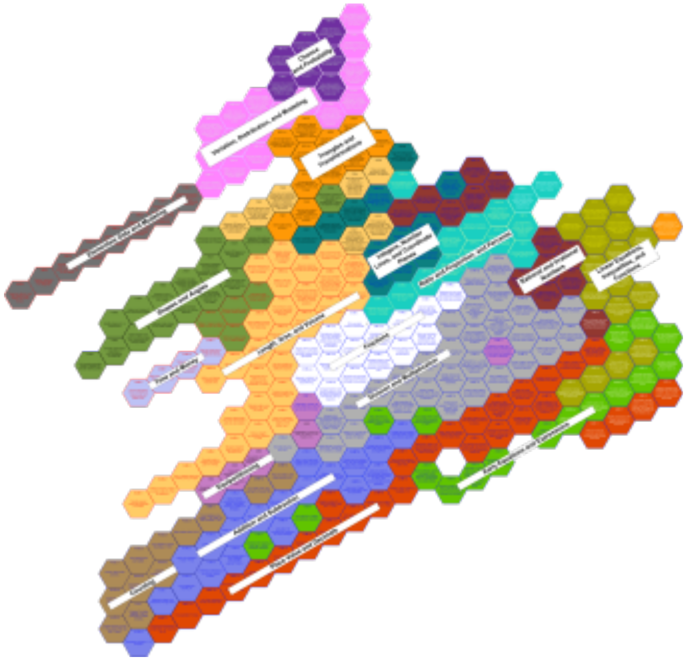

TurnOnCCMath.net

TurnOnCCMath.net: Learning Trajectories to Interpret the Common Core Standards

NCTM Regional Conference, Baltimore, MD; October 17-18, 2013

Alan Maloney, Ph. D.

The Friday Institute for Educational Innovation
College of Education
North Carolina State University

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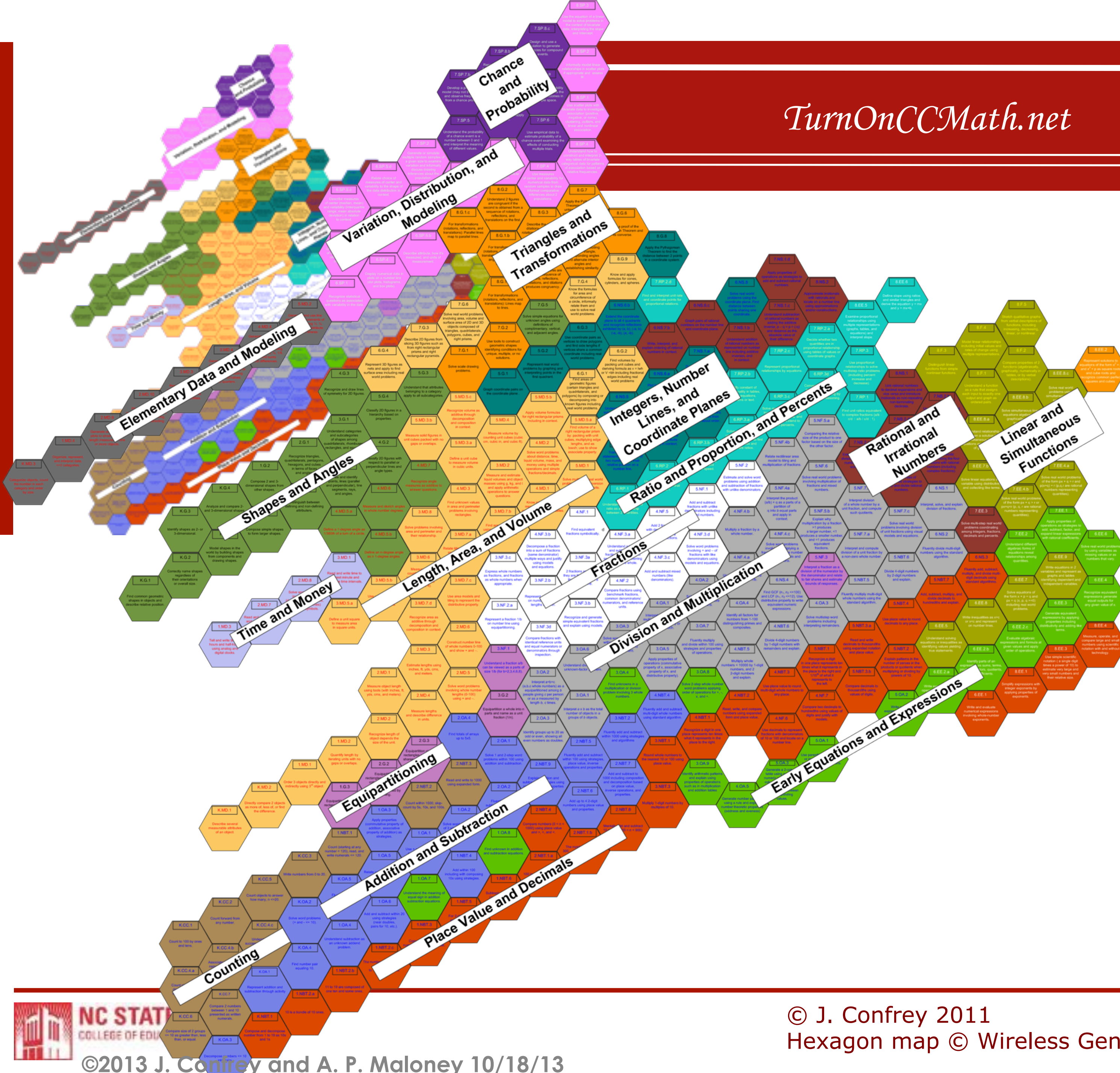
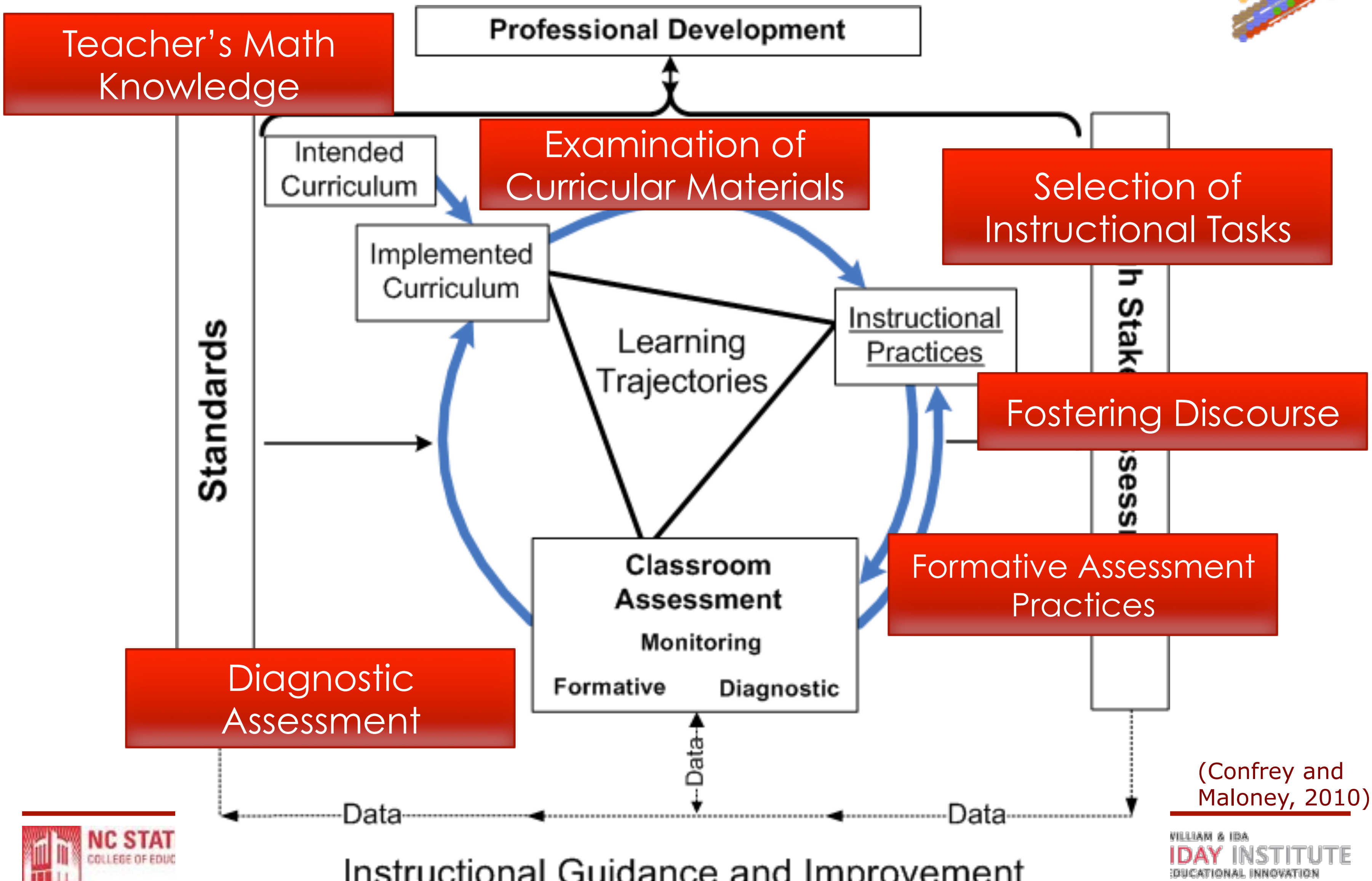


© J. Confrey 2011
Hexagon map © Wireless Generation 2011

Instructional Core—
Learning Trajectories as Boundary Objects



(Confrey and Maloney, 2010)

Project Team (NC State University)

Jere Confrey (NCSU), Principal Investigator (emerita)

Alan Maloney (NCSU), Principal Investigator

- Dicky Ng, Postdoctoral Research Associate
- Nicole Fonger, Postdoctoral Research Associate
- Dung Tran, Postdoctoral Research Associate
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- Ko-Sze Lee, PhD, Postdoctoral Research Associate
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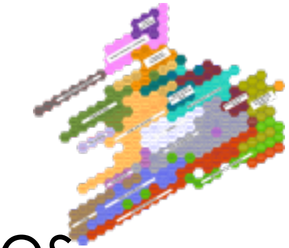
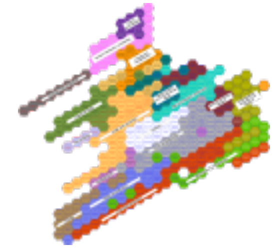
- Kenny Nguyen, PhD (NCSU), Postdoctoral Research Associate (emeritus)
- Drew Corley (NCSU), Graduate Research Assistant (emeritus)
- Shirley Varela, Project Coordinator (emerita)

Session Overview



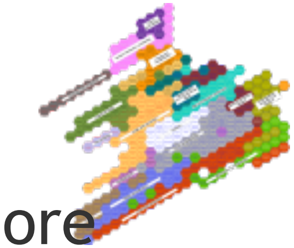
- The CCSS-M context: oh them challenges
- Making sense of the CCSS-M as a teacher: Why Learning Trajectories
- TurnOnCCMath Hexagon Map and Descriptors
- Teacher Experiences with LTs and TOCC
- MOOC-Ed initiative

