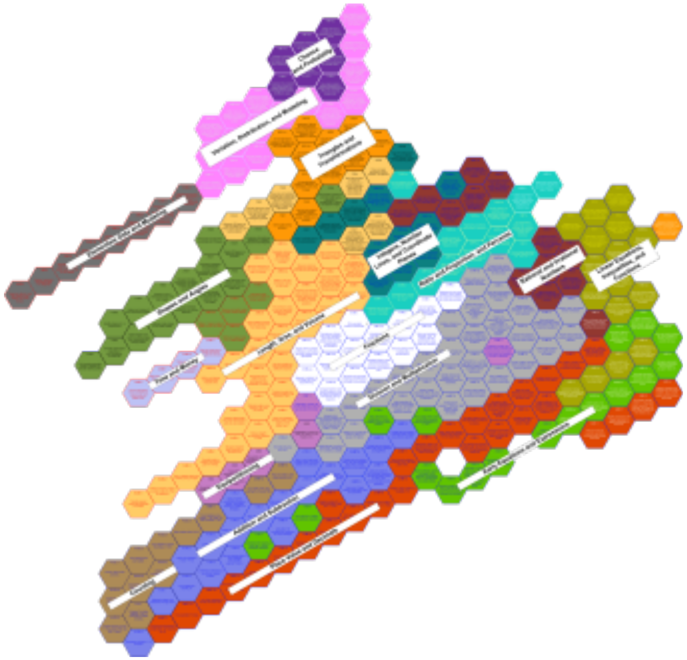


TurnOnCCMath.net

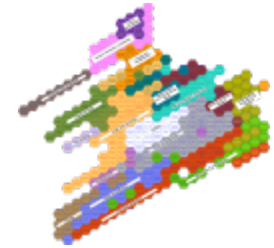
Navigating Middle School Statistics Learning in the CCSS-M

NCTM Regional Conference, Baltimore, MD; October 17-18, 2013

Tamar Avineri **Alan Maloney, Ph. D.**

The Friday Institute for Educational Innovation
College of Education
North Carolina State University

What We Have Planned Today



- [(Discussion of) statistical reasoning challenges woven throughout]
- Statistical Literacy, Learning Trajectories, and the CCSS-M
- TurnOnCCMath.net—quick introduction to the resource
- Highlights of the VDM LT for 6th, 7th, and 8th grade topics
- Summary of Descriptor Elements
- LTs and teachers…

NC STATE UNIVERSITY
COLLEGE OF EDUCATION

10/18/13

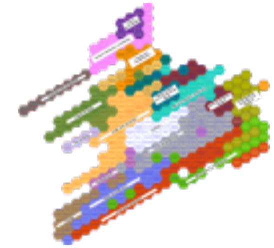
THE WILLIAM & IDA FRIDAY INSTITUTE FOR EDUCATIONAL INNOVATION

The Oldest Person You Know

(Prudential advertisement available on YouTube

http://www.youtube.com/watch?v=C3qj88J7-jA*)*

Statistical Literacy



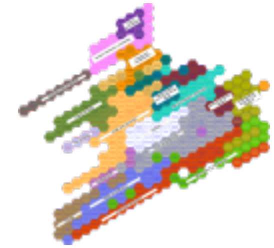
"[Statistical literacy] empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to survive in the modern world."

(Steen, 2001)

"Sound statistical reasoning skills take a long time to develop. They cannot be honed to the level needed in the modern world through one high-school course. The surest way to help students attain the necessary skill level is to begin the statistics education process in the elementary grades and keep strengthening and expanding students' statistical thinking skills throughout the middle- and high-school years."

(GAISE K-12 Report, Franklin et al., 2007, p. 3)

Progression in Statistical Reasoning for the 21st Century



Data and Data Representation, Variation

Distribution (shape, spread, central tendency)

Measures and Informal Inference

Chance and Probability

Association and Bivariate Statistics

Probability--Conditional and Theoretical

Inference--bringing probability and statistics merged

Bivariate relationships in more detail

Deeper into inference and hypothesis testing

Big Ideas in Statistical Reasoning (K-8)

Move from data-as-individual-points to the aggregate level

Students' informal and graphical representations

Expect Variation (and Distribution) in the real world

Types of Variation (measurement and experimental)

Statistical Investigation Cycle

(Randomness)

Distribution: Shape, Center, Spread, Measures

Connections between Sampling and Populations

Distinguish between distributions, sample; and Inference about populations

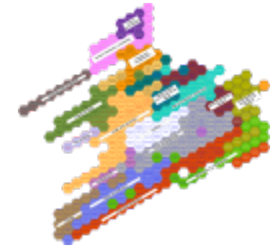
Association

Statistical Investigation Cycle

All student should engage in statistical investigation, which includes four phases:

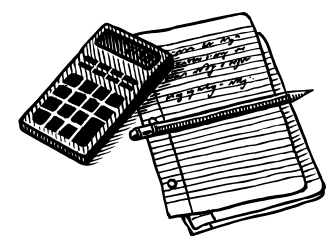
- Formulate questions
- Collect relevant data
- Analyze the data to answer the question
- Interpret results

Statistical Investigation
An example

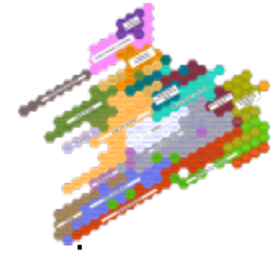


Suppose the students in two classes, as part of the President's Challenge for physical fitness, all participated in a (standing) long-jump competition. Each student then charted the distance of his/her single best jump on a single number line.

