VISUALIZING FUNCTIONS

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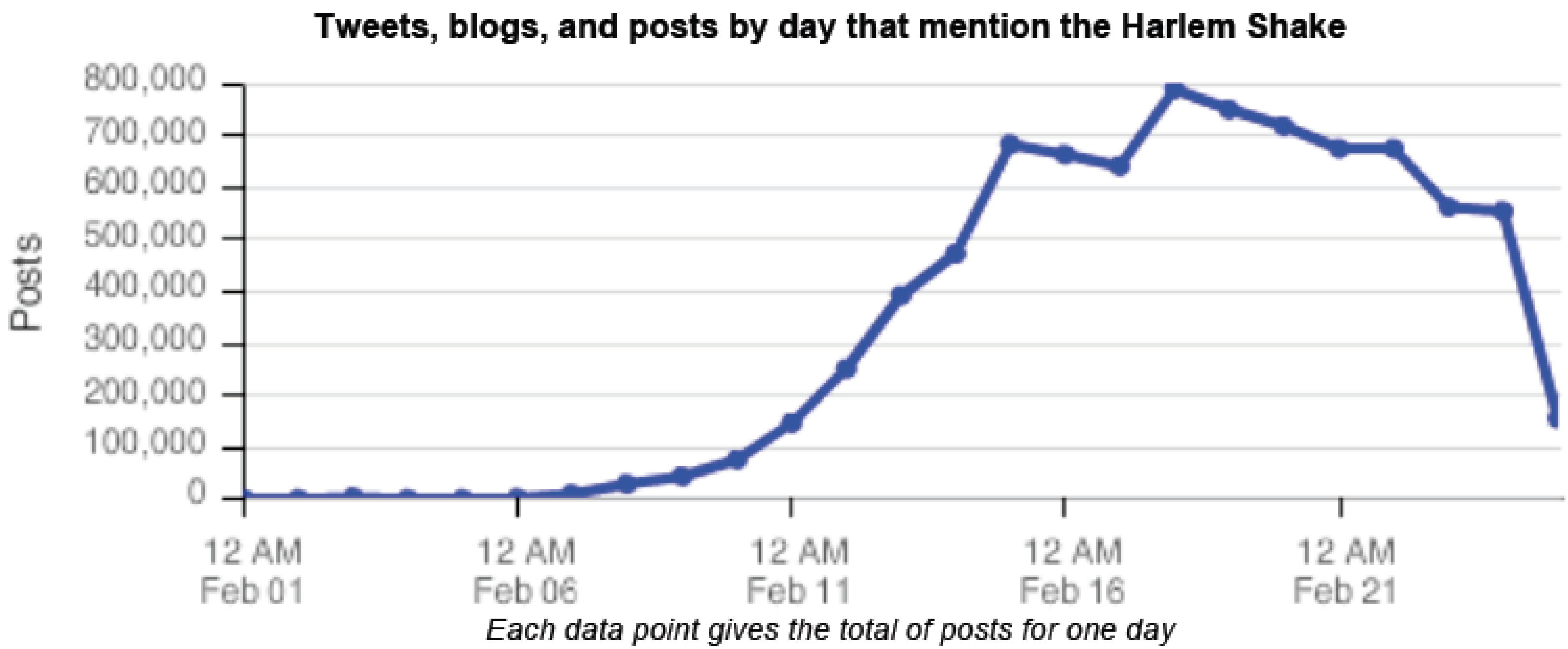
Warm-Up



Yummymath.org

What did that have to do with Functions?

Does each time have only one number of posts?

Domain/Range

Linear/Nonlinear

Rate of change

Definition of a function

Real-world application

Non-symbolic

Data based

What is a function?

What do think of when you think about teaching functions?

What does CCSSM say?

Connect to ratio and proportion

Start with linear functions in grade 8

Multiple Representations

Functions Progression (and more!)

http://ime.math.arizona.edu/progressions/

Our Goals for Today

- Use function notation and evaluation of functions in non-traditional ways
- Look for generalizations in transformations of functions
- Use multiple representations to understand families of functions
- Model the Student Mathematical Practices

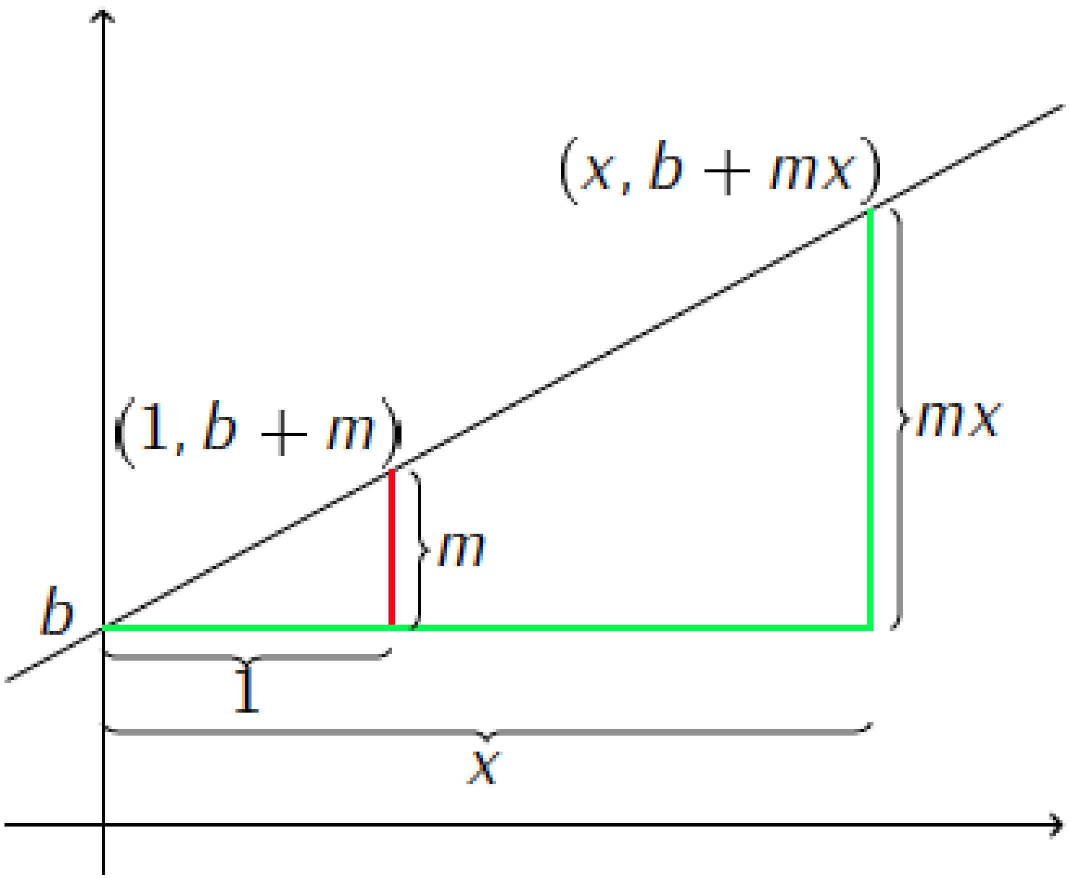
Is it Algebra I?

The main focus in Grade 8 is linear functions, those of the form, $y = mx + b$ where m and b are constants.

A linear function is an important piece of reasoning connecting algebra with geometry in Grade 8.

Algebraic thinking outside an Algebra I class.