Significance of the Common Core



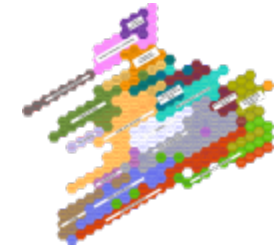
- Expectations--and opportunities--for higher level reasoning and skills
- Structured to incorporate student learning research and to support learning trajectories/progressions
- Economies of scale: *coherence* of standards *across* states, w.r.t. curriculum, assessment, PD
- May buffer some of the effects of educational dislocation due to student mobility
- May turn out to support equity *and* customization
- (digital learning and use of real-time data: increasing, and eventually in all schools)

This is NOT Business-as-Usual



1. Standards are more demanding; new and/or more intensified topic treatment, pacing
2. Requires teaching focus on conceptual learning instead of procedural instruction
3. To assess more complex reasoning: New assessments, new problem types
4. Continuing urgency to address student performance gaps (disequities of various kinds)
5. New technologies and expanded data gathering and access transform instruction, and career and college expectations

More demanding standards and new topics

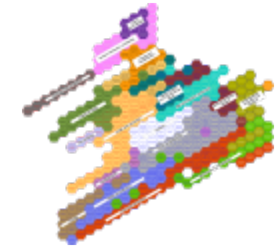


A. Eight Mathematical Practices made explicit: the *doing* of mathematics

B. Changes in Content and Grade Expectations (K-5, 6-8, 9-12): Earlier (or Later), and <u>More conceptual, and more demanding</u>

C. (Interdisciplinary Content: Reading and Writing in Science and Technical Subjects)

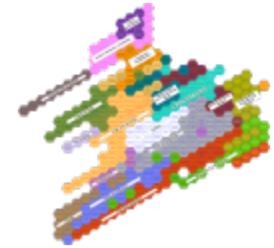
Mathematical Practices



- Practices must be interwoven with content. They must be addressed in tandem.

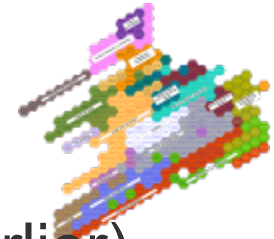
- Practices may provide a way to observe classrooms to see how the content is made understandable, challenging and engaging to students, even for observers with no expertise in mathematics instruction.

Changes by Grade Bands: grades K-5



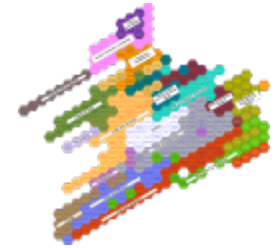
- Numeration and operation intensified, and introduced earlier
  - Early place value foundations in grade K
  - Regrouping as composing / decomposing, in grade 2
  - Decimals to hundredths in grade 4
- All three types of measurement *simultaneously*
  - Non-standard, English and Metric
- Emphasis on fractions as *numbers*
- Emphasis on number line as visualization / structure

Changes by Grade Bands: grades 6-8



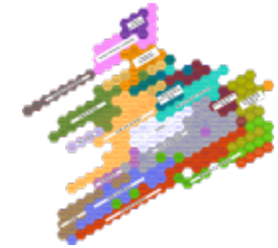
- Ratio and Proportion focused on in grade 6, 7 (not earlier)
  - Ratio, unit rates, converting measurement, tables of values, graphing, missing value problems
- Percents introduced in grade 6
- Statistics introduced in grade 6:
  - Statistical variability (distribution; measures of central tendency; measure of variation: interquartile range, mean and absolute deviation; data shape)
- Rational numbers in grade 7
- Grade 8: One-third of high-school algebra for all students

Changes by Grade Bands: grades 6-8



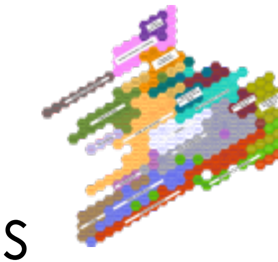
- Much higher expectations at middle grades,
  - 4th to 8th grade NAEP: where greatest performance drop-off occurs.
  - Collectively, our student performance, are weakest in middle grades
  - Very little time to repeat topics each school year.
- Much less elementary preparation for major topics in middle grades (despite research): ratio, statistics, probability
- Much more pressure on elementary school teachers to “get the job done.”
- (Greater need to understand student learning, in order to support, improve, and accelerate it…)

Summary of Changes



- Many changes in timing and intensity of topics.
- Greater explicit emphasis on conceptual understanding and mathematical practices
- New assessments will test more complex reasoning.
- Middle grades will be critical to student success.
- Elementary grades instruction critical to [--everything!—but most immediately--] middle grades preparation and success.
- Specificity of standards should support improved equity of learning (if adequate opportunities to learn)
- Central hosting, technology services models, and diversity of content offers many opportunities and pitfalls. *But it is happening*.

Session Overview



- The CCSS-M context: oh them challenges
- **Making sense of the CCSS-M as a teacher: Learning Trajectories**
- TurnOnCCMath Hexagon Map and Descriptors:
  - Unpack the Standards
  - Meld Learning Trajectories and the Common Core
  - Design, Structure, and Content
  - Examples from several LTs: LAV, VDM, RPP
- Teacher Experiences with LTs and TOCC
- Resources to Come at TOCC

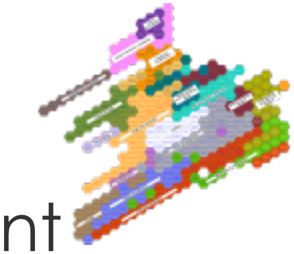
The Job: *Meaningful* implementation of the CCSS-M. The Goal: major improvements in student reasoning and achievement across the grades.

This is a *generational project*. And a legacy to young teachers.

But How to Make Sense of the CCSS-M to support fundamental improvement in student conceptual learning and reasoning? How t--

- How to **Support Continuity**—year-to-year
- How to deal with Grain Size Variation in Standards?
- How to **Interpret**: Read? Visual Representation? Study?
- Inservice teachers: Transition? “Step up”? Retrofit?
- Preservice teachers: Begin teaching careers with the CCSS-M *assumed* (and what does this mean?)

Our Goals and Commitment



- Research←→Practice: Understanding Student Learning
- Help educators make sense of the CCSS-M, using Learning Trajectories
- Resources that educators can turn to and dive deeper with time
- Multiple levels: website, presentations, powerpoints, MOOCs
- (Set out groundwork for pre-service teachers)
- Continual Improvement: Analytics and Your Assistance
- Community of Support

Learning Trajectories: what?

To learn mathematics learning is *not* synonymous with the structure of the discipline of mathematics.

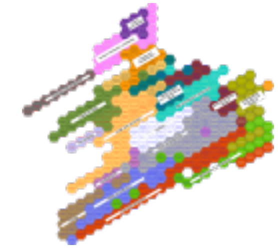
To Connect (a) research on student learning of mathematics to (b) mathematics instruction.

Potential for improved instructional planning, anticipation of student strategies, representations, and misconceptions

CCSS-M built to incorporate learning progressions:

"One promise of common state standards is that over time, they will allow research on learning progressions to inform and improve the design of Standards to a much greater extent than is possible today." CCSS 2010, p.5

Learning trajectory/progression--



…a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e. activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time

(Confrey et al., 2009)

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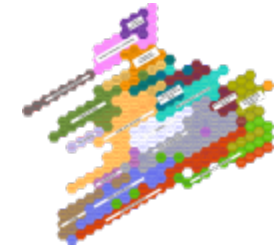
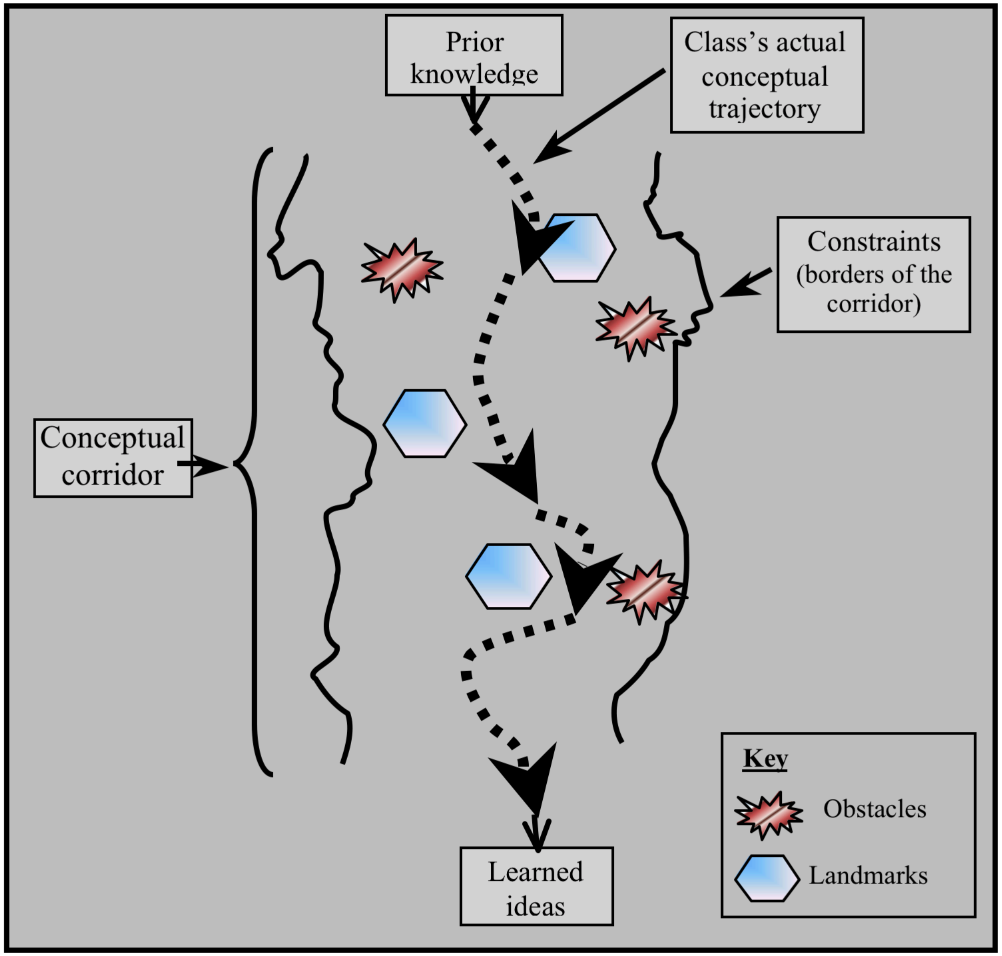
Learning trajectory/progression--



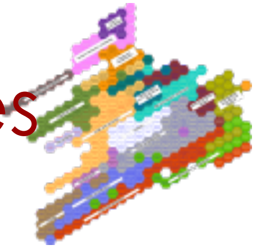
In brief--Research-based descriptions of student learning, especially their conceptual development, through instruction, across time (years)

