Transformational Geometry And Common Core: Are you Ready?

A long time advocate of transformational geometry

Johnny W. Lott
jlott@mso.umt.edu

Common Core State Standards for Mathematics

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity (p.75)

Common Core and Technology

- Represent transformations in the plane using, e.g., transparencies and geometry software
- Given a geometric figures and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software
- Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) (p. 76)

Common Core Standards for Geometry

- Not typical geometry
- Little teacher preparation
- Few collegiate courses
- Adequate knowledge of tools
- Little preparation for proofs
- An end in and of itself?

What is not typical?

- With a true axiomatic approach, it is very different from normal.
- Some theorems are easier; some are not.
- The FIRST standard under congruence is EXPERIMENT WITH TRANSFORMATIONS IN THE PLANE!

Teacher Preparation?

- Most teacher preparation programs have exactly one geometry class, taken by teacher prep candidates, not math majors.
- Most such geometry classes are not transformational geometry.
- The transformational geometry in a teacher program is primarily buried in a linear algebra class.

Little consensus on geometry course

- 40% co courses start with elementary axioms and emphasize Euclidean geometry or a mix of Euclidean and non-Euclidean geometry
- 23% employ a survey approach
- 20% use an analytic or projective geometry approach
- 63% of the courses use primarily instructor lecture
- 36% have substantial amount of group work during class
- "A typical geometry course did not emerge from the data." (30 syllabi and 108 questionnaire responses)

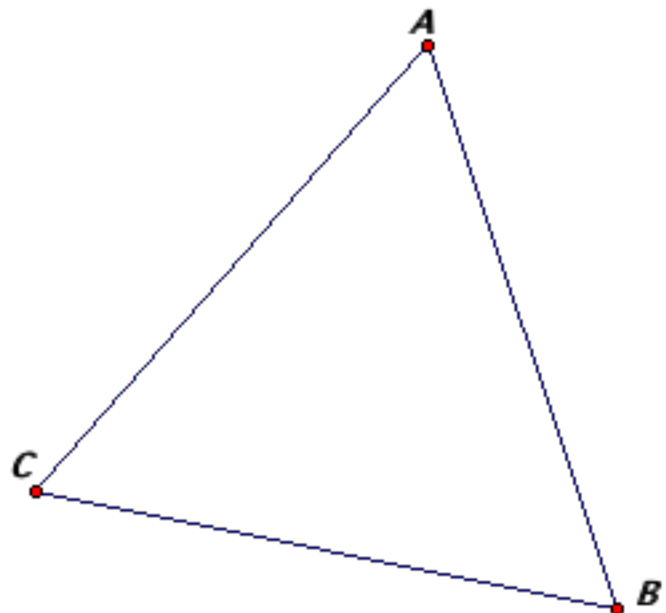
Technology in Geometry for Pre-service Teachers

- "…most secondary mathematics education programs do not incorporate learning how to use GSP in the classroom into the teacher education curriculum."

(Da Ponte, Oliveira, and Varandas, 2002)

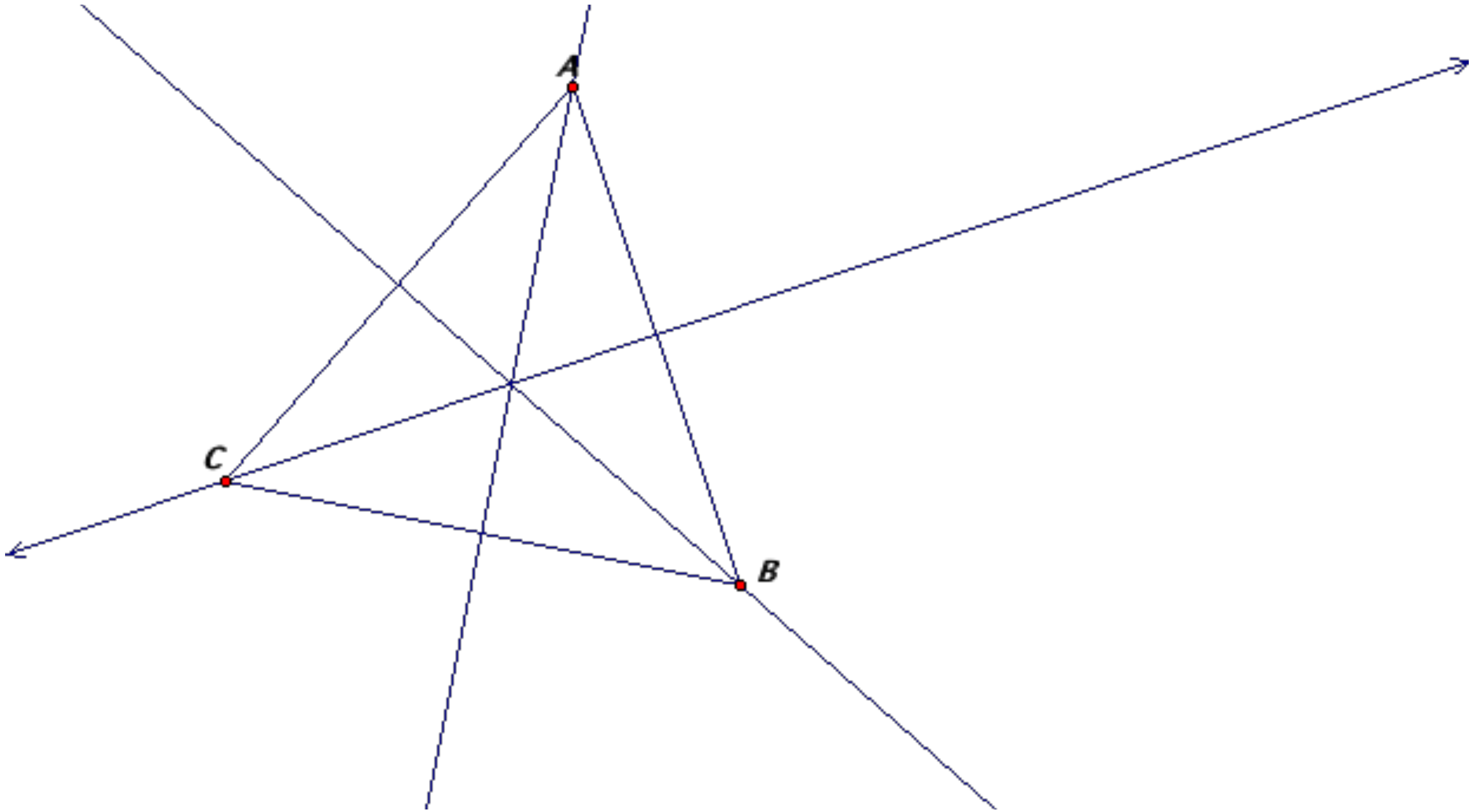
Construct an equilateral triangle

First think about what you have:



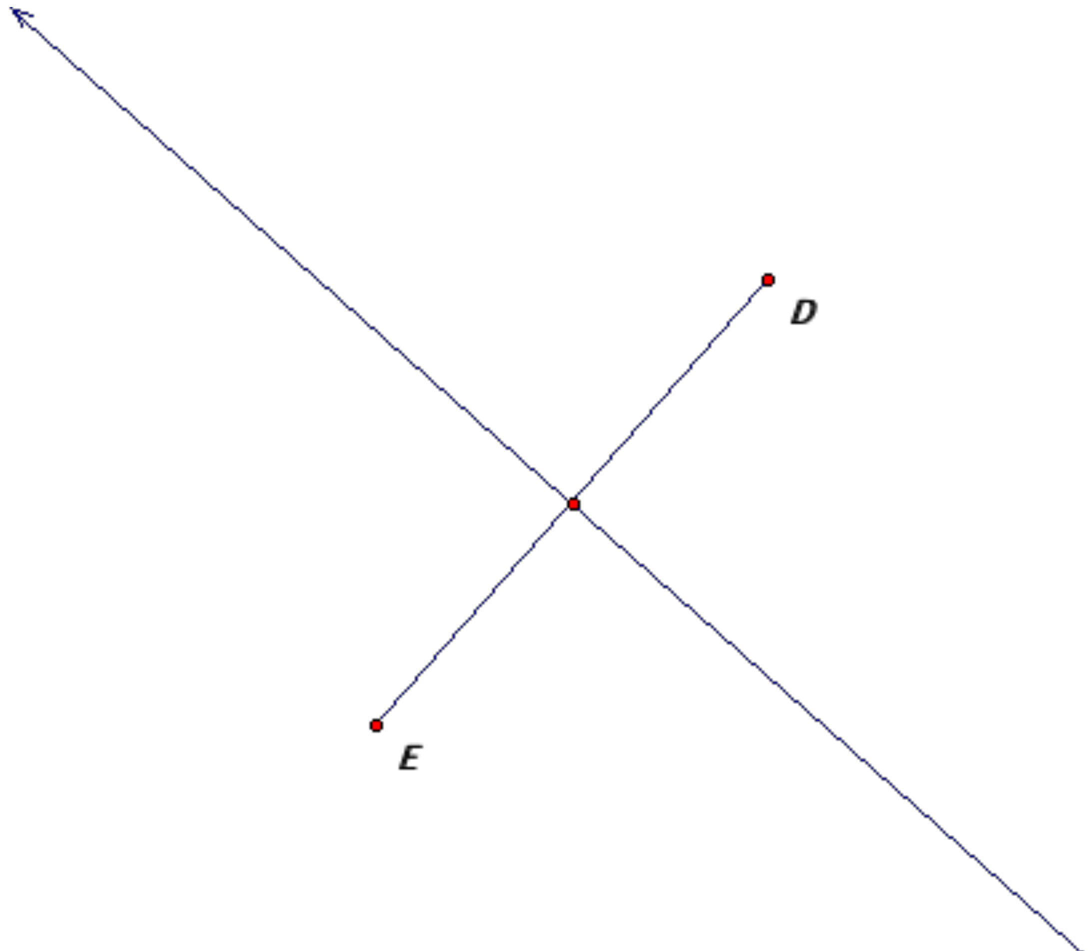
Where are lines of symmetry?