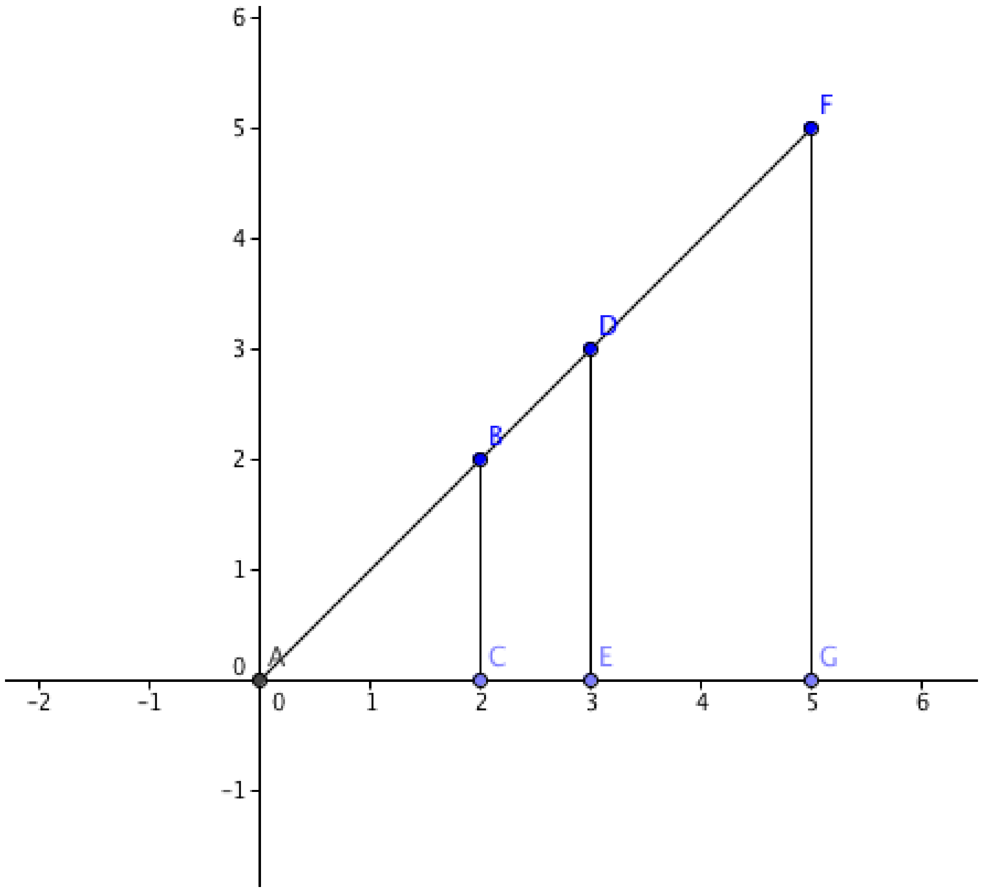
Linear Functions



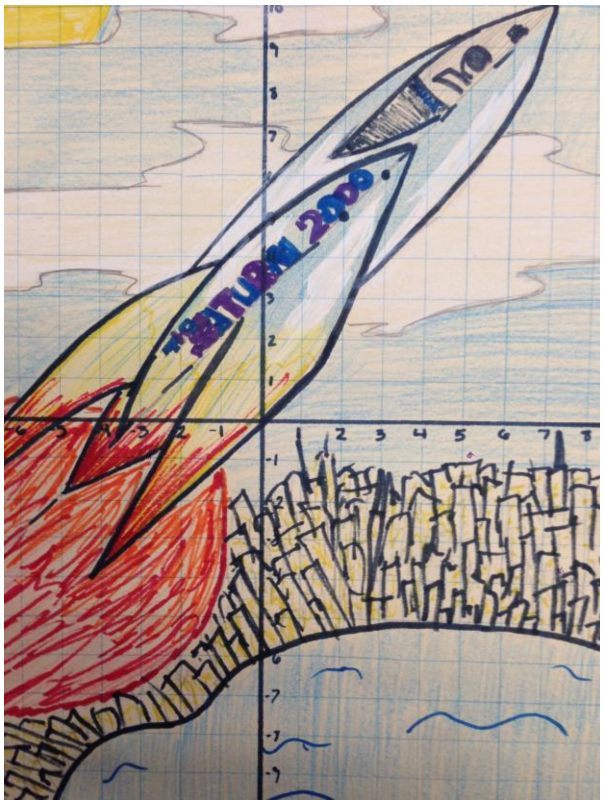
Progressions for the Common Core State Standards in Mathematics (draft)

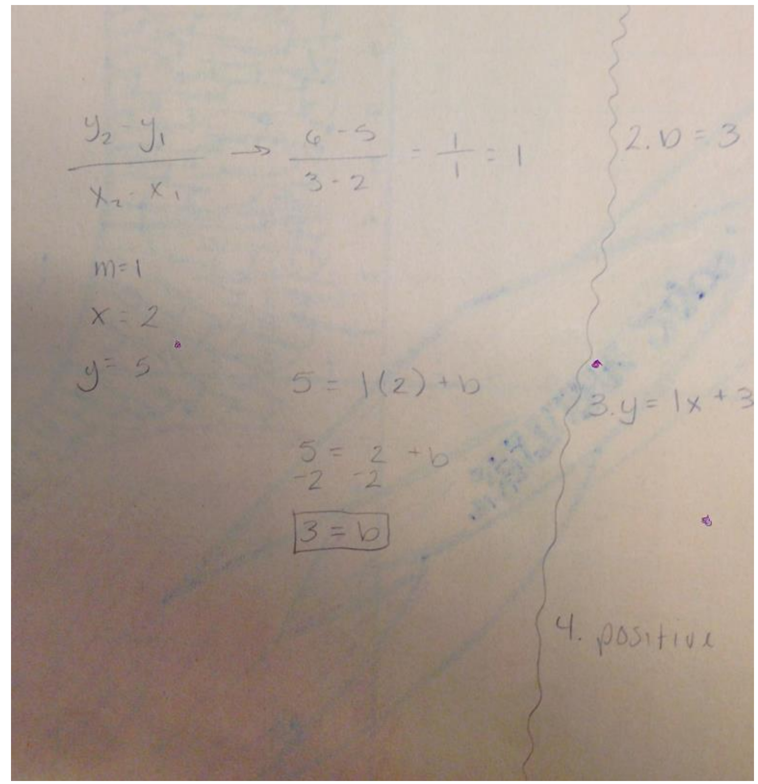
Slope and Linear Functions

- Transformations can help students think about algebraic concepts.
- Here ΔABC is dilated to create ΔADE.
- How can this help students think about slope?
- How can this help students think about collinearity?



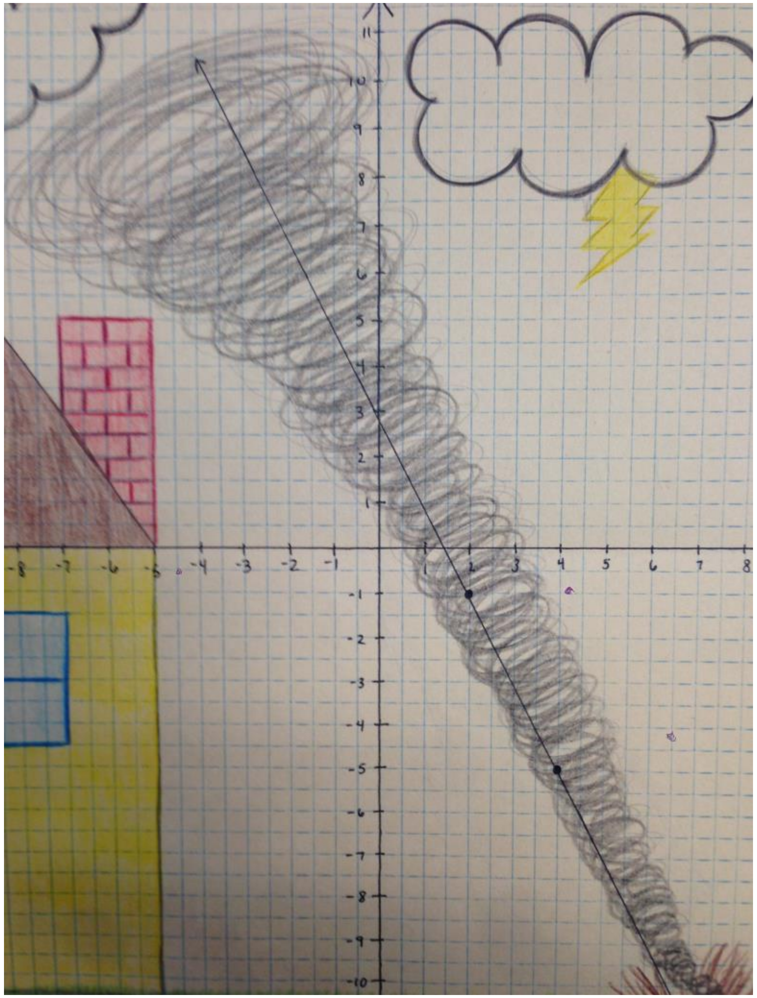
Looking at functions

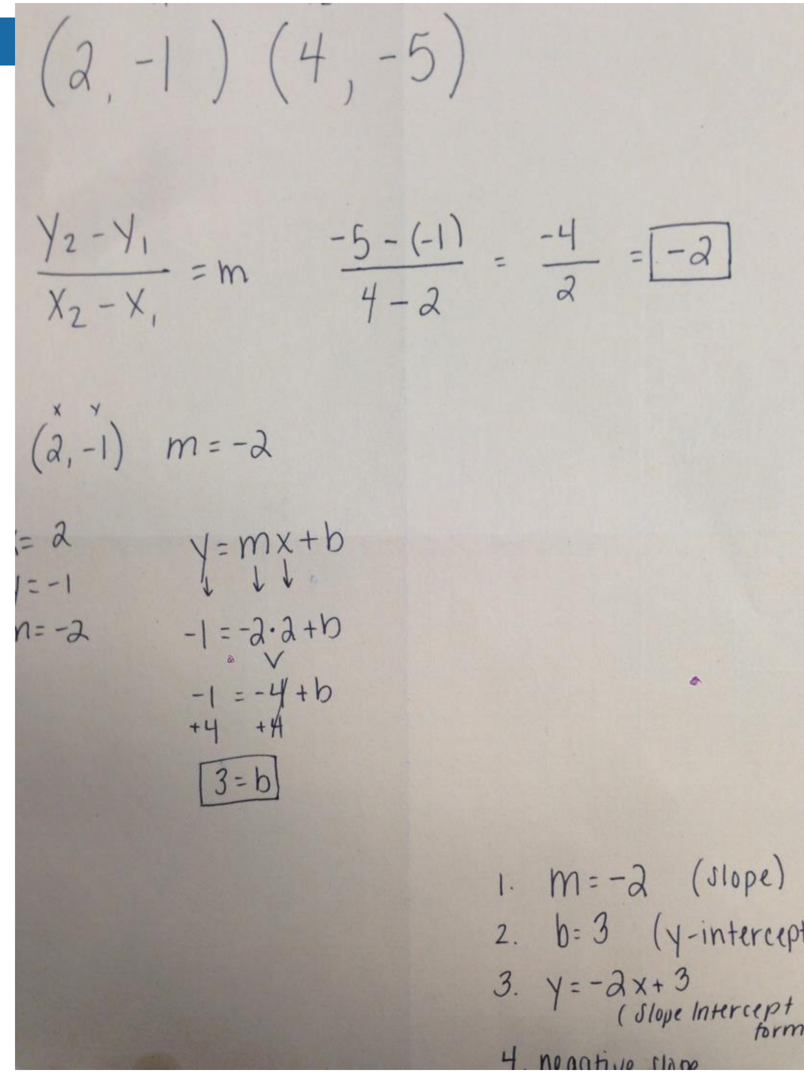




From the class of Chelsea Matthews, Maple Heights High School, Ohio

Looking at functions





From the class of Chelsea Matthews, Maple Heights High School, Ohio

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

CCSSI, 6-12.