Learning Trajectory.
within a Conceptual Corridor



Confrey (2006) Design Studies Chapter, Cambridge Handbook of the Learning Sciences

Underlying Features of Learning Trajectories

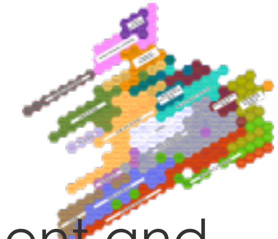


1. Emphasize big ideas that develop gradually over time (across grades)

2. Describe the transition from students' *prior knowledge* to more sophisticated target understandings (*domain goal understanding*)

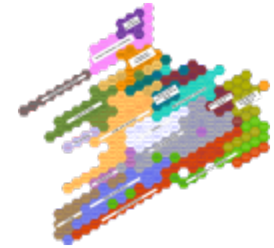
3. Identify intermediate understandings, indicate how they can contribute to growth in conceptual understanding, in order to recognize and build on these

Value of Learning Trajectories --to Teachers



- Support understanding of student conceptual development and intermediate understandings
- Identify clusters of related concepts at grade level
- Unifying theory of student learning in the domain
- Suggest rich uses of classroom assessment
- Clarify what to expect about students' preparation from last year, and what will be expected of your students next year.
- Support managing the range of preparation and needs of students—the more you understand about *student* learning of concepts and skills, the more readily you can identify tasks and discourse that supports improved proficiency
- Support cross-grade (vertical) instructional collaboration and coordination

Session Overview



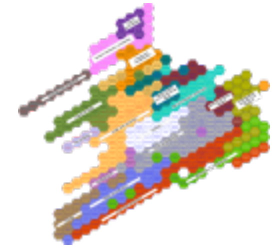
- The CCSS-M context: oh them challenges
- Making sense of the CCSS-M as a teacher: Why Learning Trajectories
- **TurnOnCCMath Hexagon Map and Descriptors:**
  - **Unpack the Standards**
  - **Meld Learning Trajectories and the Common Core**
  - **Visualize Continuity and Structure in the CCSS-M**
  - **Design, Structure, and Content**
  - **Examples from several LTs: EQP, DVM, LAV, VDM, RPP**
- Teacher Experiences with LTs and TOCC
- Resources to Come at TOCC

Standards, as seen in CCSS-M Document

- [1.] Grade
  - [2.] **Domains**: larger groups of related standards. Standards from different domains may sometimes be closely related
    - [3.] **Clusters** of groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
      - [4.] **Individual standards** that define what students should understand and be able to do.

CCSSO, 2010

Unpacking the CCSS-M: TurnOnCCMath



Learning Trajectories to Interpret the Common Core

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Hexagon map of CCSS-M

Descriptors

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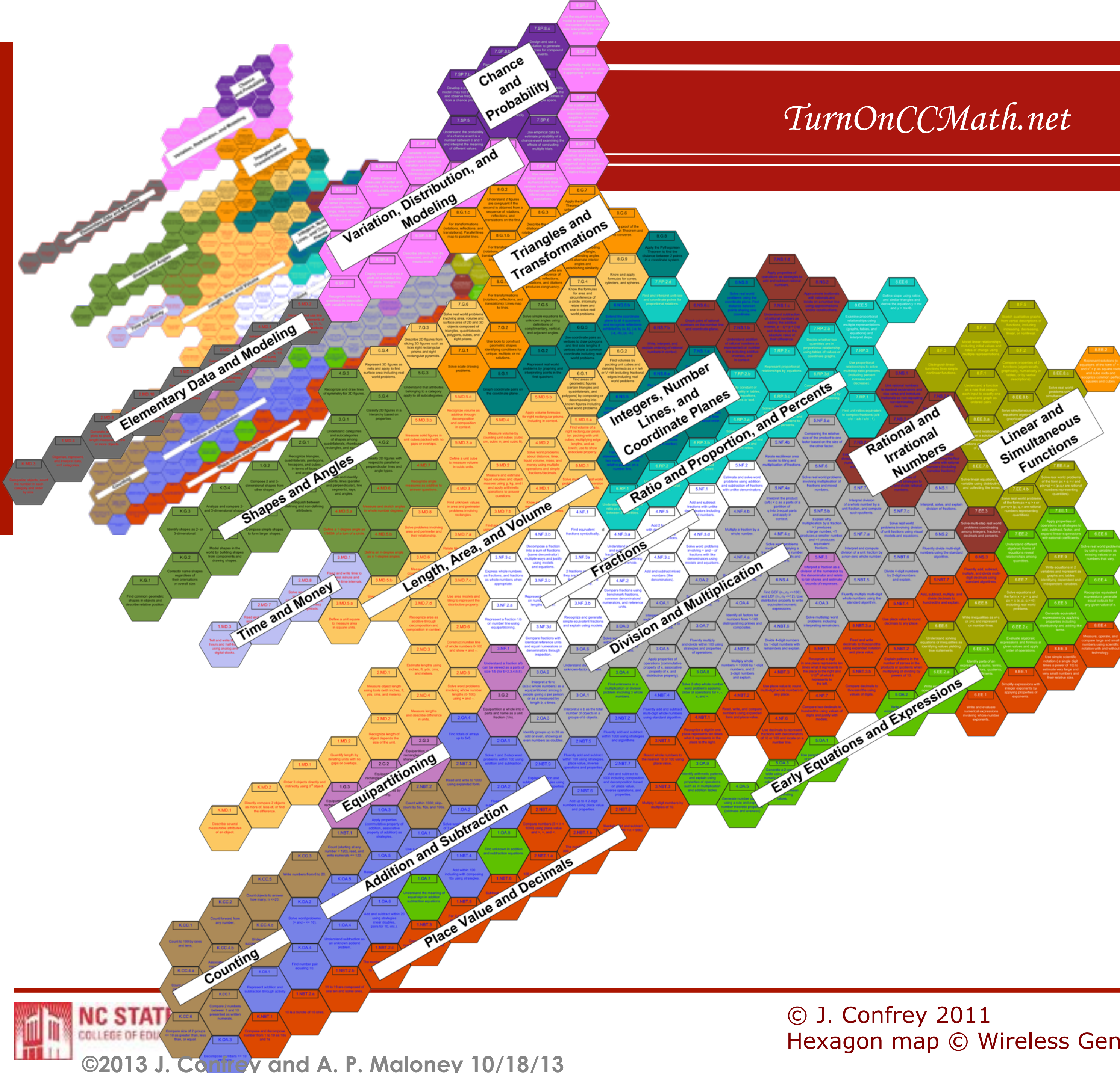
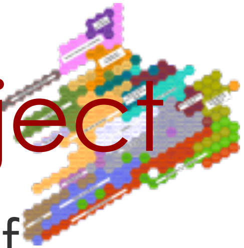
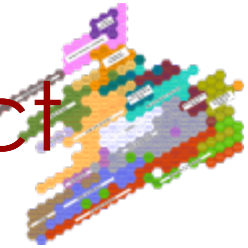


Turn On Common Core Math project



1. Goal: Interpret the CCSS-M from standpoint of student learning.
2. (CCSS-M is NOT a Curriculum) Standards become landmarks within a trajectory of conceptual development
3. Embed the CC Standards within Learning Trajectories to articulate student learning developing over time (across grades) consistent with CCSS-M

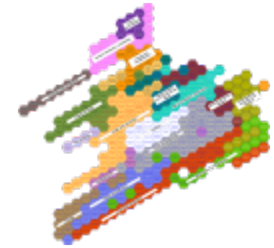
TurnOnCommonCoreMath project



4. A common resource for
   a. teachers,
   b. professional development leaders,
   c. teacher educators,
   d. researchers.

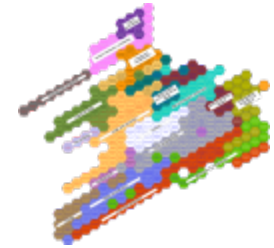
5. Based on:
   a. research synthesis
   b. service on CCSS-M national validation committee
   c. experience revising NC Standards
   d. hexagon representations of state standards (with Wgen)
   e. multiple iterations of standards charts

List of 18 K-8 Learning Trajectories

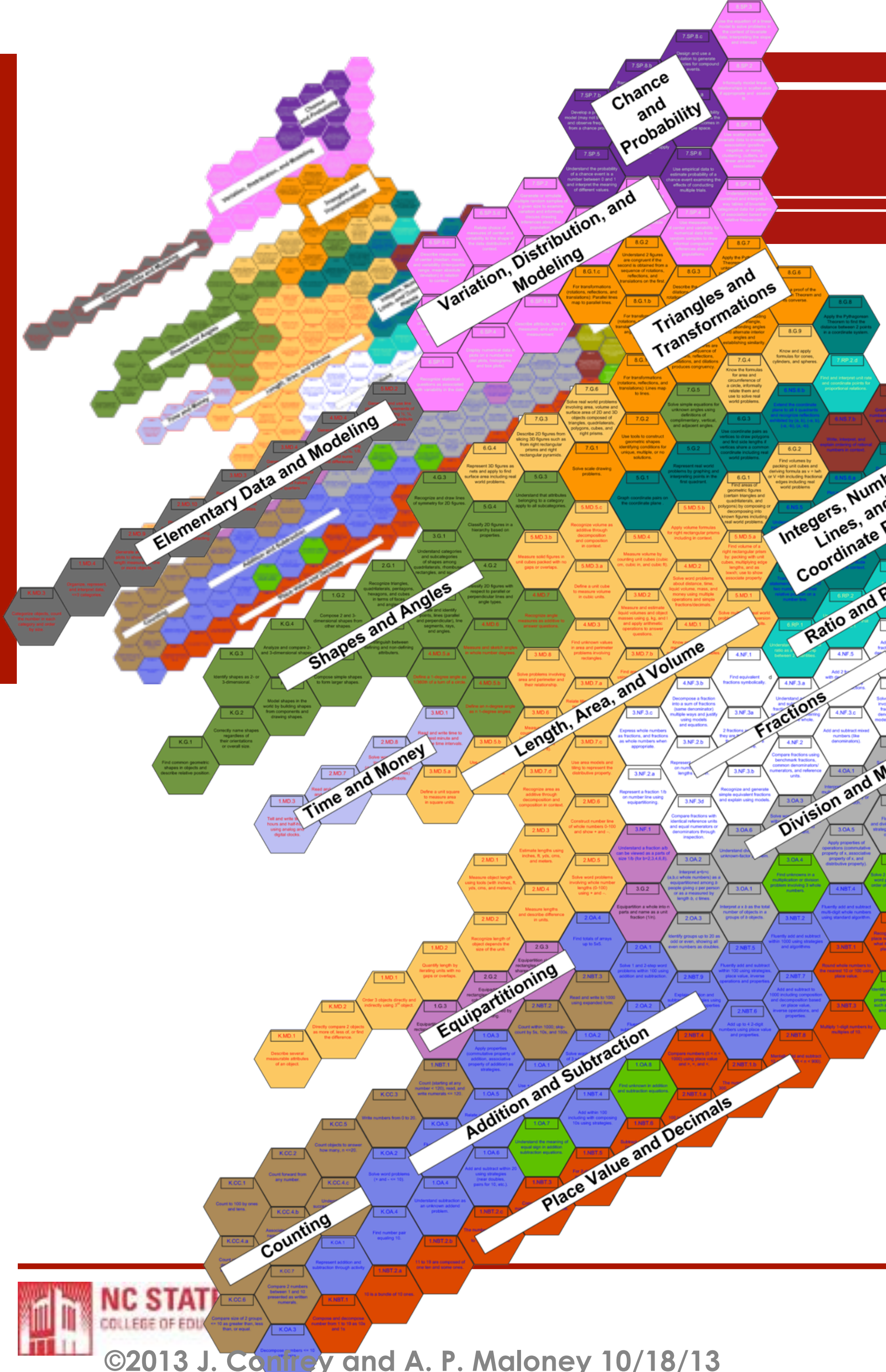


- Counting
- Place Value and Decimals
- Addition and Subtraction
- Equipartitioning
- Time and Money
- Length, Area and Volume
- Fractions
- Multiplication and Division
- Ratio and Proportion, and Percent