Equilateral Triangle with Lines of Symmetry



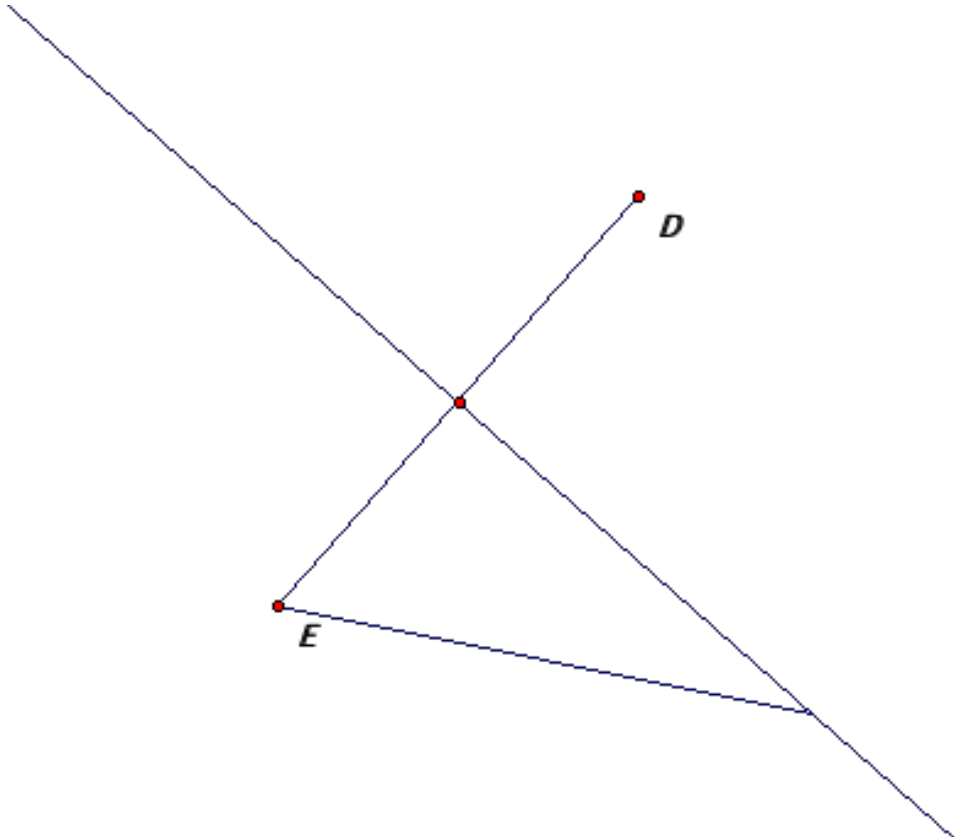
Construct an Equilateral Triangle Using Transformations

- Find midpoint; construct perpendicular bisector;



Construct Equilateral Triangle using Transformations

Using E as center, rotate segment DE -60 degrees. D, E, and new intersection point determine the equilateral triangle. Why?

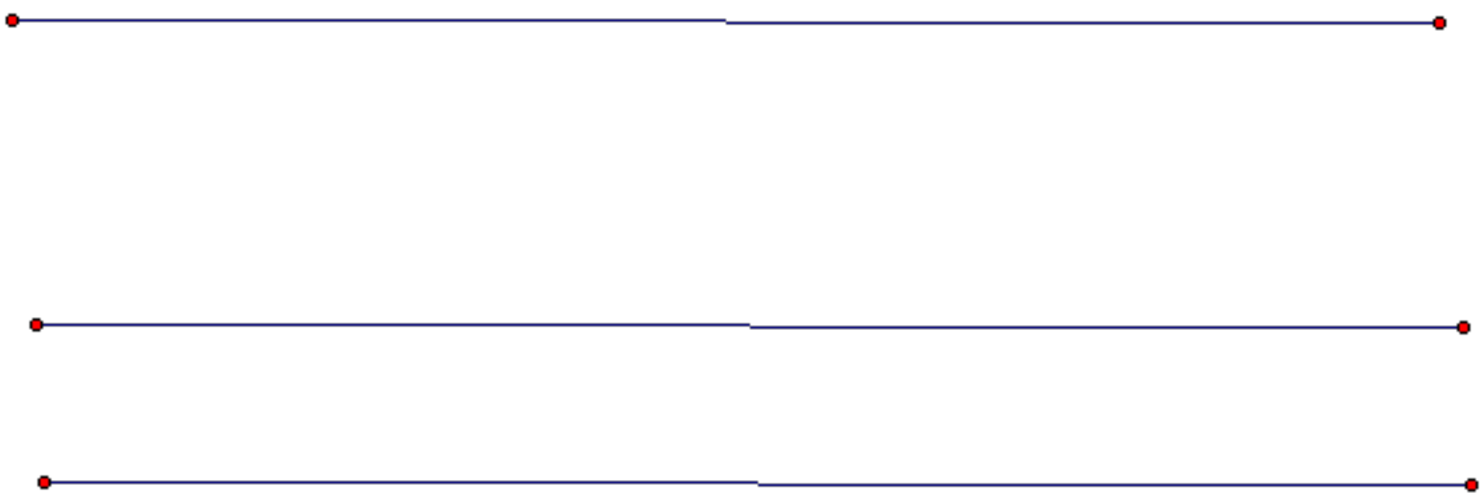


Equilateral Triangle Constructions

- Other methods
  - Use reflecting lines
  - Use rotation of same segment twice
  - Others?

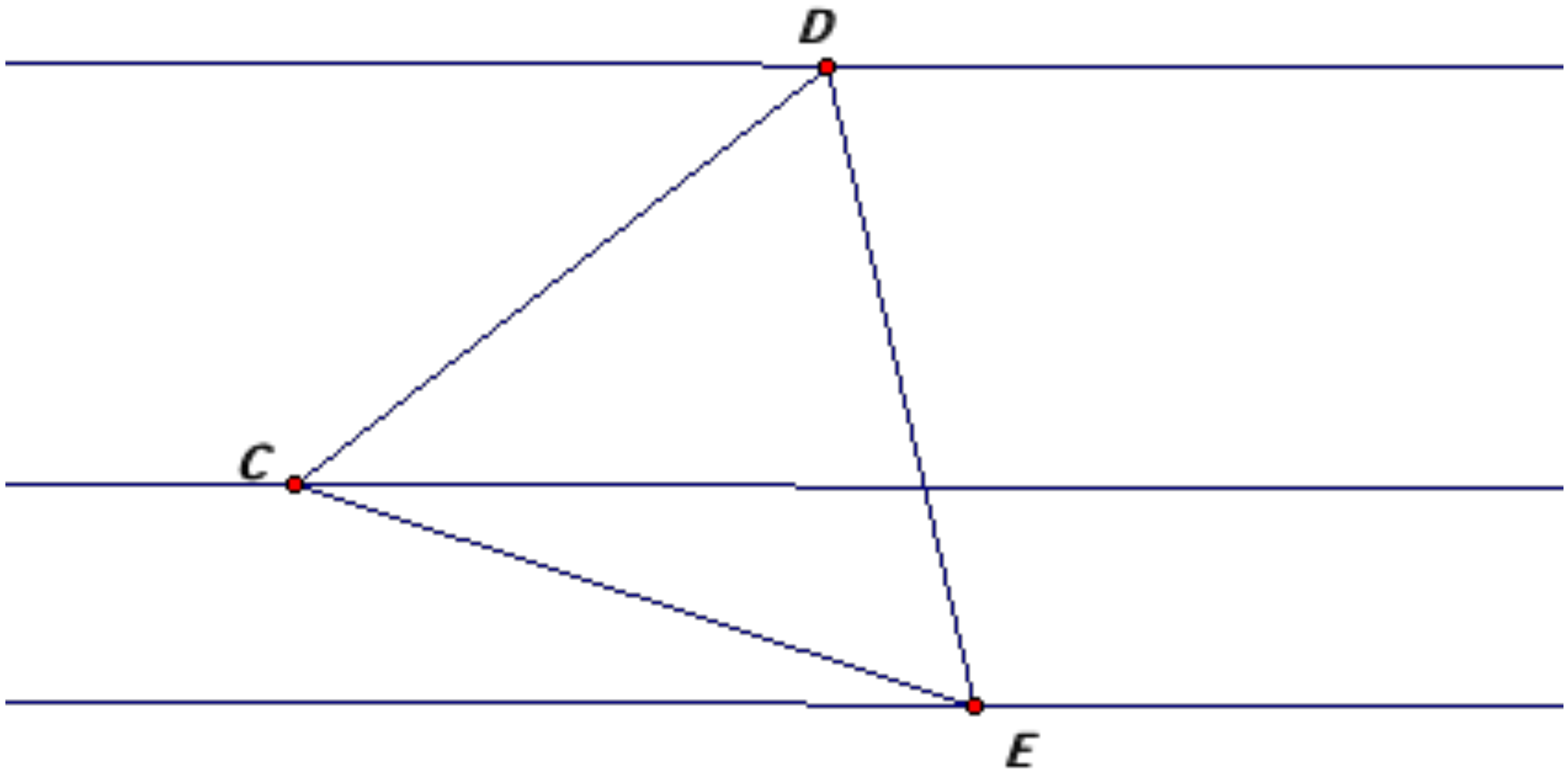
Experiment with transformations in the plane

- A farmer wanted to place three scarecrows in a garden in such a way that one would be in each of the furrows indicated below and so that each was the same distance from the other. Where could the scarecrows be placed?