“My Function Project”

You will come up with an original function idea. Remember a function is a rule where **every input value has exactly one output value.** For example the rule *f*(body part)= clothing won’t work because

f(foot)=sock and *f*(foot)=shoe

Function definition	Rule is a function (one output for every input) 2 pts	Rule is not a function and explains why 1 pt	Rule is not a function, no explanation 0 pts
Function notation	Rule uses correct function notation 2 pts	Rule has some function notation 1 pt	Rule does not use function notation 0 pts
Domain and Range	Defines domain and range accurately 2 pts	Either domain or range is defined or accurate 1 pt.	Does not address domain nor range or neither is accurate 0 pts
Examples	Provides at least 5 complete examples 2 pts	Provides fewer than 3 examples 1 pt	Provides no examples 0 pts
Practice Problems	Provides at least 5 practice problems 2 pts	Provides fewer than 3 practice problems 1 pt	Provides no practice problems 0 pts
Presentation	Poster has no errors and is neat, has color 0 pts	Poster is not neat or uses no color -1 pt	Poster is not neat and uses no color -2 pts

“My Function Project”

Function definition	Rule is a function (one output for every input) 2 pts	Rule is not a function and explains why 1 pt	Rule is not a function, no explanation 0 pts
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FUNCTION RULE PROJECT

By: Nasyria Taylor

Rule:

f(person) = their birthday

- I know this is a function because for every input (person) it can only have 1 output (their birthday).

Try some for your life:

f(mom) = ______

f(brother) = ______

f(dad) = ______

f(uncle) = ______

f(aunt) = ______

f(cousin) = ______

EVALUATING EXAMPLES:

f(Tonya) = October 17

f(Larry) = March 16

f(Farqu) = October 22

f(Nasyria) = December 21

f(Sylvia) = February 29

SOLVING EXAMPLES:

f(~~Tonya~~ person) = October 17, ~~time = 6:43 am~~ person = Tonya

f(person) = March 16, ~~time = 2:18 pm~~ person = Larry

f(person) = October 22, person = Farqu

f(person) = December 21, person = Nasyria

f(person) = February 29, person = Sylvia

Horoscope Function

Rule:

f(Date) = horoscope Sign

I know this is a function because for every date input (DATE) I can only have 1 output (h.Sign).

Evaluating Examples:

f(3/30) = Aries
f(4/25) = Taurus
f(6/18) = Gemini
f(7/4) = Cancer
f(8/3) = Leo
f(9/15) = Virgo
f(10/15) = Libra
f(11/17) = Scorpio
f(12/20) = Sagittarius
f(12/28) = Capricorn
f(1/19) = Aquarius
f(2/24) = Pisces

Solving Examples:

f(date) = Cancer, Date = June 21 - July 22
f(date) = Leo, Date = July 23 - August 22
f(date) = Libra, Date = Sept 23 - Oct 22
f(Date) = Scorpio, Date = Oct 23 - NOV 21
f(date) = Capricorn, Date = Dec 22 - Jan 19
f(date) = Aquarius, Date = Jan 20 - Feb 18

ractice Problems:

(7/6) = ______
f(date) = Capricorn, Date = ______
f(1/20) = ______
(2/26) = ______
f(Date) = Libra, Date = ______
f(Date) = Aries, Time = ______

Siara R.
math

PRESIDENTIAL FUNCTION

Rule:

f(term served)=president

→ I know this is a function because for every input (term served), I can only have one output (president)

Evaluating Examples:

f(1789-1797)=George washington
f(1809-1817)=James Madison
f(1817-1825)=James Monroe
f(1837-1841)=Martin Van Buren
f(1853-1857)=Franklin Pierce

Solving Examples:

f(term served)=Abraham Lincoln, term served= 1861-1865
f(term served)=Rutherford B. Hayes; term served=1877-1881
f(term served)=Ronald Reagan; term served = 1981-1989
f(term served)=Theodore Roosevelt; term served= 1901-1909
f(term served)=Richard Nixon; term served= 1969-1974

Try Some On Your Own!

f(2001-2009)=

f(term served)=John Quincy Adams, term served=

(1989-1993)=

f(1929-1933)=

(term served)=William Mckinley, term served=

f(1797-1801)=

“My Function Project”

NCTM Baltimore 2013
Bring one back to your classroom

f(________)= ___________

f(“Breaking Bad” Characters)= Number of lies they tell

Domain: {Walt, Jesse, Skylar, Hank, Marie, Walt Jr. Saul, Fring, Mike…}
Range: $\{ y \geq 0\}$

f(Walt Jr.)=0,
f(Walt)=_____

Is this a function?

In what other ways can this function be written?

Is this function linear?

“Transformations Gallery Walk”