- What questions would the class pose to try and answer?
- How would the class configure the data to answer the question?
- How would they analyze the data?
- How would they interpret those results?



The Problematic



- Students typically begin with finding descriptive statistics before they gain statistical reasoning skills.

  "(The knowledge of) computational rules not only does not imply any real understanding of the basic underlying concept, but…may actually inhibit the acquisition of more adequate (relational) understanding."

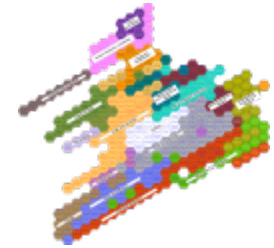
  (Pollatsek, Lima, & Well, 1981, as quoted in Garfield and Ben-Zvi, 2010, p. 189)

- The CCSSM is not curriculum, and does not dictate the *path* of student learning, or pedagogy, to develop proficiencies in statistical reasoning.

  "These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B." *(CCSS-M, p. 5)*

- CCSSM and GAISE call for the teaching of statistical thinking/reasoning to be a steady growth of conceptually-based reasoning beginning in the middle grades and continuing through college.

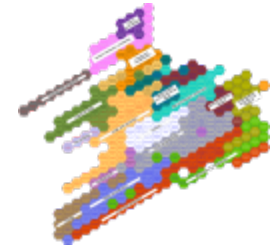
Learning Trajectories



Research-based descriptions of student learning, especially their conceptual development, across time (years)

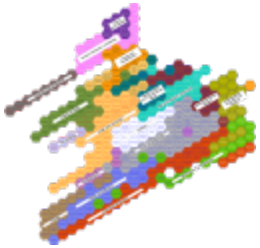
The learning trajectory provides an evidence-based characterization of student growth from informal reasoning/ ideas to structured, more formalized proficiency in statistical modeling/inference.

Underlying Features of Learning Trajectories



- Emphasize big ideas that develop gradually over time
- Describe the transitions from students' prior knowledge to more sophisticated target understandings (domain goal understanding)
- Identify intermediate understandings, how they can contribute to conceptual growth, and ways to recognize and build on these

Show TOCC.net

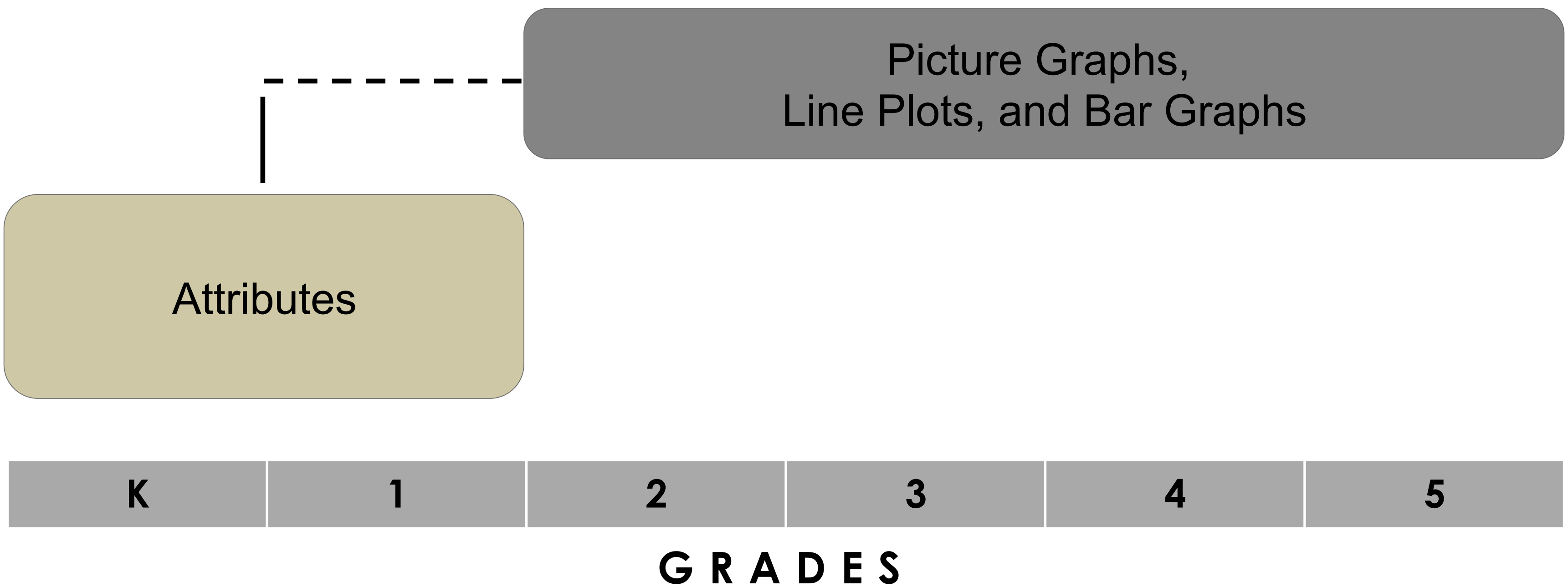


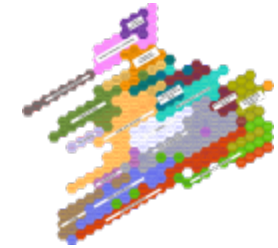
- Quick rundown of purpose of TOCC, navigate to map and LTs

