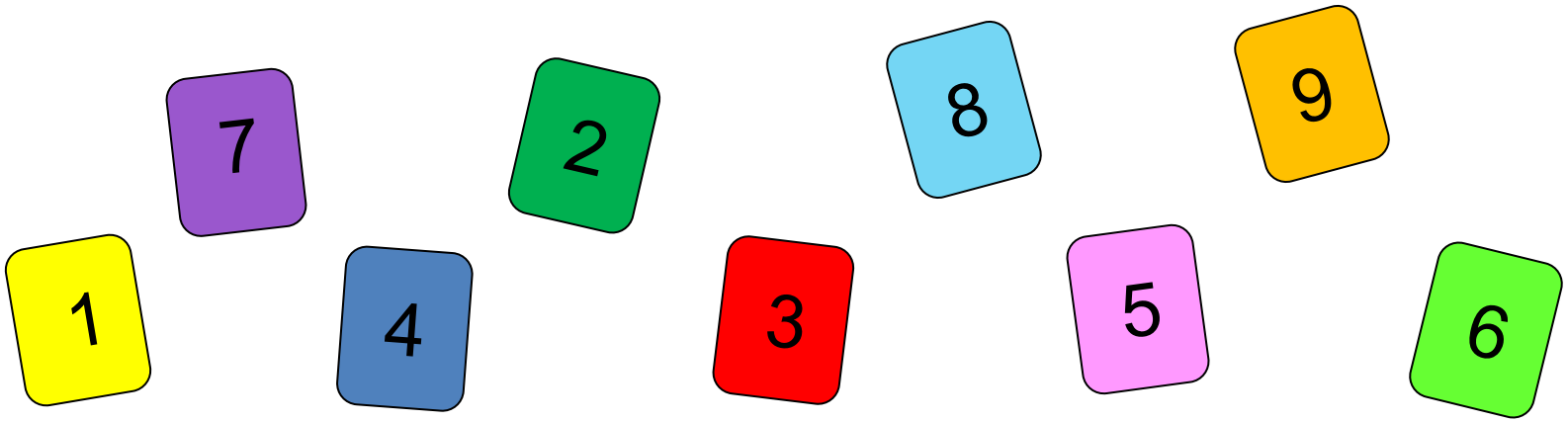
SKUNK Questions

What other questions could you ask to get your students thinking?

Think-pair-share

Game of Nine Cards

- **Materials:** Nine cards numbered 1–9
- **Object:** To get a combination of **exactly** three cards in your hand that add up to 15



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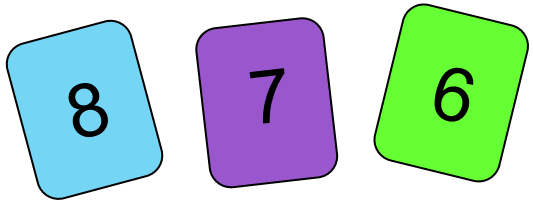
Game of Nine Cards

- Sample Game:

Player 1



Player 2

Player 1 Wins: 2 + 9 + 4 = 15

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Game of Nine Cards

Now what?

You Play!

Mister Mc Math

Mrs. Bennefield

Piedmont, Alabama

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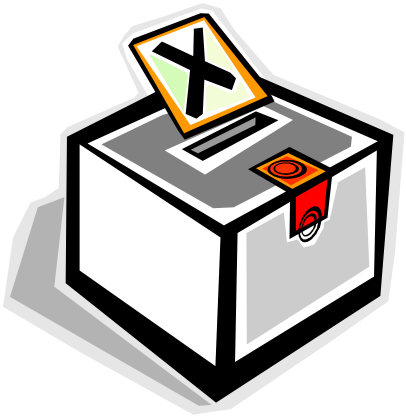
Questions to Consider …



- Who is more likely to win — the first player or the second player? Why?
- Will someone always win? Lose?
- What can you do to ensure that you don't lose? (Or is that not possible?)

- Is there a "best" card to choose?
- Why do we use a sum of 15?

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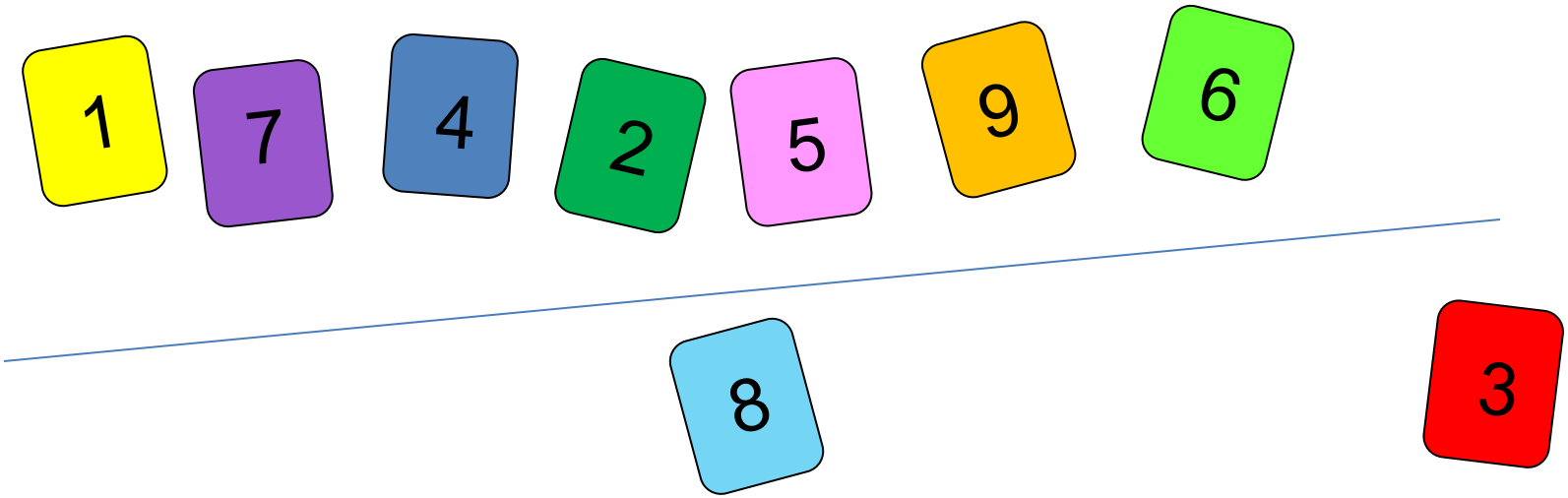
A *Winning* Strategy?

- You play first, **pick 8**.
- Your opponent then **chooses 3**.
- What are the **three numbers** that you can choose to ensure a win?



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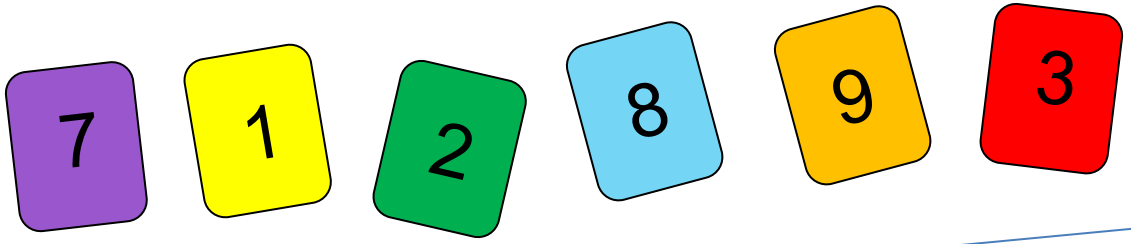
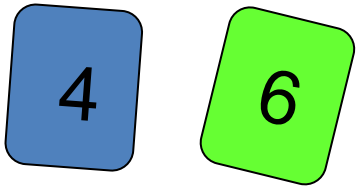
A *Winning* Strategy?