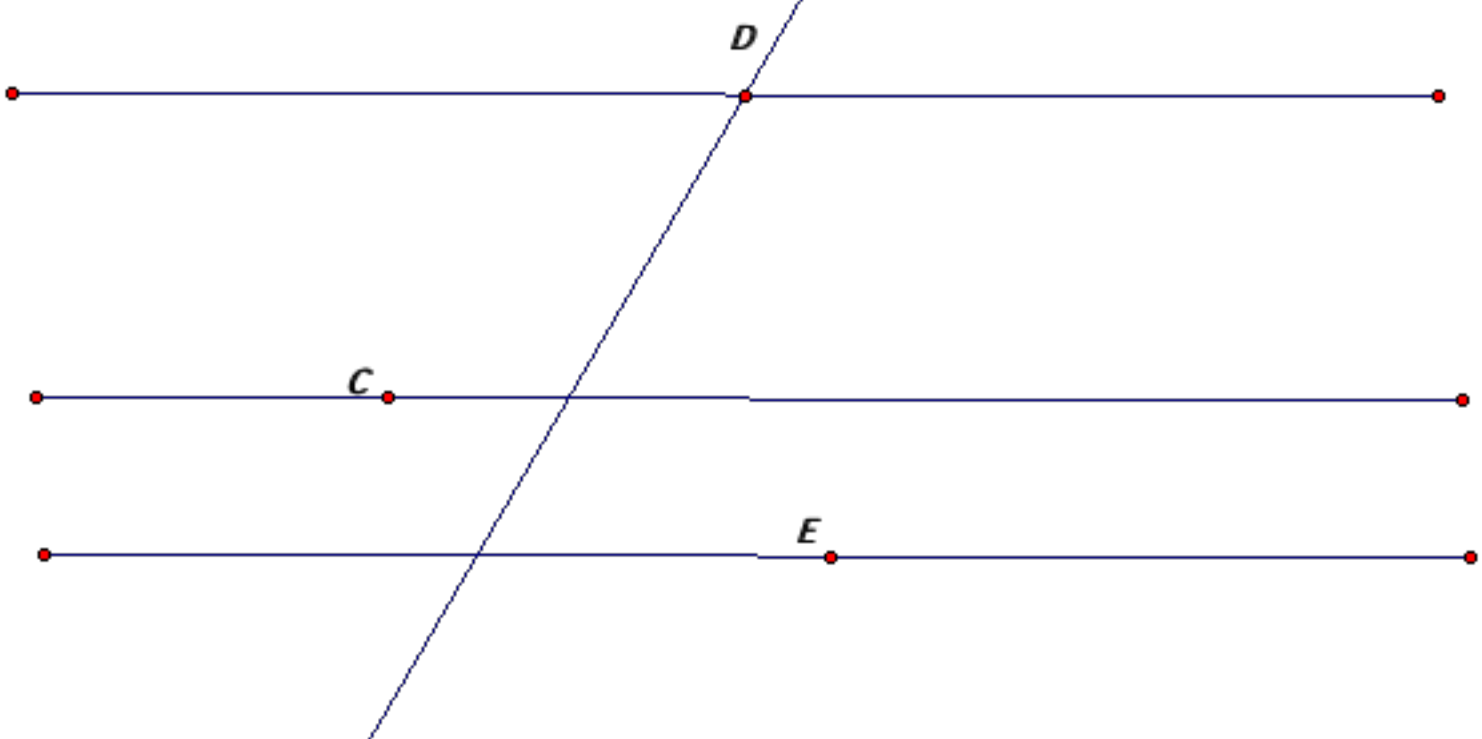
Imagine the problem done

If done, triangle CDE would be an equilateral triangle. Using C as center, rotate bottom furrow 60 degrees. What point would you find?



Construct solution

- Choose point C
- Rotate bottom furrow 60 degrees with C as center to determine D
- With D as center rotate C 60 degrees to find E

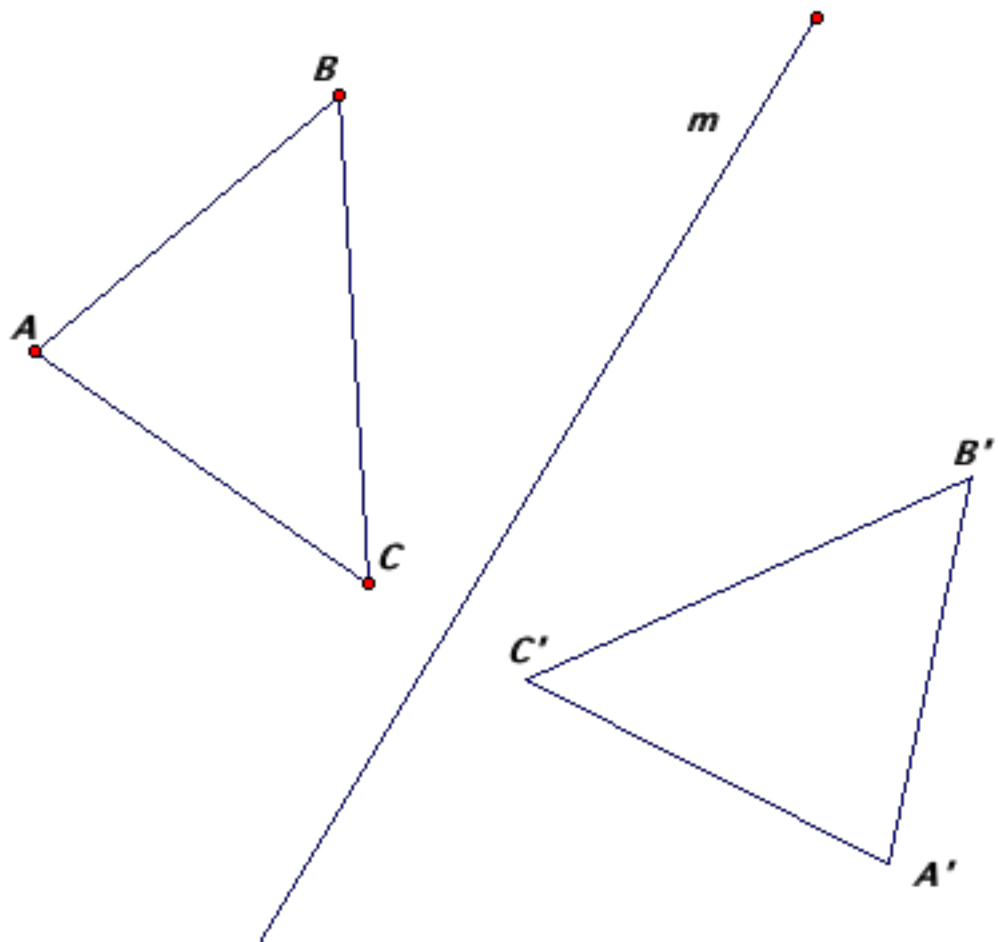


Understand congruence in terms of rigid motions

- We envision moving one figure and placing it atop a congruent figure and they fit perfectly.
- Intuitively, we flip them, turn them, or slide them.
- This is in lower grades.
- Mathematically, there are actually four moves that are required: reflection (flip), rotation (turn), translation (slide), and glide reflection.

Reflection (flip) Review

- Look at triangle *ABC* reflected in line *m* to obtain triangle *A'B'C'*. What do we know?