Unpacking the CCSS-M: Descriptor Elements

1. **Underlying Cognitive or Conceptual Principles:** components of cognitive framework for making meaning; “big ideas”
2. **Student Strategies, Inscriptions (Representations), and Misconceptions:** how students make their reasoning and intermediate understandings visible
3. **Mathematical Distinctions and Multiple Models:** emerging distinctions, and models for reasoning, that support increasingly sophisticated and nuanced building of the big ideas
4. **Coherent Structure**: recurring themes or frameworks for reasoning, which can be fostered deliberately in instruction to support student investigation and reflection.
5. **Bridging Standards:** identify intermediate understandings, address CCSS-M grain size variations, and signal major instructional gaps that might not otherwise be addressed, for student progress and transitions.

Elementary Data and Modeling





Variation, Distribution, and Modeling Learning Trajectory

Grades 6-8

Variation, Distribution, and Modeling

Bivariate Data, Scatter Plots and Basic Linear Regression

Comparing Two Data Sets

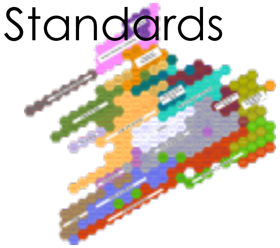
Sampling And Early Inference

Describing the Distribution of a Set of Data

6	7	8

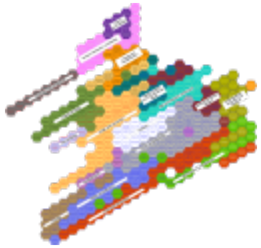
GRADES

Variation, Distribution, and Modeling Learning Trajectory Levels and CC Standards



Describing the Distribution of a Set of Data	Summarize data sets, justify measures w.r.t. investigation context, measurement method/error, measures sensitivity	6.Sp.5.a, b, d
	Summarize numerical data sets (measures, patterns and deviations) in relation to data displays	6.SP.4 Part 2, 6.Sp.5.c
	Measures of variation	6.SP.3
	Measures of central tendency	6.SP.3
	Describe distributions: shape, center, spread	6.SP.2
	Display numerical data, single population, using scaled axis; compare different displays	6.SP.4 Part 1
	Distinguish between statistical and mathematical questions; statistical investigation	6.SP.1
	Recognize and distinguish sources of variation	6.SP.1

Statistical Questions



Deterministic: Find the equation of a line that passes through two given points

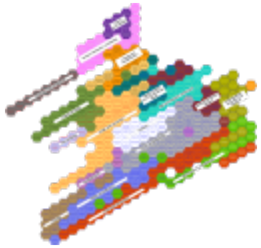
Statistical: How can you describe the relationship between the height and weight of a group of students?

Statistical Questions

"The formulation of a statistics question requires an understanding of the difference between a question that anticipates a deterministic answer and a question that anticipates an answer based on data that vary."

(GAISE K-12 Report, Franklin et al., 2007, p.11)

