

# VISUALIZING FUNCTIONS

Evelyn Baracaldo

Affiliation: Math for America

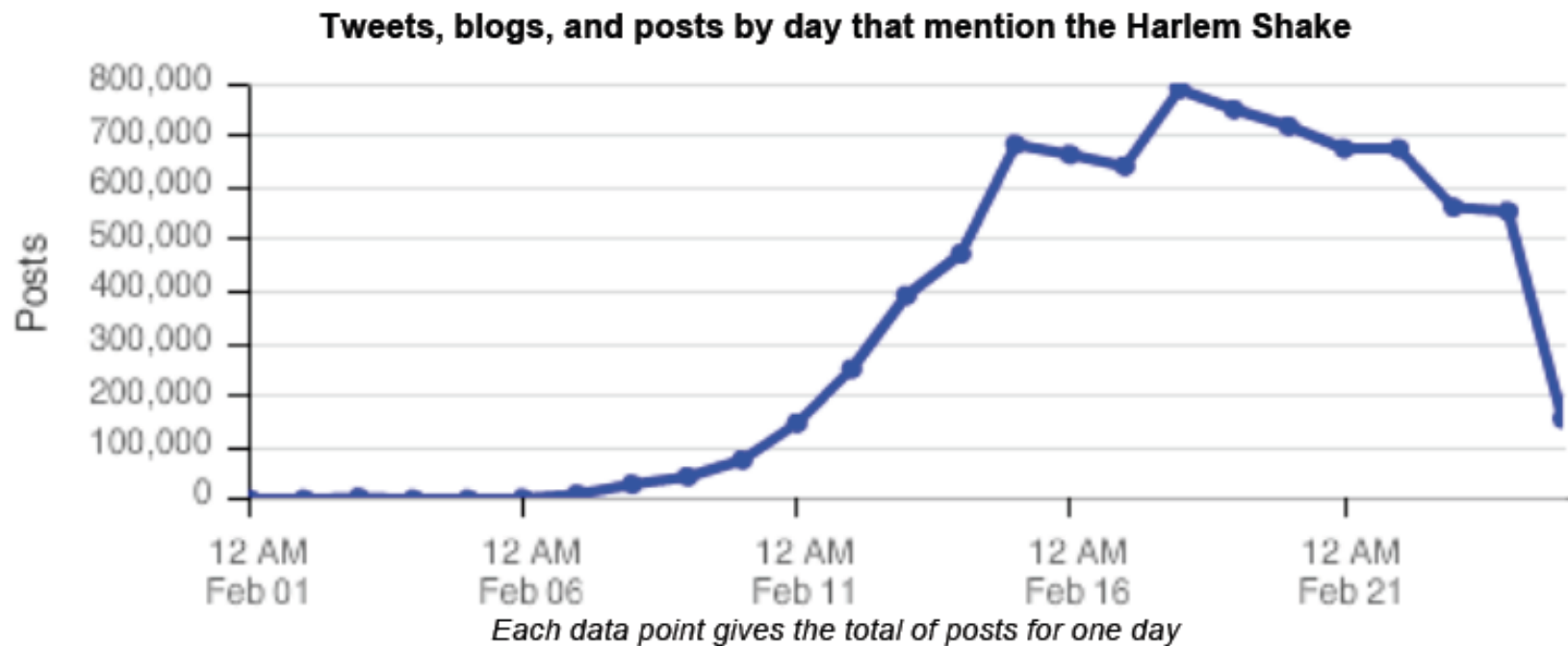
[evelyn.baracaldo@gmail.com](mailto:evelyn.baracaldo@gmail.com)

Fred Dillon

Ideastream

[fred.dillon@ideastream.org](mailto:fred.dillon@ideastream.org)

# Warm-Up



# What did that have to do with Functions?

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Does each time have only one number of posts?

Domain/Range

Linear/Nonlinear

Rate of change

Definition of a function

Real-world application

Non-symbolic

Data based

# What is a function?

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What do think of when you think about teaching functions?

# What does CCSSM say?

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Connect to ratio and proportion

Start with linear functions in grade 8

Multiple Representations

# Functions Progression (and more!)

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<http://ime.math.arizona.edu/progressions/>

# Our Goals for Today

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- Use function notation and evaluation of functions in non-traditional ways
- Look for generalizations in transformations of functions
- Use multiple representations to understand families of functions
- Model the Student Mathematical Practices

# Is it Algebra I?

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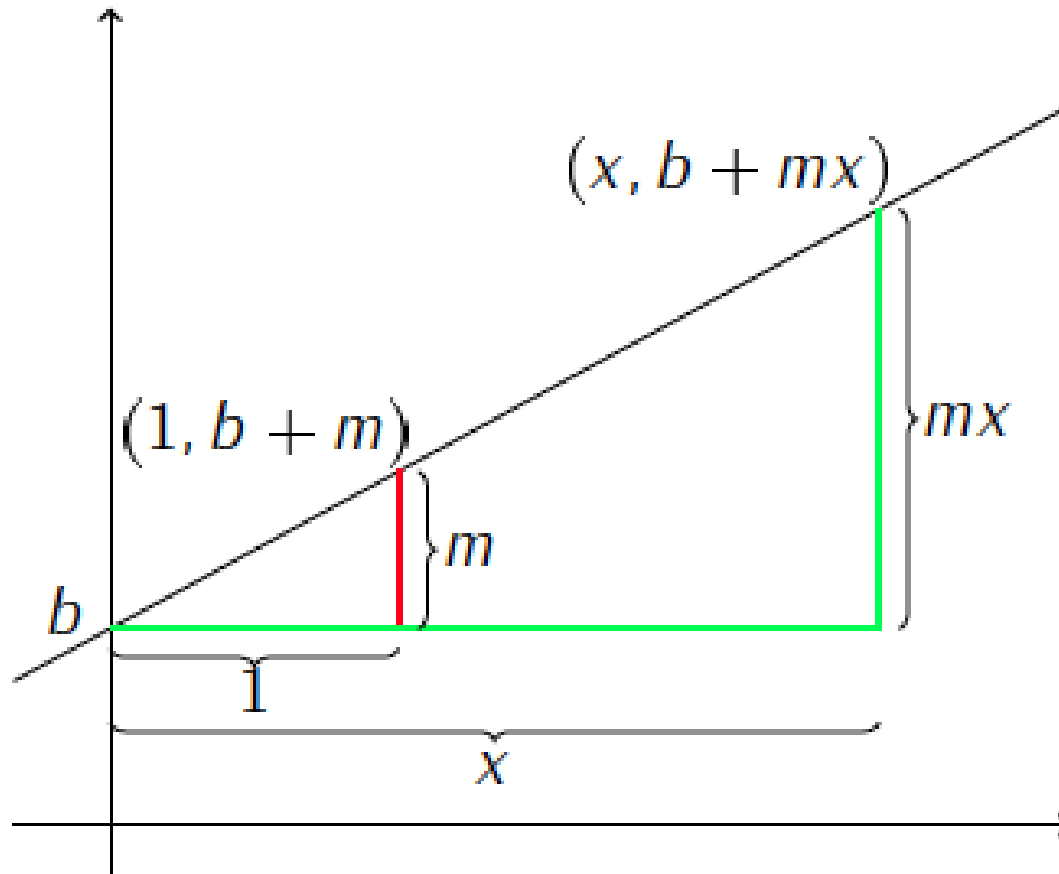
The main focus in Grade 8 is linear functions, those of the form,  $y = mx + b$  where  $m$  and  $b$  are constants.

A linear function is an important piece of reasoning connecting algebra with geometry in Grade 8.

Algebraic thinking outside an Algebra I class.