- Your opponent plays first, **picks 6**.
- You **choose 5**.
- Your opponent **picks 4**.
- Which **two numbers** should you ***not*** pick?



Yours



His or Hers

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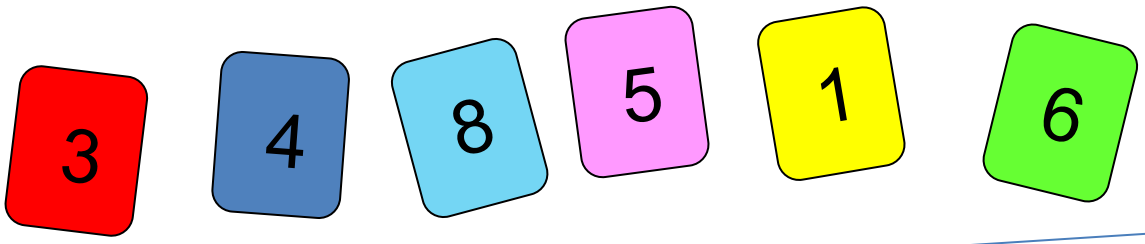
A *Winning* Strategy?

- Your opponent plays first, **picks 7**.
- Then you **choose 2**.
- Your opponent **picks 9**.
- Which **three numbers** should you ***not*** pick?



Yours

His or Hers

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More Sophisticated Yet?

- If your opponent plays first and picks an **even** number, what number should you choose to **avoid a loss**?

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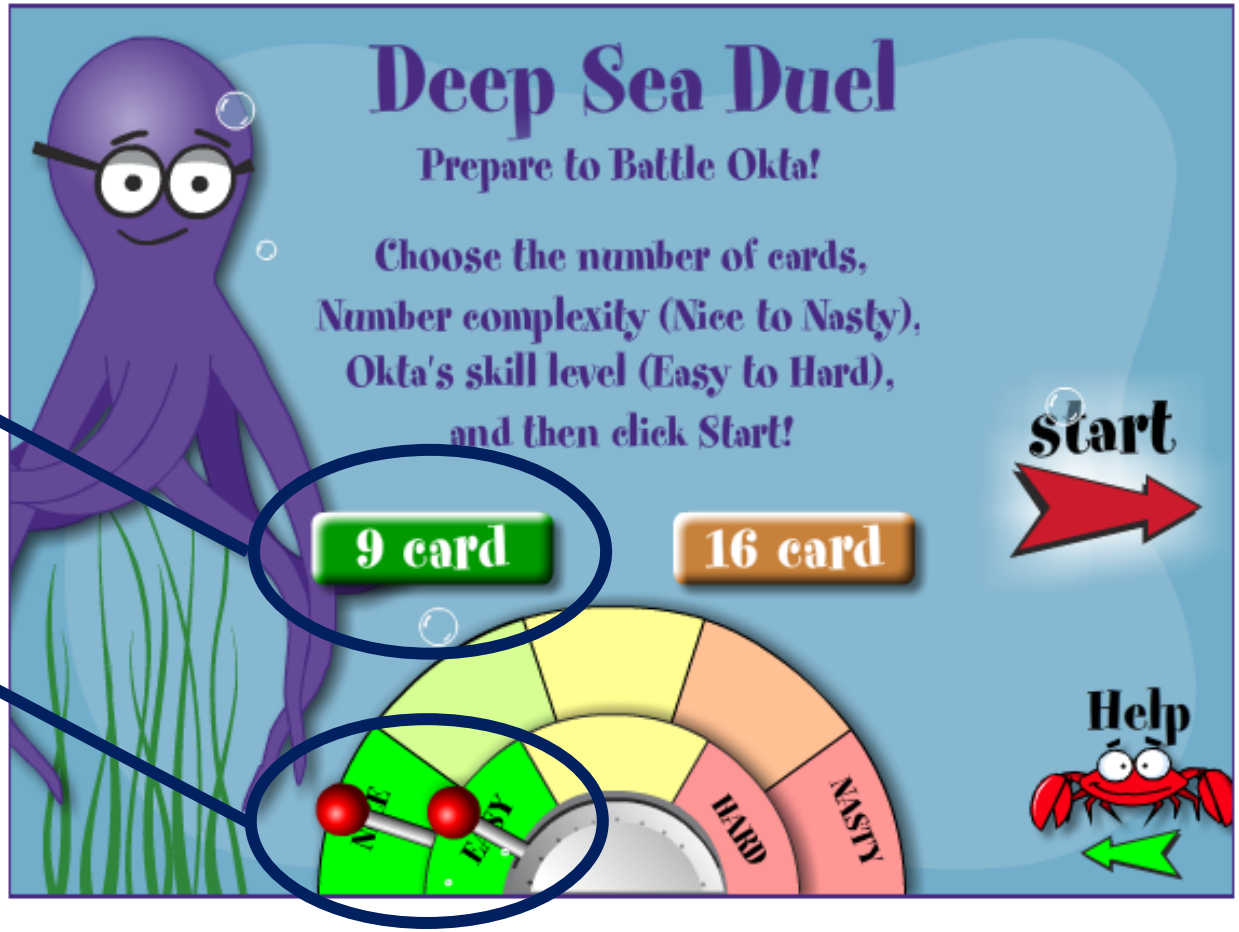
Another App from Under the Sea



9 Card

Nice / Easy

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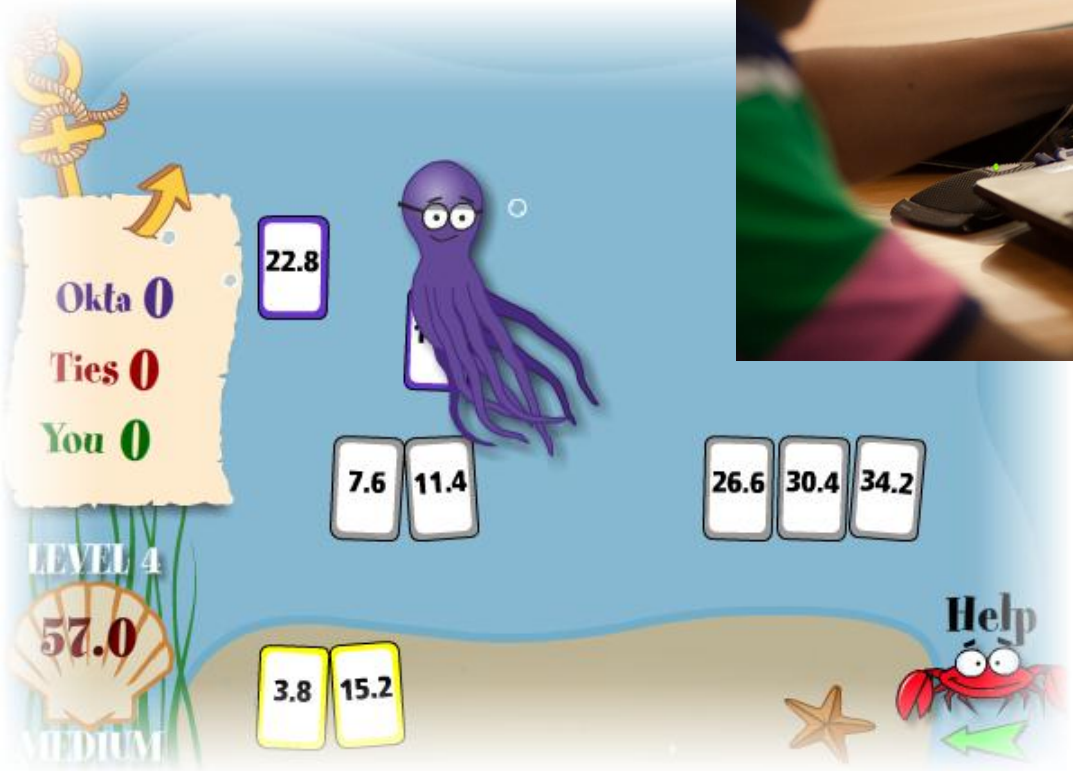
Tablet, phone, PC