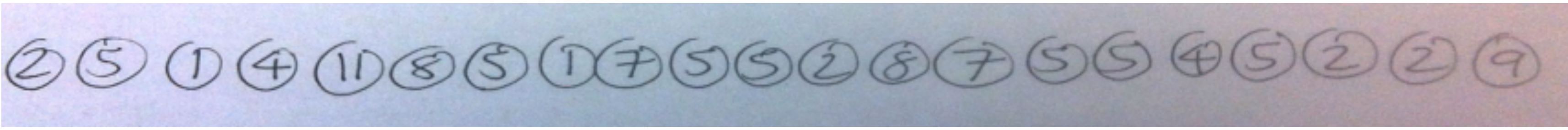
Long Jump Example



- Deterministic Questions

- Statistical Questions

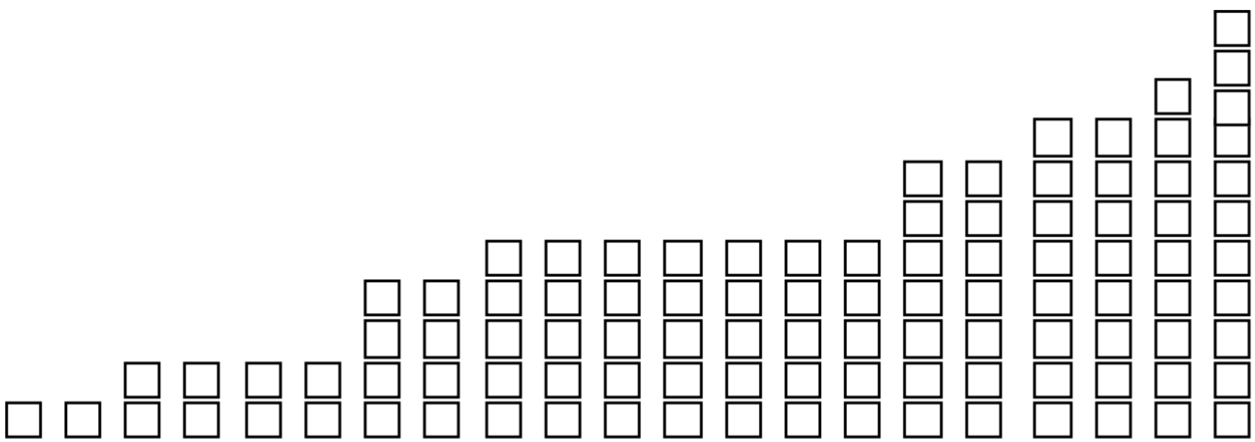
Student Inscriptions of Data



Case plot

1 1 2 2 2 2 4 4 5 5 5 5 5 5 5 7 7 8 8 9 11