Transformations Group Work A:
$f(x+2)$

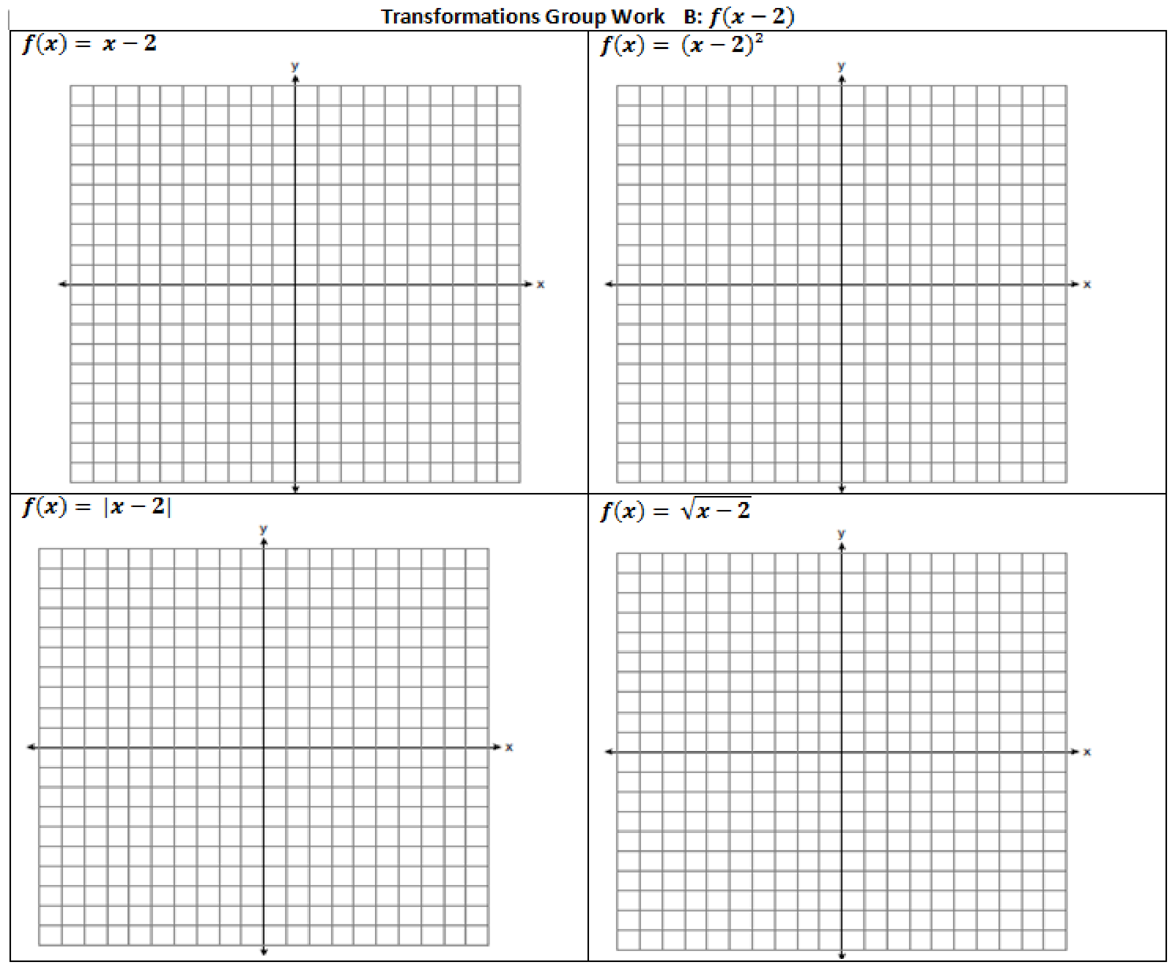
Transformations Group Work B:
$f(x-2)$

Transformations Group Work C:
$f(x)\ +2$

Transformations Group Work D:
$f(x)\ -2$

Transformations Group Work E:
$-f(x)$

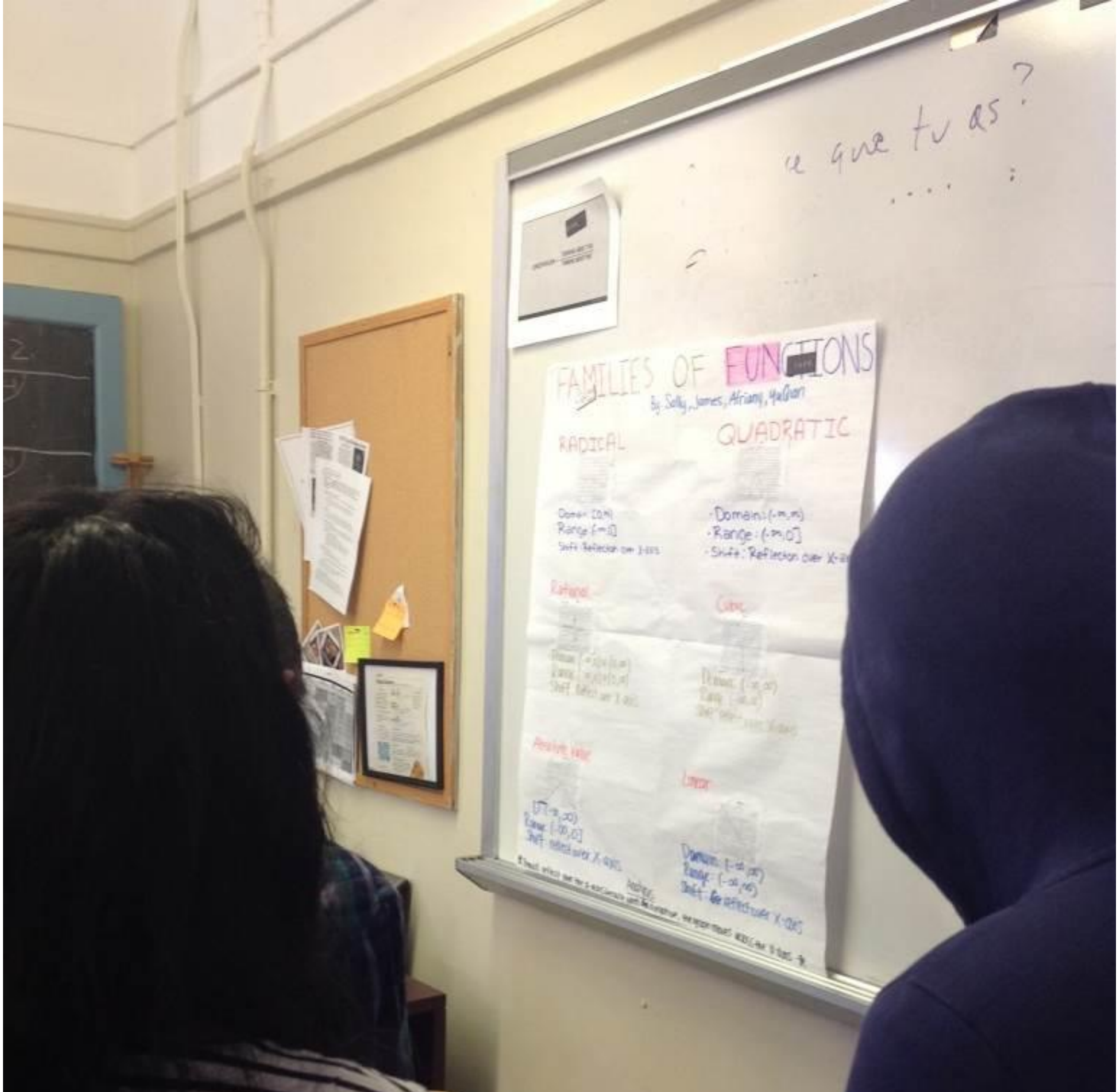
Transformations Group Work F:
$f(-x)$

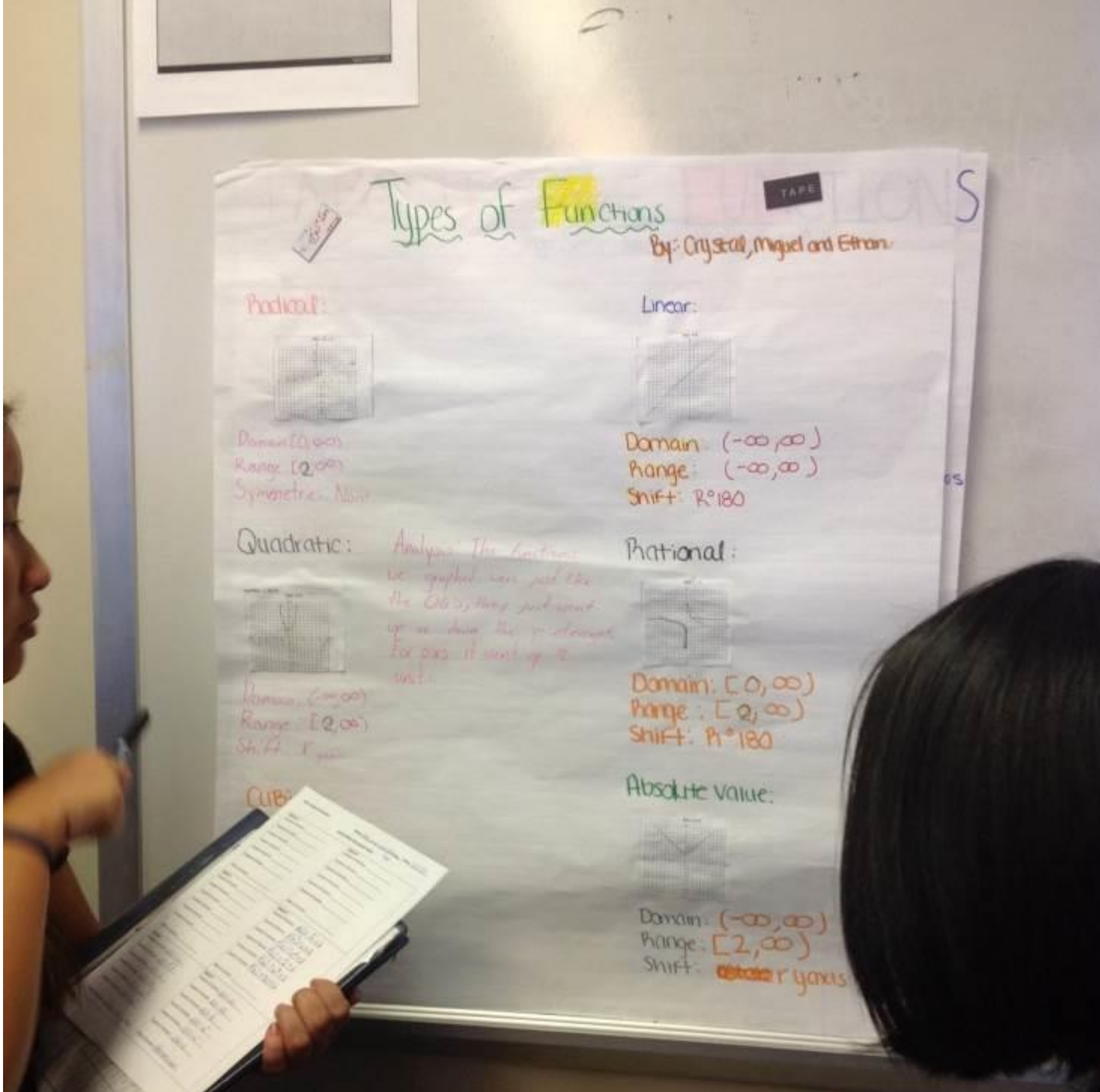


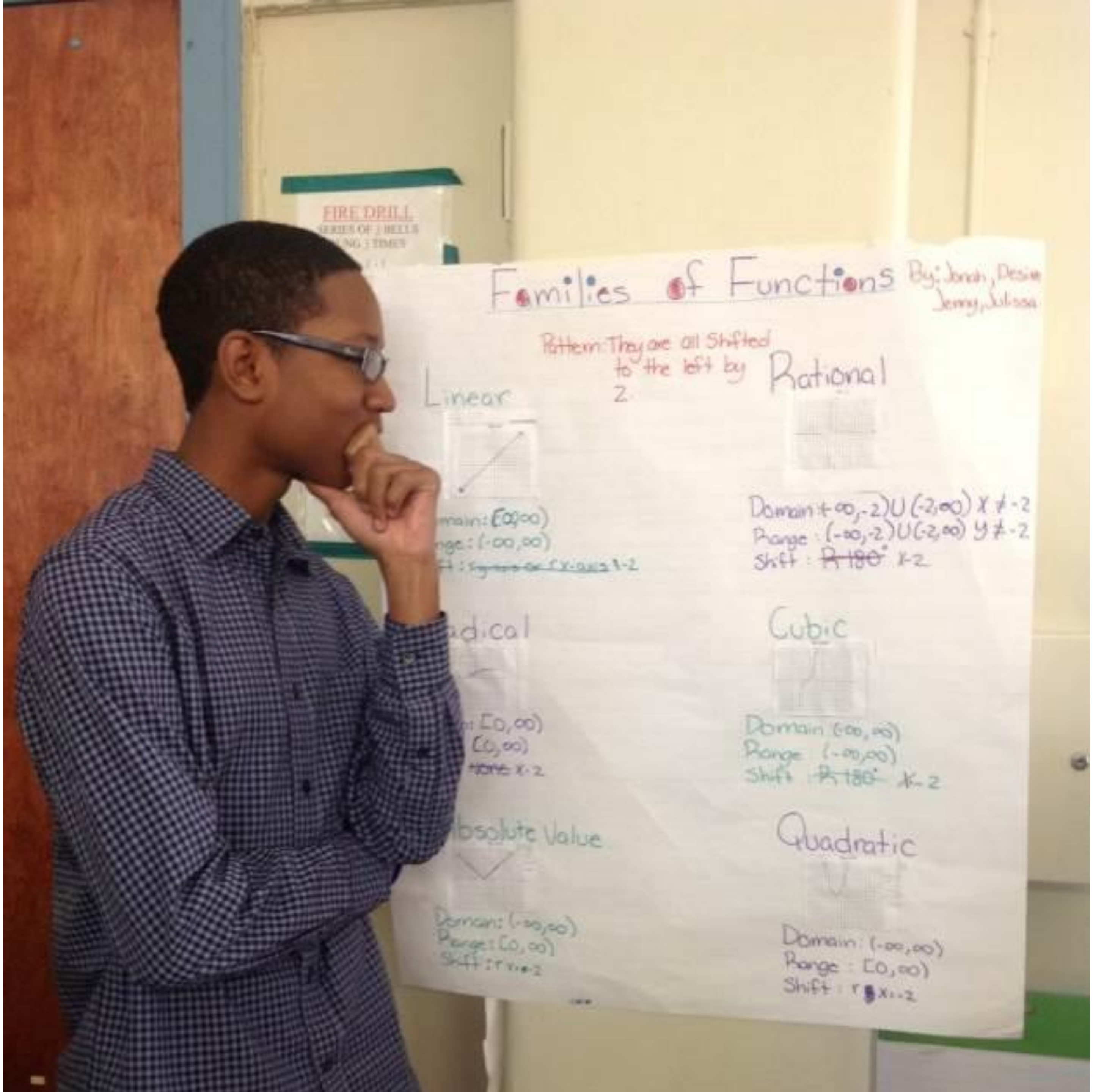
“Transformations Gallery Walk”