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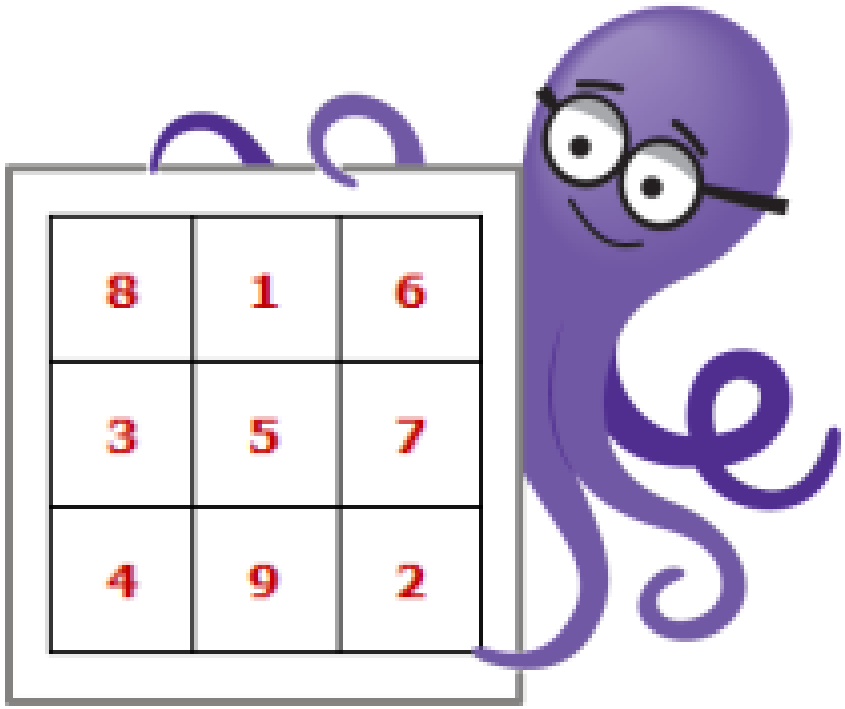
A Hint from Under the Sea



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Modifying the Game of Nine Cards

- Label the nine cards as follows:

5, 12, 19, 26, 33, 40, 47, 54, 61

The winner must get three cards that total 99.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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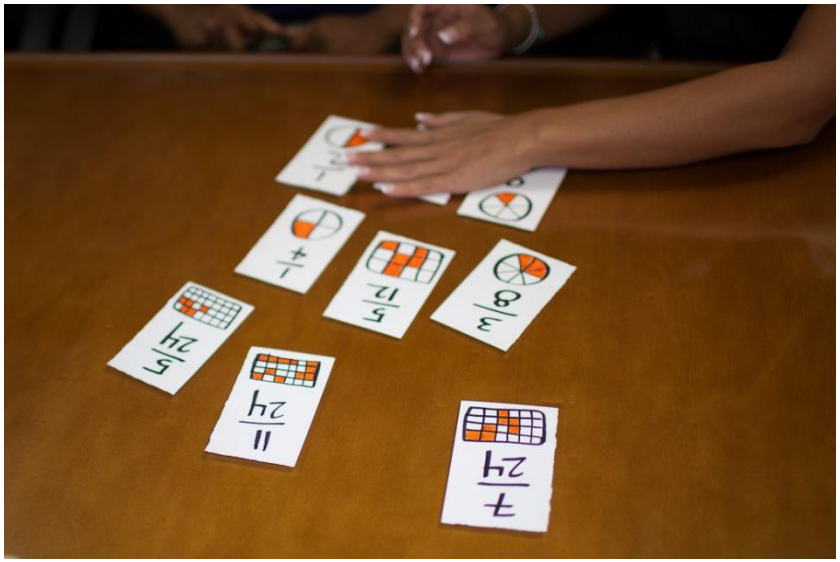
Modifying the Game of Nine Cards

- Label your nine cards with fractions:

1/6, 5/24, 1/4, 7/24, 1/3, 3/8,

5/12, 11/12, 1/2

The winner must get three cards that total 1.



Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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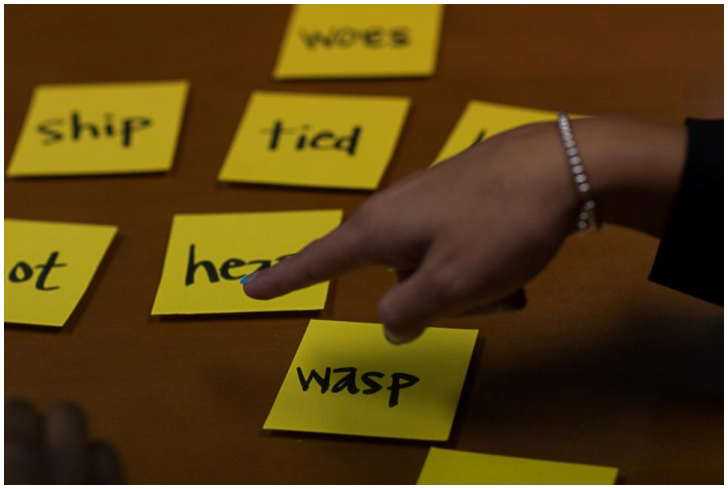
Modifying the Game of Nine Cards

- Use words! Label the cards as follows:

TIED, HOT, HEAR, TANK, WASP,

WOES, SHIP, HORN, BRIM

The winner must get three cards that bear the same letter.



Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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Modifying the Game of Nine Cards

- Use exponents!
- Label the nine cards as follows:

$$x,\ x^2,\ x^3,\ \ldots,\ x^9$$

The winner must the *product* get x^{15}.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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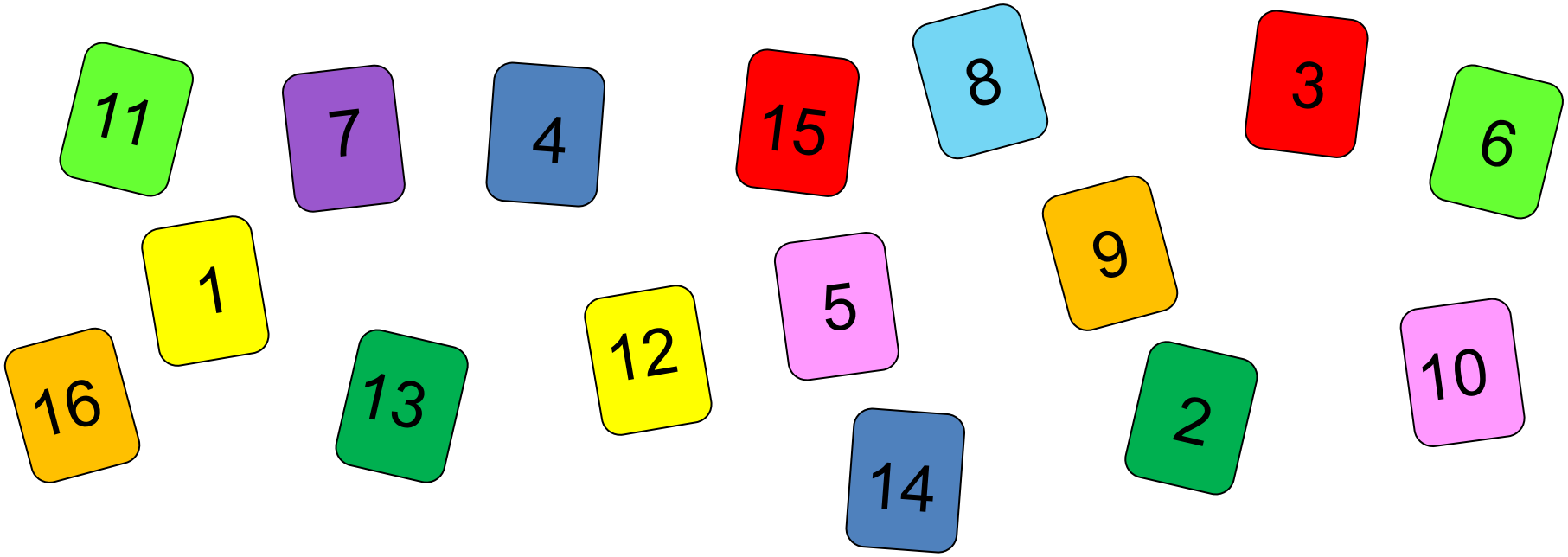
From NINE Cards to SIXTEEN

The winner would use the sum of *four* cards to win.



Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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You tell me!

What sum should the winner need to win?

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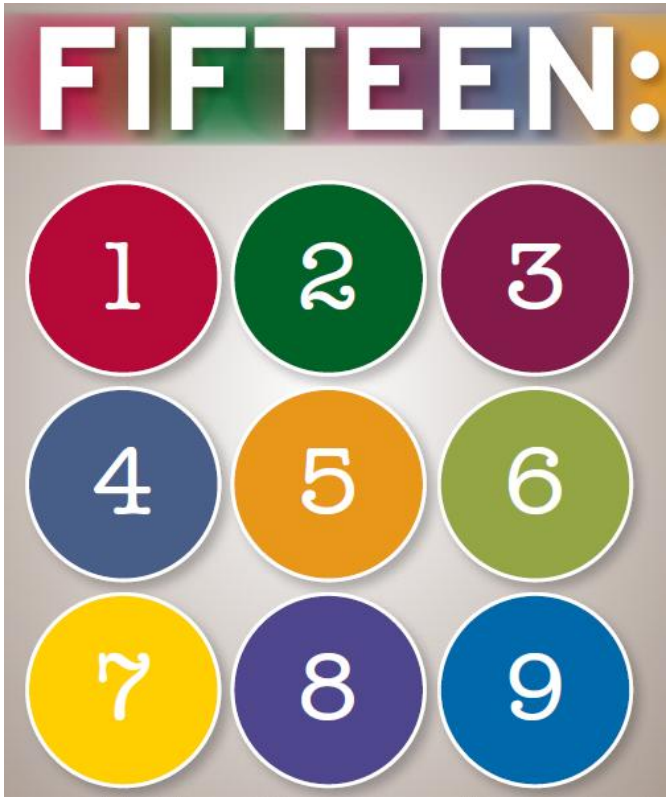
Another Extension

- The winner is the first player to obtain the sum of exactly 15 from any **TWO OR MORE** cards.
- Does your strategy change? How so?



- Double the deck & double the sum.
- How does THAT change the game?

Yeo, Joseph. [Title removed in order to not give away punch line of strategy.] *Mathematics Teacher,* August 2012.

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Reminder: What is the Goal?

- How does your strategy from the first version of the game of 9 cards compare to the strategy for these modifications?

- REFLECT: ***How*** did I come up with these other versions for the game of 9 cards?

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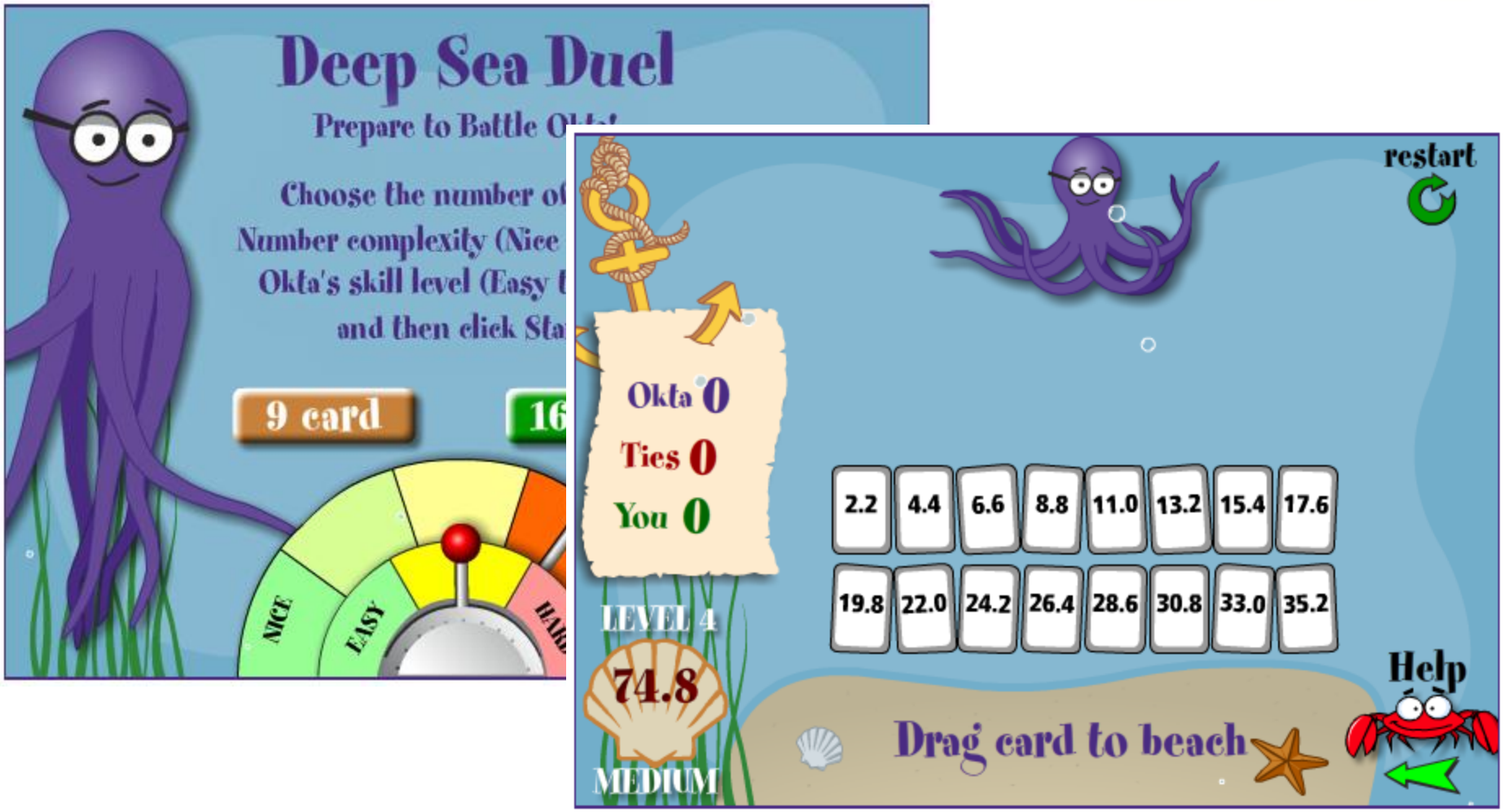
Options & Modifications in App