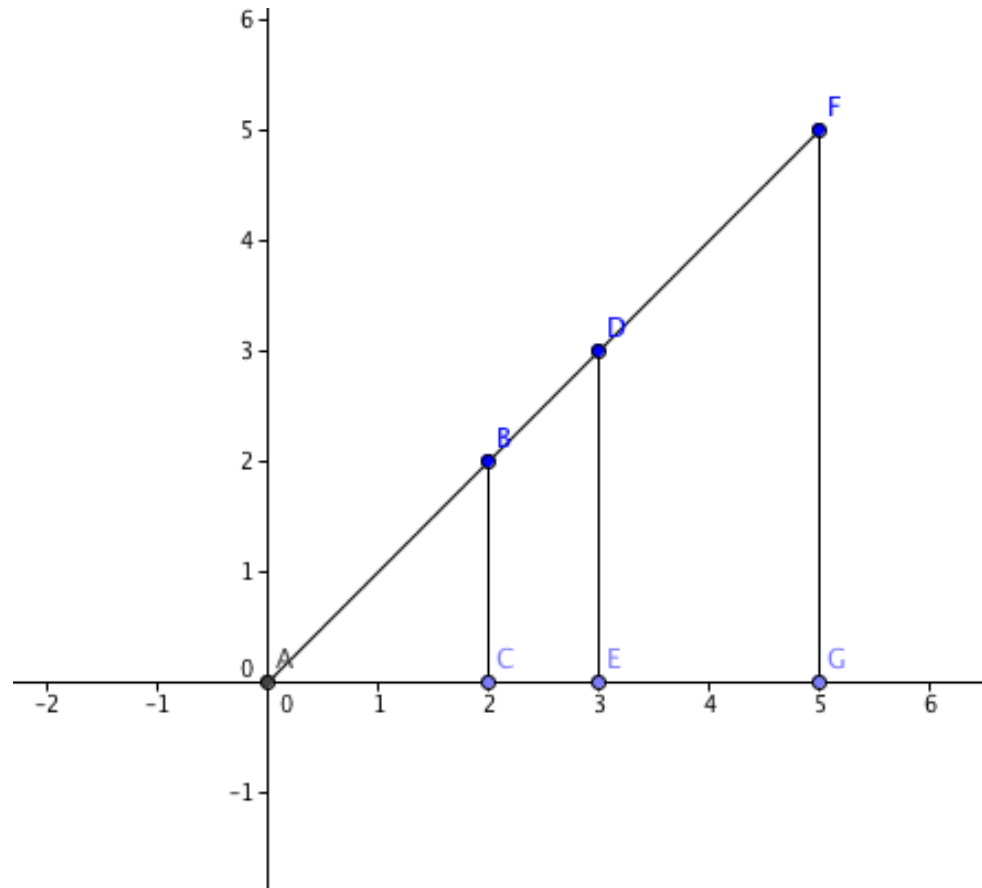
# Linear Functions



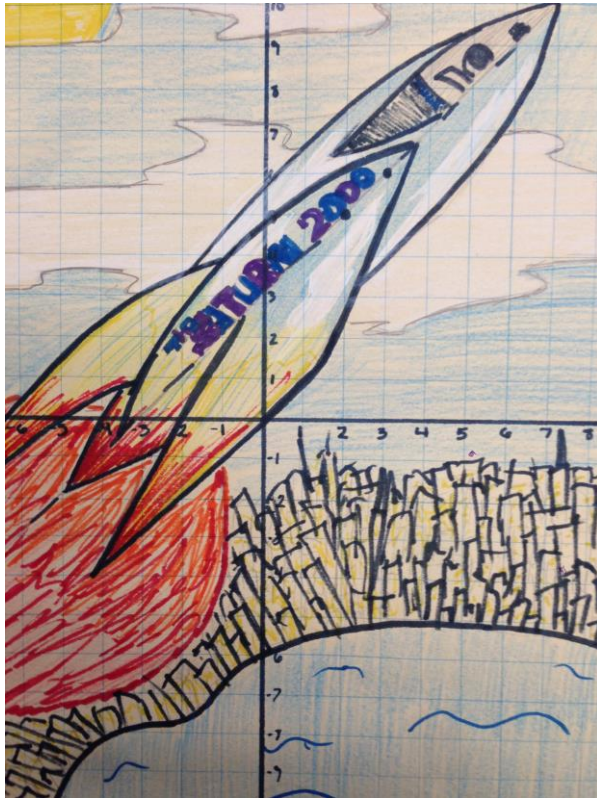
Progressions for  
the Common  
Core State  
Standards in  
Mathematics  
(draft)

# Slope and Linear Functions

- Transformations can help students think about algebraic concepts.
- Here  $\triangle ABC$  is dilated to create  $\triangle ADE$ .
- How can this help students think about slope?
- How can this help students think about collinearity?



# Looking at functions



$$\frac{y_2 - y_1}{x_2 - x_1} \rightarrow \frac{6 - 5}{3 - 2} = \frac{1}{1} = 1$$

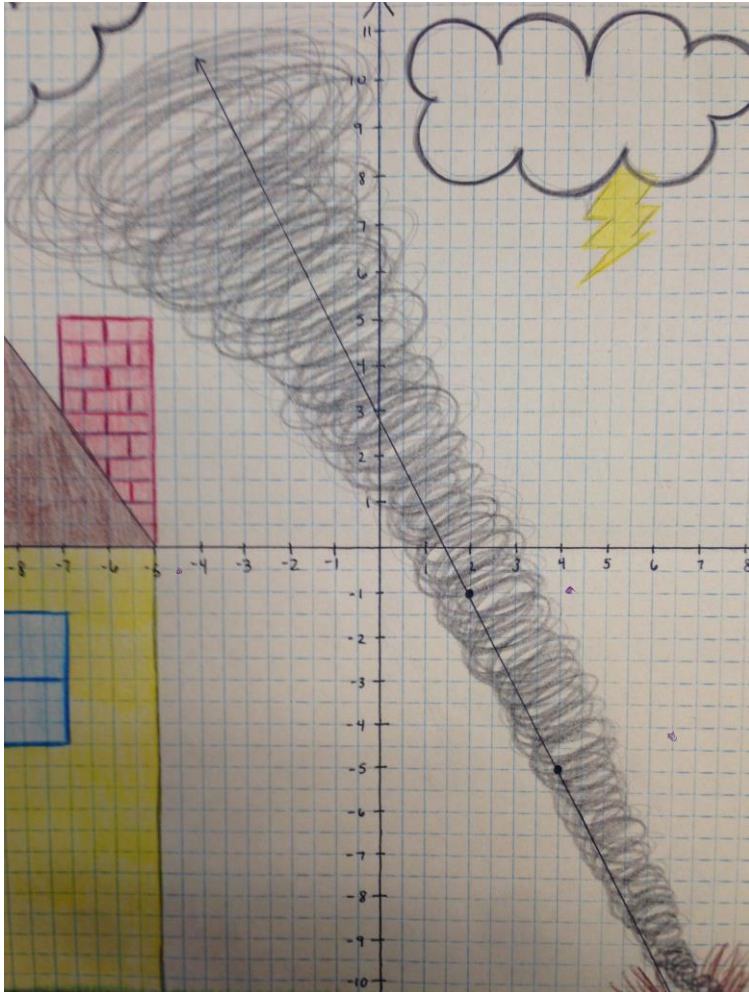
$m = 1$   
 $x = 2$   
 $y = 5$

$$5 = 1(2) + b$$
$$5 = 2 + b$$
$$-2 \quad -2$$
$$\boxed{3 = b}$$

2.  $b = 3$   
3.  $y = 1x + 3$   
4. positive

From the class of Chelsea Matthews, Maple Heights High School, Ohio

# Looking at functions



$(2, -1) (4, -5)$

$$\frac{y_2 - y_1}{x_2 - x_1} = m \quad \frac{-5 - (-1)}{4 - 2} = \frac{-4}{2} = \boxed{-2}$$

$\begin{matrix} x & y \\ (2, -1) & m = -2 \end{matrix}$

$\begin{matrix} m = -2 \\ y = -1 \\ x = 2 \end{matrix}$

$$y = mx + b$$
$$-1 = -2 \cdot 2 + b$$
$$-1 = -4 + b$$
$$+4 \quad +4$$
$$\boxed{3 = b}$$

1.  $m = -2$  (slope)
2.  $b = 3$  (y-intercept)
3.  $y = -2x + 3$   
(Slope Intercept form)
4. negative slope

From the class of Chelsea Matthews, Maple Heights High School, Ohio

# Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# “My Function Project”

You will come up with an original function idea. Remember a function is a rule where **every input value has exactly one output value**. For example the rule  $f(\text{body part}) = \text{clothing}$  won't work because

$$f(\text{foot}) = \text{sock and } f(\text{foot}) = \text{shoe}$$

Function definition	Rule is a function (one output for every input) 2 pts	Rule is not a function and explains why 1 pt	Rule is not a function, no explanation 0 pts
Function notation	Rule uses correct function notation 2 pts	Rule has some function notation 1 pt	Rule does not use function notation 0 pts
Domain and Range	Defines domain and range accurately 2 pts	Either domain or range is defined or accurate 1 pt.	Does not address domain nor range or neither is accurate 0 pts
Examples	Provides at least 5 complete examples 2 pts	Provides fewer than 3 examples 1 pt	Provides no examples 0 pts
Practice Problems	Provides at least 5 practice problems 2 pts	Provides fewer than 3 practice problems 1 pt	Provides no practice problems 0 pts
Presentation	Poster has no errors and is neat, has color 0 pts	Poster is not neat or uses no color -1 pt	Poster is not neat and uses no color -2 pts

# “My Function Project”

Function definition	Rule is a function (one output for every input) 2 pts	Rule is not a function and explains why 1 pt	Rule is not a function, no explanation 0 pts
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# FUNCTION RULE PROJECT

By: Nasyria Taylor

## Rule:

$f(\text{person}) = \text{their birthday}$

• I know this is a function because for every input (person) it can only have 1 output (their birthday).