NOTES FROM THE GALLERY WALK

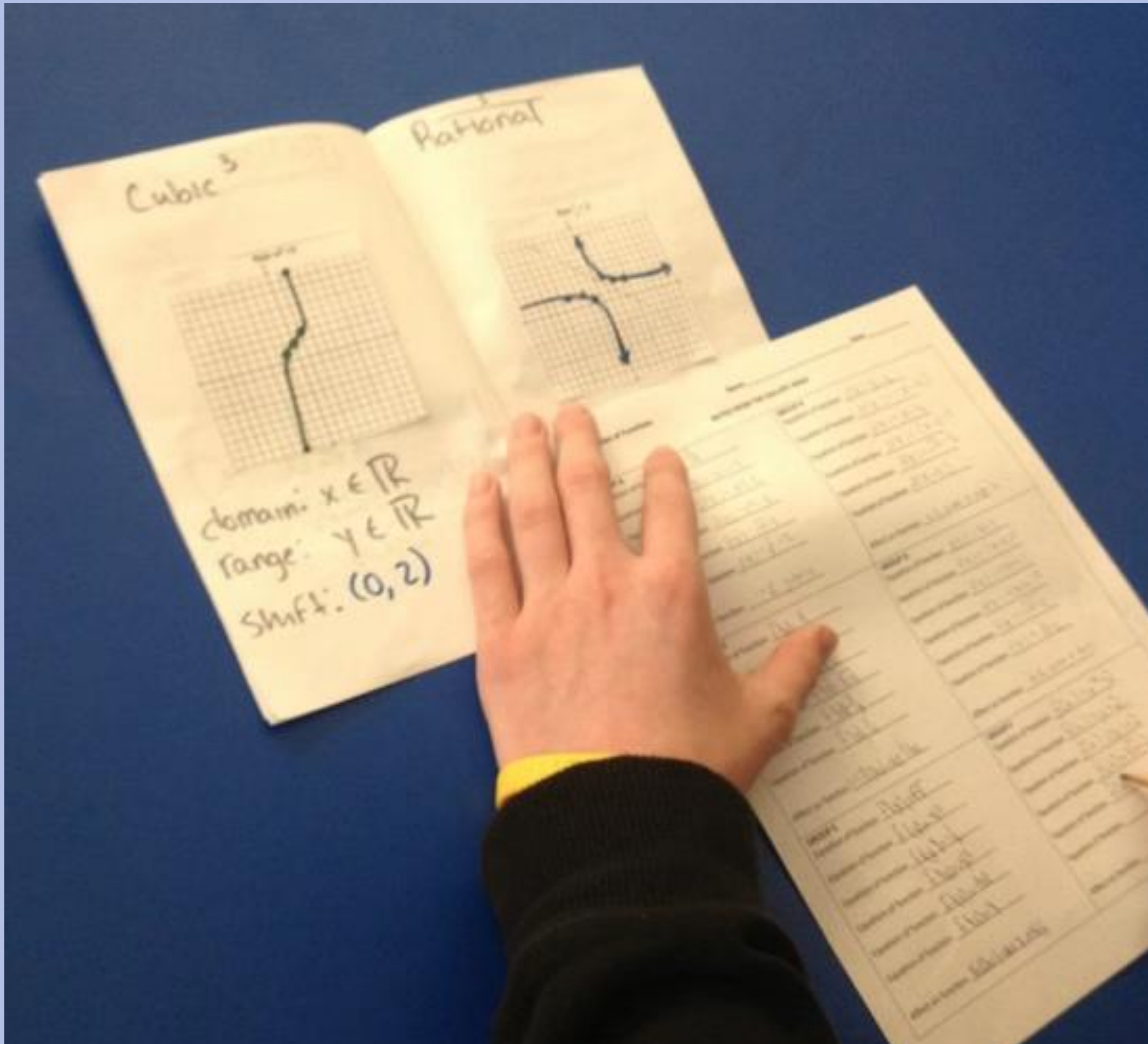
GROUP A	**GROUP B**
Equation of function: ____________________	Equation of function: ____________________
Equation of function: ____________________	Equation of function: ____________________
Equation of function: ____________________	Equation of function: ____________________
Equation of function: ____________________	Equation of function: ____________________
Equation of function: ____________________	Equation of function: ____________________
Equation of function: ____________________	Equation of function: ____________________
Effect on function: ______________________	Effect on function: ______________________


FAMILIES OF
RADICAL
QUADRATIC


TAPE
Types of Functions
By: Crystal, Miguel and Ethan
Radical:
Linear:
Domain: (-∞,∞)
Range: (-∞,∞)
Shift: R°180
Quadratic:
Rational:
Domain: [0,∞)
Range: [2,∞)
Shift: R°180
Absolute Value:
Domain: (-∞,∞)
Range: [2,∞)
Shift:



FIRE DRILL
Families of Functions
Pattern: They are all Shifted to the left by 2
Linear
Rational
Domain: (-∞,-2)U(-2,∞) x ≠ -2
Range: (-∞,-2)U(-2,∞) y ≠ -2
Shift: x-2
Cubic
Domain (-∞,∞)
Range (-∞,∞)
Shift: x-2
Absolute Value
Domain: (-∞,∞)
Range: [0,∞)
Quadratic
Domain: (-∞,∞)
Range: [0,∞)


Cubic
Rational
domain: x ∈ ℝ
range: y ∈ ℝ
Shift: (0, 2)

Function "YMCA"

Absolute Value

Quadratic

Cubic

Rational

Square Root

Thinking about Composition

(illustrativemathematics.com)

Suppose the swine flu, influenza H1N1, is spreading on a school campus. The following table shows the number of students, n, that have the flu d days after the initial outbreak. The number of students who have the flu is a function of the number of days, $n = f(d)$.

d **(days)**	0	2	6	8	12	16	24
$n = f(d)$ **(number of students infected)**	3	9	16	30	55	45	32

There is a school store on campus. As the number of students who have the flu increases, the number of tissue boxes, b, sold at the school store also increases. The number of tissue boxes sold on a given day is a function of the number of students who have the flu, $b = g(n)$, on that day.

n **(number of students infected)**	0	3	8	9	12	16	18	30	32	38	45	50	55
$b = g(n)$ **(number of tissue boxes sold)**	1	4	8	12	13	18	24	33	34	40	45	51	57

a. Find g(f(0)) and state the meaning of this value in the context of the flu epidemic. Include units in your answer.

b. Fill in the chart below using the fact that $b = g(f(d))$.

d **(days)**	0	2	6	8	12	16	24
b **(number of tissue boxes sold)**							

c. For each of the following expressions, explain its meaning in the context of the problem, and if possible, give an approximation of its value. Justify your answer.

i. $g(f(16))$

ii. $g(f(18))$

iii. $f(g(9))$

Thinking about Composition

(illustrativemathematics.com)

3.) Suppose the swine flu, influenza H1N1, is spreading on a school campus. The following table shows the number of students, n, that have the flu d days after the initial outbreak. The number of students who have the flu is a function of the number of days, $n=f(d)$.

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$b=g(n)$ (number of tissue boxes sold)	1	4	8	12	13	18	24	33	34	40	45	51	57

Find $g(f(0))$ and state the meaning of this value in the context of the flu epidemic. Include units in your answer.