List of 18 K-8 Learning Trajectories

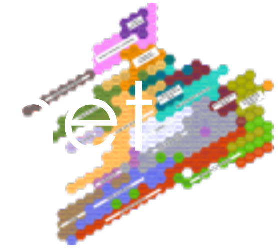


- Shapes and Angles
- Triangles and Transformations
- Elementary Data and Modeling
- Variation, Distribution and Modeling
- Chance and Probability
- Integers, Number lines, and Coordinate Planes
- Rational and Irrational Numbers
- Early Equations and Expressions
- Linear and Simultaneous Functions



CCSS-M Description	Descriptor
K.MD.B Indirectly compare two objects by representing the attribute with, for example, another object and then directly comparing.	This Bridging Standard is introduced here to describe how students' learning of measurements emerges. At the heart of the measurement learning trajectory is the movement from identifying attributes, to representing attributes, directly and indirectly comparing attributes, and finally unitizing attributes using constructed units and wisely choosing common units. [...]
1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.	When two fixed objects cannot be placed adjacently for direct comparison, a third object can be used for indirect comparison. If the third object is longer than the other two, the lengths of the other two can be marked on the third object and be compared. If the length of the third object falls between the other two, then ordering is established. If the third object is shorter than the other two, students would need to develop a way to begin a measurement process (see the Standard 1.MD.2 later). Note that this standard is slightly more advanced than the earlier Bridging Standard K.MD.A in which, for example, strips were created to represent the lengths of two objects. In this standard only one mediating object is used for comparison.

Quick Tour--



1. Two map views : grade level and LT
2. One standard per hexagon (to the #.AB.#.a level)
3. Hexagon: abbreviated text of standard, full-text visible
4. Descriptors:
    1. Structural Overview
    2. Full text of standard
    3. Extended discussion of standard's content and implications for student learning
    4. Bridging standards
    5. Refers to supporting LTs and Standards

Unpacking the CCSS-M: LT Descriptor Elements

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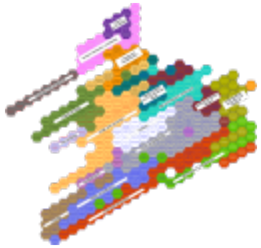
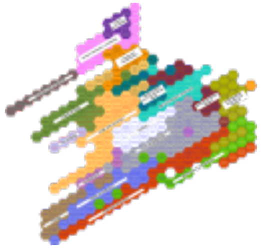


1. Underlying cognitive or conceptual principles (big ideas)
2. Student strategies, representations (inscriptions), and misconceptions
3. Mathematical distinctions and multiple models
4. Coherent structure
5. Bridging standards

Unpacking the CCSS-M: Descriptor Elements

1. **Underlying Cognitive or Conceptual Principles:** components of cognitive framework for making meaning; “big ideas”
2. **Student Strategies, Inscriptions (Representations), and Misconceptions:** how students make their reasoning and intermediate understandings visible
3. **Mathematical Distinctions and Multiple Models:** emerging distinctions, and models for reasoning, that support increasingly sophisticated and nuanced building of the big ideas
4. **Coherent Structure**: recurring themes or frameworks for reasoning, which can be fostered deliberately in instruction to support student investigation and reflection.
5. **Bridging Standards:** identify intermediate understandings, address CCSS-M grain size variations, and signal major instructional gaps that might not otherwise be addressed, for student progress and transitions.

LT Descriptors: Elements



1. **Underlying cognitive or conceptual principles (big ideas):** components of cognitive framework for making meaning; “big ideas”
2. Student strategies, representations (inscriptions), and misconceptions
3. Mathematical distinctions and multiple models
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5. Bridging standards

Equipartitioning

Multiple Wholes

Single (divisible) Wholes

Co-Splitting

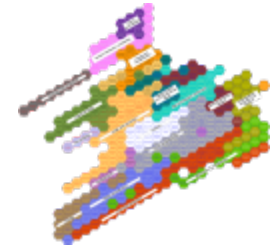
Evenly Divisible Collections

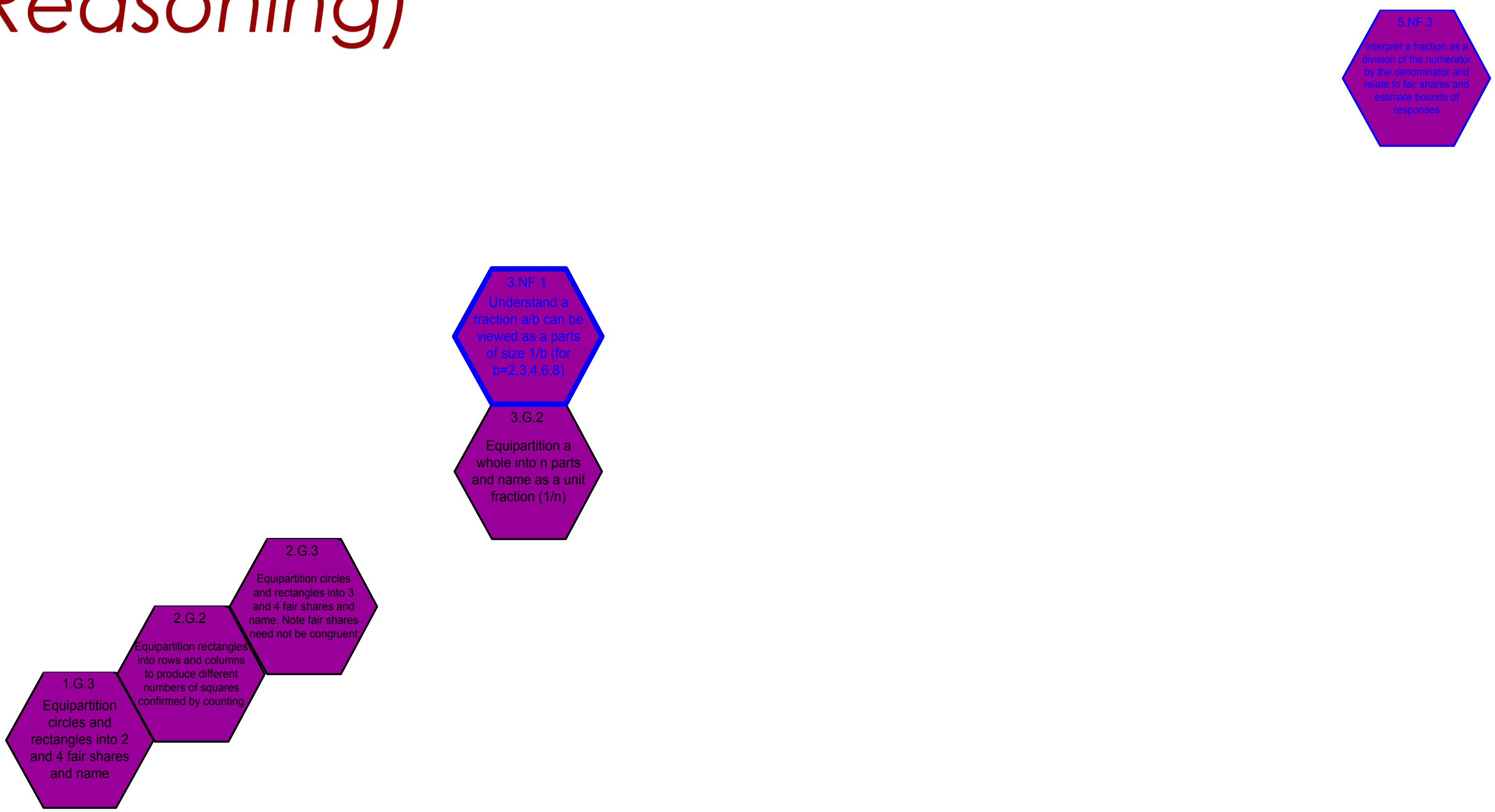
…developed from student learning research on splitting, sharing, one-to-one correspondence, and fractions, including Confrey, Pothier and Sawada, Mitchelmore, Bryant, Pitkethly, Davis, Lamon, and others.

K	1	2	3	4	5

GRADES

Equipartitioning LT: A Common Foundation for moving around in multiplicative space (Rational Number Reasoning)

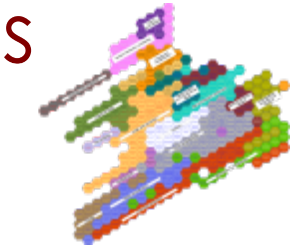




Equipartitioning/Splitting Construct

- Equipartitioning/Splitting: cognitive behaviors that have the goal of producing equal-sized <u>groups</u> (from collections) or <u>pieces</u> (from continuous wholes) as "*fair shares*" for each of a set of individuals.
- Equipartitioning/Splitting is <u>not</u> breaking, fracturing, fragmenting, or segmenting in which there is the creation of unequal parts.
- Equipartitioning/Splitting is the foundation of division and multiplication, ratio, rate, and fraction.
- Concept draws on student experience with fair sharing, and develops gradually over multiple years.

Cognitive and conceptual principles (Equipartitioning LT)



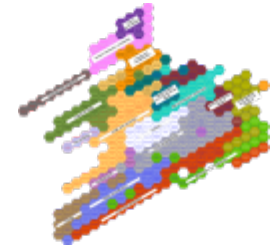
"Cases" of Equipartitioning: task contexts in which fair sharing is developed and analyzed by students: collections, single wholes, multiple wholes.

Proficiencies in splitting (proto-division) into shares and reassembly (inverse, proto-multiplication) of shares are distinct but related for collections and single wholes. Reasoning from both cases is brought to bear in equipartitioning multiple wholes.

PEEQ: Property of Equality of Equipartitioning: Congruent wholes, split in different directions but with the same splitting number: equal (not congruent) parts from the two wholes

Co-Splitting: A foundation for *ratio*: Performing the same split on each of two quantities (objects and sharers) to create two new quantities with the same fair share relationship.