Try some for your life:

$f(\text{mom}) = \underline{\hspace{2cm}}$

$f(\text{brother}) = \underline{\hspace{2cm}}$

$f(\text{dad}) = \underline{\hspace{2cm}}$

$f(\text{uncle}) = \underline{\hspace{2cm}}$

$f(\text{aunt}) = \underline{\hspace{2cm}}$

$f(\text{cousin}) = \underline{\hspace{2cm}}$

## EVALUATING EXAMPLES:

$f(\text{Tonya}) = \text{October 17}$

$f(\text{Larry}) = \text{March 16}$

$f(\text{Farqu}) = \text{October 22}$

$f(\text{Nasyria}) = \text{December 21}$

$f(\text{Sylvia}) = \text{February 29}$

## SOLVING EXAMPLES:

$f(\overset{\text{person}}{\text{Tonya}}) = \text{October 17, time} = \text{6:43 am}$

$f(\overset{\text{person}}{\text{Larry}}) = \text{March 16, time} = \text{2:18 pm}$  person = Larry

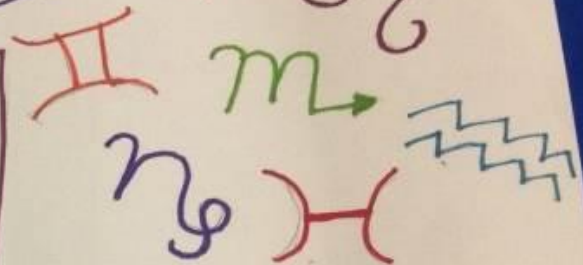
$f(\text{person}) = \text{October 22, person} = \text{Farqu}$

$f(\text{person}) = \text{December 21, person} = \text{Nasyria}$

$f(\text{person}) = \text{February 29, person} = \text{Sylvia}$



# Horoscope Function



## Rule:

$f(\text{Date}) = \text{horoscope Sign}$

I know this is a function because for every date input (DATE) I can only have 1 output (h. Sign).

## Evaluating Examples:

$f(3/30) = \text{Aries}$	$f(10/15) = \text{Libra}$
$f(4/25) = \text{Taurus}$	$f(11/17) = \text{Scorpio}$
$f(6/18) = \text{Gemini}$	$f(12/20) = \text{Sagittarius}$
$f(7/4) = \text{Cancer}$	$f(12/28) = \text{Capricorn}$
$f(8/3) = \text{Leo}$	$f(1/19) = \text{Aquarius}$
$f(9/15) = \text{Virgo}$	$f(2/24) = \text{Pisces}$

## Solving Examples:

$f(\text{date}) = \text{Cancer, Date} = \text{June 21} - \text{July 22}$   
 $f(\text{date}) = \text{Leo, Date} = \text{July 23} - \text{August 22}$   
 $f(\text{date}) = \text{Libra, Date} = \text{Sept 23} - \text{Oct 22}$   
 $f(\text{Date}) = \text{Scorpio, Date} = \text{Oct 23} - \text{NOV 21}$   
 $f(\text{date}) = \text{Capricorn, Date} = \text{Dec 22} - \text{Jan 19}$   
 $f(\text{date}) = \text{Aquarius, Date} = \text{Jan 20} - \text{Feb 18}$

## Practice Problems:

$f(7/6) = \underline{\hspace{2cm}}$

$f(\text{date}) = \text{Capricorn, Date} = \underline{\hspace{2cm}}$

$f(1/20) = \underline{\hspace{2cm}}$

$f(2/26) = \underline{\hspace{2cm}}$

$f(\text{Date}) = \text{Libra, Date} = \underline{\hspace{2cm}}$

$f(\text{Date}) = \text{Aries, Time} = \underline{\hspace{2cm}}$

# PRESIDENTIAL FUNCTION

## Rule:

$$f(\text{term served}) = \text{president}$$

→ I know this is a function because for every input (term served), I can only have one output (president)

### Evaluating Examples:

- $f(1789-1797) = \text{George Washington}$
- $f(1809-1817) = \text{James Madison}$
- $f(1817-1825) = \text{James Monroe}$
- $f(1837-1841) = \text{Martin Van Buren}$
- $f(1853-1857) = \text{Franklin Pierce}$

### Solving Examples:

- $f(\text{term served}) = \text{Abraham Lincoln, term served} = 1861-1865$
- $f(\text{term served}) = \text{Rutherford B. Hayes, term served} = 1877-1881$
- $f(\text{term served}) = \text{Ronald Reagan, term served} = 1981-1989$
- $f(\text{term served}) = \text{Theodore Roosevelt, term served} = 1901-1909$
- $f(\text{term served}) = \text{Richard Nixon, term served} = 1969-1974$

Try Some On Your Own!

$f(2001-2009) =$

$1989-1993 =$

$\text{term served} = \text{William McKinley, term served} =$

$f(\text{term served}) = \text{John Quincy Adams, term served} =$

$f(1929-1933) =$

$f(1797-1801) =$

# “My Function Project”

NCTM Baltimore 2013  
Bring one back to your classroom

$$f(\text{_____}) = \text{_____}$$

$f(\text{“Breaking Bad” Characters}) = \text{Number of lies they tell}$

*Domain: {Walt, Jesse, Skylar, Hank, Marie, Walt Jr. Saul, Fring, Mike...}*

*Range: {  $y \geq 0$ }*

$f(\text{Walt Jr.}) = 0,$

$f(\text{Walt}) = \text{_____}$



Is this a function?

●●○○ AT&T 8:36 PM 93%

Back **NEW** Scramble friends™ with

### Select a Token Package

	Small <b>16</b>		<b>\$0.99</b>
	Medium <b>36</b>	GET <b>12%</b> MORE VALUE	<b>\$1.99</b>
	Large <b>95</b>	GET <b>18%</b> MORE VALUE	<b>\$4.99</b>
	X-Large <b>200</b>	GET <b>25%</b> MORE VALUE	<b>\$9.99</b>
	Deluxe <b>550</b>	GET <b>37%</b> MORE VALUE	<b>\$24.99</b>

In what other ways can this function be written?



Is this function linear?

The screenshot shows the 'NEW Scramble Friends with' app interface. At the top, there is a 'Back' button and the app title. Below the title is a section titled 'Select a Token Package' with a gold coin icon. There are five package options listed in a vertical stack, each with an icon of gold coins, a label, a quantity, a price, and a discount badge. The packages are: Small (16 tokens, \$0.99), Medium (36 tokens, \$1.99, 12% more value), Large (95 tokens, \$4.99, 18% more value), X-Large (200 tokens, \$9.99, 25% more value), and Deluxe (550 tokens, \$24.99, 37% more value).

Package	Quantity	Price	Discount
Small	16	\$0.99	
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Large	95	\$4.99	18% MORE VALUE
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Deluxe	550	\$24.99	37% MORE VALUE

# “Transformations Gallery Walk”

**Transformations Group Work A:**

$$f(x + 2)$$

**Transformations Group Work B:**

$$f(x - 2)$$

**Transformations Group Work C:**

$$f(x) + 2$$

**Transformations Group Work D:**

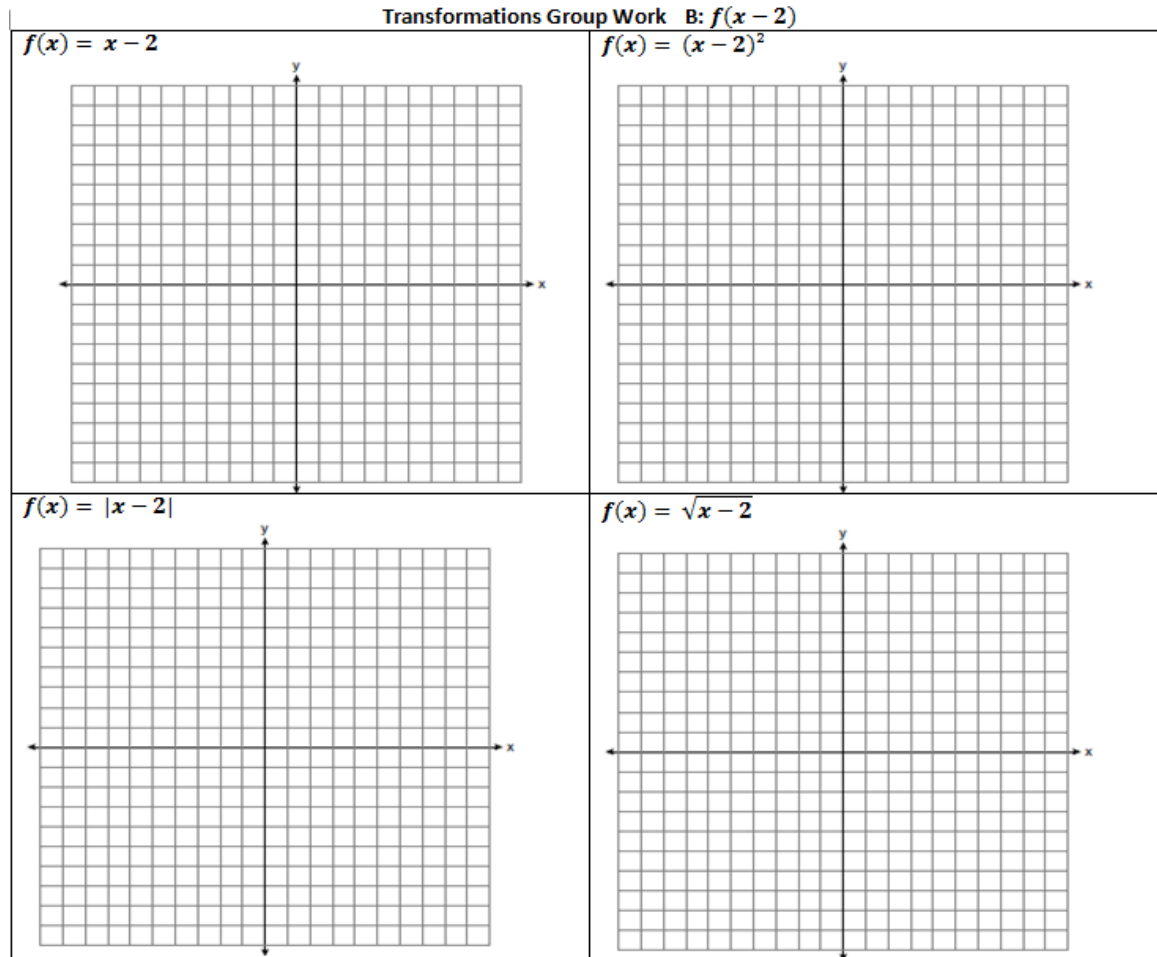
$$f(x) - 2$$

**Transformations Group Work E:**

$$-f(x)$$

**Transformations Group Work F:**

$$f(-x)$$



# “Transformations Gallery Walk”

## NOTES FROM THE GALLERY WALK

### GROUP A

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Effect on function: \_\_\_\_\_

### GROUP B

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Equation of function: \_\_\_\_\_

Effect on function: \_\_\_\_\_



are que tu as?  
.....