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Learning is fun. Get addicted!

Deep Sea Duel is FREE online at Illuminations and Google Play and the App Store for phones and tablets.

Upgrade that includes a 2-Player Version and even MORE modifications coming soon!

What a constraint!

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Let's Make a Deal

The Problem

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A Present!

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Stick or Switch

The Options:

- Strategy 1: Stick with the original door
- Strategy 2: Flip a coin, stick if it shows heads, switch if it shows tails
- Strategy 3: Switch to the other door

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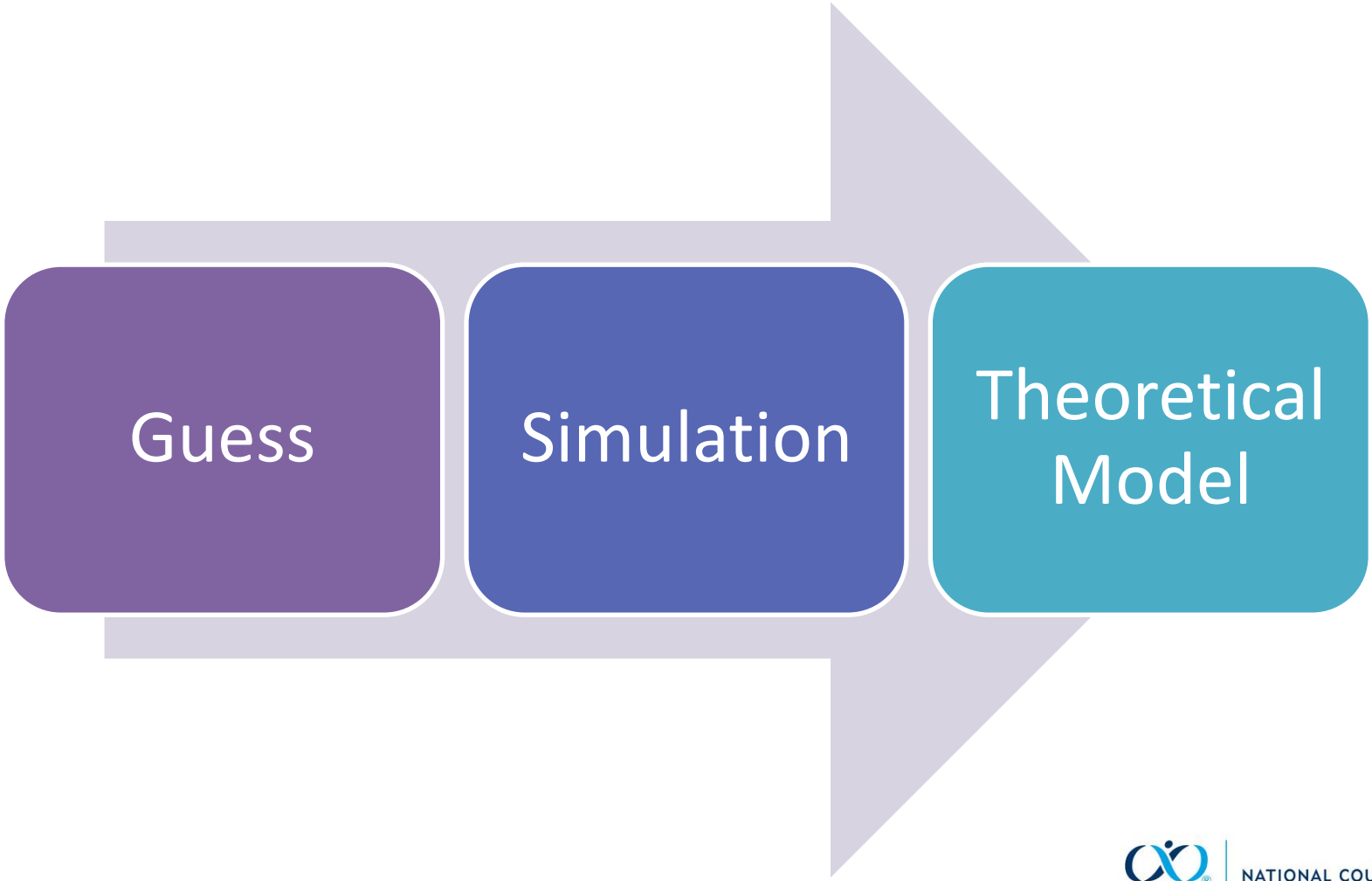
Which Strategy Do You Choose?