Complementary Approaches to Unpacking the CCSS-M



1. Underlying cognitive and conceptual principles

2. **Student strategies, representations (inscriptions), and misconceptions:** how students make their reasoning and intermediate understandings visible

3. Mathematical distinctions and multiple models

4. Coherent structure

5. Bridging standards

Variation, Distribution, and Modeling

Bivariate Data, Scatter Plots and Basic Linear Regression

Comparing Two Data Sets

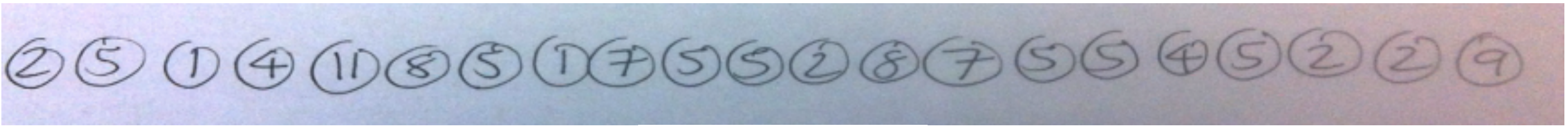
Sampling And Early Inference

Describing the Distribution of a Set of Data

6	7	8

GRADES

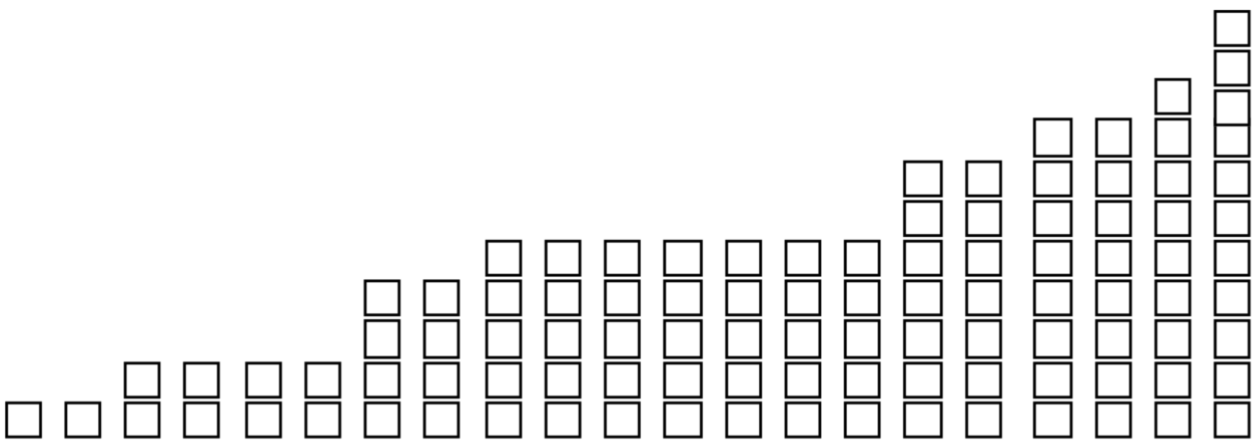
Student Inscriptions of Data



Case plot

1 1 2 2 2 2 4 4 5 5 5 5 5 5 5 7 7 8 8 9 11

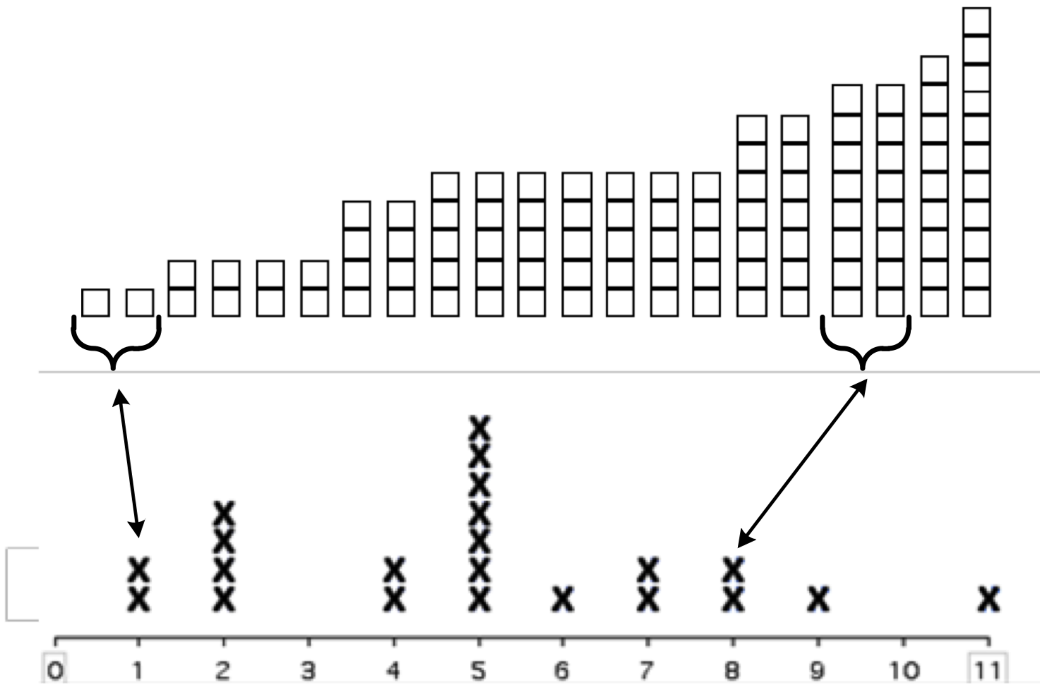
Ordered Cases



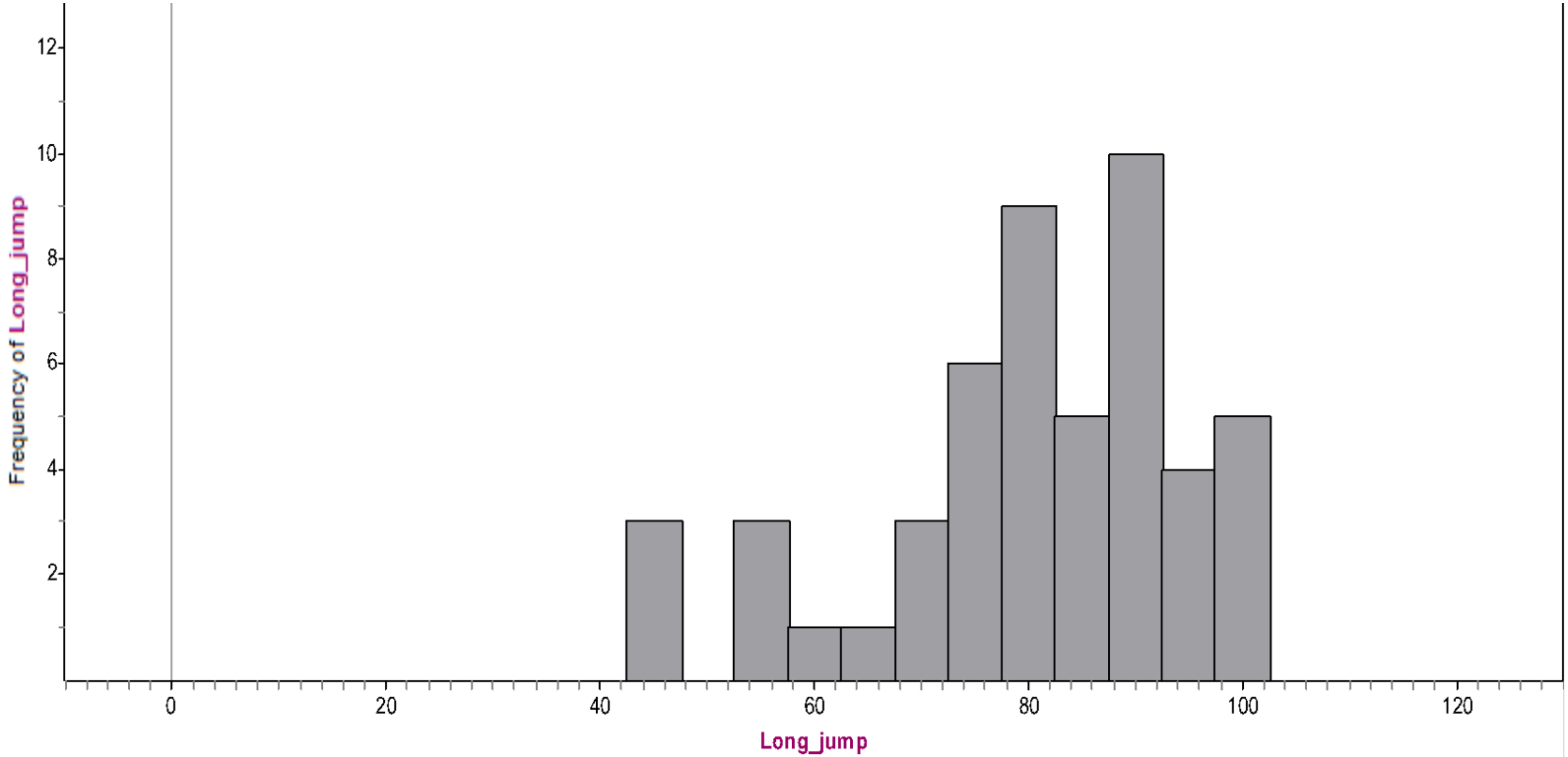
Case Value Plot

Dot Plots

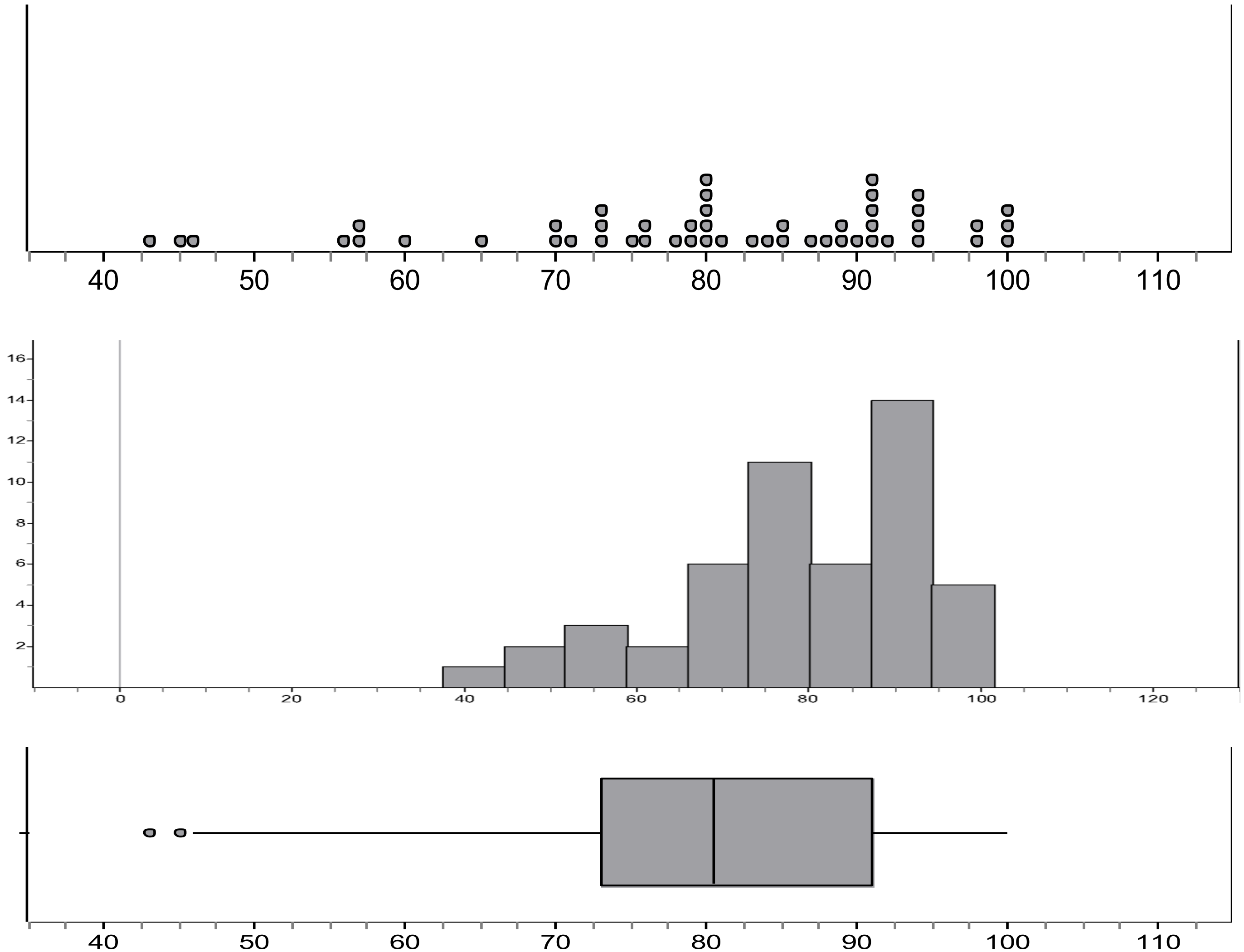
Movement from a case value plot to a dot plot



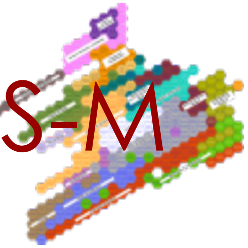
Distributions of Data



Distributions of Data



Elements of LTs in Unpacking CCSS-M



1. Underlying cognitive and conceptual principles
2. Student strategies, representations (inscriptions), and misconceptions
3. **Mathematical distinctions and multiple models**
   emerging distinctions, and models for reasoning, that support increasingly sophisticated and nuanced building of the big ideas
4. Coherent structure
5. Bridging standards

Division and Multiplication

Fractions

Multi-digit Whole Numbers

Factors and Multiples

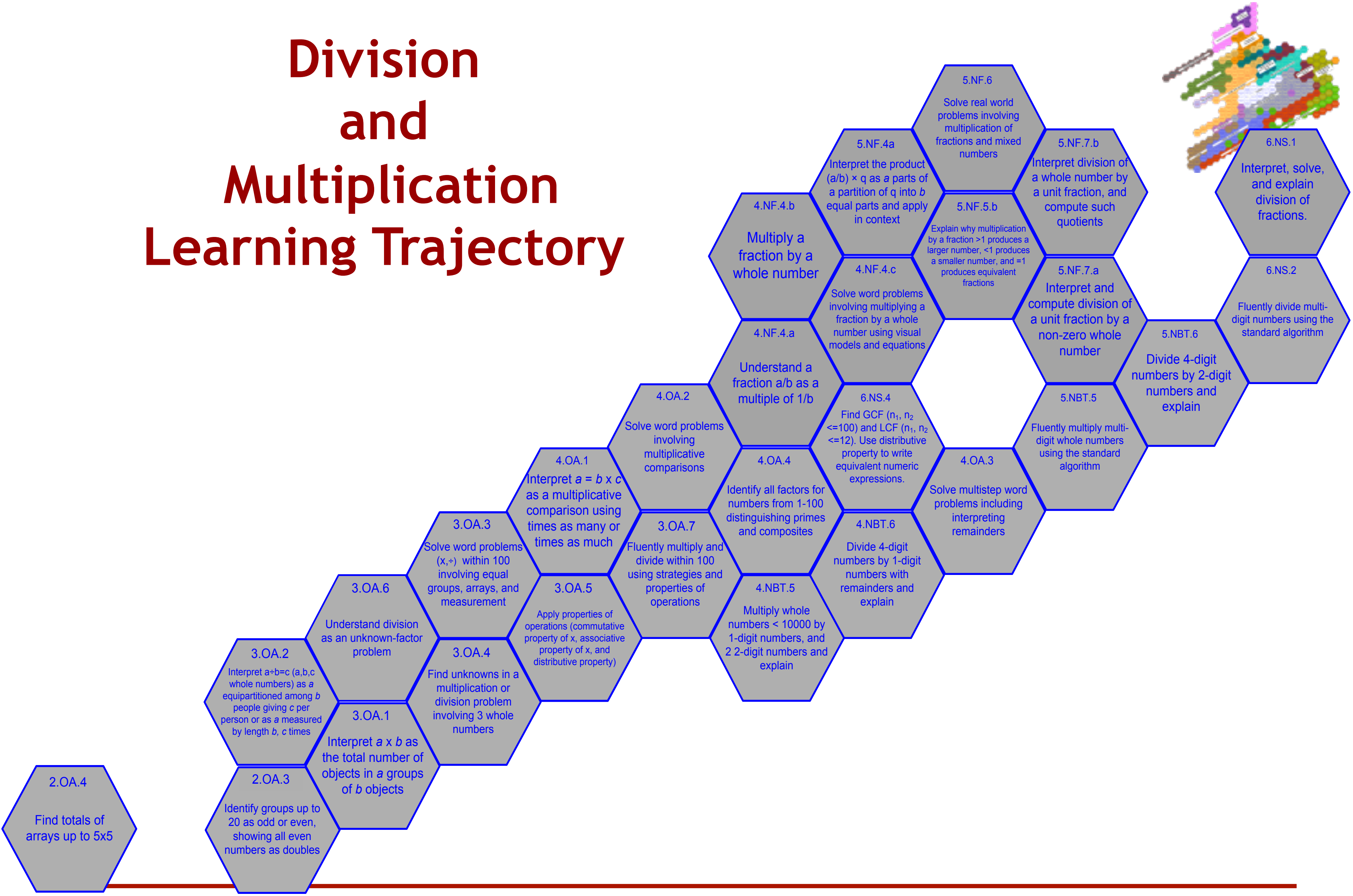
Problem Types,
Properties and Strategies

Models of Division and
Multiplication