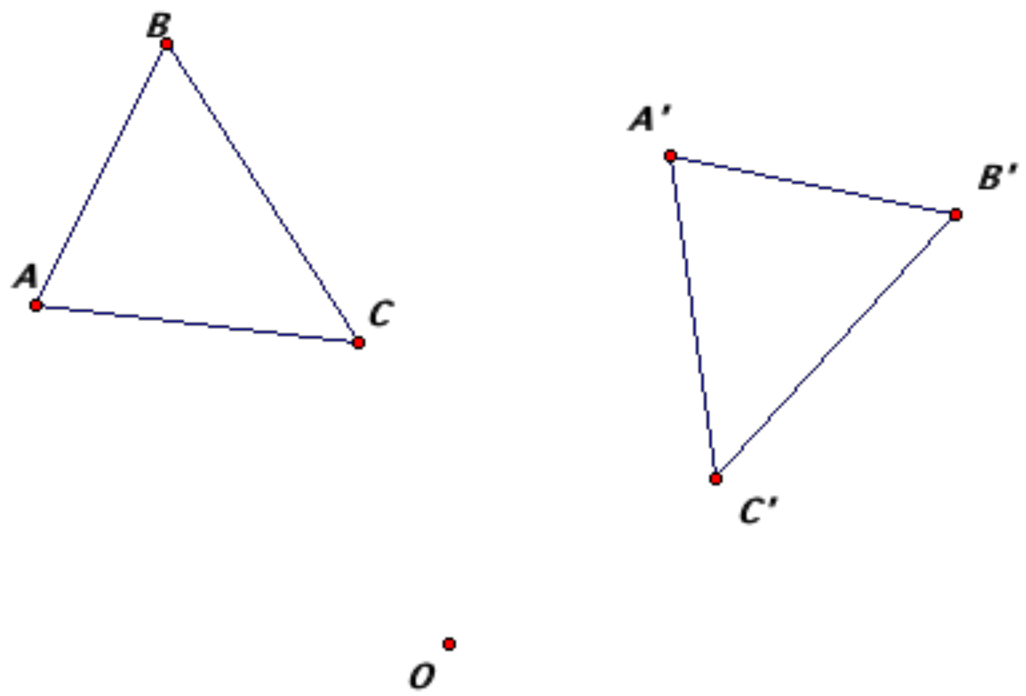


Reflection (flip) Review

- *m* is the perpendicular bisector of segment connecting point and its image if the point is not on *m*
- *The orientation of the original is reversed in a reflection.*

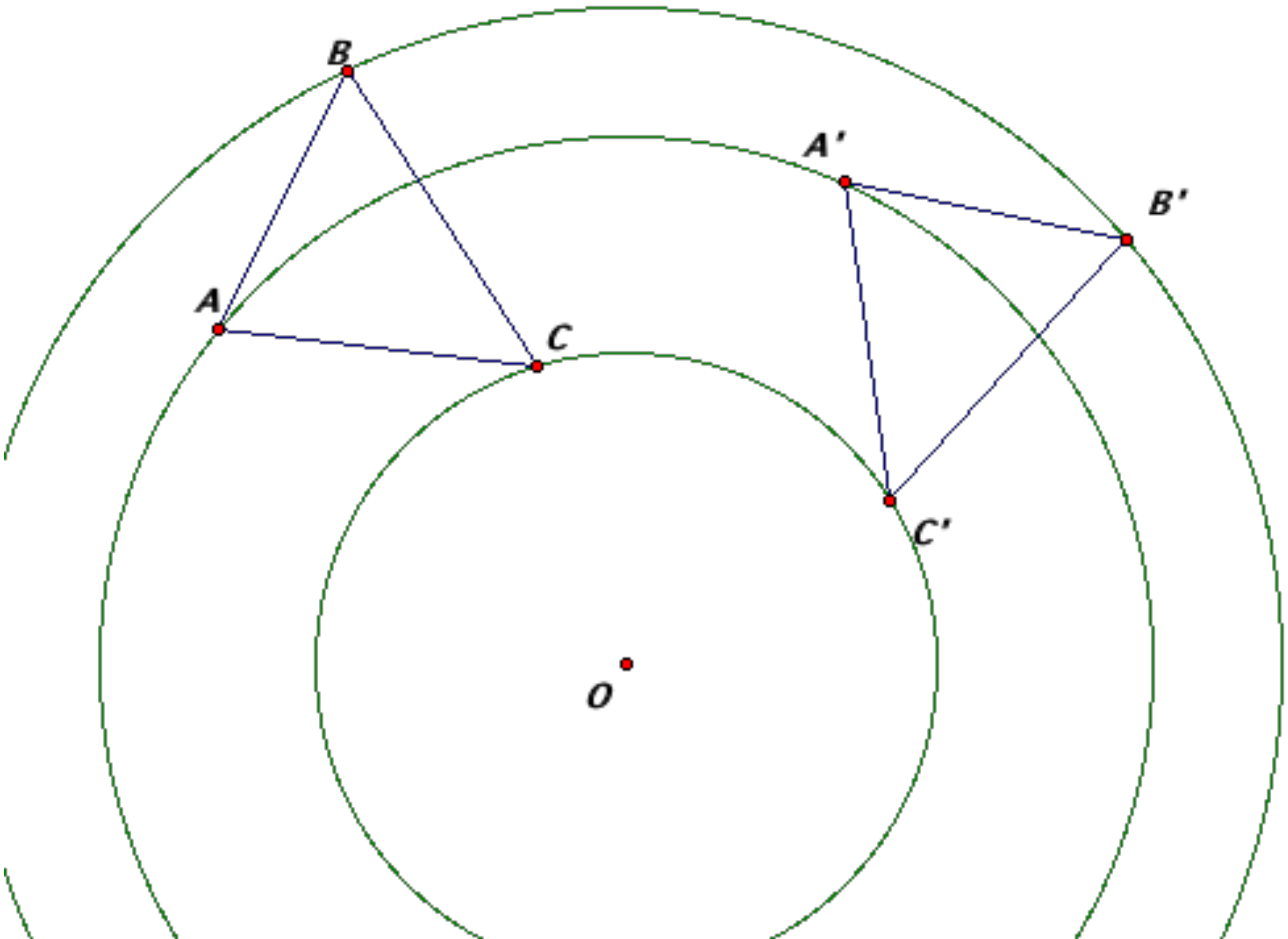
Rotation (Turn) Review

- Consider triangle ABC rotated in point *O*
- What do we know?



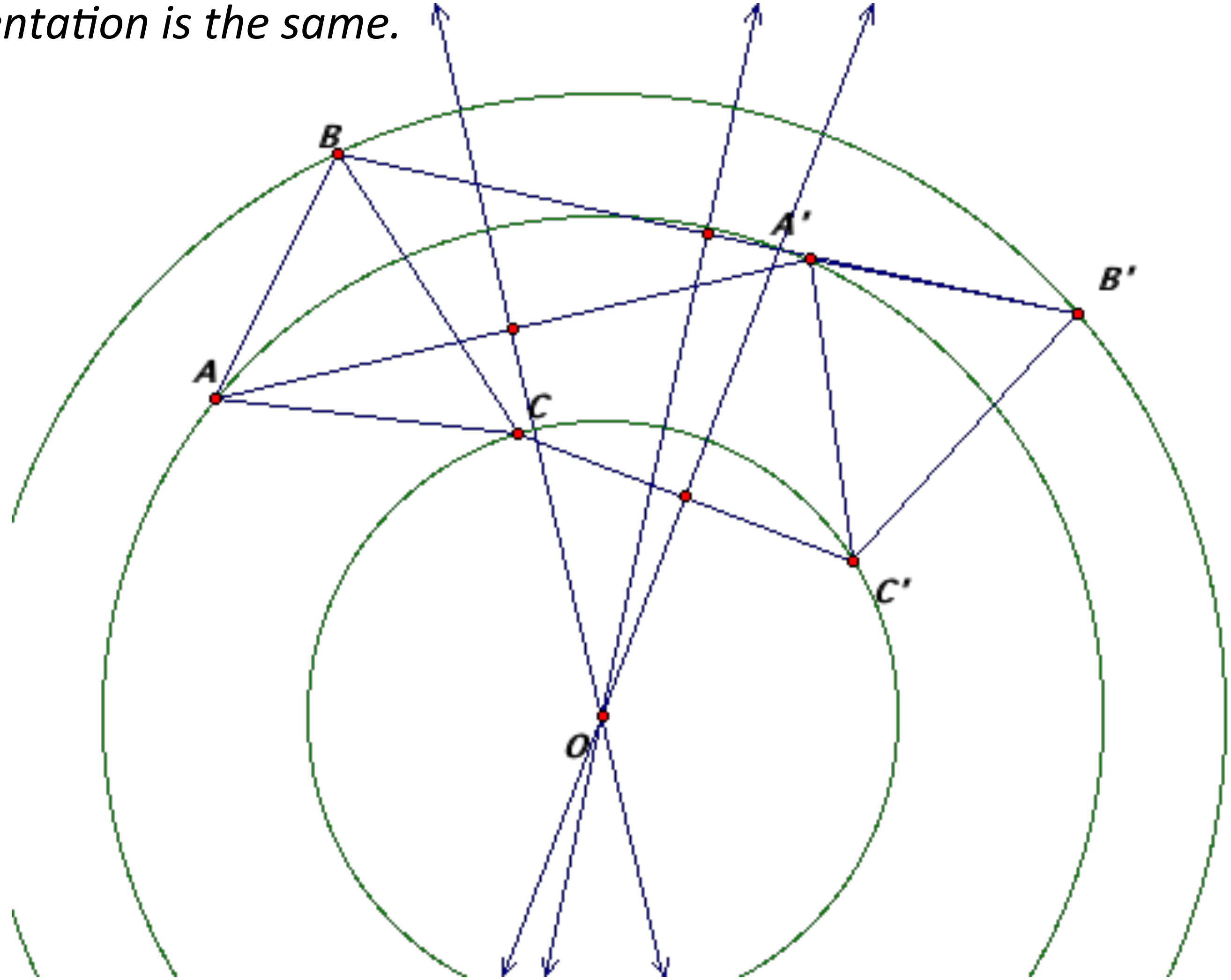
Rotation (Turn) Review

- What do we know?