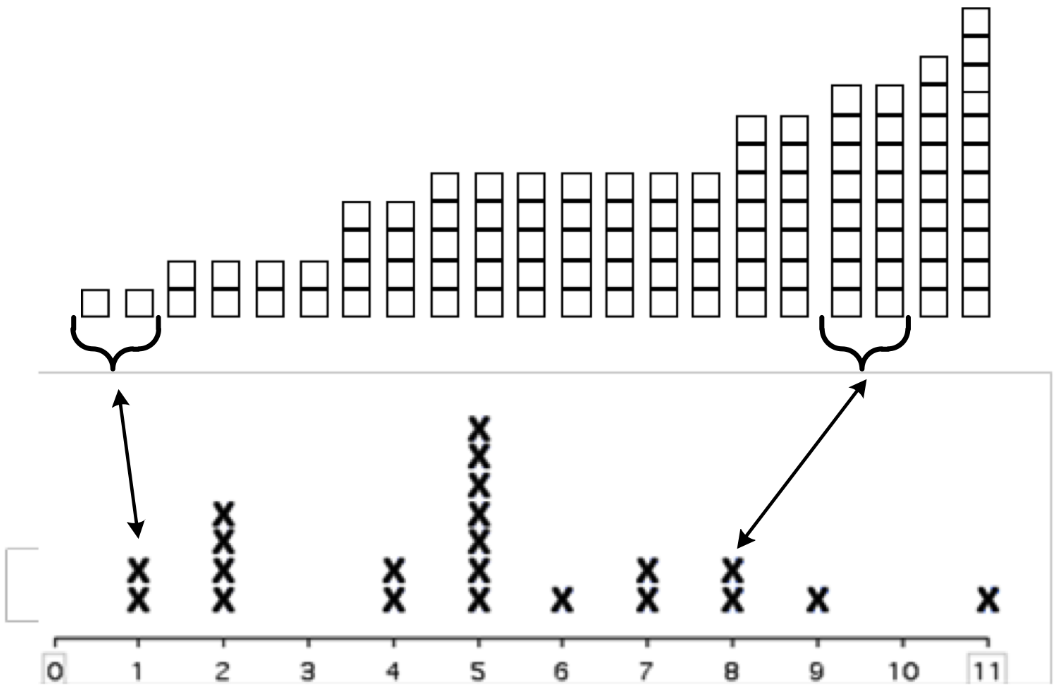
Ordered Cases



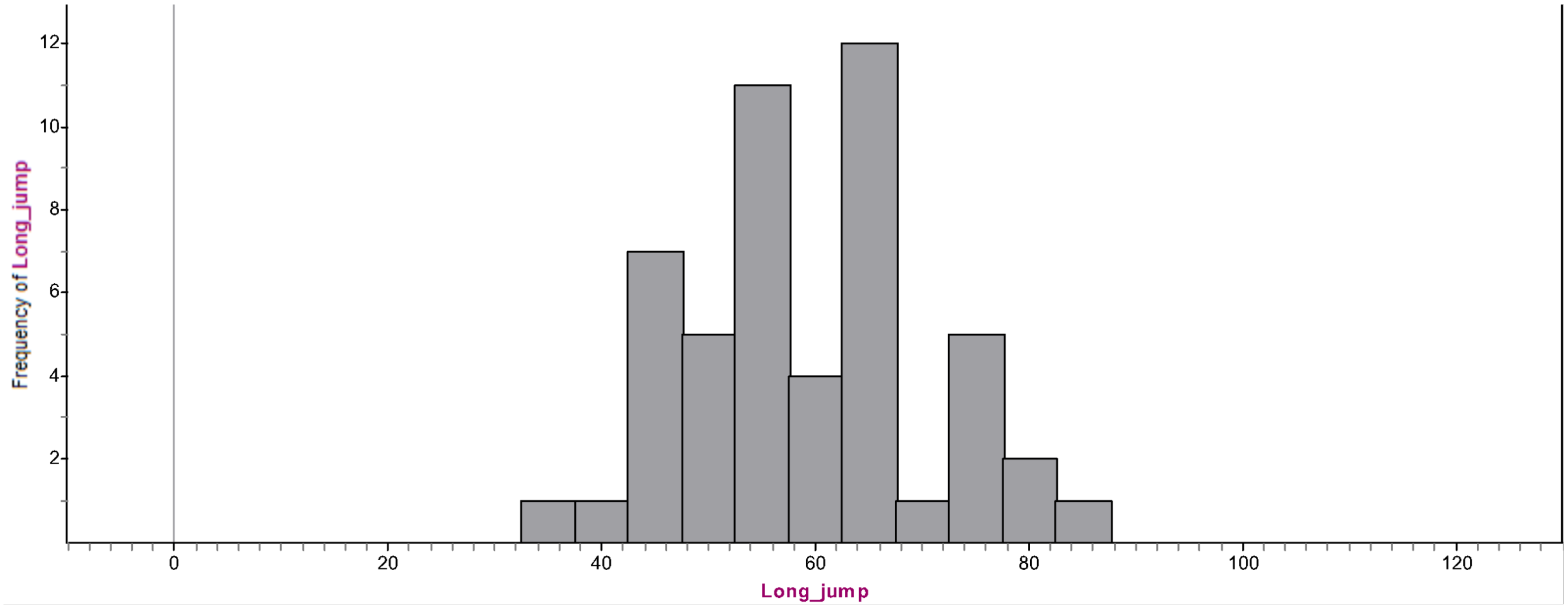
Case Value Plot

Dot Plots

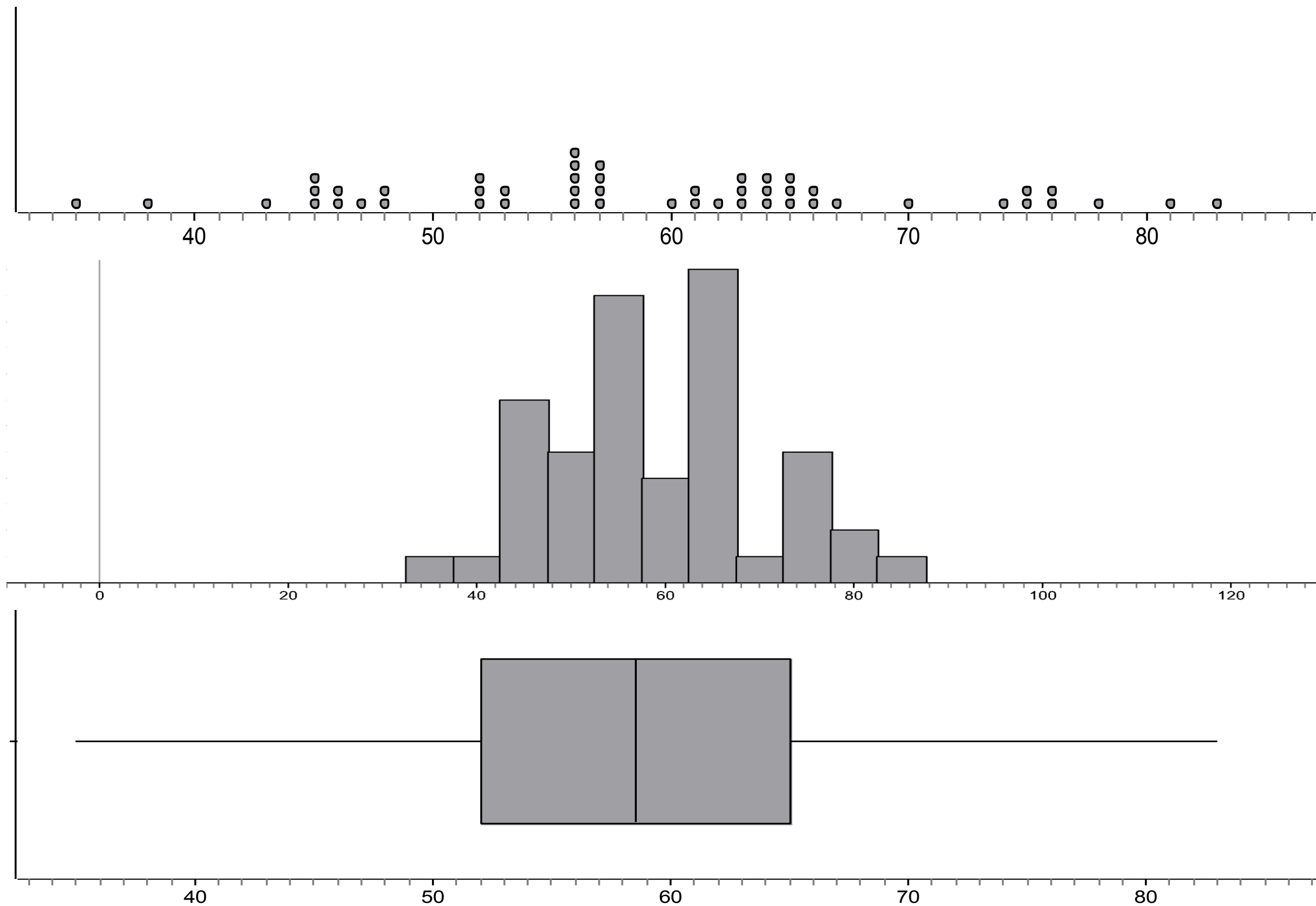
Movement from a case value plot to a dot plot