# FAMILIES OF FUNCTIONS

By Solly, James, Ariani, Hafsan

## RADICAL

Domain:  $[0, \infty)$   
Range:  $[-1, \infty)$   
Shift: Reflection over  $x=0$

## QUADRATIC

Domain:  $(-\infty, \infty)$   
Range:  $[-1, \infty)$   
Shift: Reflection over  $x=0$

## Cubic

Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$   
Shift: Reflection over  $x=0$

## Cubic

Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$   
Shift: Reflection over  $x=0$

## Absolute Value

Domain:  $(-\infty, \infty)$   
Range:  $[-1, \infty)$   
Shift: Reflection over  $x=0$

## Linear

Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$   
Shift: Reflection over  $x=0$

# Types of Functions

By: Crystal, Miguel and Ethan

## Polynomial:



Domain:  $(-\infty, \infty)$   
Range:  $[2, \infty)$   
Symmetric: None

## Linear:



Domain:  $(-\infty, \infty)$   
Range:  $(-\infty, \infty)$   
Shift:  $R^\circ 180$

## Quadratic:



Domain:  $(-\infty, \infty)$   
Range:  $[2, \infty)$   
Shift:  $R^\circ 180$

Analysis: The functions we graphed can just be the  $x$  axis, they just went up or down. The  $y$  doesn't. For  $x$  axis it went up & down.

## Rational:



Domain:  $(0, \infty)$   
Range:  $[2, \infty)$   
Shift:  $R^\circ 180$

## Cube:

## Absolute Value:



Domain:  $(-\infty, \infty)$   
Range:  $[2, \infty)$   
Shift:  $R^\circ 180$  y-axis

# Families of Functions

By: Jonah, Deshae, Jenny, Liliana

Pattern: They are all shifted to the left by 2.

## Linear



Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, \infty)$   
 Shift:  $\uparrow$  2



Domain:  $(-\infty, -2) \cup (-2, \infty)$   $x \neq -2$   
 Range:  $(-\infty, 2) \cup (2, \infty)$   $y \neq 2$   
 Shift:  $\uparrow$  2

## Radical



Domain:  $[0, \infty)$   
 Range:  $[0, \infty)$   
 Shift:  $\uparrow$  2

## Cubic



Domain:  $(-\infty, \infty)$   
 Range:  $(-\infty, \infty)$   
 Shift:  $\uparrow$  2