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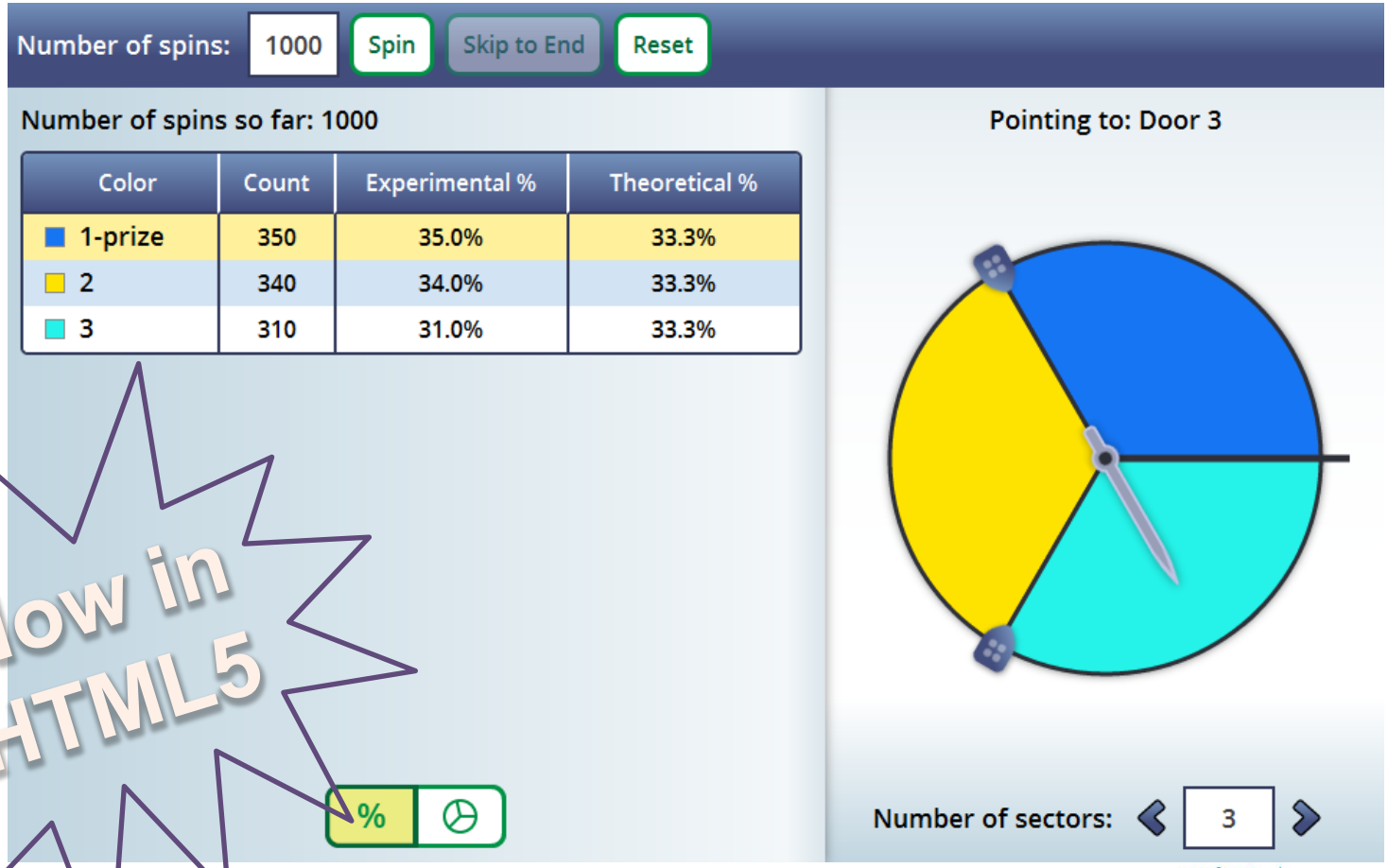
Simulate: Adjustable Spinner

Illuminations.nctm.org/Adjustable Spinner



Now in HTML5

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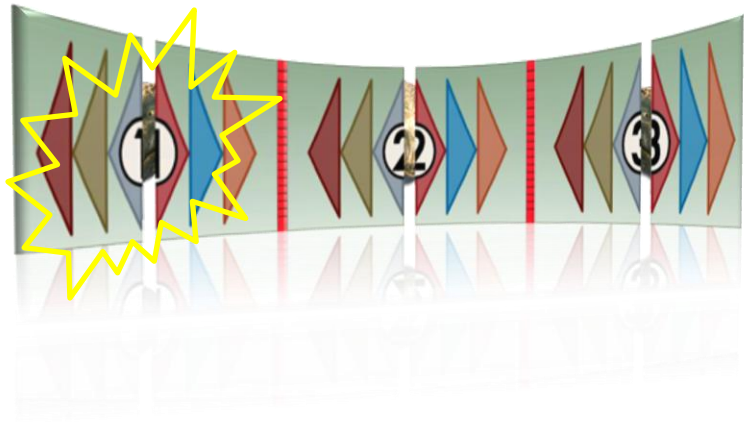
Simulation

Strategy 1: STICK

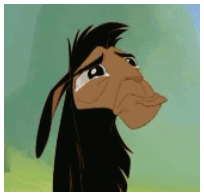


Number of spins so far: 1000

Color	Count	Experimental %	Theoretical %
1-prize	350	35.0%	33.3%
2	340	34.0%	33.3%
3	310	31.0%	33.3%

Choose Door 1. What does Monty do? What do you do? Do you win or lose?

Choose Door 2. What does Monty do? What do you do? Do you win or lose?



Choose Door 3. What does Monty do? What do you do? Do you win or lose?

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Simulation

Strategy 2: FLIP A COIN

Number of spins so far: 1000

Color	Count	Experimental %	Theoretical %
1-prize	350	35.0%	33.3%
2	340	34.0%	33.3%
3	310	31.0%	33.3%

Choose Door 1. What does Monty do?

Door 2: What do you do? Flip a coin → Stick. Do you win or lose? ✔
Flip a coin → Switch. Do you win or lose? ✘
Door 3: What do you do? Flip a coin → Stick. Do you win or lose? ✔
Flip a coin → Switch. Do you win or lose? ✘

Choose Door 2. What does Monty do? What do you do?

Flip a coin → Stick. Do you win or lose? ✘
Flip a coin → Switch. Do you win or lose? ✔

Choose Door 3. What does Monty do? What do you do? Do you win or lose?

Flip a coin → Stick. Do you win or lose? ✘
Flip a coin → Switch. Do you win or lose? ✔

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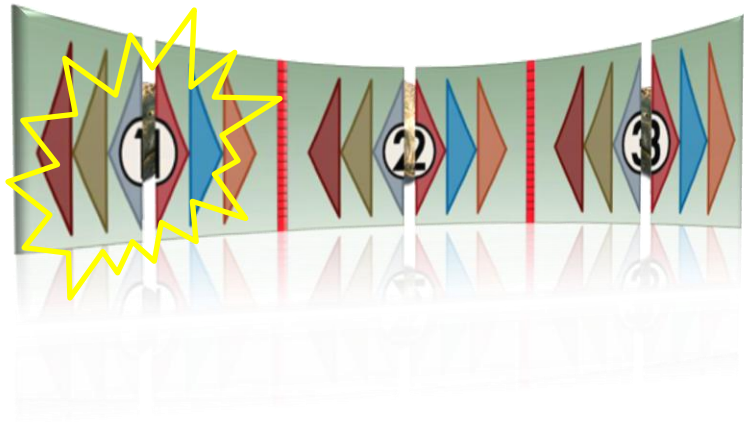
Simulation

Strategy 3: SWITCH



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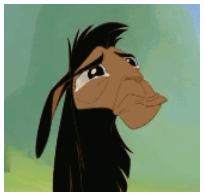
Choose Door 1. What does Monty do? What do you do? Do you win or lose?



Choose Door 2. What does Monty do? What do you do? Do you win or lose?

Choose Door 3. What does Monty do? What do you do? Do you win or lose?