If nobody is infected only 1 tissue boxes sold because of the normal stuff like crying and runny nose

Thinking about Composition

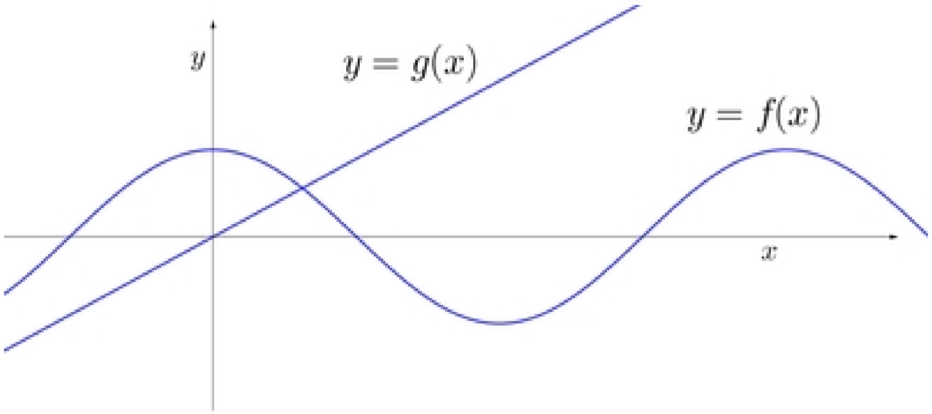
(illustrativemathematics.com)

Fill in the chart below using the fact that $b=g(f(d))$.

d (days)	0	2	6	8	12	16	24
b (number of tissue boxes sold)	4	12	18	53	57	45	34

Thinking about Functions

Use the graph (for example, by marking specific points) to illustrate the statements in (a)–(d). If possible, label the coordinates of any points you draw.



a. $f(0) = 2$

b. $f(-3) = f(3) = f(9) = 0$

c. $f(2) = g(2)$

d. $g(x) > f(x)$ for $x > 2$

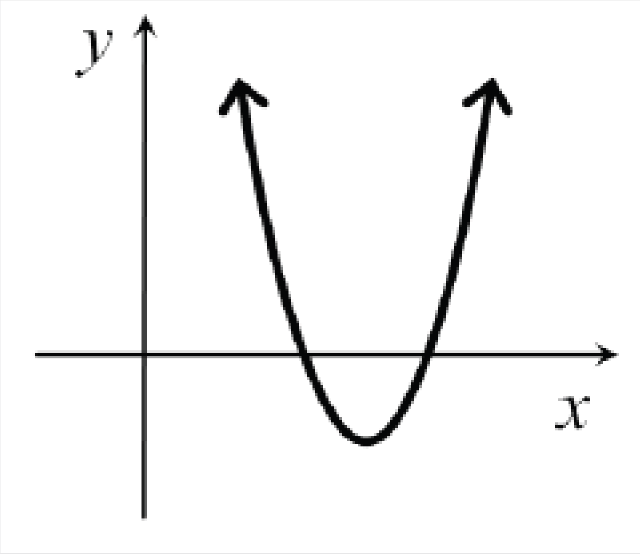
commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_functions_2013_07_02.pdf

Looking at Functions

Which Equation?

Which of the following could be an expression for the function whose graph is shown below? Explain.

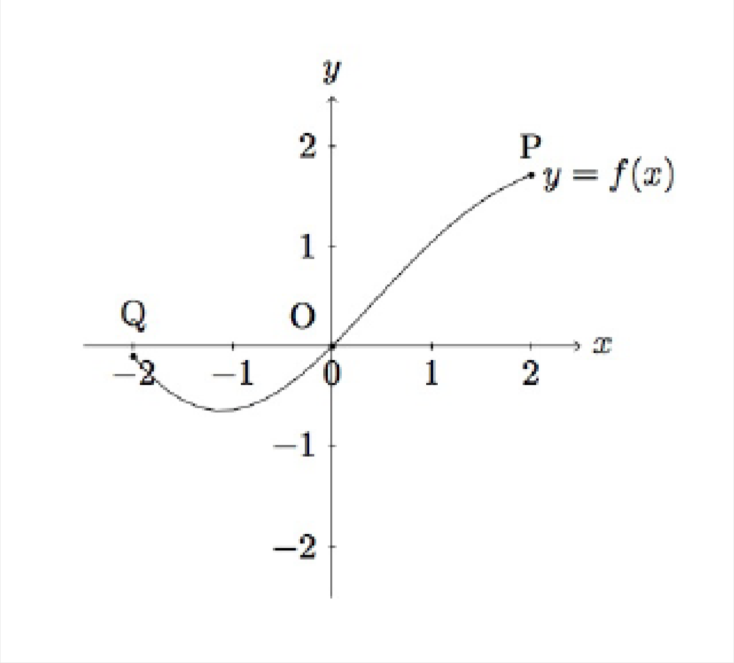
(a) $(x+12)^2+4$
(b) $-(x-2)^2-1$
(c) $(x+18)^2-40$
(d) $(x-10)^2-15$
(e) $-4(x+2)(x+3)$
(f) $(x+4)(x-6)$
(g) $(x-12)(-x+18)$
(h) $(20-x)(30-x)$



Progressions for the Common Core State Standards in Mathematics (draft)

Transforming Functions

The figure shows the graph of a function f whose domain is the interval $-2 \leqslant x \leqslant 2$.



(a) In (i)–(iii), sketch the graph of the given function and compare with the graph of f. Explain what you see.

(i) $g(x) = f(x) + 2$

(ii) $h(x) = -f(x)$

(iii) $p(x) = f(x + 2)$

Progressions for the Common Core State Standards in Mathematics (draft)

Creating Functions

A square is built with the following pattern:

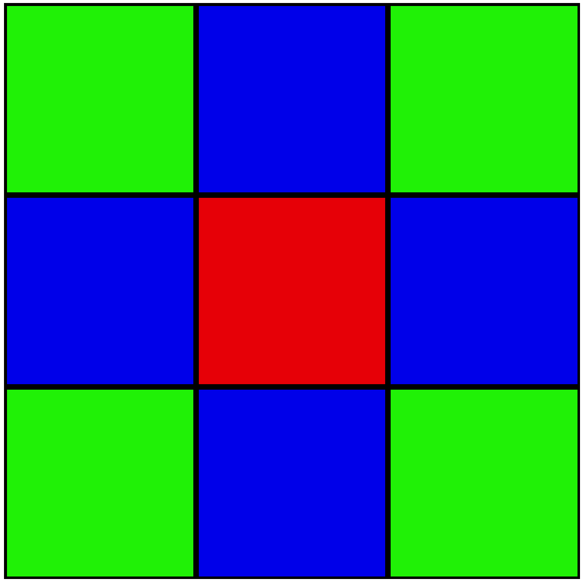
G	B	G
B	R	B
G	B	G

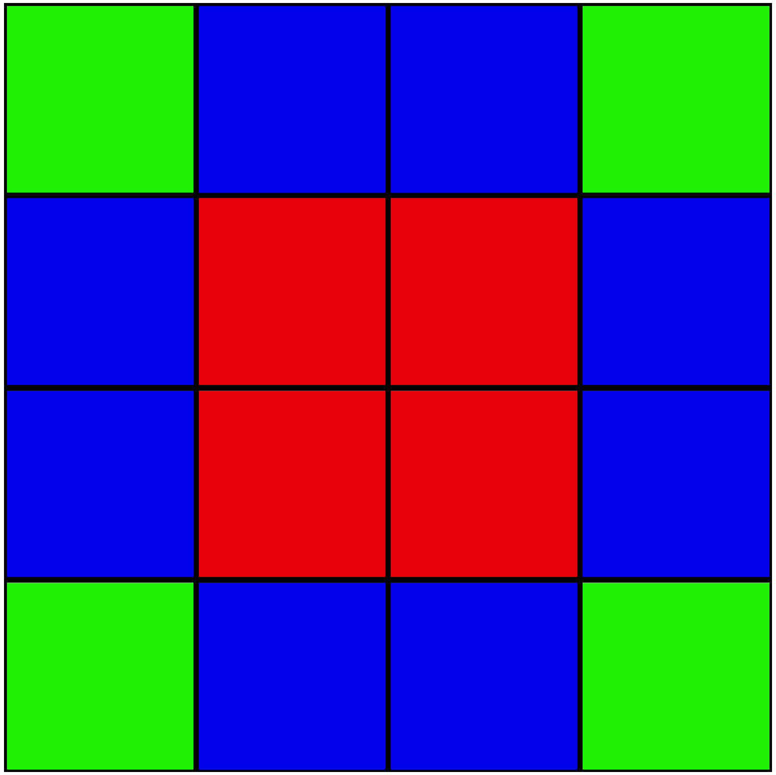
G = Green color for all corners

B = Blue color for perimeter squares that are not corners

R = Red color for all squares that are not on the perimeter.

Coloring Squares





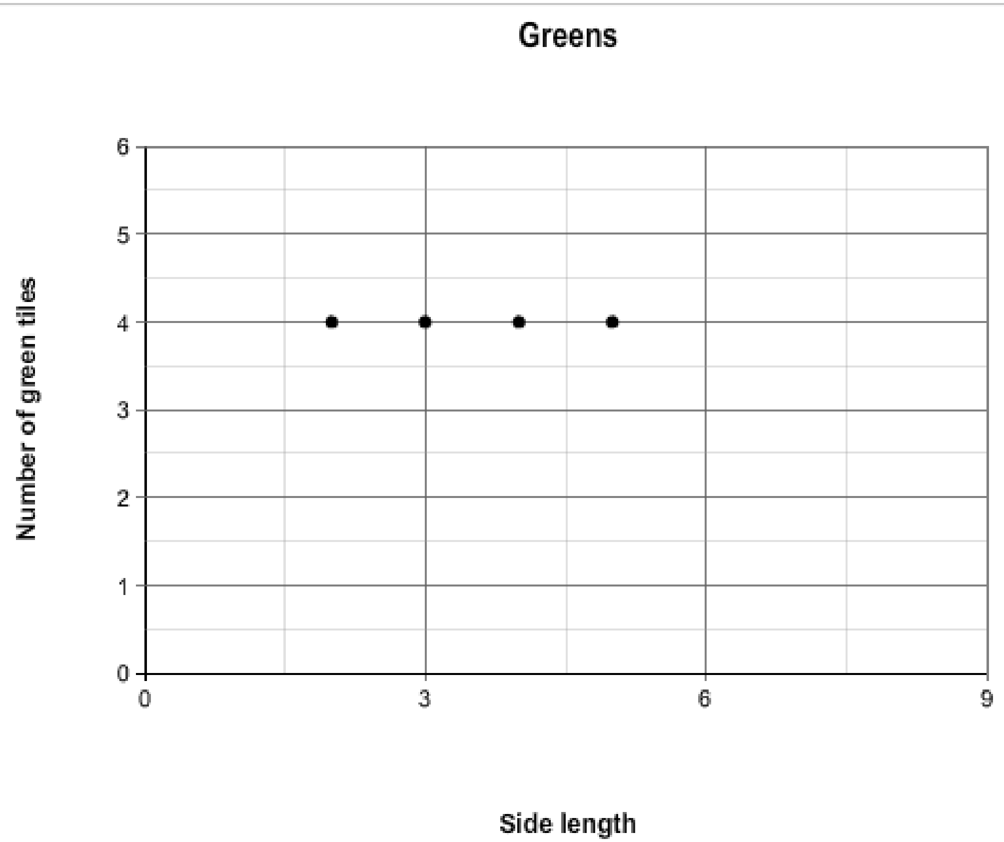
Coloring Squares

Shape	#G	#B	#R
N=3	4	4	1
4			
5			
6			
7			
8			

As a small group, complete this table for other squares.