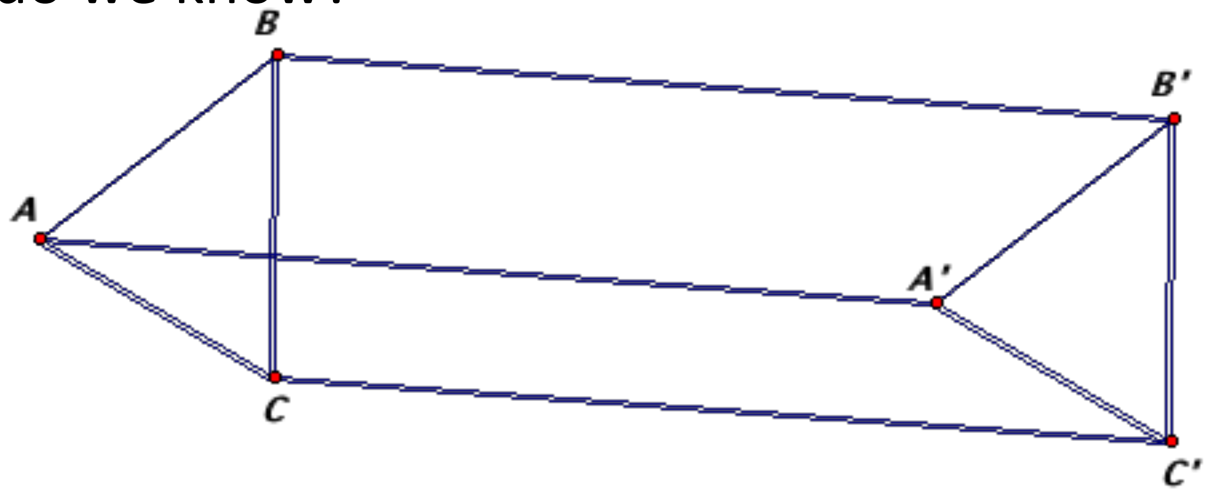
Rotation (Turn) Review

- *O* is the center of three circles passing through *A, B,* and *C* and their images.
- The perpendicular bisectors of segments *AA’*, *BB’*, and *CC’* intersect in point *O.*
- *The orientation is the same.*



Translation (slide) Review

- Triangle *ABC* is translated to triangle *A'B'C'* through slide that takes *C* to *C'*.
- What do we know?

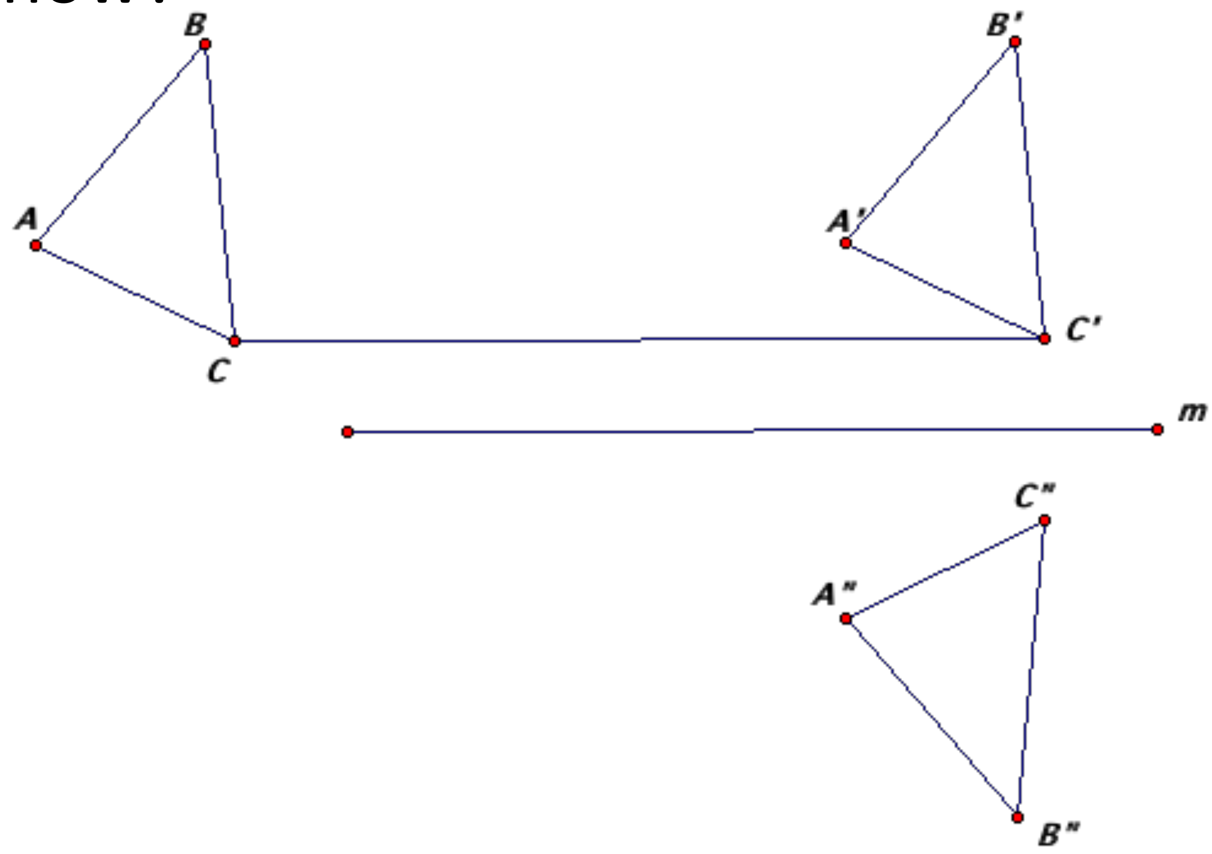


Segment *AA'* is congruent to segment *BB'* is congruent to segment *CC'*.

The orientation is the same.

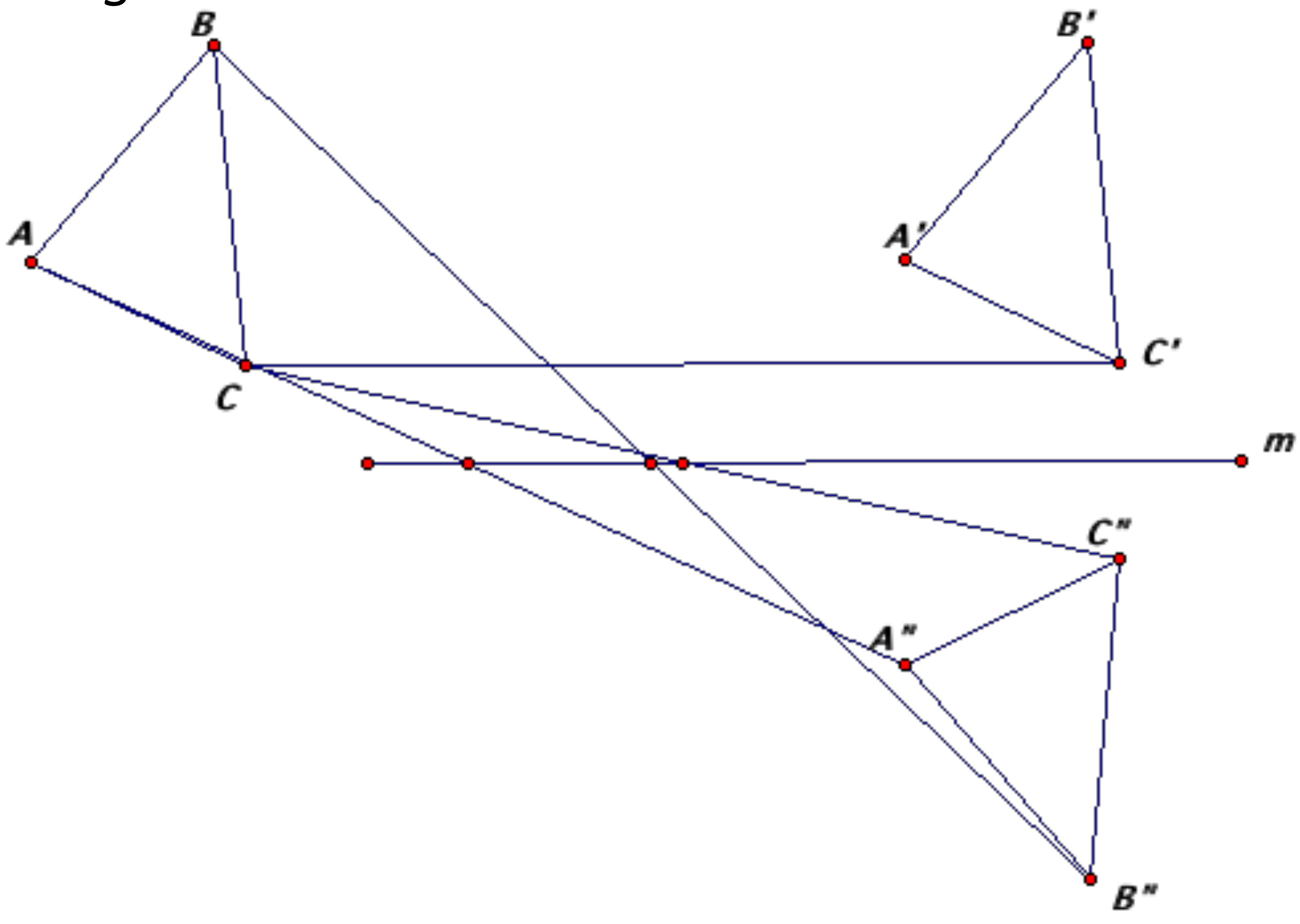
Glide Reflection (Review?)

