Tackle the Common Core: Make it Real!!

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Lorie McFee
North Buncombe High
Weaverville, NC
Email: lorie.mcfee@gmail.com

Jennifer North Morris
Math Coach/Professional Development Specialist
Tucson, AZ
Email: Jennifer@north-morris.net

Outline of Workshop

1. Growing Tall
   - Aleisha: Month 1: 60 cm; Month 2: 100 cm
   - Bernice: Month 1: 25 cm; Month 2: 50 cm
2. Standards of Mathematical Practice
   - Student vs Teacher Behaviors
   - Instructional Strategies
3. Pulse Rates
4. Bouncing Tennis Balls
5. Largest Triangle

A complete handout with investigations and materials used in the workshop are available through the NCTM or at the following website (or scan QR code): http://north-morris.net.temp.guardedhost.com/jennifer-north-morris.html

Investigation 2.4: Growing Tall

Dr. Robert Horton, Clemson University
Fostering Mathematics Series

<u>Did you know</u> that if you are considering fast growing wild trees in the United States the Eastern Cottonwood tree is one of the fastest if not the fastest of those trees. Under the right conditions it can grow as much as 10-15 feet per year. It does not sustain that kind of growth for its entire life, but it is common for it to grow about 5 feet per year and approximately 1 inch in diameter on average during a 25 year period.
www.funtrivia.com/askft Question13317.html

Aleisha and Bernice both are growing plants. A month ago, Aleisha's plant was 60 centimeters tall; it is now 100 centimeters tall. A month ago Bernice's plant was 25 centimeters tall; it is now 50 centimeters tall. Though they are great friends, they are also very competitive with each other. Each claims that her plant is growing faster.

A. Provide a convincing argument that Aleisha's plant is growing faster than Bernice's. Demonstrate your argument numerically and graphically. Also show why your argument is correct by describing what will happen over several months if the established patterns would continue.

B. Provide a convincing argument that Bernice's plant is growing faster than Aleisha's. Demonstrate your argument numerically and graphically. Also show why your argument is correct by describing what will happen over several months if the established patterns would continue.

C. Summarize mathematically why both have a legitimate claim. If you had to choose which plant is growing faster, which one would you pick?

Mathematical Practice:

Student Behaviors	Teacher Behaviors

COMMON CORE STATE STANDARDS FOR MATHEMATICS

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize* - to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents - and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimations and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying the units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Investigation 1.4: **Pulse Rates**

Did You Know?

A human's system of blood vessels, which includes arteries, veins, and capillaries, is over 60,000 miles long. That's long enough to go around the world more than twice!

Reference: http://my.clevelandclinic.org/heart/heartworks/heartfacts.aspx

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When we engage in aerobic exercise, our pulses quicken. The following data represents the pulses of several people, first, at rest, and, second, after running.

PERSON	PULSE AT REST	PULSE AFTER RUNNING
1	82	150
2	76	132
3	72	115
4	49	83
5	85	130
6	72	136
7	68	136
8	70	120
9	78	129
10	84	140
11	90	160
12	64	120
13	71	125
14	50	90
15	59	92

Source: http://www.statsci.org/data/oz/ms212.txt

A What type of relationship might you expect there to be between the two pulse rates? Explain your thinking.

B Why are pie graphs and bar graphs inappropriate for these data? Explain.

PULSE RATES (CONTINUED)

C Construct median-box graphs (box plots) for both sets of pulses on the same axes. Describe the plots and describe the information that they provide.

D Construct a scatterplot of the data using the pulse rate at rest as the independent variable. What trends do you notice?

E Calculate a best-fit line for the data. Draw it on your scatterplot and wdiscuss what you find.

Name: ______________________________ Block: ______________

Group Members: ____

What's My Rule?

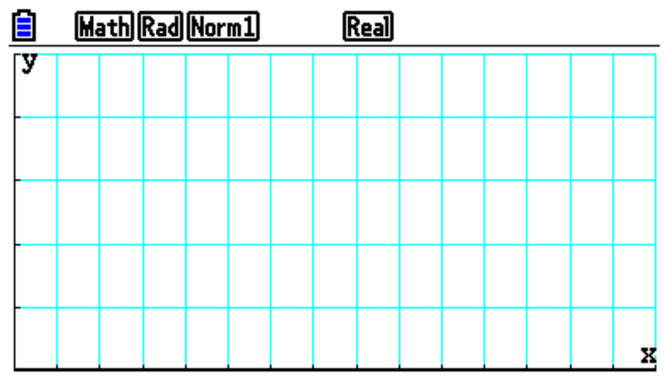
1. Describe the situation through words and pictures. What is the story? What is the pattern?

2. What do we want to find out?

3. What are your variables?

4. Create a table.

5. Graph your data.

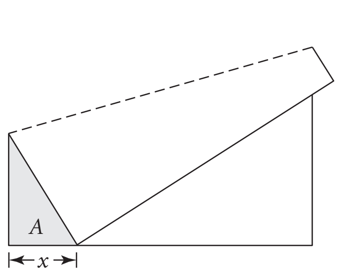


6. Write the equation that represents the relationship of your situation. Describe how you figured this out.

7. What did you find out? Explain.

Investigation • The Largest Triangle

Name ______________________ **Period** __________ **Date** ______________



Take a sheet of notebook paper and orient it so that the longest edge is closest to you. Fold the upper-left corner so that it touches some point on the bottom edge. Find the area, *A*, of the triangle formed in the lower-left corner of the paper. What distance, *x*, along the bottom edge of the paper produces the triangle with the greatest area?

Work with your group to find a solution. You may want to use strategies you've learned in several lessons in this chapter. Write a report that explains your solution and your group's strategy for finding the largest triangle. Include any diagrams, tables, or graphs that you used. You might use the sample table and grid provided here.

Distance along bottom (base of triangle) (cm) *x*	Height of triangle (cm)	Area (cm^2) *y*