## Absolute Value



Domain:  $(-\infty, \infty)$   
 Range:  $[0, \infty)$   
 Shift:  $\uparrow$  2

## Quadratic



Domain:  $(-\infty, \infty)$   
 Range:  $[0, \infty)$   
 Shift:  $\uparrow$  2

Cubic 3



Domain:  $x \in \mathbb{R}$   
Range:  $y \in \mathbb{R}$   
Shift:  $(0, 2)$

Rational



Function	Domain	Range	Asymptotes	Graph
$f(x) = x^3$	$\mathbb{R}$	$\mathbb{R}$	None	
$f(x) = x^2$	$\mathbb{R}$	$[0, \infty)$	$x = 0$	
$f(x) = \frac{1}{x}$	$\mathbb{R} \setminus \{0\}$	$\mathbb{R} \setminus \{0\}$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2}$	$\mathbb{R} \setminus \{0\}$	$(0, \infty)$	$x = 0, y = 0$	
$f(x) = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$	$x = 0$	
$f(x) = \sqrt[3]{x}$	$\mathbb{R}$	$\mathbb{R}$	None	
$f(x) = \frac{1}{x-1}$	$\mathbb{R} \setminus \{1\}$	$\mathbb{R} \setminus \{0\}$	$x = 1, y = 0$	
$f(x) = \frac{1}{x+2}$	$\mathbb{R} \setminus \{-2\}$	$\mathbb{R} \setminus \{0\}$	$x = -2, y = 0$	
$f(x) = \frac{1}{x^2+1}$	$\mathbb{R}$	$(0, 1]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1}$	$\mathbb{R} \setminus \{-1, 1\}$	$\mathbb{R} \setminus \{0\}$	$x = -1, x = 1, y = 0$	
$f(x) = \frac{1}{x^2+4}$	$\mathbb{R}$	$(0, 1/4]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-4}$	$\mathbb{R} \setminus \{-2, 2\}$	$\mathbb{R} \setminus \{0\}$	$x = -2, x = 2, y = 0$	
$f(x) = \frac{1}{x^2+9}$	$\mathbb{R}$	$(0, 1/9]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-9}$	$\mathbb{R} \setminus \{-3, 3\}$	$\mathbb{R} \setminus \{0\}$	$x = -3, x = 3, y = 0$	
$f(x) = \frac{1}{x^2+16}$	$\mathbb{R}$	$(0, 1/16]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-16}$	$\mathbb{R} \setminus \{-4, 4\}$	$\mathbb{R} \setminus \{0\}$	$x = -4, x = 4, y = 0$	
$f(x) = \frac{1}{x^2+25}$	$\mathbb{R}$	$(0, 1/25]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-25}$	$\mathbb{R} \setminus \{-5, 5\}$	$\mathbb{R} \setminus \{0\}$	$x = -5, x = 5, y = 0$	
$f(x) = \frac{1}{x^2+36}$	$\mathbb{R}$	$(0, 1/36]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-36}$	$\mathbb{R} \setminus \{-6, 6\}$	$\mathbb{R} \setminus \{0\}$	$x = -6, x = 6, y = 0$	
$f(x) = \frac{1}{x^2+49}$	$\mathbb{R}$	$(0, 1/49]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-49}$	$\mathbb{R} \setminus \{-7, 7\}$	$\mathbb{R} \setminus \{0\}$	$x = -7, x = 7, y = 0$	
$f(x) = \frac{1}{x^2+64}$	$\mathbb{R}$	$(0, 1/64]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-64}$	$\mathbb{R} \setminus \{-8, 8\}$	$\mathbb{R} \setminus \{0\}$	$x = -8, x = 8, y = 0$	
$f(x) = \frac{1}{x^2+81}$	$\mathbb{R}$	$(0, 1/81]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-81}$	$\mathbb{R} \setminus \{-9, 9\}$	$\mathbb{R} \setminus \{0\}$	$x = -9, x = 9, y = 0$	
$f(x) = \frac{1}{x^2+100}$	$\mathbb{R}$	$(0, 1/100]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-100}$	$\mathbb{R} \setminus \{-10, 10\}$	$\mathbb{R} \setminus \{0\}$	$x = -10, x = 10, y = 0$	
$f(x) = \frac{1}{x^2+121}$	$\mathbb{R}$	$(0, 1/121]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-121}$	$\mathbb{R} \setminus \{-11, 11\}$	$\mathbb{R} \setminus \{0\}$	$x = -11, x = 11, y = 0$	
$f(x) = \frac{1}{x^2+144}$	$\mathbb{R}$	$(0, 1/144]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-144}$	$\mathbb{R} \setminus \{-12, 12\}$	$\mathbb{R} \setminus \{0\}$	$x = -12, x = 12, y = 0$	
$f(x) = \frac{1}{x^2+169}$	$\mathbb{R}$	$(0, 1/169]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-169}$	$\mathbb{R} \setminus \{-13, 13\}$	$\mathbb{R} \setminus \{0\}$	$x = -13, x = 13, y = 0$	
$f(x) = \frac{1}{x^2+196}$	$\mathbb{R}$	$(0, 1/196]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-196}$	$\mathbb{R} \setminus \{-14, 14\}$	$\mathbb{R} \setminus \{0\}$	$x = -14, x = 14, y = 0$	
$f(x) = \frac{1}{x^2+225}$	$\mathbb{R}$	$(0, 1/225]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-225}$	$\mathbb{R} \setminus \{-15, 15\}$	$\mathbb{R} \setminus \{0\}$	$x = -15, x = 15, y = 0$	
$f(x) = \frac{1}{x^2+256}$	$\mathbb{R}$	$(0, 1/256]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-256}$	$\mathbb{R} \setminus \{-16, 16\}$	$\mathbb{R} \setminus \{0\}$	$x = -16, x = 16, y = 0$	
$f(x) = \frac{1}{x^2+289}$	$\mathbb{R}$	$(0, 1/289]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-289}$	$\mathbb{R} \setminus \{-17, 17\}$	$\mathbb{R} \setminus \{0\}$	$x = -17, x = 17, y = 0$	
$f(x) = \frac{1}{x^2+324}$	$\mathbb{R}$	$(0, 1/324]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-324}$	$\mathbb{R} \setminus \{-18, 18\}$	$\mathbb{R} \setminus \{0\}$	$x = -18, x = 18, y = 0$	
$f(x) = \frac{1}{x^2+361}$	$\mathbb{R}$	$(0, 1/361]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-361}$	$\mathbb{R} \setminus \{-19, 19\}$	$\mathbb{R} \setminus \{0\}$	$x = -19, x = 19, y = 0$	
$f(x) = \frac{1}{x^2+400}$	$\mathbb{R}$	$(0, 1/400]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-400}$	$\mathbb{R} \setminus \{-20, 20\}$	$\mathbb{R} \setminus \{0\}$	$x = -20, x = 20, y = 0$	
$f(x) = \frac{1}{x^2+441}$	$\mathbb{R}$	$(0, 1/441]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-441}$	$\mathbb{R} \setminus \{-21, 21\}$	$\mathbb{R} \setminus \{0\}$	$x = -21, x = 21, y = 0$	
$f(x) = \frac{1}{x^2+484}$	$\mathbb{R}$	$(0, 1/484]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-484}$	$\mathbb{R} \setminus \{-22, 22\}$	$\mathbb{R} \setminus \{0\}$	$x = -22, x = 22, y = 0$	
$f(x) = \frac{1}{x^2+529}$	$\mathbb{R}$	$(0, 1/529]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-529}$	$\mathbb{R} \setminus \{-23, 23\}$	$\mathbb{R} \setminus \{0\}$	$x = -23, x = 23, y = 0$	
$f(x) = \frac{1}{x^2+576}$	$\mathbb{R}$	$(0, 1/576]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-576}$	$\mathbb{R} \setminus \{-24, 24\}$	$\mathbb{R} \setminus \{0\}$	$x = -24, x = 24, y = 0$	
$f(x) = \frac{1}{x^2+625}$	$\mathbb{R}$	$(0, 1/625]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-625}$	$\mathbb{R} \setminus \{-25, 25\}$	$\mathbb{R} \setminus \{0\}$	$x = -25, x = 25, y = 0$	
$f(x) = \frac{1}{x^2+676}$	$\mathbb{R}$	$(0, 1/676]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-676}$	$\mathbb{R} \setminus \{-26, 26\}$	$\mathbb{R} \setminus \{0\}$	$x = -26, x = 26, y = 0$	
$f(x) = \frac{1}{x^2+729}$	$\mathbb{R}$	$(0, 1/729]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-729}$	$\mathbb{R} \setminus \{-27, 27\}$	$\mathbb{R} \setminus \{0\}$	$x = -27, x = 27, y = 0$	
$f(x) = \frac{1}{x^2+784}$	$\mathbb{R}$	$(0, 1/784]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-784}$	$\mathbb{R} \setminus \{-28, 28\}$	$\mathbb{R} \setminus \{0\}$	$x = -28, x = 28, y = 0$	
$f(x) = \frac{1}{x^2+841}$	$\mathbb{R}$	$(0, 1/841]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-841}$	$\mathbb{R} \setminus \{-29, 29\}$	$\mathbb{R} \setminus \{0\}$	$x = -29, x = 29, y = 0$	
$f(x) = \frac{1}{x^2+900}$	$\mathbb{R}$	$(0, 1/900]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-900}$	$\mathbb{R} \setminus \{-30, 30\}$	$\mathbb{R} \setminus \{0\}$	$x = -30, x = 30, y = 0$	
$f(x) = \frac{1}{x^2+961}$	$\mathbb{R}$	$(0, 1/961]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-961}$	$\mathbb{R} \setminus \{-31, 31\}$	$\mathbb{R} \setminus \{0\}$	$x = -31, x = 31, y = 0$	
$f(x) = \frac{1}{x^2+1024}$	$\mathbb{R}$	$(0, 1/1024]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1024}$	$\mathbb{R} \setminus \{-32, 32\}$	$\mathbb{R} \setminus \{0\}$	$x = -32, x = 32, y = 0$	
$f(x) = \frac{1}{x^2+1089}$	$\mathbb{R}$	$(0, 1/1089]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1089}$	$\mathbb{R} \setminus \{-33, 33\}$	$\mathbb{R} \setminus \{0\}$	$x = -33, x = 33, y = 0$	
$f(x) = \frac{1}{x^2+1156}$	$\mathbb{R}$	$(0, 1/1156]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1156}$	$\mathbb{R} \setminus \{-34, 34\}$	$\mathbb{R} \setminus \{0\}$	$x = -34, x = 34, y = 0$	
$f(x) = \frac{1}{x^2+1225}$	$\mathbb{R}$	$(0, 1/1225]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1225}$	$\mathbb{R} \setminus \{-35, 35\}$	$\mathbb{R} \setminus \{0\}$	$x = -35, x = 35, y = 0$	
$f(x) = \frac{1}{x^2+1296}$	$\mathbb{R}$	$(0, 1/1296]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1296}$	$\mathbb{R} \setminus \{-36, 36\}$	$\mathbb{R} \setminus \{0\}$	$x = -36, x = 36, y = 0$	
$f(x) = \frac{1}{x^2+1369}$	$\mathbb{R}$	$(0, 1/1369]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1369}$	$\mathbb{R} \setminus \{-37, 37\}$	$\mathbb{R} \setminus \{0\}$	$x = -37, x = 37, y = 0$	
$f(x) = \frac{1}{x^2+1444}$	$\mathbb{R}$	$(0, 1/1444]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1444}$	$\mathbb{R} \setminus \{-38, 38\}$	$\mathbb{R} \setminus \{0\}$	$x = -38, x = 38, y = 0$	
$f(x) = \frac{1}{x^2+1521}$	$\mathbb{R}$	$(0, 1/1521]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1521}$	$\mathbb{R} \setminus \{-39, 39\}$	$\mathbb{R} \setminus \{0\}$	$x = -39, x = 39, y = 0$	
$f(x) = \frac{1}{x^2+1600}$	$\mathbb{R}$	$(0, 1/1600]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1600}$	$\mathbb{R} \setminus \{-40, 40\}$	$\mathbb{R} \setminus \{0\}$	$x = -40, x = 40, y = 0$	
$f(x) = \frac{1}{x^2+1681}$	$\mathbb{R}$	$(0, 1/1681]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1681}$	$\mathbb{R} \setminus \{-41, 41\}$	$\mathbb{R} \setminus \{0\}$	$x = -41, x = 41, y = 0$	
$f(x) = \frac{1}{x^2+1764}$	$\mathbb{R}$	$(0, 1/1764]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1764}$	$\mathbb{R} \setminus \{-42, 42\}$	$\mathbb{R} \setminus \{0\}$	$x = -42, x = 42, y = 0$	
$f(x) = \frac{1}{x^2+1849}$	$\mathbb{R}$	$(0, 1/1849]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1849}$	$\mathbb{R} \setminus \{-43, 43\}$	$\mathbb{R} \setminus \{0\}$	$x = -43, x = 43, y = 0$	
$f(x) = \frac{1}{x^2+1936}$	$\mathbb{R}$	$(0, 1/1936]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-1936}$	$\mathbb{R} \setminus \{-44, 44\}$	$\mathbb{R} \setminus \{0\}$	$x = -44, x = 44, y = 0$	
$f(x) = \frac{1}{x^2+2025}$	$\mathbb{R}$	$(0, 1/2025]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2025}$	$\mathbb{R} \setminus \{-45, 45\}$	$\mathbb{R} \setminus \{0\}$	$x = -45, x = 45, y = 0$	
$f(x) = \frac{1}{x^2+2116}$	$\mathbb{R}$	$(0, 1/2116]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2116}$	$\mathbb{R} \setminus \{-46, 46\}$	$\mathbb{R} \setminus \{0\}$	$x = -46, x = 46, y = 0$	
$f(x) = \frac{1}{x^2+2209}$	$\mathbb{R}$	$(0, 1/2209]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2209}$	$\mathbb{R} \setminus \{-47, 47\}$	$\mathbb{R} \setminus \{0\}$	$x = -47, x = 47, y = 0$	
$f(x) = \frac{1}{x^2+2304}$	$\mathbb{R}$	$(0, 1/2304]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2304}$	$\mathbb{R} \setminus \{-48, 48\}$	$\mathbb{R} \setminus \{0\}$	$x = -48, x = 48, y = 0$	
$f(x) = \frac{1}{x^2+2401}$	$\mathbb{R}$	$(0, 1/2401]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2401}$	$\mathbb{R} \setminus \{-49, 49\}$	$\mathbb{R} \setminus \{0\}$	$x = -49, x = 49, y = 0$	
$f(x) = \frac{1}{x^2+2500}$	$\mathbb{R}$	$(0, 1/2500]$	$x = 0, y = 0$	
$f(x) = \frac{1}{x^2-2500}$	$\mathbb{R} \setminus \{-50, 50\}$	$\mathbb{R} \setminus \{0\}$	$x = -50, x = 50, y = 0$	

# Function “YMCA”

Absolute Value

Quadratic

Cubic

Rational

Square Root



# Thinking about Composition

(illustrativemathematics.com)

Suppose the swine flu, influenza H1N1, is spreading on a school campus. The following table shows the number of students,  $n$ , that have the flu  $d$  days after the initial outbreak. The number of students who have the flu is a function of the number of days,  $n = f(d)$ .