- Triangle *ABC* is translated to triangle *A'B'C'* and then reflected over a line parallel to translation to obtain triangle *A"B"C"*.
- What do we know?

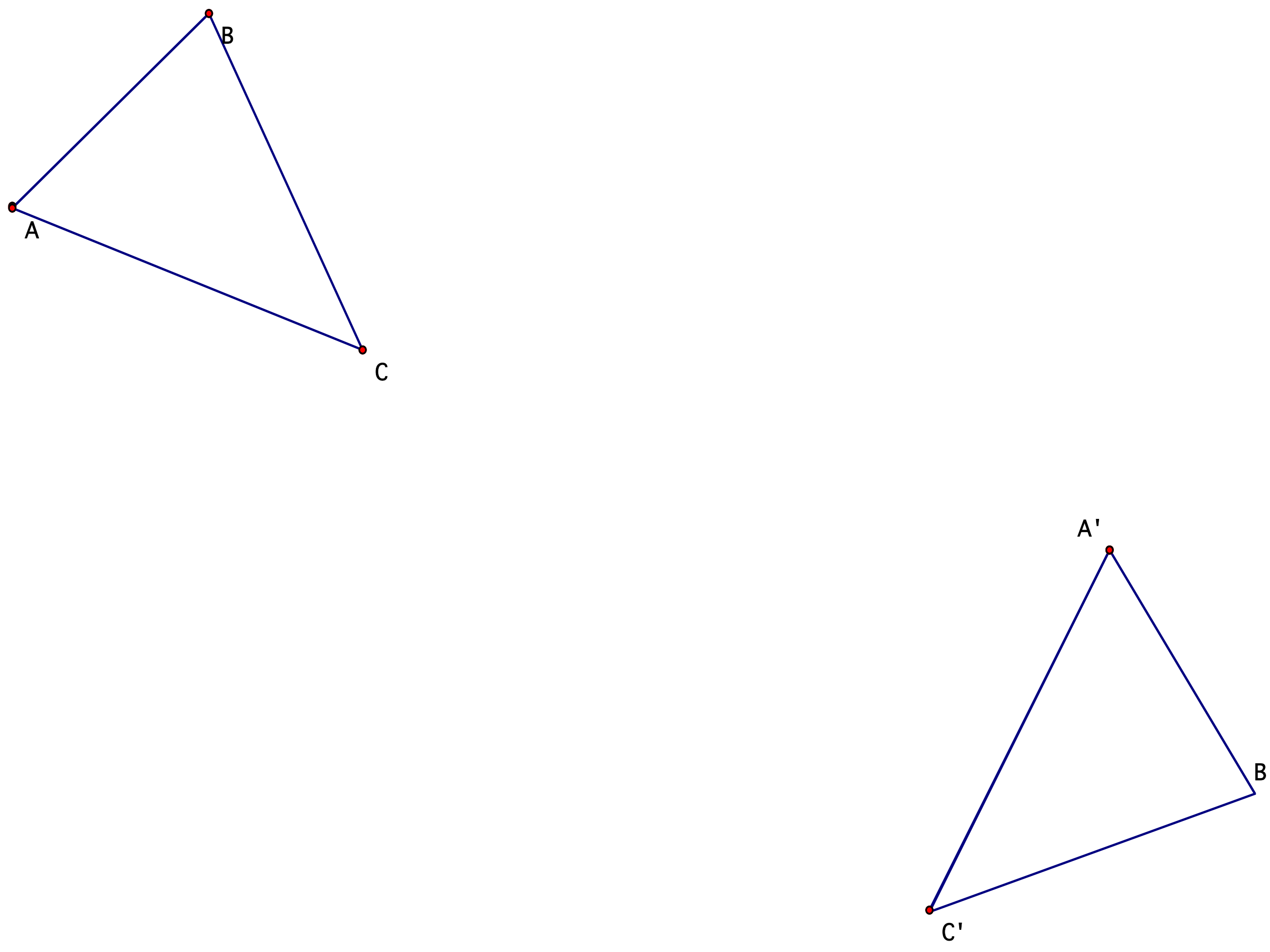


Glide Reflection

The midpoints of segments *AA", BB",* and *CC"* appear to be on line *m. (WHY takes a bit more than the time we have.)*
The orientation changes.



Given two congruent triangles, can we show that they are congruent?



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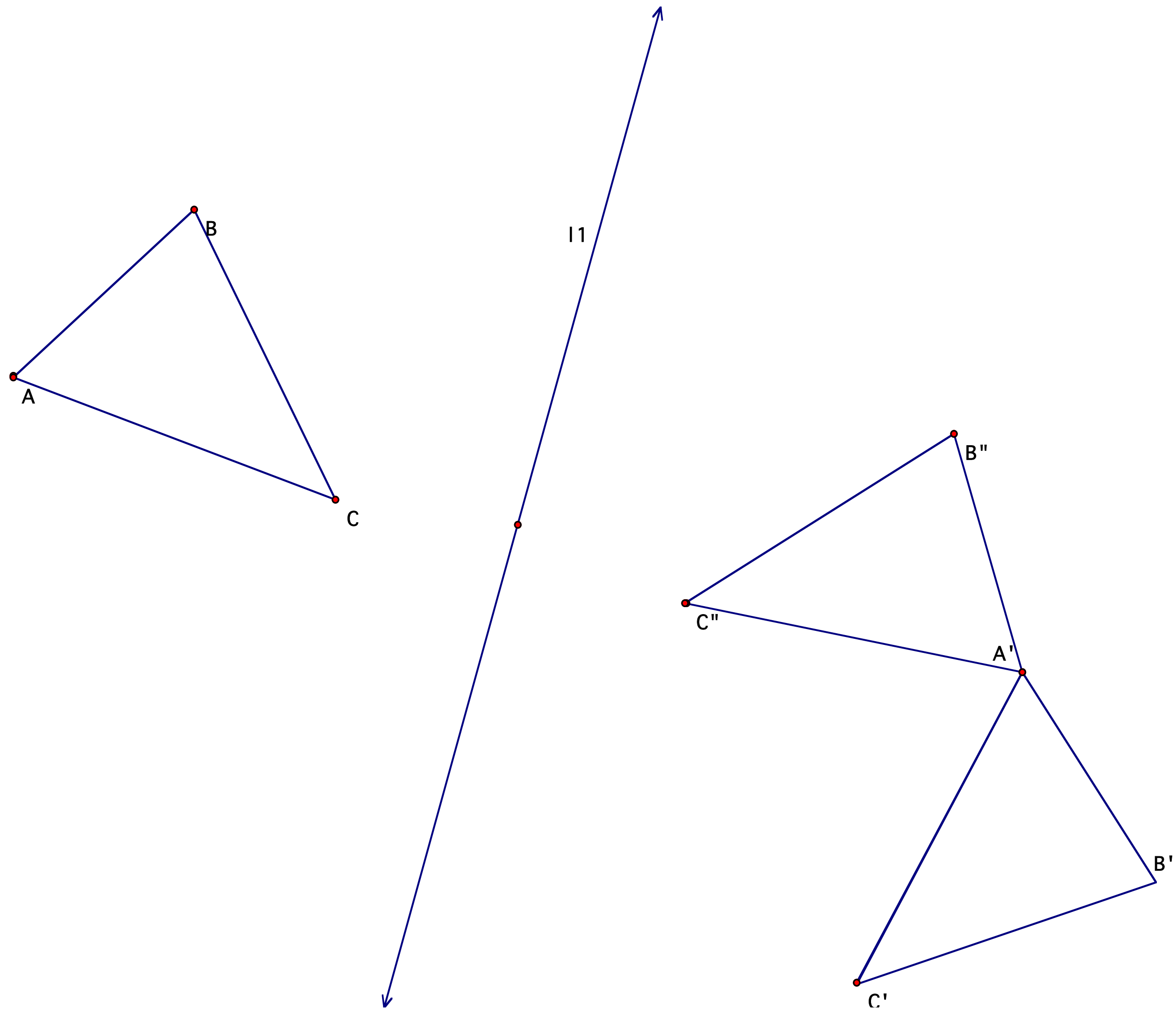