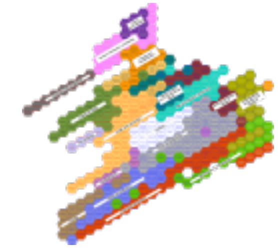
2	3	4	5	6

GRADES

Division and Multiplication Learning Trajectory



Three Word Problem Types



- Two dimension situations (two unit types):
  - Equipartitioning/Sharing: numbers of items, persons, number PER person
  - Rate: miles, time, miles PER hour
- One-dimension situations (one unit type):
  - Quotative/Measurement: How many groups of items,
  - Scaling: how much bigger or smaller (multiplicative compare)
- New dimension and new unit situations
  - One unit type → a second unit type: linear x linear →area.
  - Two different units → a third completely different unit: Cartesian products
  - Arrays as transitional strategies

Mathematical Distinctions and Multiple Models: Division and Multiplication LT

Problem types as *models* for division and multiplication, characterized with respect to changes in the referent (unit) in the operation.

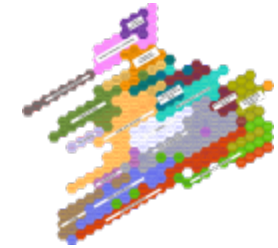
Model 1: Referent-Transforming

Model 2: Referent-Preserving (scaling)

Model 3: Referent-Composing

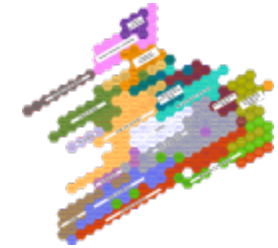
Referent: the quantity to which the numerical values refer (unit) (reinterprets these terms as used by Schwartz, 1988)

Division/Multiplication Models 1 through 3, summary



Model 1: Referent-Transforming	Model 2: Referent-Preserving	Model 3: Referent-Composing
Fair sharing	Unit conversion	(Arrays)
Rate	Scaling	Area
Equal Groups		Cartesian Product

Complementary Approaches to Unpacking the CCSS-M



1. Underlying cognitive and conceptual principles

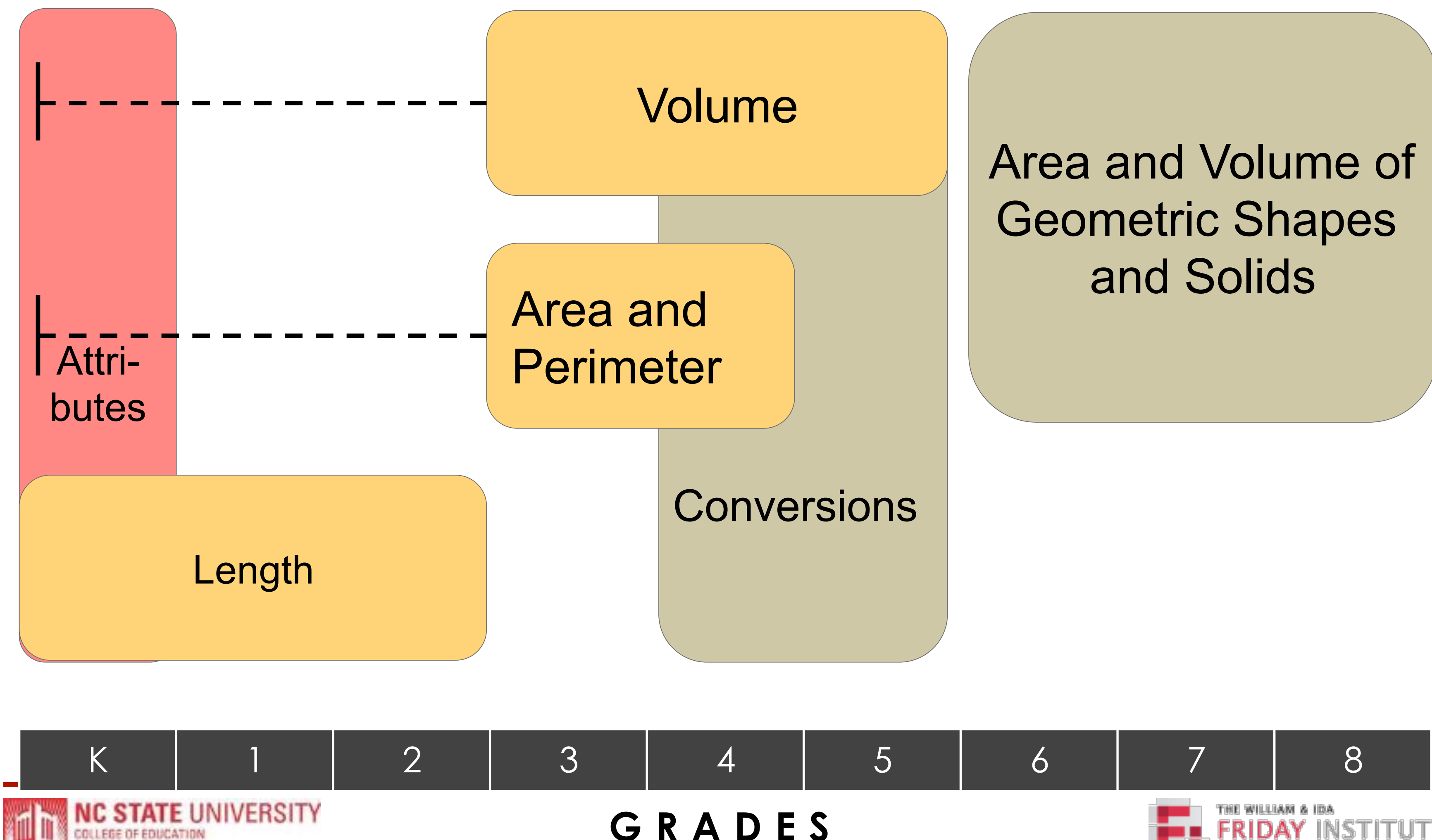
2. Student strategies, representations (inscriptions), and misconceptions