Distributions of Data



Distributions of Data



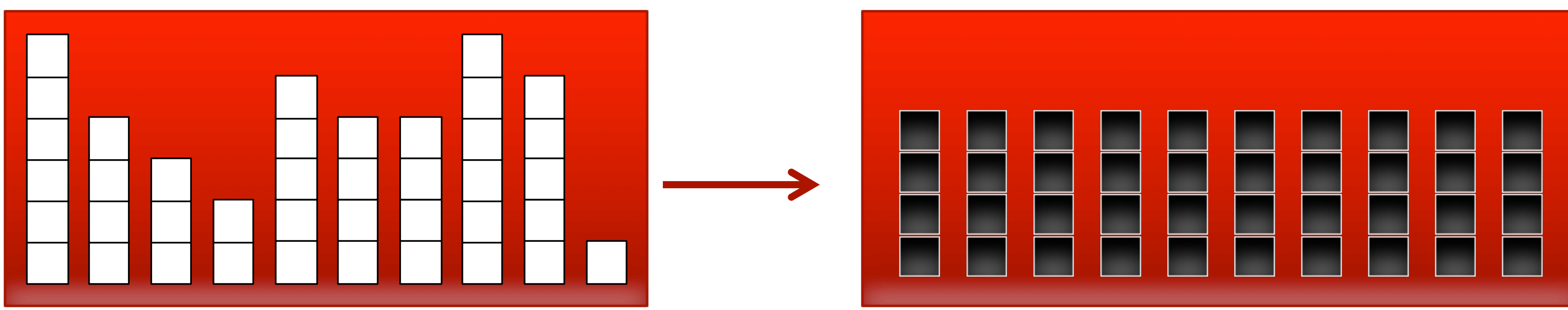
Conceptual Discussion of Central Tendency

In your groups, discuss examples of questions/tasks you've posed for your students that build (on) conceptions of central tendency.

Measures of Central Tendency

Students are asked, “What if we combined all our pies and redistributed them one at time to each of the ten students until all the pies are gone? How many pies would each student get?”

Students redistribute the cubes one at a time until each student has the same number. Each student would get 4 pies.



Variation, Distribution, and Modeling

		Bivariate Data, Scatter Plots and Basic Linear Regression
	Comparing Two Data Sets	
	Sampling And Early Inference	
Describing the Distribution of a Set of Data		
6	**7**	**8**

GRADES

Variation, Distribution, and Modeling Learning Trajectory Levels and CC Standards

Comparing Distributions of Multiple Data Sets	**Draw informal comparative inferences about distributions of different sized samples, using measures of center and variation**	**7.SP.4**
	Draw inferences about differences between distributions using measures of center and variation	**7.SP.3**
	Draw inferences comparing two populations by visually comparing distributions of same-sized samples	**7.SP.3**
Sampling and Early Inference	**Draw inferences about a population, in relation to sample characteristics and sampling method**	**7.SP.2**
	Describe measures of samples in relation to size of sample and sampling method	**7.SP.2**
	Generate samples to represent a population	**7.SP.1**
Describing the Distribution of a Set of Data	Summarize data sets, justify measures w.r.t. investigation context, measurement method/error, measures sensitivity	6.Sp.5.a, b, d
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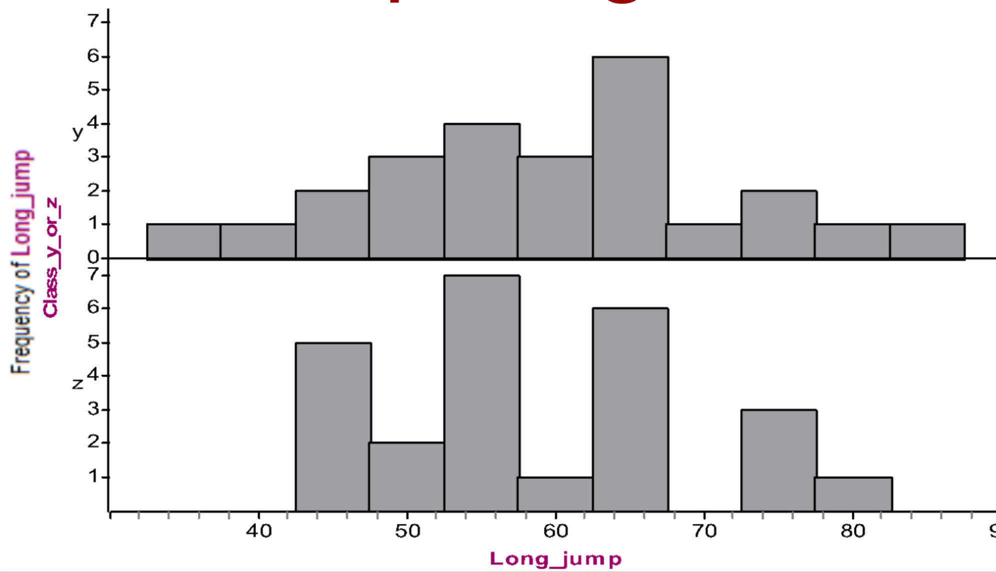
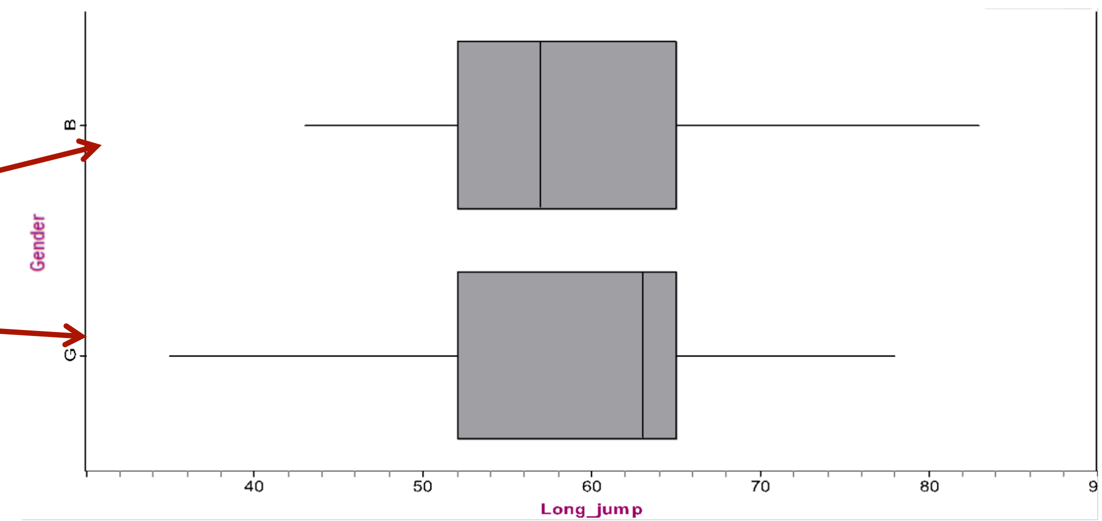
	Distinguish between statistical and mathematical questions; statistical investigation	6.SP.1
	Recognize and distinguish sources of variation	6.SP.1