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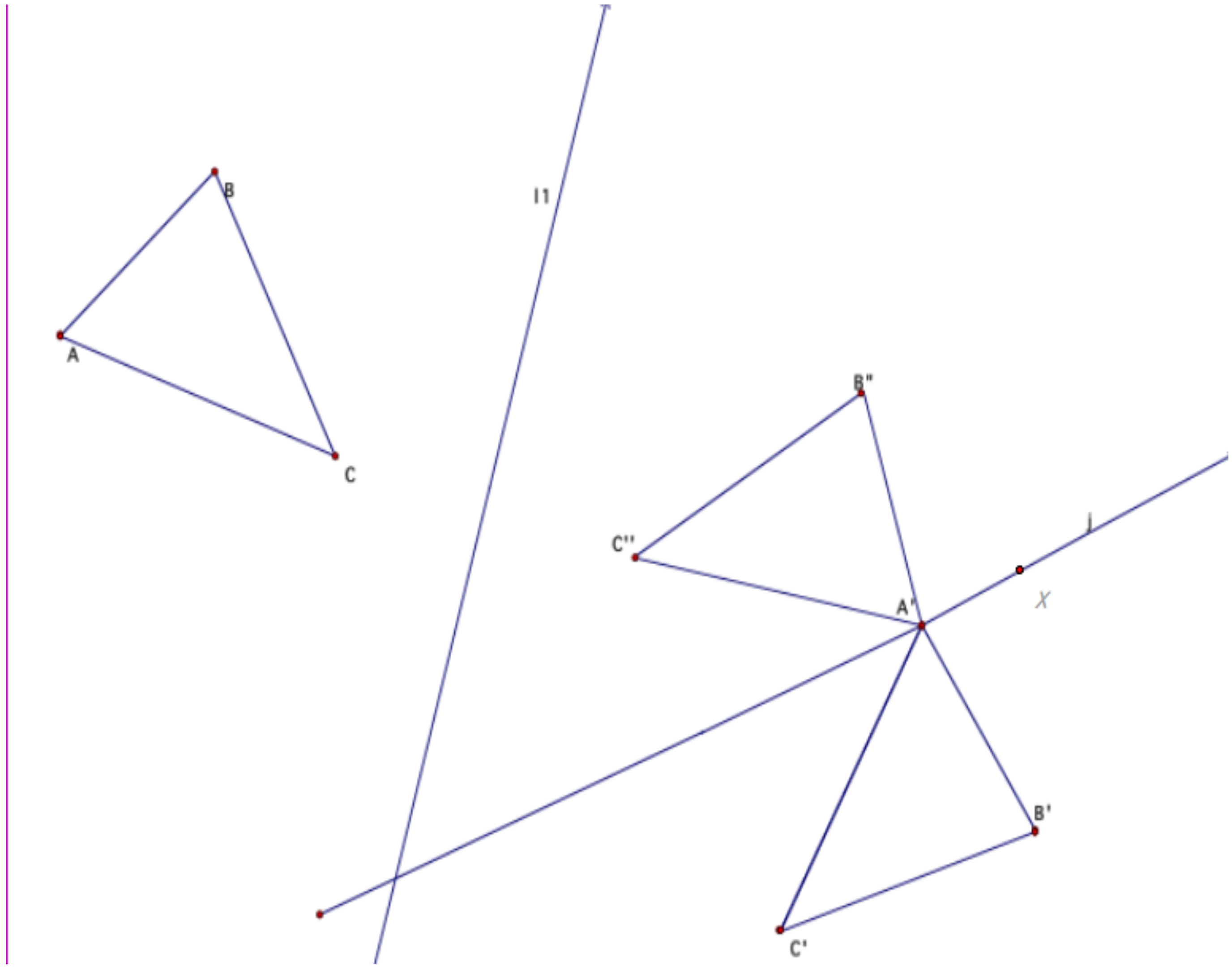

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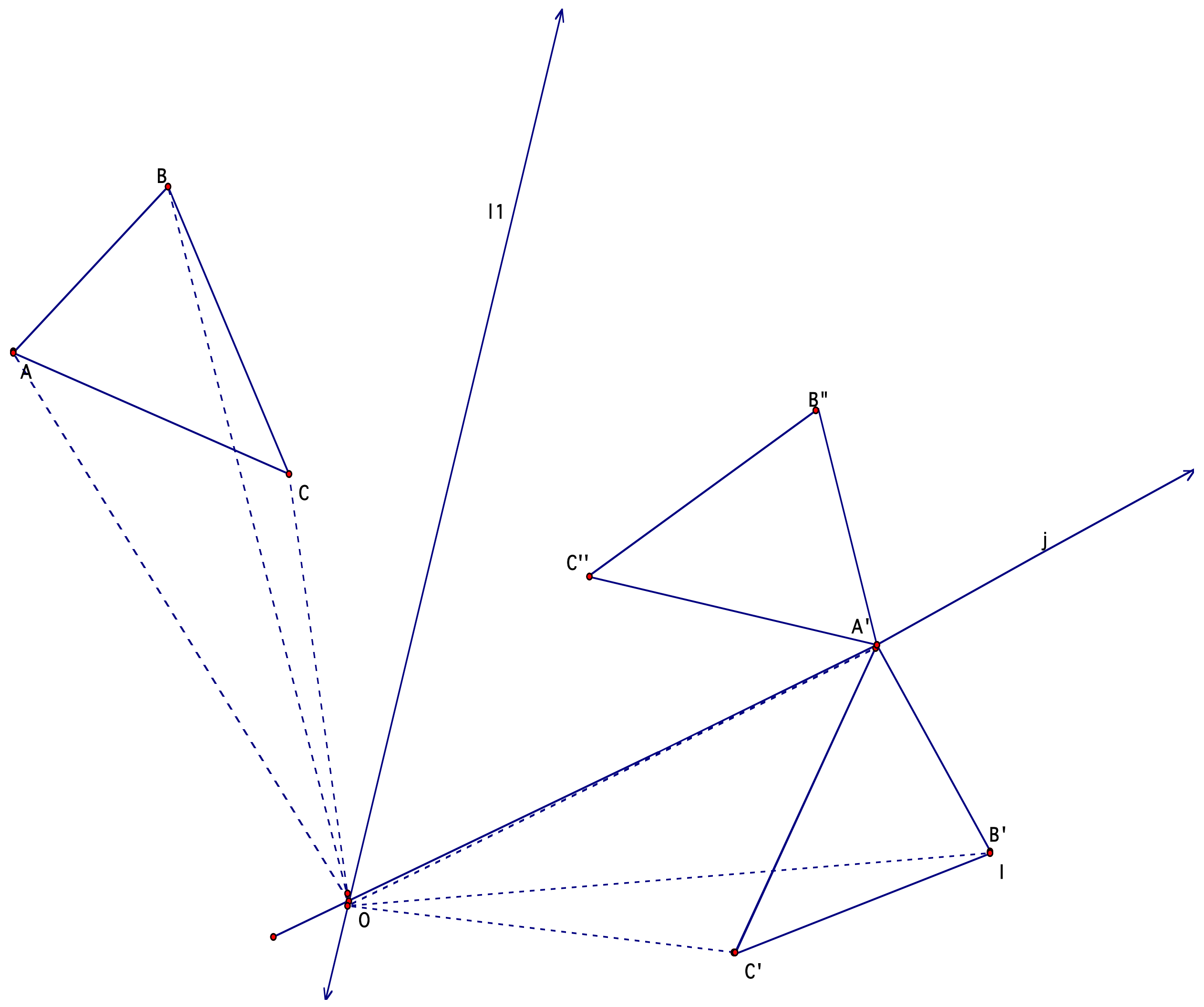
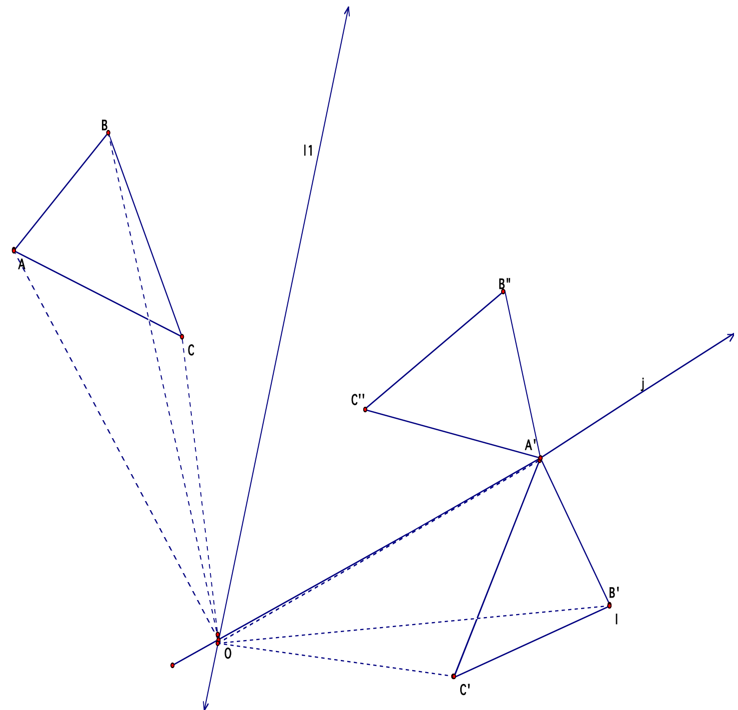


Given two congruent triangles, can we show that they are congruent?

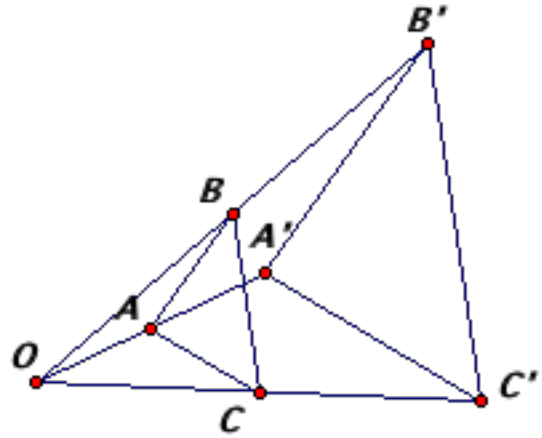
What have we done?
Is the resulting composition of motions either a reflection, rotation, translation or glide reflection?