3. Mathematical distinctions and multiple models

4. **Coherent structure:** recurring themes or frameworks for reasoning, which can be fostered deliberately in instruction to support student investigation and reflection.

5. Bridging standards

Length, Area and Volume



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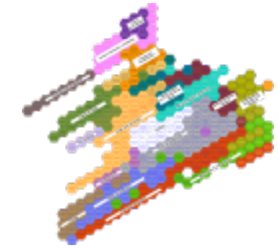
COLLEGE OF EDUCATION

THE WILLIAM & IDA FRIDAY INSTITUTE FOR EDUCATIONAL INNOVATION

Coherent Structure for Mathematical Reasoning: Length, Area, and Volume learning trajectory



Additive principle

Compensatory principle

Measure with no-gaps or overlaps

Indirectly compare

Directly compare

Conservation Principle

Define attributes

Increasing sophistication of reasoning

These form a module that is accomplished by students first for *length*, then *area*, then *volume*.

Additive principle

Compensatory principle

Measure: no gaps or overlaps

Indirectly compare

Directly compare

Conservation Principle

Define attributes

Length

Multiplicative principle

Additive principle

Compensatory principle

Measure: no gaps or overlaps

Indirectly compare

Directly compare

Conservation Principle

Define attributes

Area, Volume

Variation, Distribution, and Modeling

Bivariate Data, Scatter Plots and Basic Linear Regression

Comparing Two Data Sets

Sampling And Early Inference

Describing the Distribution of a Set of Data

6	7	8

GRADES

Student strategies, representations (inscriptions), and misconceptions: Variation, Distribution, and Modeling LT

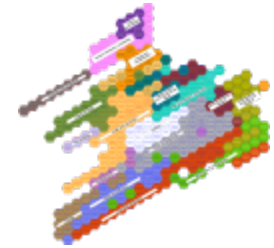
Coherent structure for statistical reasoning

Students engage in investigations that include the four steps in which they:

1. Pose a (meaningful) question
2. Collect relevant data
3. Analyze the data to answer the question
4. Interpret the results

A repeated structure that applies to every level of statistical investigation, from elementary up.

Complementary Approaches to Unpacking the CCSS-M



1. Underlying cognitive and conceptual principles

2. Student strategies, representations, and misconceptions

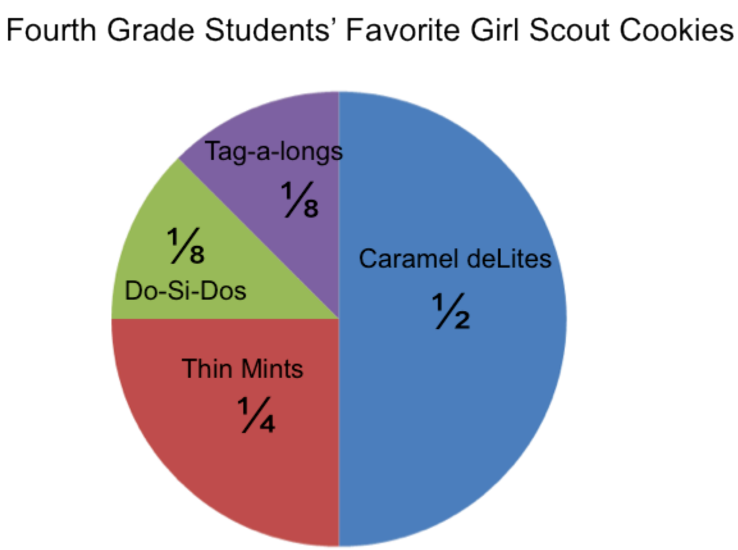
3. Mathematical distinctions and multiple models

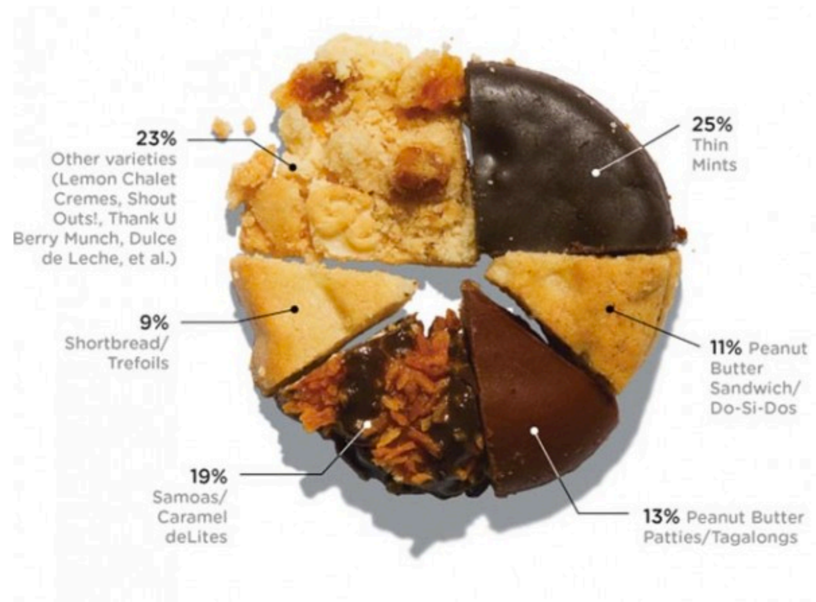
4. Coherent structure

5. **Bridging standards. Examples from Ratio and Proportion, and Percents LT:** identify intermediate understandings, address CCSS-M grain size variations, and signal major instructional gaps that might not otherwise be addressed, for student progress and transitions.

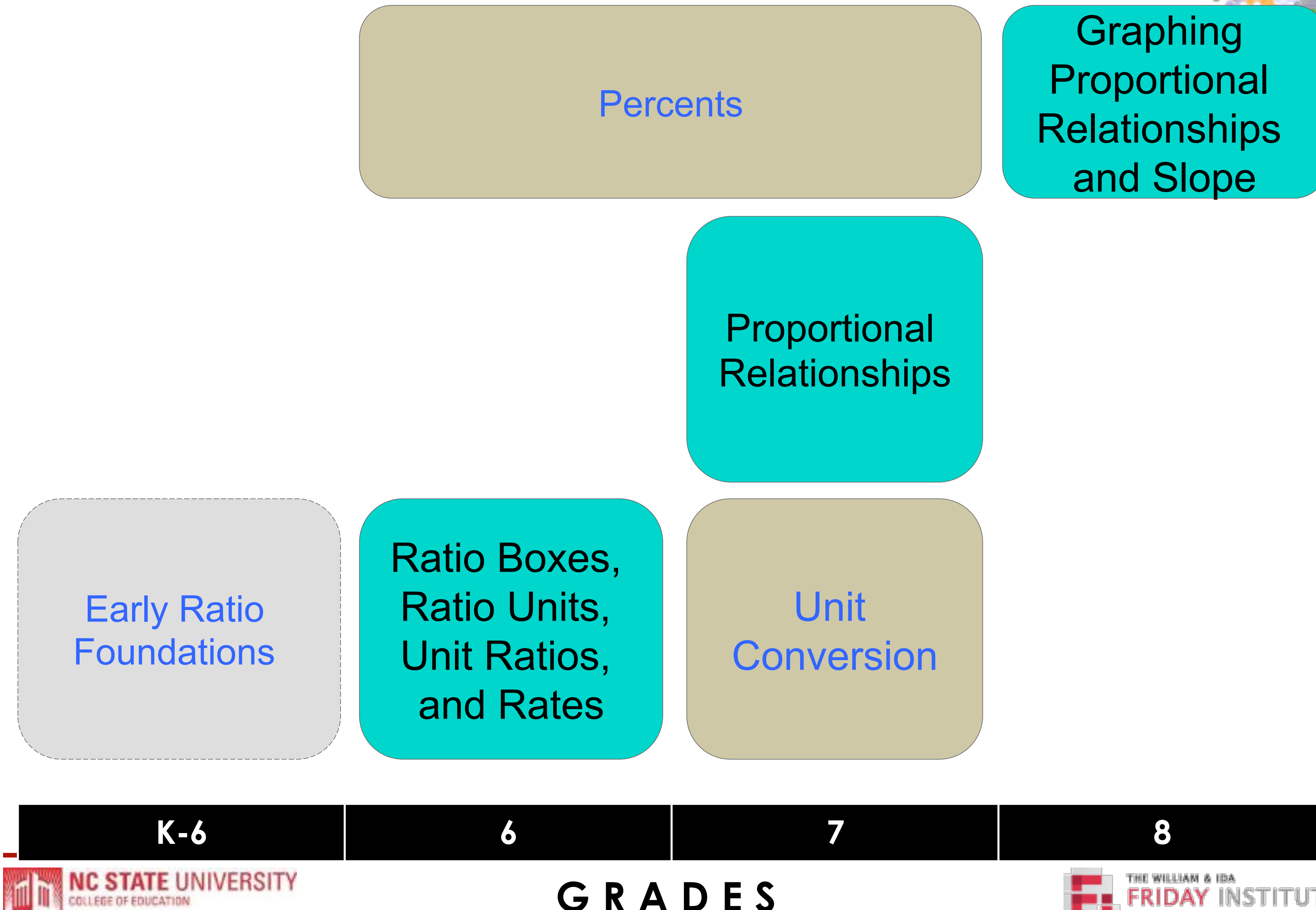
Circle Graphs…

***4.EDM.A** Interpret circle graphs (pie graphs) to map the relative frequency of data to the fractional part of the whole, limited to fractional parts of halves, fourths, and eighths.*