Comparing Two Distributions



Comparing two 6th grade classes



Comparing the boys and the girls of the combined 6th grade classes (on the same scaled axis)

Variation, Distribution, and Modeling

Bivariate Data, Scatter Plots and Basic Linear Regression

Comparing Two Data Sets

Sampling And Early Inference

Describing the Distribution of a Set of Data

6	7	8

GRADES

Bivariate Data, Scatter Plots, and Basic Linear Regression	**Describe and interpret association of two categorical variables**	**8.SP.4**
	Use linear models to interpret association in continuous bivariate data, and to make predictions	**8.SP.3**
	Model association in continuous bivariate data using straight line, including criteria for best fit of line	**8.SP.2**
	Use scatterplots to display, and qualitatively describe and interpret association in, continuous bivariate data	**8.SP.1**
Comparing Distributions of Multiple Data Sets	Draw informal comparative inferences about distributions of different sized samples, using measures of center and variation	7.SP.4
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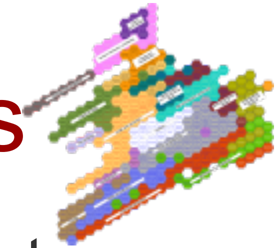
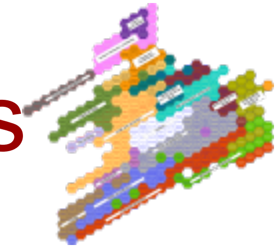
Bivariate Data and Associations



- If you had data sets containing values of two different variables, would you suspect any relationship between the two variables?
- How would you use the data to develop evidence of some kind of relationship between the two variables?
- (How would you conduct a statistical investigation to find out whether your hunch about two features was true…?)