$d$ (days)	0	2	6	8	12	16	24
$n = f(d)$ (number of students infected)	3	9	16	30	55	45	32

There is a school store on campus. As the number of students who have the flu increases, the number of tissue boxes,  $b$ , sold at the school store also increases. The number of tissue boxes sold on a given day is a function of the number of students who have the flu,  $b = g(n)$ , on that day.

$n$ (number of students infected)	0	3	8	9	12	16	18	30	32	38	45	50	55
$b = g(n)$ (number of tissue boxes sold)	1	4	8	12	13	18	24	33	34	40	45	51	57

- Find  $g(f(0))$  and state the meaning of this value in the context of the flu epidemic. Include units in your answer.
- Fill in the chart below using the fact that  $b = g(f(d))$ .

$d$ (days)	0	2	6	8	12	16	24
$b$ (number of tissue boxes sold)							

- For each of the following expressions, explain its meaning in the context of the problem, and if possible, give an approximation of its value. Justify your answer.
  - $g(f(16))$
  - $g(f(18))$
  - $f(g(9))$

# Thinking about Composition

(illustrativemathematics.com)

3.) Suppose the swine flu, influenza H1N1, is spreading on a school campus. The following table shows the number of students,  $n$ , that have the flu  $d$  days after the initial outbreak. The number of students who have the flu is a function of the number of days,  $n=f(d)$ .

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$n$ (number of students infected)	0	3	8	9	12	16	18	30	32	38	45	50	55
$b=g(n)$ (number of tissue boxes sold)	1	4	8	12	13	18	24	33	34	40	45	51	57

Find  $g(f(0))$  and state the meaning of this value in the context of the flu epidemic. Include units in your answer.

If nobody is infected only 1 tissue box is sold because of the normal state like crying and sneezing nose

# Thinking about Composition

(illustrativemathematics.com)

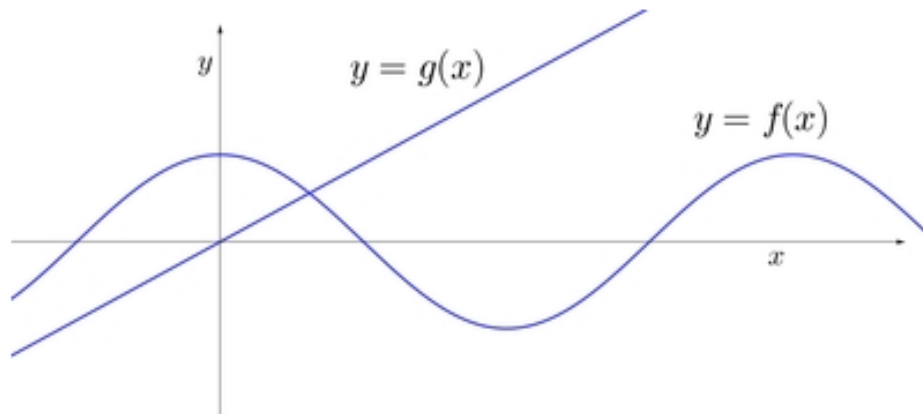
Fill in the chart below using the fact that  $b = g(f(d))$ .

$d$ (days)	0	2	6	8	12	16	24
$b$ (number of tissue boxes sold)	4	12	18	53	57	15	34



# Thinking about Functions

Use the graph (for example, by marking specific points) to illustrate the statements in (a)–(d). If possible, label the coordinates of any points you draw.



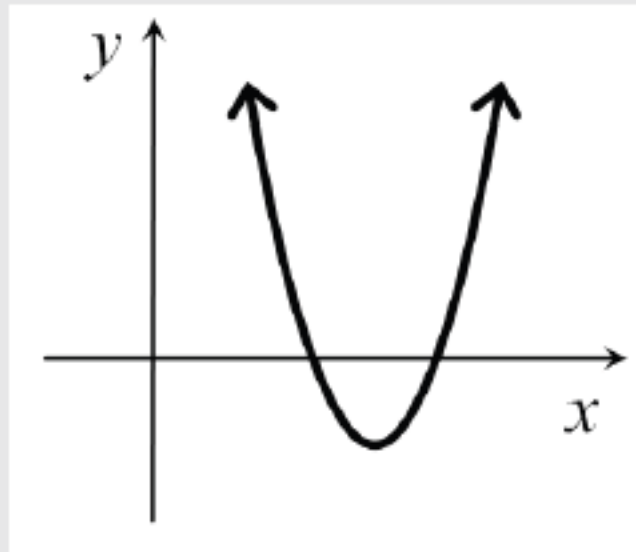
- $f(0) = 2$
- $f(-3) = f(3) = f(9) = 0$
- $f(2) = g(2)$
- $g(x) > f(x)$  for  $x > 2$

# Looking at Functions

## Which Equation?

Which of the following could be an expression for the function whose graph is shown below? Explain.

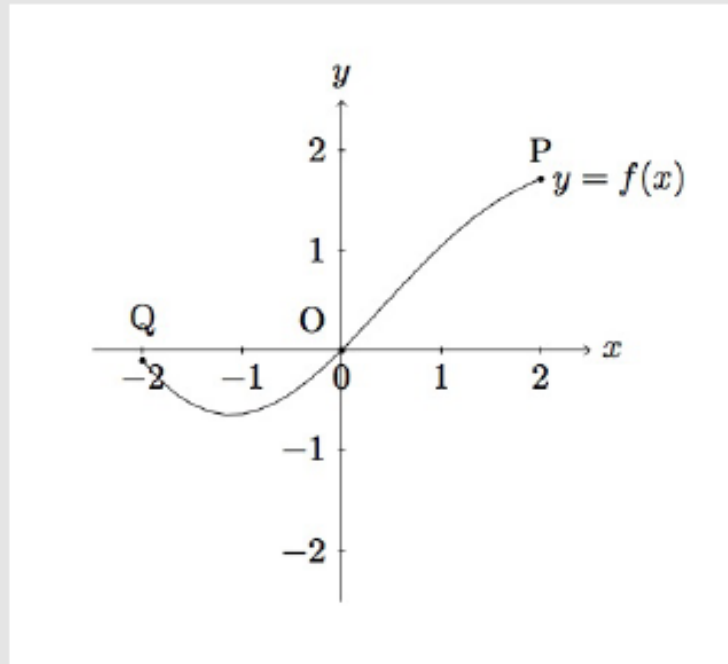
- |                         |                        |
|-------------------------|------------------------|
| (a) $(x + 12)^2 + 4$    | (b) $-(x - 2)^2 - 1$   |
| (c) $(x + 18)^2 - 40$   | (d) $(x - 10)^2 - 15$  |
| (e) $-4(x + 2)(x + 3)$  | (f) $(x + 4)(x - 6)$   |
| (g) $(x - 12)(-x + 18)$ | (h) $(20 - x)(30 - x)$ |



Progressions for  
the Common  
Core State  
Standards in  
Mathematics  
(draft)

## Transforming Functions

The figure shows the graph of a function  $f$  whose domain is the interval  $-2 \leq x \leq 2$ .



(a) In (i)–(iii), sketch the graph of the given function and compare with the graph of  $f$ . Explain what you see.

(i)  $g(x) = f(x) + 2$

(ii)  $h(x) = -f(x)$

(iii)  $p(x) = f(x + 2)$

Progressions  
for the  
Common Core  
State  
Standards in  
Mathematics  
(draft)

# Creating Functions

A square is built with the following pattern:

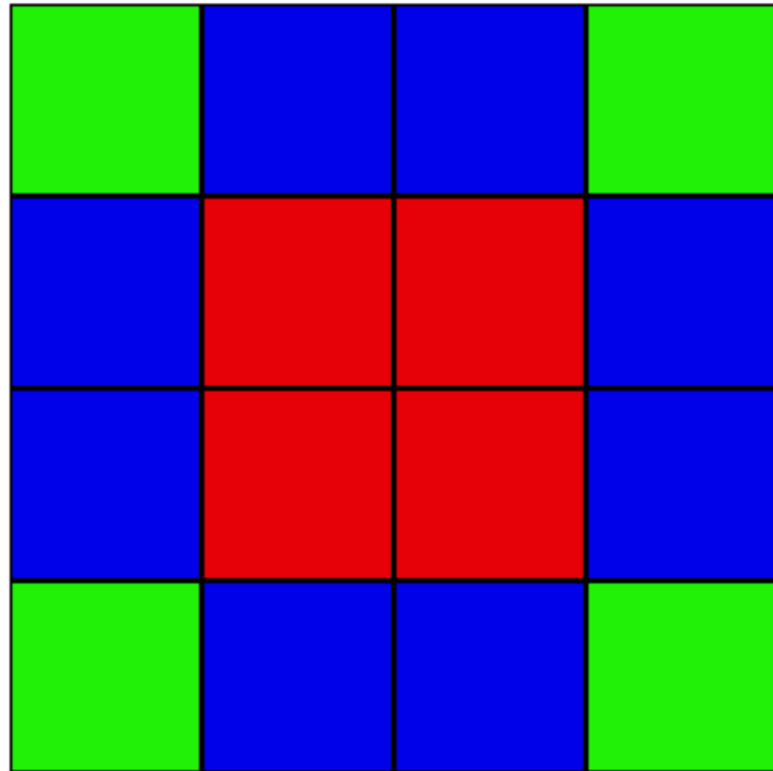
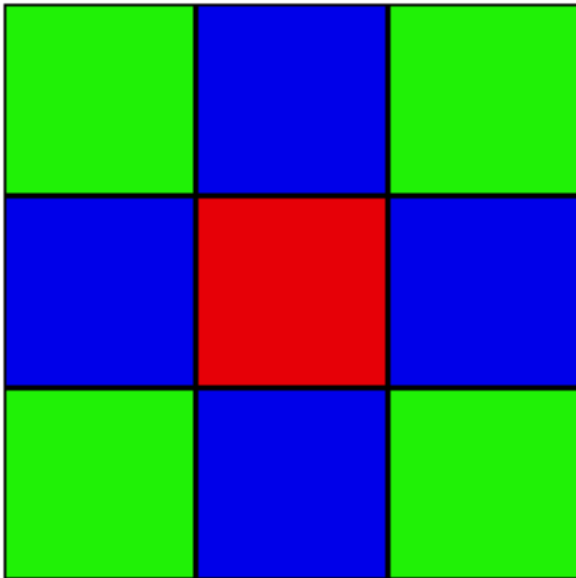
G	B	G
B	R	B
G	B	G

G = Green color for all corners

B = Blue color for perimeter squares that are not corners

R = Red color for all squares that are not on the perimeter.

# Coloring Squares



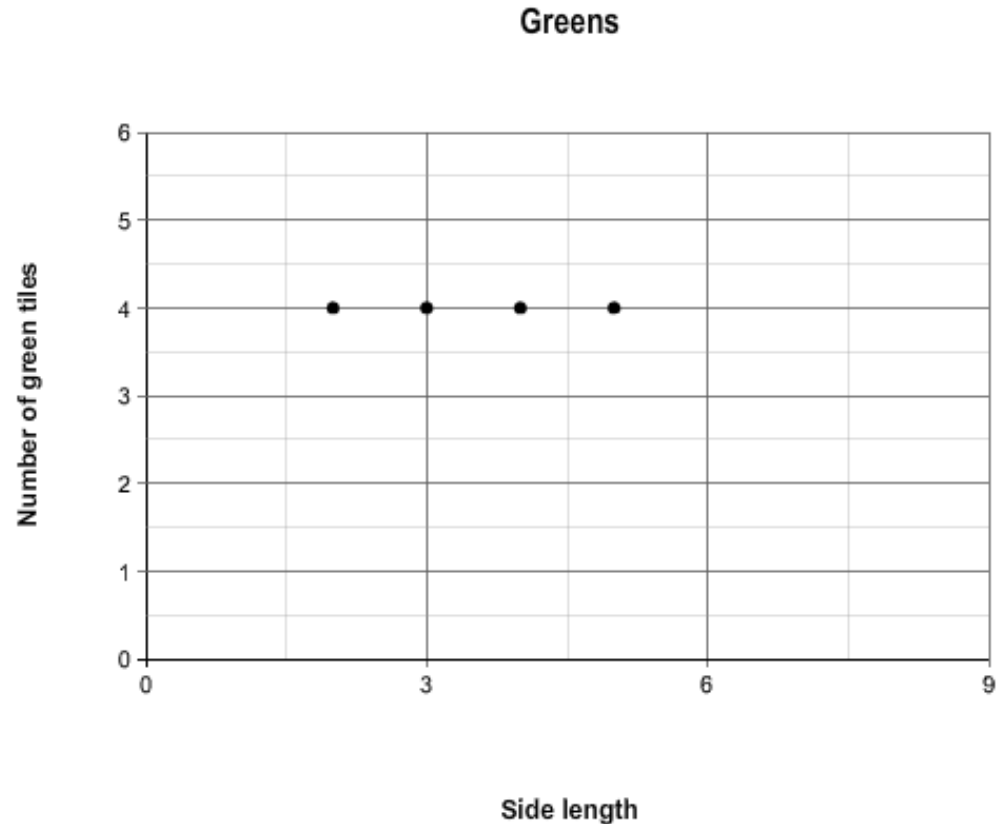
# Coloring Squares

Shape	#G	#B	#R
N=3	4	4	1
4			
5			
6			
7			
8			

As a small group, complete this table for other squares.

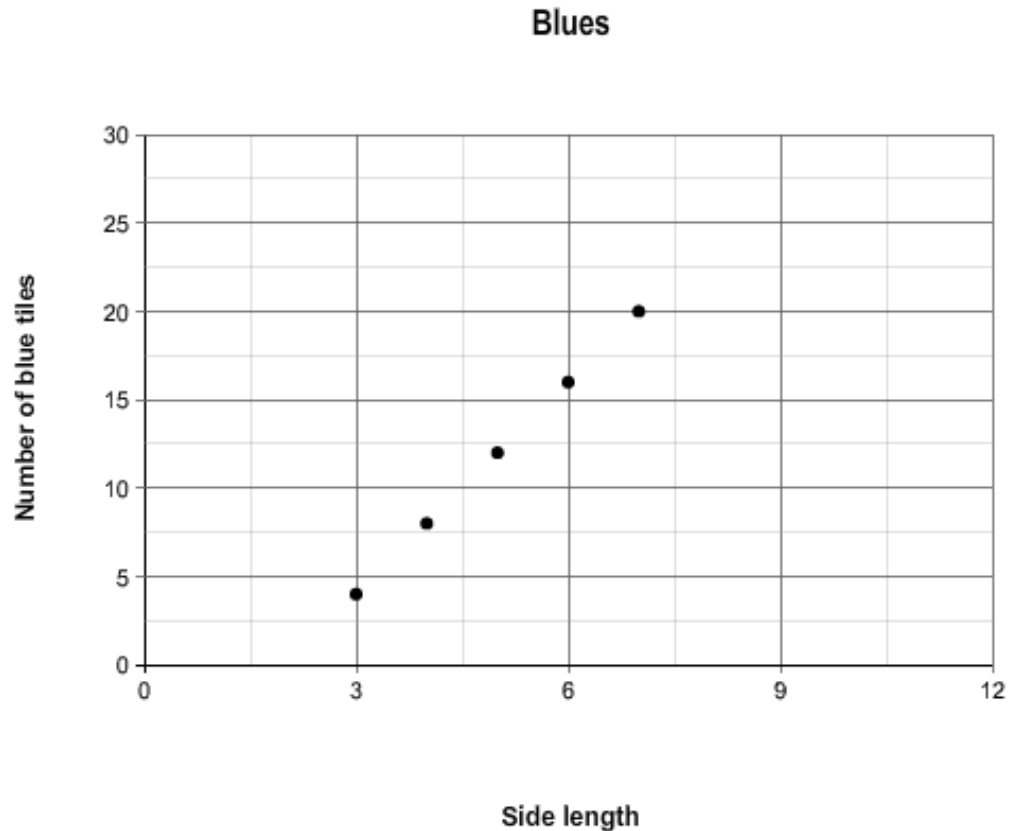
# Coloring Squares

Describe  
the graph  
of the  
Green  
squares...



# Coloring Squares

Describe  
the graph  
of the blue  
squares.