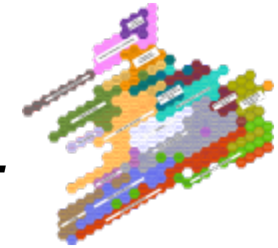




Ratio and Proportion, and Percents



Bridging Standards: Ratio and Proportion, and Percents LT



Section 1: A New Form of Equivalence

Part 1: Using ratio tables

Part 2: Ratio Units and Unit Ratios

Part 3: Graphing

Part 4: Building Up

Part 5: Ratio Boxes

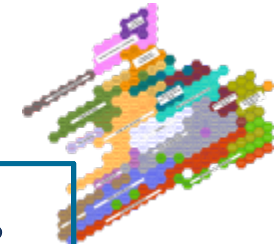
Ratio Boxes, Ratio Units, Unit Ratios, and Rates

Ratio and Proportion, and Percents

***6.RP.1**: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.*

***6.RPP.A**: (Bridging Standard) Express the concept of a ratio using a table of values based on doubling, tripling, etc., and splitting using a variety of factors in both columns of the ratio table.*

Ratio and Proportion, and Percents



***Standard 6.RPP.B** (Bridging Standard): Understand the concepts of ratio unit and unit ratio. Relate these concepts for a given table of values and show them on a graph.*

This bridging standard was added because it is important for students to identify and work with both *ratio units* and *unit ratios* in ratio tables and graphs.

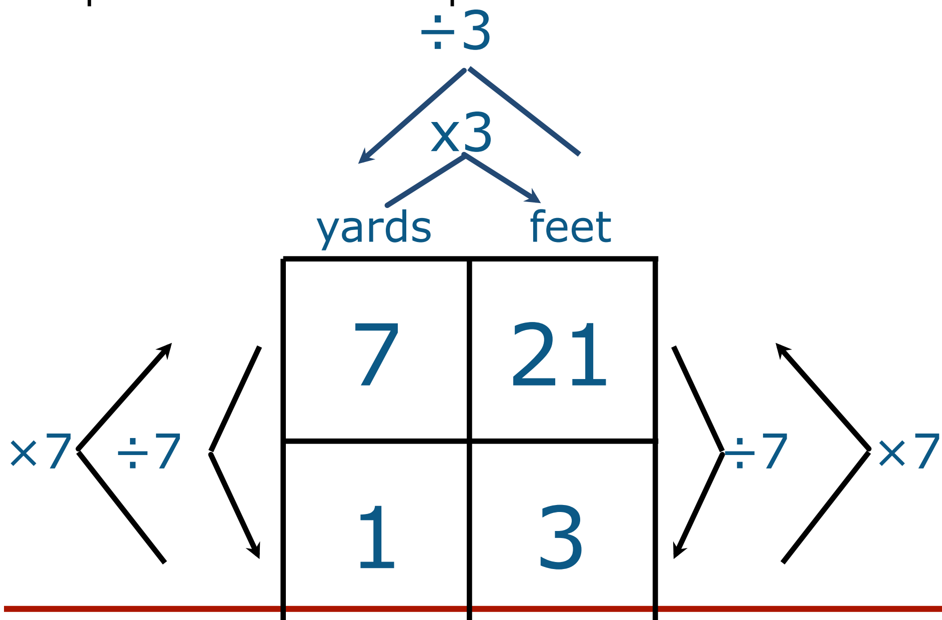
- A *ratio unit*, or "base ratio," is the smallest pair of whole numbers that defines the ratio; "littlest recipe;" smallest whole-number ratio…leads to slope, rational numbers, constant of proportionality

- A *unit ratio* is a ratio in which one of the two values is equal to *one*. 1:*n* OR *n*:1 unit rate; constant of proportionality

Ratio and Proportion, and Percents

***Standard 6.RPP.C** (Bridging Standard): Use a ratio box to describe the relationships and explain how to move multiplicatively: a.) between the two quantities (correspondence), and b.) within the two quantities (covariation).*

This bridging standard was added because it is important for students understand the multiplicative relationships in a ratio box in order to solve missing ratio-value problems later.



<u>Correspondence</u> rule:
the number of feet is 3 times the number of yards; the no. of yards is the no. of feet, div. by 3

<u>Covariation</u> rule:
Each time the number of yards increases by x *n* (or 1/*n*), the number of feet increases by x *n* (or 1/*n*)

Ratio and Proportion, and Percents

***6.RP.D** (Bridging Standard): Distinguish ratio relationships from non-ratio relationships in a ratio box.*

10	6
5	3

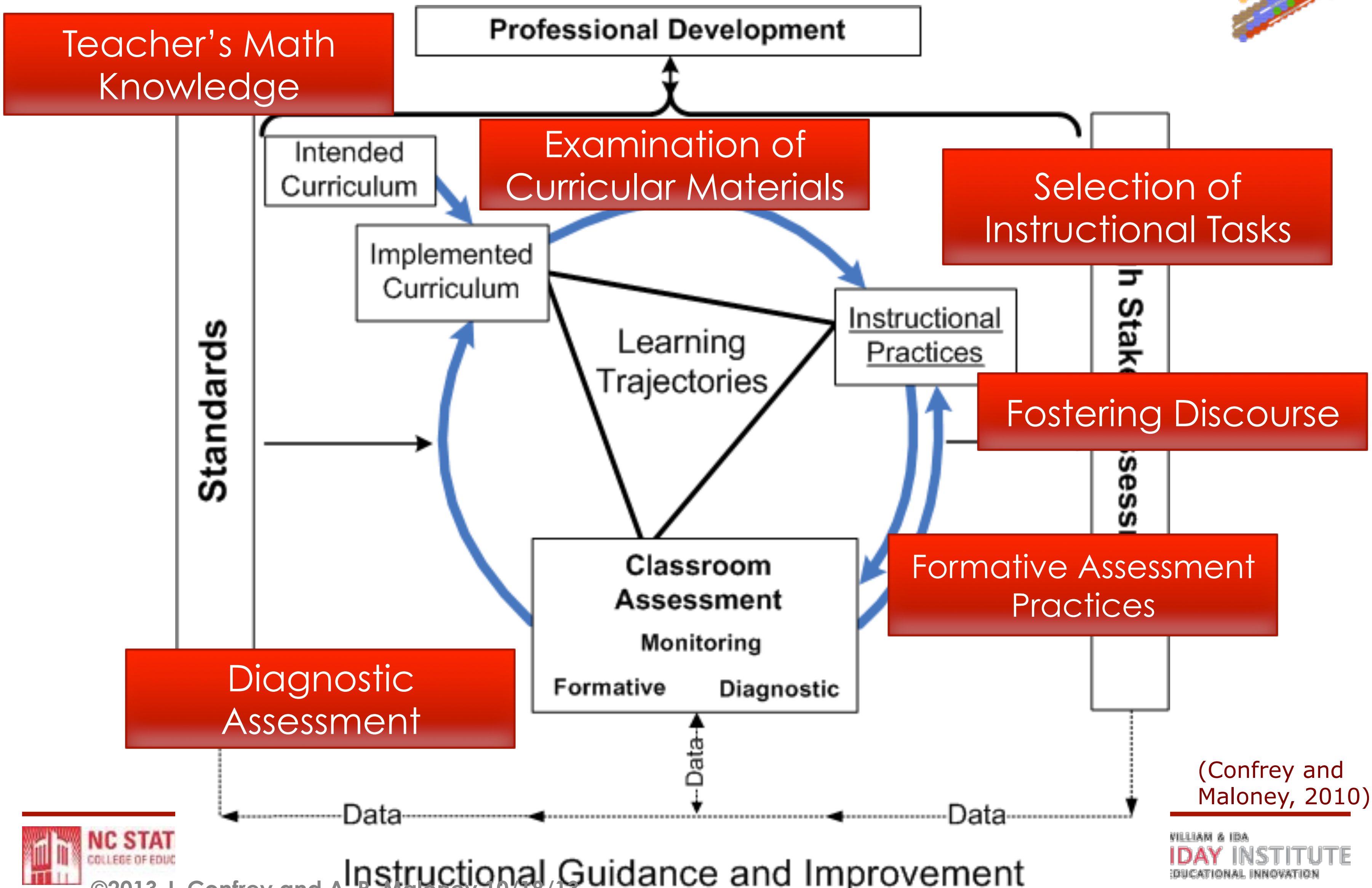
3	5
6	8

5	10
2	1

Session Overview

- The CCSS-M context: oh them challenges
- Making sense of the CCSS-M as a teacher: Why Learning Trajectories
- TurnOnCCMath Hexagon Map and Descriptors:
  - Unpack the Standards
  - Meld Learning Trajectories and the Common Core
  - Design, Structure, and Content
  - Examples from several LTs: LAV, VDM, RPP
- Teacher Experiences with LTs and TOCC
- Resources to Come at TOCC

Instructional Core—
Learning Trajectories as Boundary Objects



(Confrey and Maloney, 2010)

Teacher responses to LTs and TurnOnCCMath

Outcomes

- Increasingly evidence-based, nuanced perceptions of students' work, mathematical distinctions, argument
- Improved mathematical content knowledge(!)
- Teachers reflected on and filled in gaps in their own understanding. LTs helped them make sense of their own and their students' intermediate knowledge, and growth paths.
- The (Common Core) Standards make more sense…
- Hope for supporting student learning, even if saddled with a procedure-heavy curriculum

Challenges:

- Time and Grain Size of substantial PD efforts
  - Different approach to instruction and learning, urgency to implement CCSS, and not-business-as-usual
- Teachers as Learners: not a 'one-shot' process--need iterative work with the CCSS-M to become more expert.

MOOC-Ed Initiative: http://mooc-ed.org

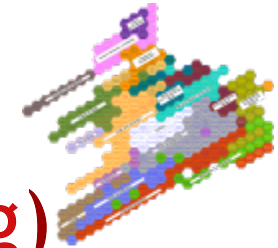
(MOOC: Massive Open Online Courses)

(Ed: for Educators)

- Mathematics Learning Trajectories for the Common Core.

- First course, running now: Equipartitioning
  - Registration open until Monday (or later)
  - Emphasizes cross-grade development of understanding
  - Forums: discussion and task design/adaptation/critique
  - LT-based instruction task design (optional)
  - Participants: teachers, teacher leaders and PD-supervisors, teacher educators. Nationwide.
  - Video based
  - Self-directed and self-paced
  - 3-5 hours per week
  - Certificate of completion
  - Free(!)

Mathematics Learning Trajectories for the Common Core (http://mooc-ed.org)



- Next MOOC-Eds:
  - Division and Multiplication (February 2014)
  - Fractions (June 2014)
  - Ratio and Proportional Reasoning
  - Statistics (6-8)