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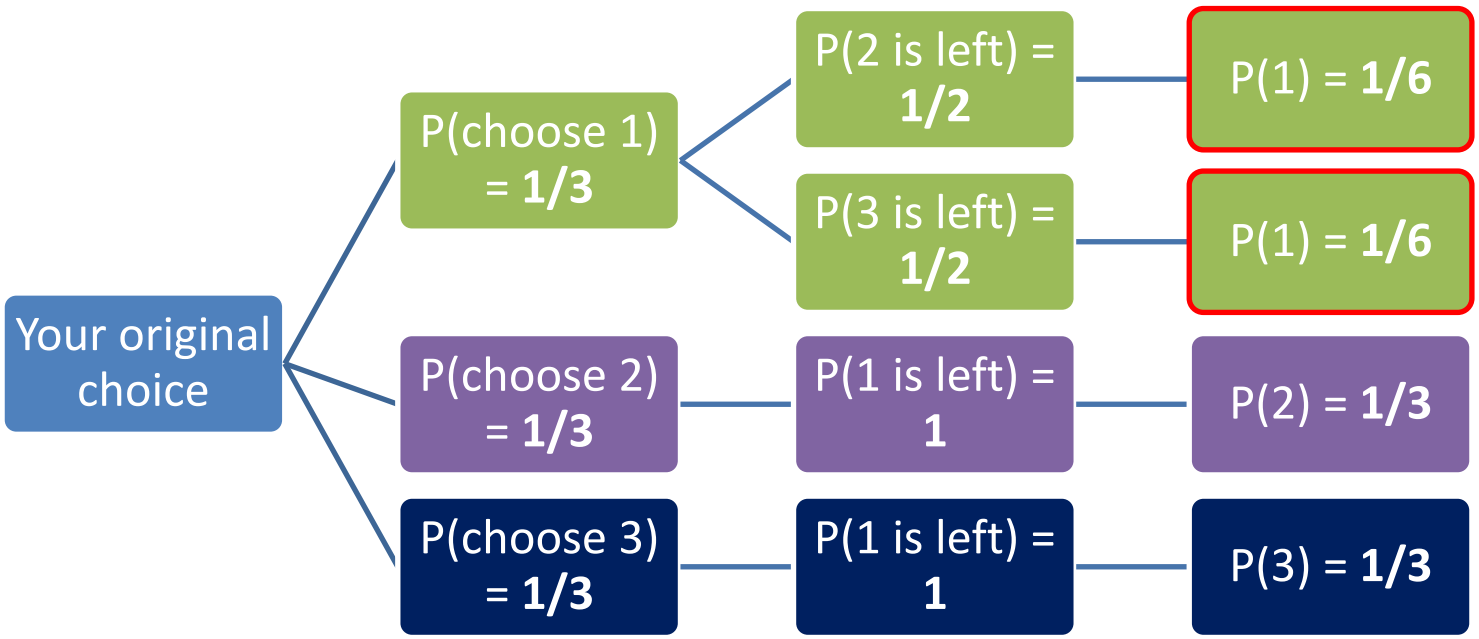
Strategy 1: Stick

Theoretical Model



P(win)= 2/6

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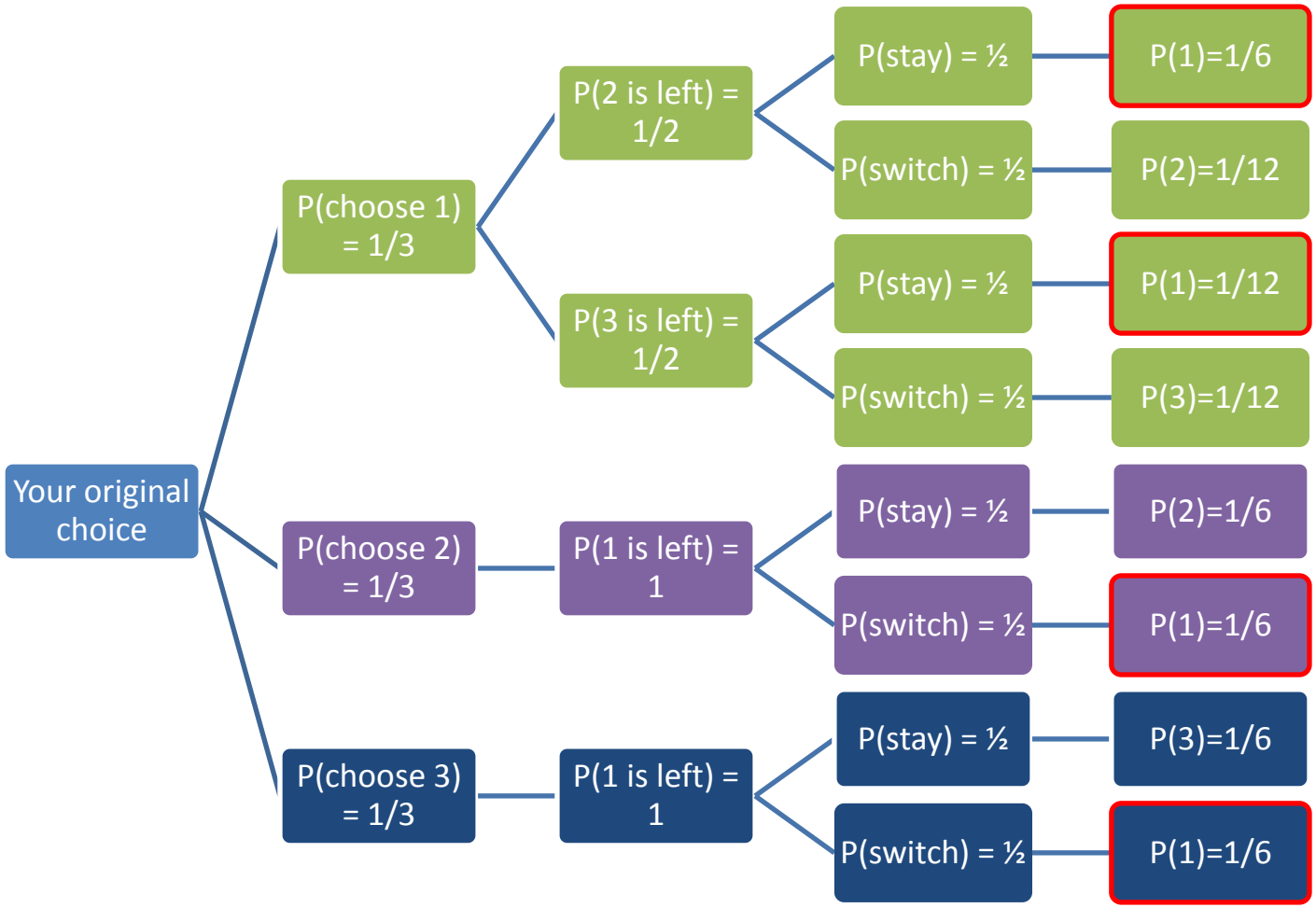
Strategy 2: Flip a Coin