Coloring Squares

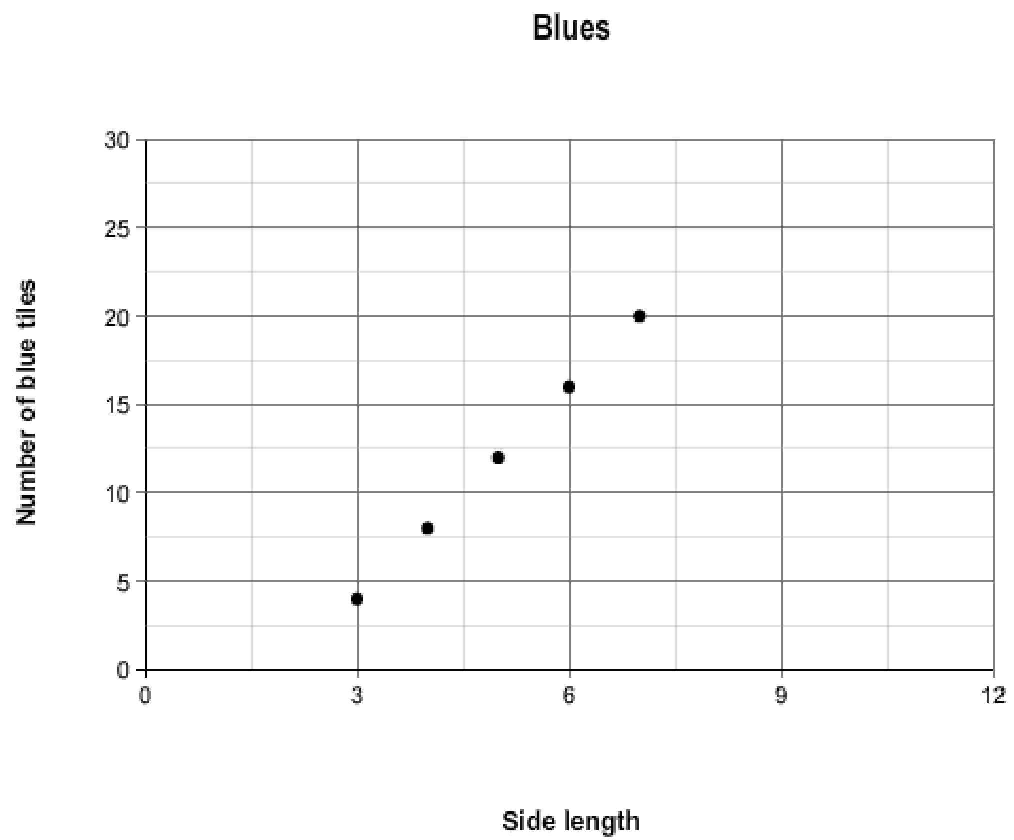
Describe the graph of the Green squares…



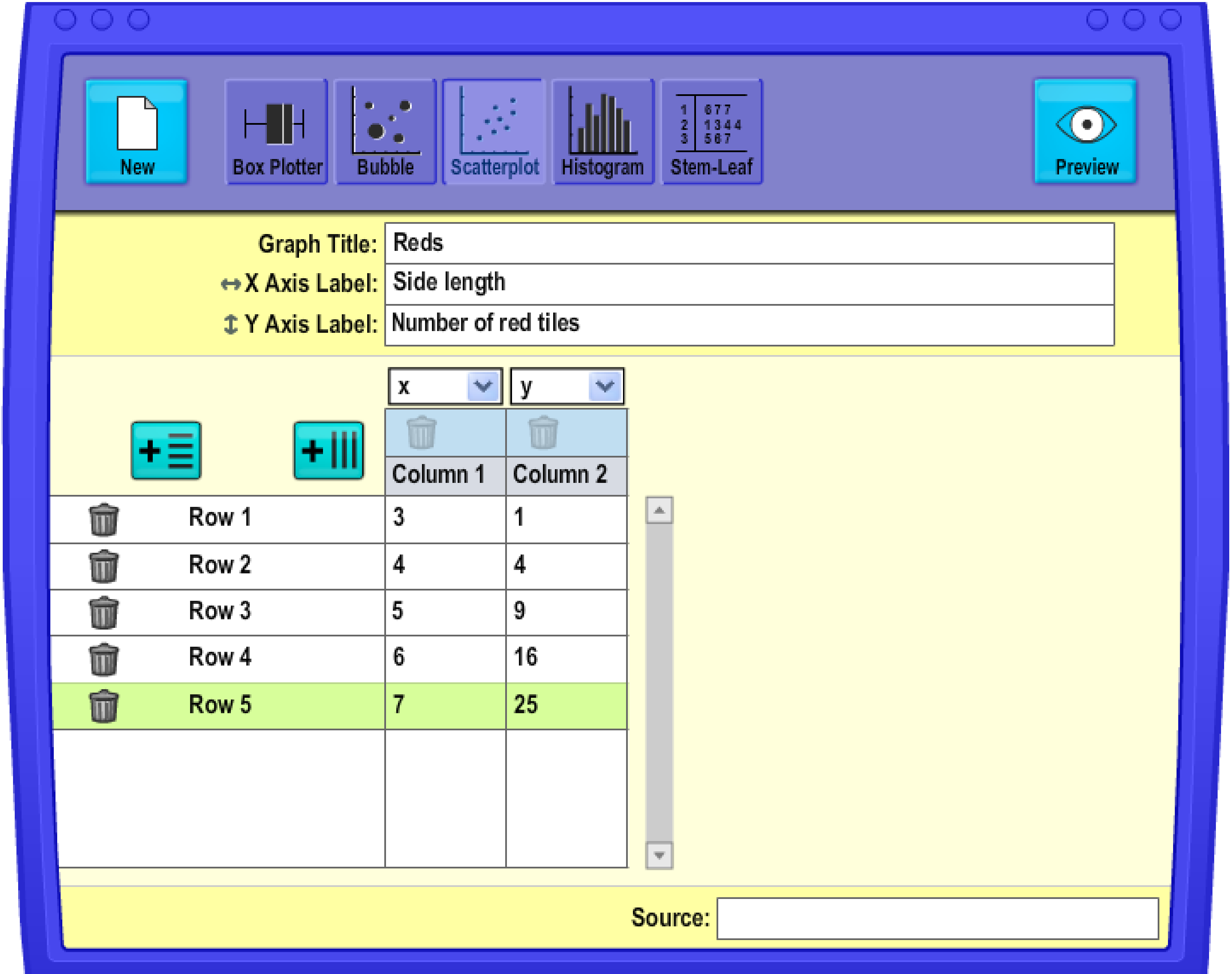
http://illuminations.nctm.org/ActivityDetail.aspx?ID=220

Coloring Squares

Describe the graph of the blue squares.