Check orientation for first clue.
Then what could we do?



Dilation

- Triangle *ABC* is dilated with center *O* and scale factor 2.
- What do we know?
  - Triangle *ABC* is similar to triangle *A'B'C'* with scale factor 2.
  - Lines through *B* and *B'*, *A* and *A'*, and *C* and *C'* all contain *O*.



A similarity is the composition of a dilation and an isometry (rigid motion).

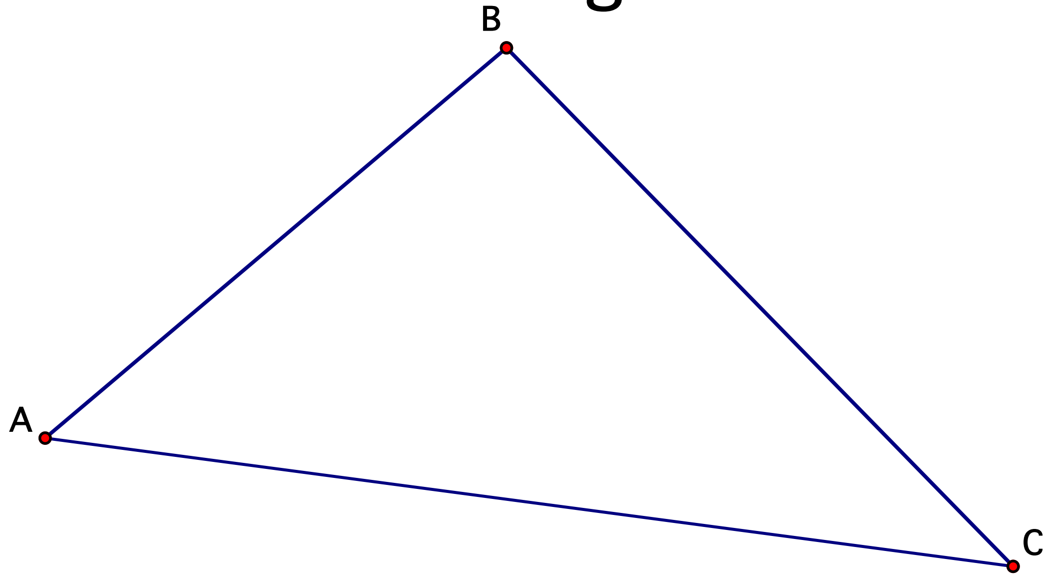
- Again takes more time than we have.
  - Need to play with technology
  - Need to question order of composition
  - Need to even think about composition of two dilations
    - What is scale factor of resulting motion?
    - What is the center of resulting motion?
    - …

Apply to problems and theorems

- Classic transformational geometry problem
- Prove that the angle bisectors of a triangle are concurrent (meet in a point).

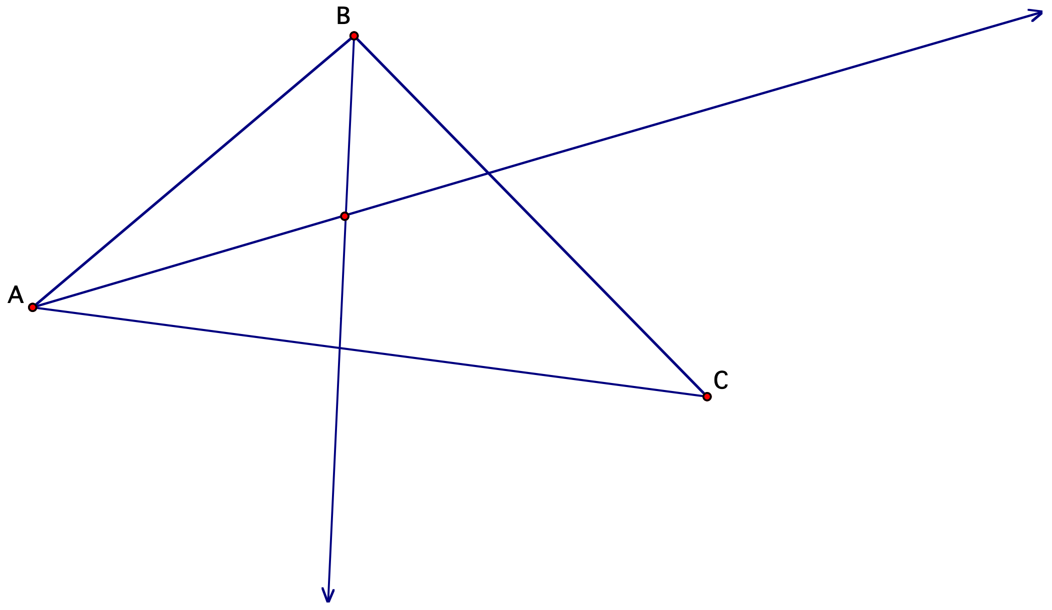
Given triangle *ABC*

- Construct bisector of angle *BAC* and *ABC.*



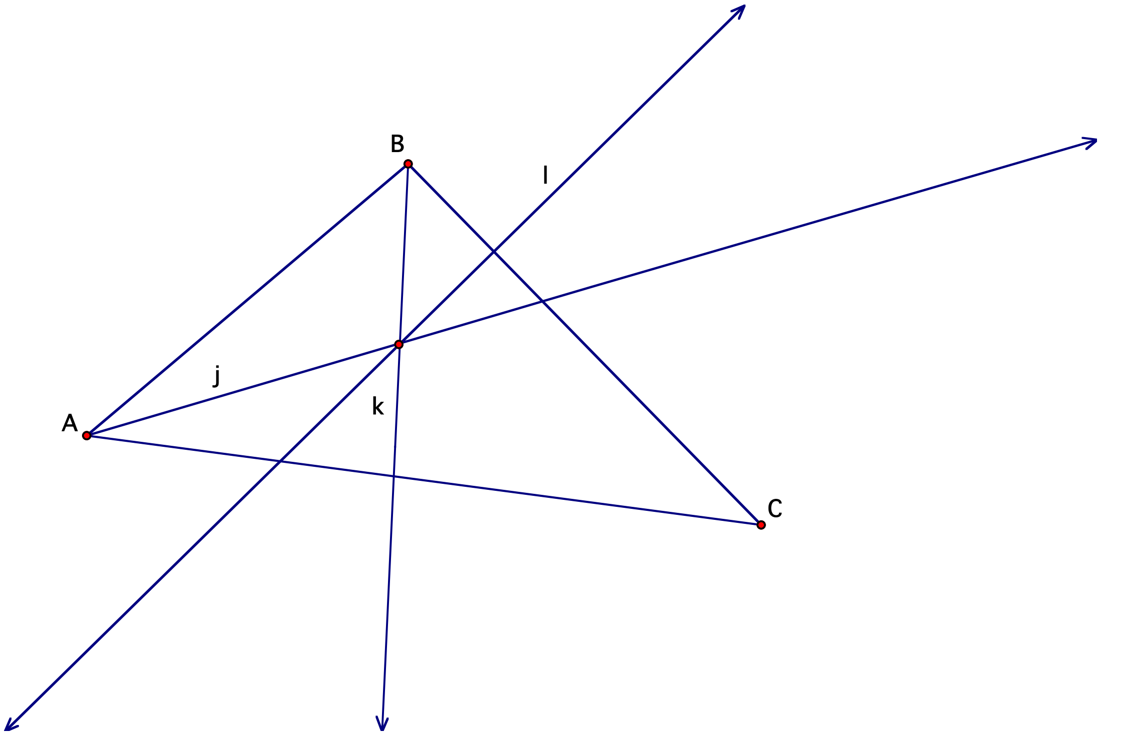
Next Step

- Find a line that maps line *BC* to itself.



And Then

- That line could be the perpendicular from the point of intersection of *j* and *k.*



Now consider the following:

- $r_l r_k r_j(\overleftrightarrow{AC}) = \overleftrightarrow{BC}$
- So whatever $r_l r_k r_j$ is results in taking side *AC* to side *BC.*
- ***And a single reflection in the angle bisector of angle ACB does exactly that.***
- ***Without proof here, it can be shown that the composition of reflections in three concurrent lines can be replaced by a reflection in another line that is concurrent with the three.***
- ***With this, the argument would be complete.***

So what do we need in teacher preparation IF we are to teach geometry from a transformational viewpoint?

- A course adapted to allow prospective teachers to
  1. experiment with transformations in the plane.
  2. learn technology and other tools to play with transformations
  3. materials to do those things
  4. use of matrices to play with transformations
  5. use of complex numbers to play with transformations

So what do we need before testing starts in 2014?

- PROFESSIONAL DEVELOPMENT FOR CURRENT TEACHERS
- MATERIALS FOR STUDENTS WITH THE TRANSFORMATIONAL GEOMETRY EMPHASIS
- TOOLS IN CLASSROOMS FOR PLAY IN TRANSFORMATIONAL GEOMETRY
- …

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