Theoretical Model



P(win)= 7/12

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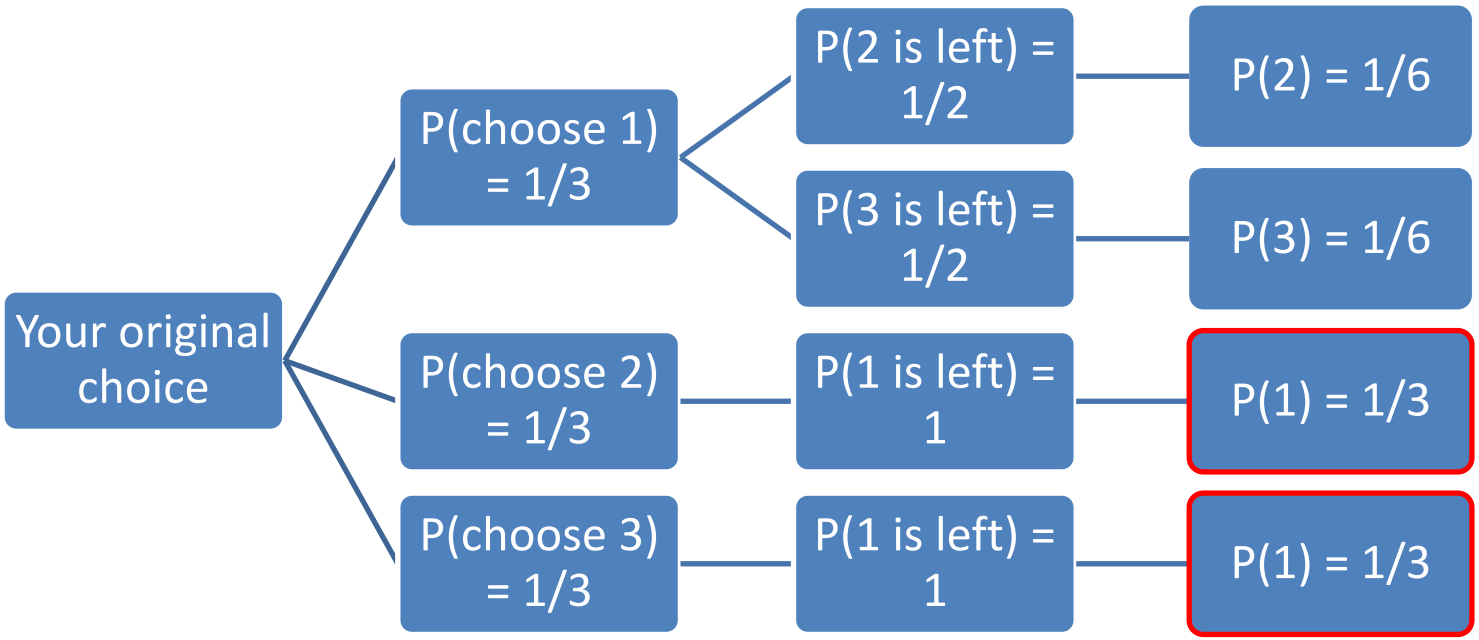
Strategy 3: Flip

Theoretical Model



P(win)= 2/3

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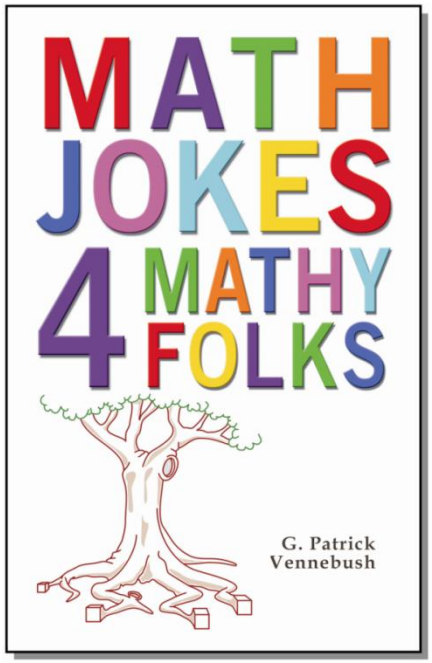
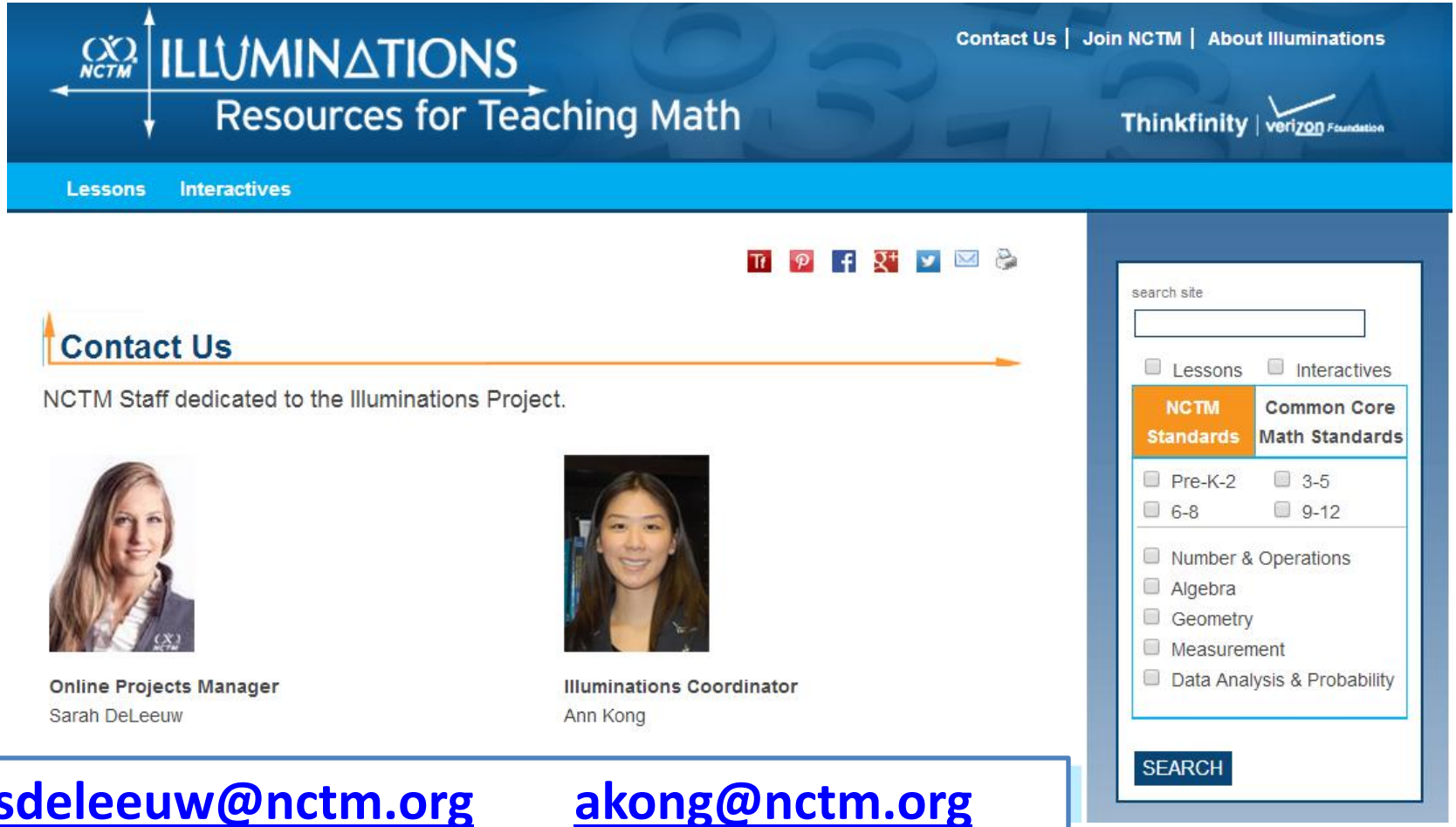
Simple and Oh So Fun



- Write a positive integer on a piece of paper.
- Show it to your neighbor.
- The winner is…

Whoever wrote the smallest integer NOT written by anyone else.

Questions? Comments ? CC?



sdeleeuw@nctm.org akong@nctm.org

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15q – 5q

THANK YOU !!!

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