New



Box Plotter



Bubble



Scatterplot



Histogram



Stem-Leaf



Preview

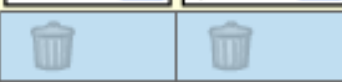
Graph Title: Reds

X Axis Label: Side length

Y Axis Label: Number of red tiles



x y



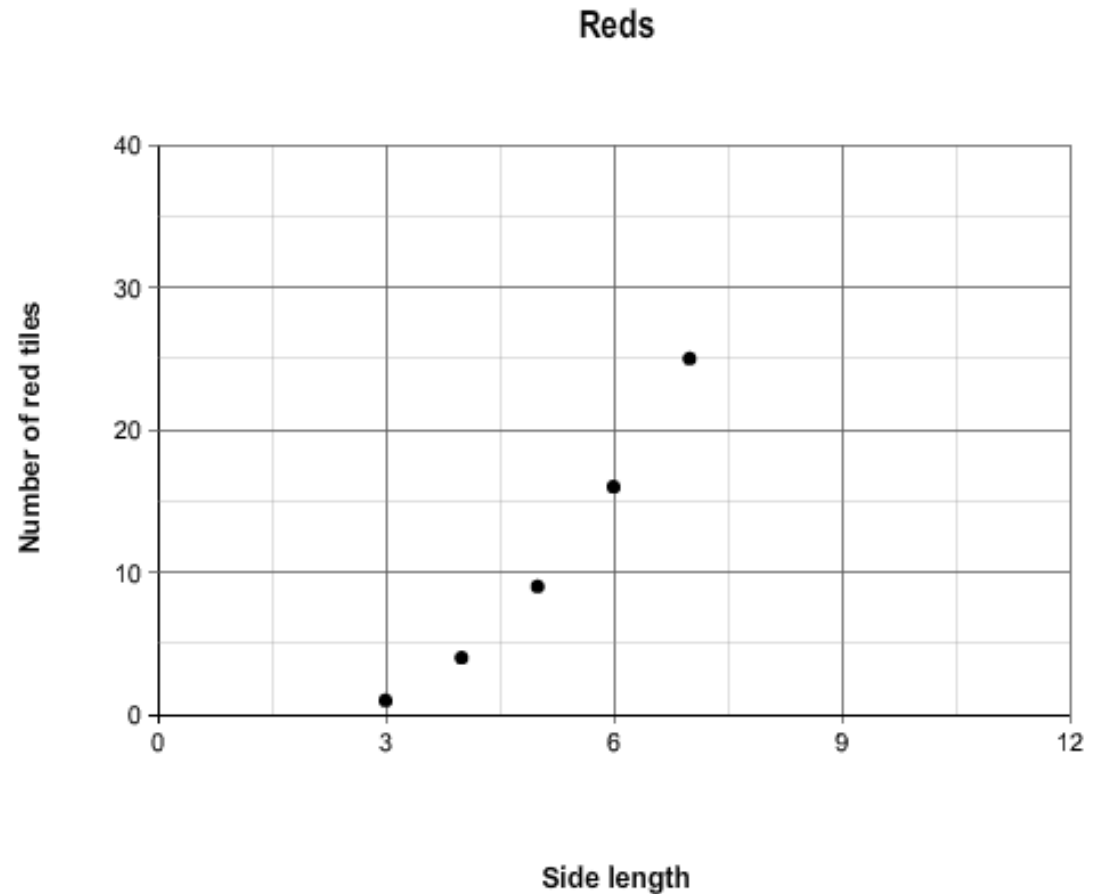
Column 1 Column 2

	Row 1	3	1
	Row 2	4	4
	Row 3	5	9
	Row 4	6	16
	Row 5	7	25

Source:

# Coloring Squares

Describe the graph of the red squares.

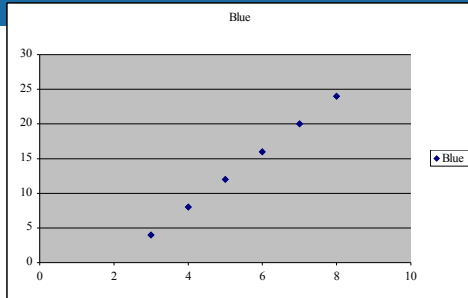


# Coloring Squares

Shape	#G	#B	#R
N=3	4	4	1
4	4	8	4
5	4	12	9
6	4	16	16
7	4	20	25
8	4	24	49
N			

Investigate the table with finite differences or a graph. Look at the rates of change!

# Coloring Squares

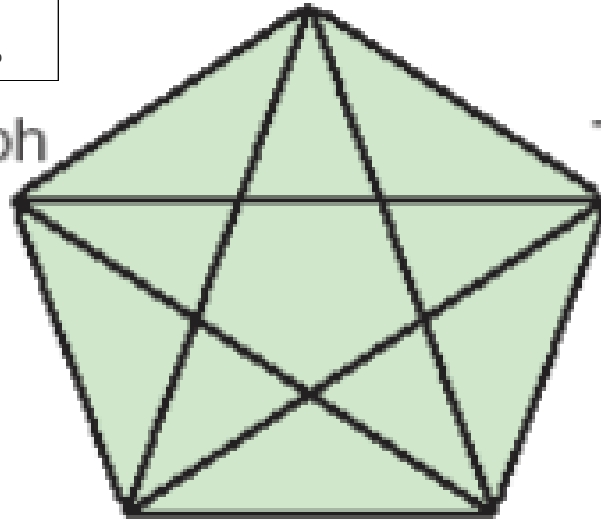


Graph

Algebraic  
formula

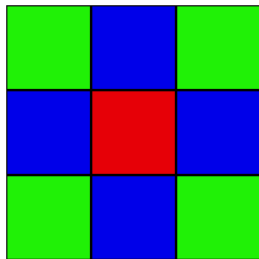
$$G = 4$$

$$R = (N-2)^2$$



Table

Shape	#G	#B	#R
N=3	4	4	1
4	4	8	4
5	4	12	9
6	4	16	16
7	4	20	25
8	4	24	49
N			

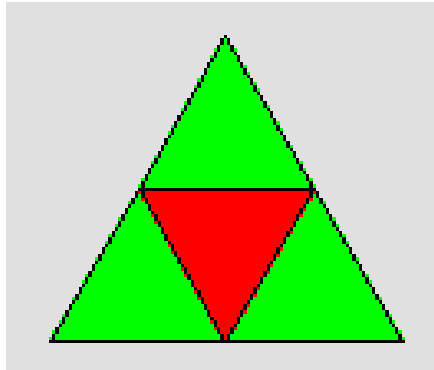


Concrete or  
pictorial  
representation

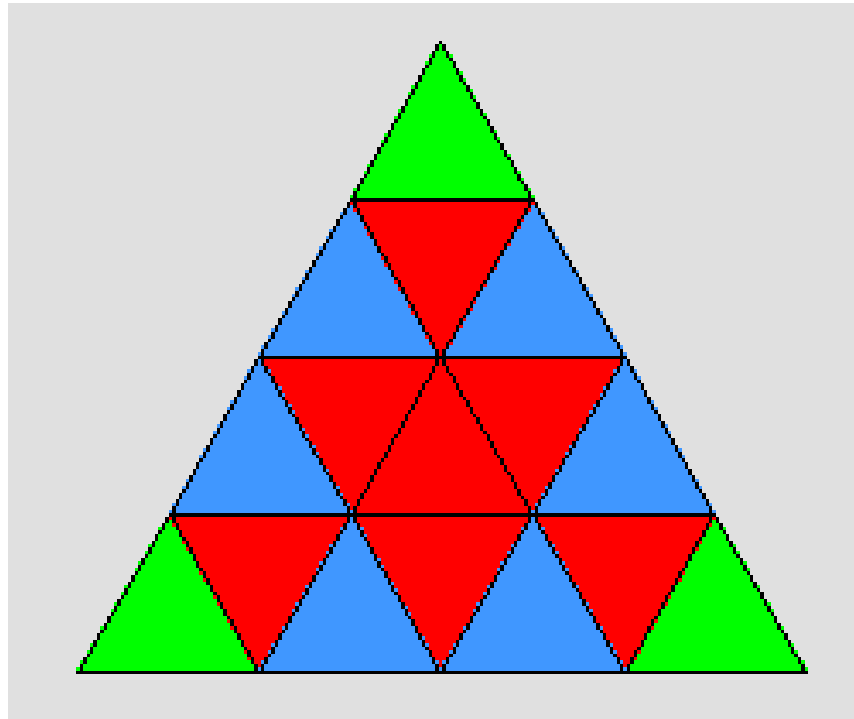
Verbal  
description

The Blue goes up by 4 each time  
the N goes up by one

# Extension – Use triangles



Side length is 2



Side length is 4

# Progressions

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## F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship

# Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by  $P(x) = 5b^x$ , where  $x$  is the time in weeks following the introduction and  $b$  is a positive unknown base.

Exactly how many fish did the fisherman release into the lake?

# Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by  $P(x)=5b^x$ , where  $x$  is the time in weeks following the introduction and  $b$  is a positive unknown base.

**Find  $b$  if you know the lake contains 33 fish after eight weeks. Show step-by-step work.**



# Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by  $P(x)=5b^x$ , where  $x$  is the time in weeks following the introduction and  $b$  is a positive unknown base.

Instead, now suppose that  $P(x)=5b^x$  and  $b=2$ . What is the weekly percent growth rate in this case? What does this mean in every-day language?

- a. The fisherman released the fish into the lake at time zero,  $t = 0$ , the exact moment of introduction. Thus, the number of fish that the fisherman released into the lake is given by:

$$P(0) = 5b^0$$

$$P(0) = 5 \cdot 1$$

$$P(0) = 5$$

This means that the fisherman released 5 fish into the lake.

- b. We know that  $x$  is the time in weeks following the introduction. Let us assume that 2 months is approximately 8 weeks, giving  $t = 8$ . Then, if the lake contains 33 fish after two months, or  $P(8) = 33$ , we can solve for  $b$ :

$$33 = 5b^8$$

$$b^8 = \frac{33}{5}$$

$$b = \left(\frac{33}{5}\right)^{\frac{1}{8}}$$

$$b \approx 1.266$$

Thus,  $b$  is approximately equal to 1.2 if the lake contains 33 fish after two months.

- c. The "weekly percent growth rate" is the percent increase of the population in one week. Since  $b = 2$ , we know that the population at any time  $x$  is given by  $P(x) = 5 \cdot 2^x$ , and that the population one week later is given by

$$P(x + 1) = 5 \cdot 2^{x+1} = (5 \cdot 2^x) \cdot 2 = 2P(x).$$

We learn that the population doubles each week, which is to say that there is a 100% weekly growth rate.

# Thank You!

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evelyn.baracaldo@gmail.com

Fred.Dillon@ideastream.org