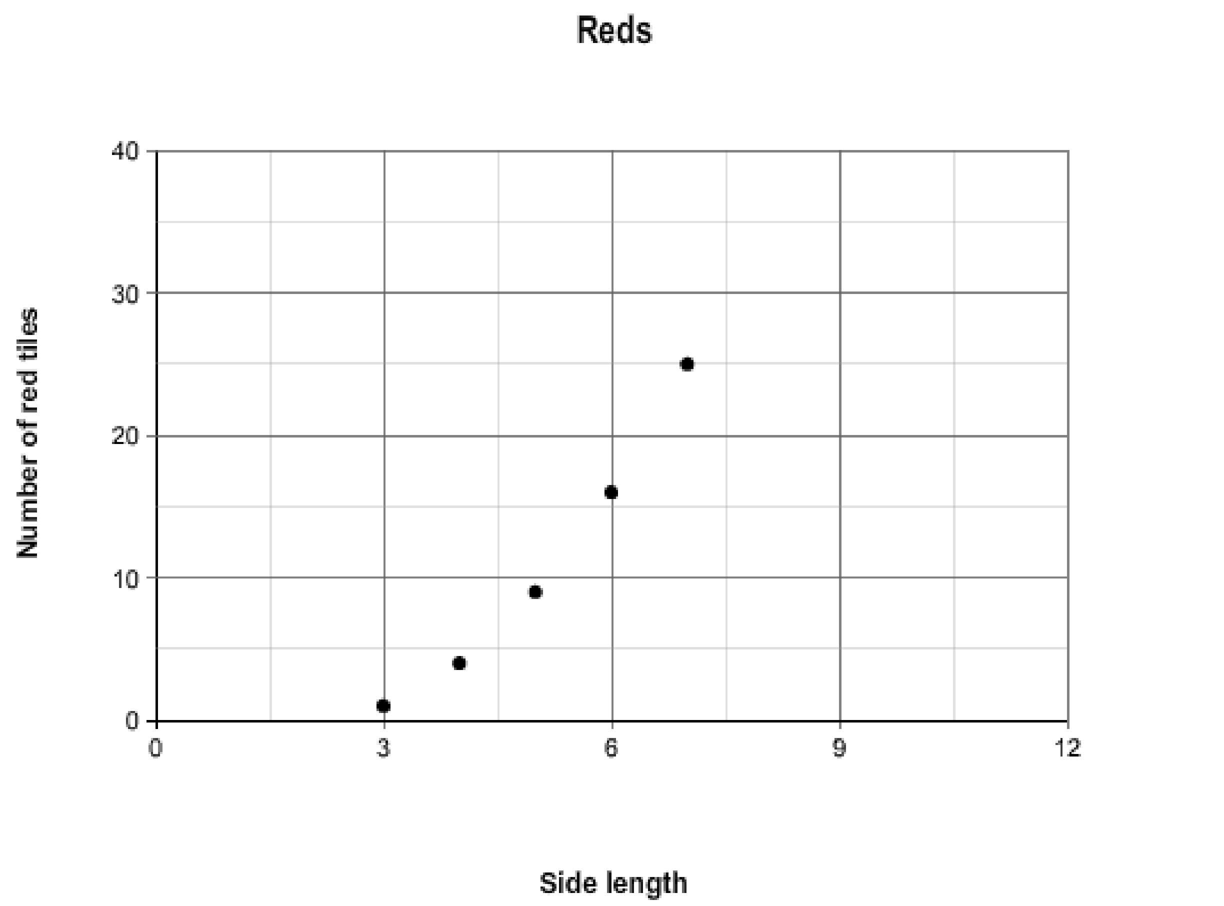


http://illuminations.nctm.org/ActivityDetail.aspx?ID=220


New
Box Plotter
Bubble
Scatterplot
Histogram
Stem-Leaf
Preview
Graph Title: Reds
X Axis Label: Side length
Y Axis Label: Number of red tiles
x
y
Column 1
Column 2
Row 1 3 1
Row 2 4 4
Row 3 5 9
Row 4 6 16
Row 5 7 25
Source:

Coloring Squares

Describe the graph of the red squares.



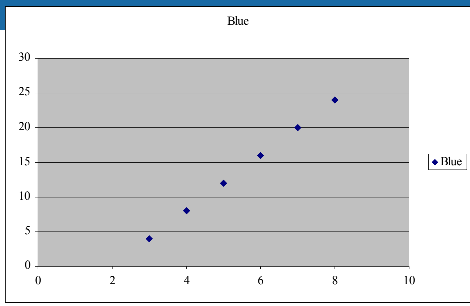
http://illuminations.nctm.org/ActivityDetail.aspx?ID=220

Coloring Squares

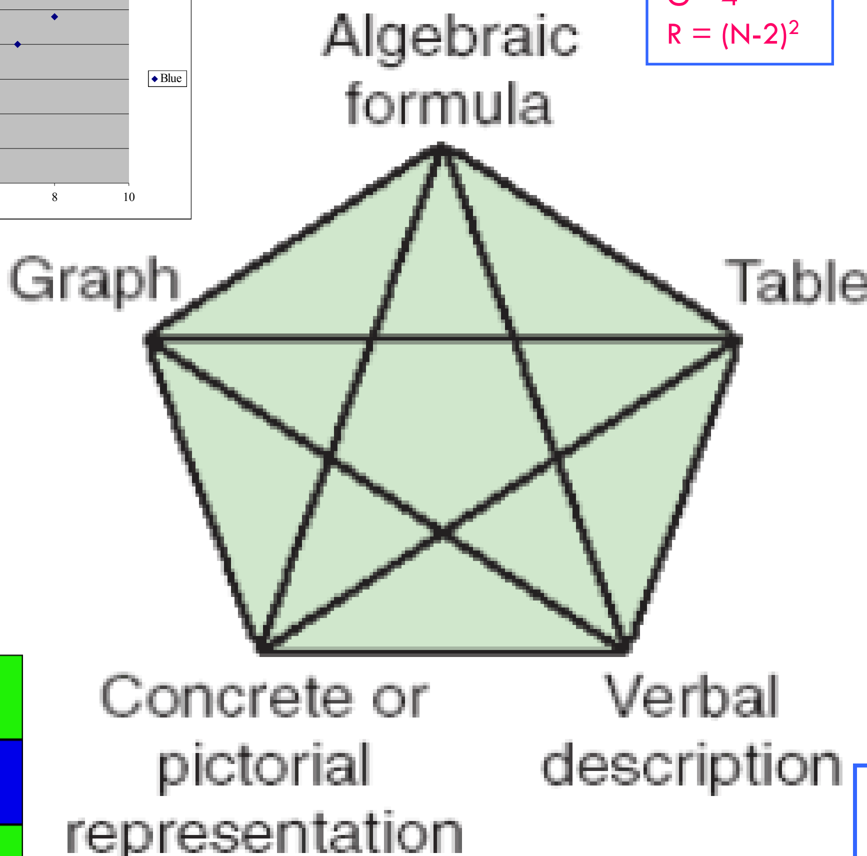
Shape	#G	#B	#R
N=3	4	4	1
4	4	8	4
5	4	12	9
6	4	16	16
7	4	20	25
8	4	24	49
N			

Investigate the table with finite differences or a graph. Look at the rates of change!

Coloring Squares



G =4
R = $(N-2)^2$



Shape	#G	#B	#R
N=3	4	4	1
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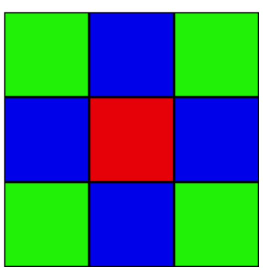
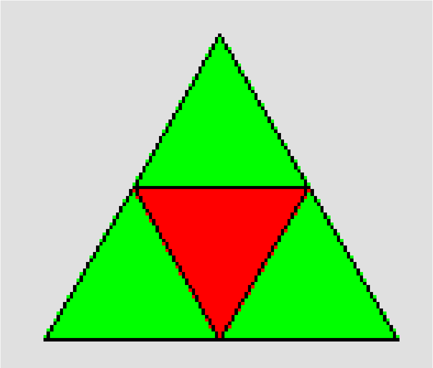
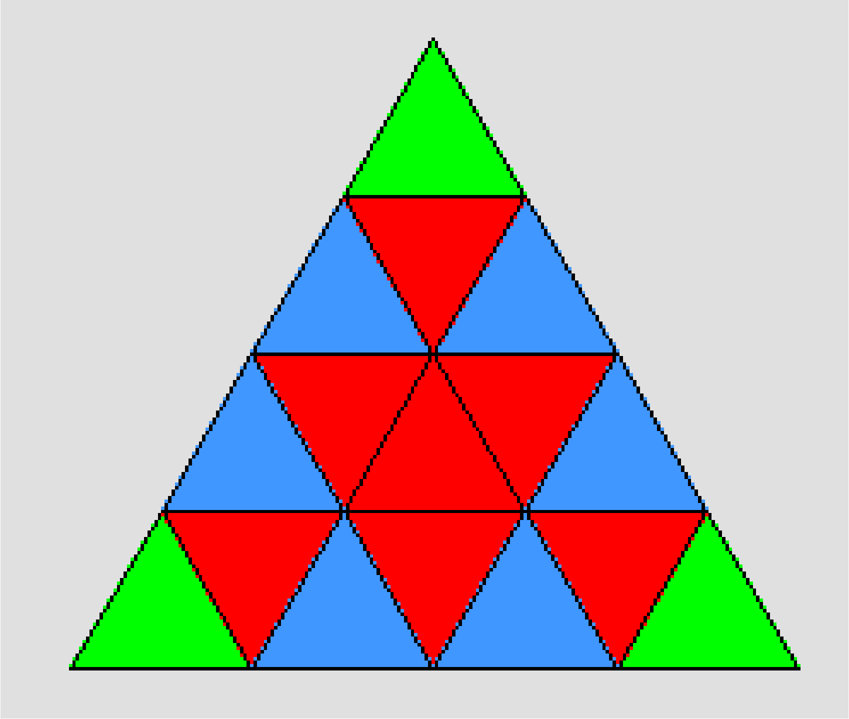


The Blue goes up by 4 each time
the N goes up by one

Extension – Use triangles



Side length is 2



Side length is 4

Progressions

F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship

Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x)=5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

Exactly how many fish did the fisherman release into the lake?

Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x)=5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

Find b if you know the lake contains 33 fish after eight weeks. Show step-by-step work.

Illustrative Mathematics

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x)=5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

Instead, now suppose that $P(x)=5b^x$ and $b=2$. What is the weekly percent growth rate in this case? What does this mean in every-day language?

a. The fisherman released the fish into the lake at time zero, $t = 0$, the exact moment of introduction. Thus, the number of fish that the fisherman released into the lake is given by:

$$P(0) = 5b^0$$
$$P(0) = 5 \cdot 1$$
$$P(0) = 5$$

This means that the fisherman released 5 fish into the lake.

b. We know that x is the time in weeks following the introduction. Let us assume that 2 months is approximately 8 weeks, giving $t = 8$. Then, if the lake contains 33 fish after two months, or $P(8) = 33$, we can solve for b:

$$33 = 5b^8$$
$$b^8 = \frac{33}{5}$$
$$b = \left(\frac{33}{5}\right)^{\frac{1}{8}}$$
$$b \approx 1.266$$

Thus, b is approximately equal to 1.2 if the lake contains 33 fish after two months.

c. The "weekly percent growth rate" is the percent increase of the population in one week. Since $b = 2$, we know that the population at any time x is given by $P(x) = 5 \cdot 2^x$, and that the population one week later is given by

$$P(x + 1) = 5 \cdot 2^{x+1} = (5 \cdot 2^x) \cdot 2 = 2P(x).$$

We learn that the population doubles each week, which is to say that there is a 100% weekly growth rate.

Thank You!

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