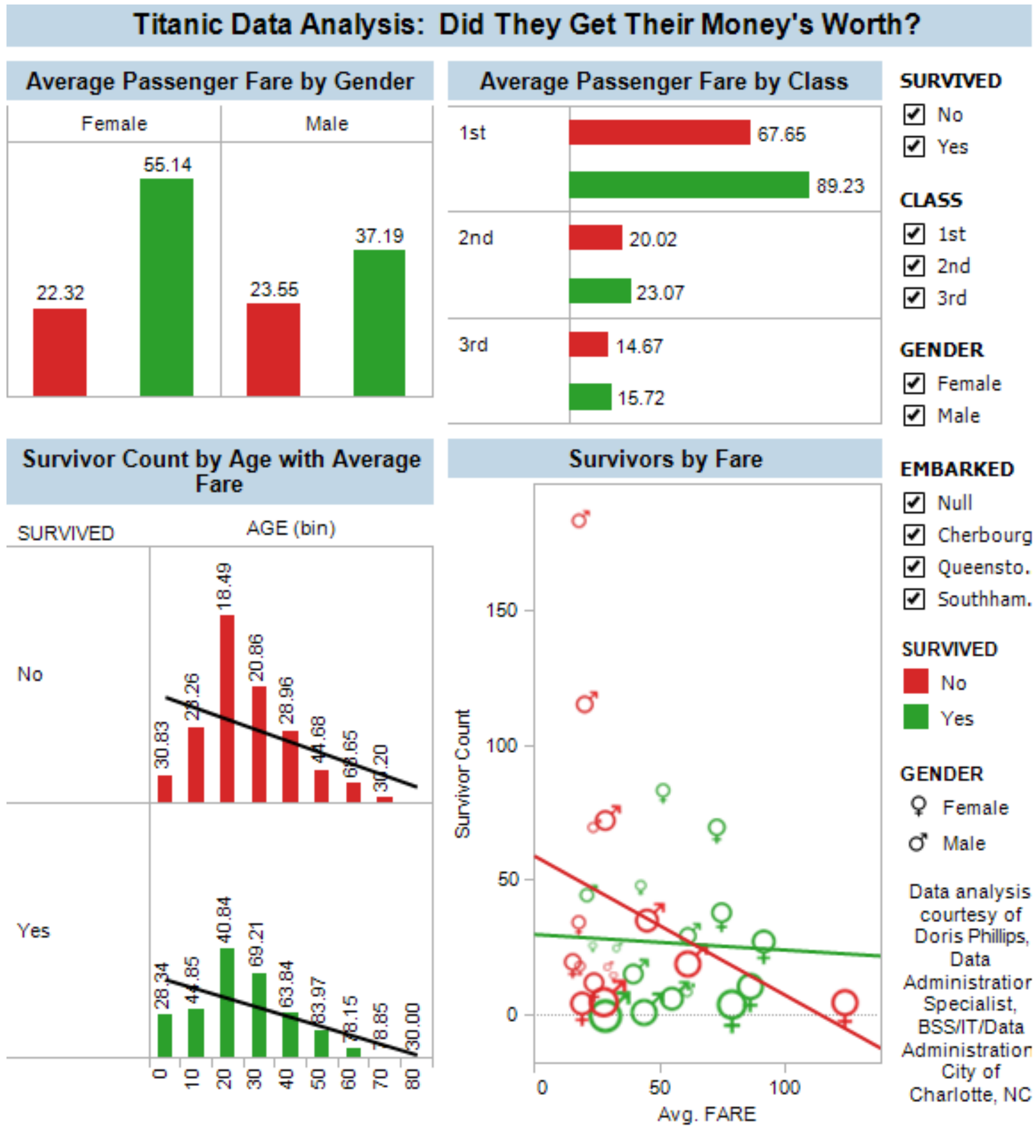
Bivariate Data and Associations



- Displaying and reasoning about bivariate data, and associations between two variables, is the main theme of the 8th grade Statistics and Probability CC Standards.

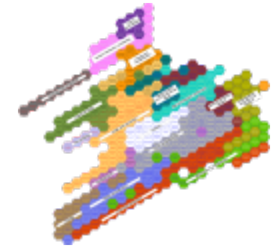
- Tables, and the steps to scatterplots, and concepts of (early) regression…

- Students draw on recently developed notions of variable and graphing of sets of points and lines in early algebra work, in their developing statistical reasoning.



http://www.tableausoftware.com/blog/titanic-data-analysis-from-tableau-customer

Titanic Data Analysis

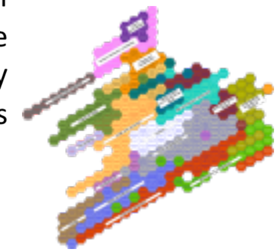


Passenger Class:	First	Second	Third	Crew
Survived	203	118	178	212
Died	122	167	528	673

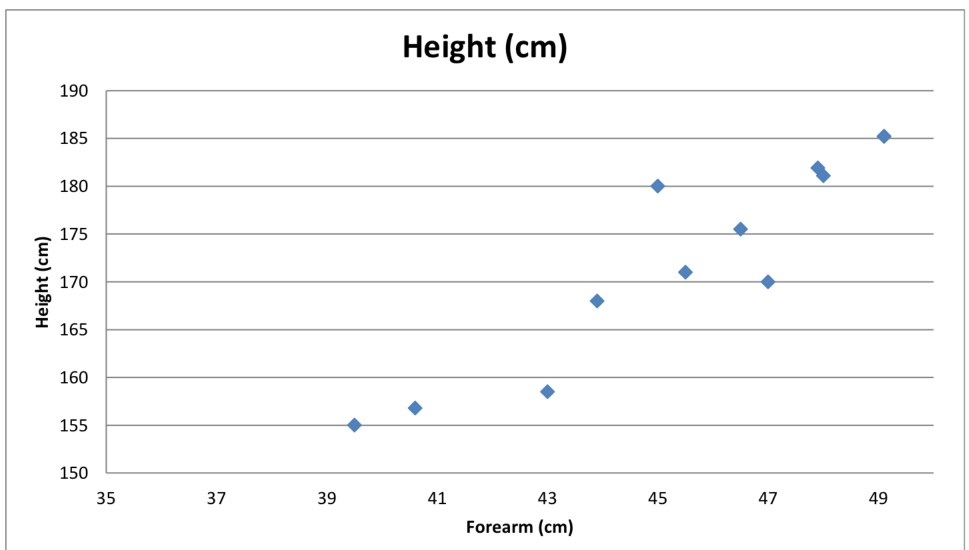
Using these data:
1. What questions could students ask, and
2. How could they conduct an investigation to answer their question(s)?

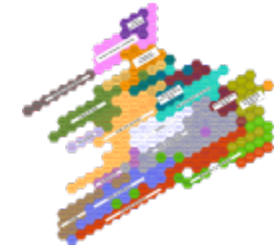
http://www.tableausoftware.com/blog/titanic-data-analysis-from-tableau-customer

Relationships among students' physical features can be a basis of a meaningful statistical investigation involving fitting a line to a scatterplot. For example, students in a class may be interested in exploring the relationship between their forearm length and their height. They begin by recording the data in a table and plotting it on a scatterplot such as in the ones below.



Forearm (cm	Height (cm)
45	180
49.1	185.2
39.5	155
43.9	168
47	170
49.1	185.2
48	181.1
47.9	181.9
40.6	156.8
45.5	171
46.5	175.5
43	158.5





Forearm (cm	Height (cm)
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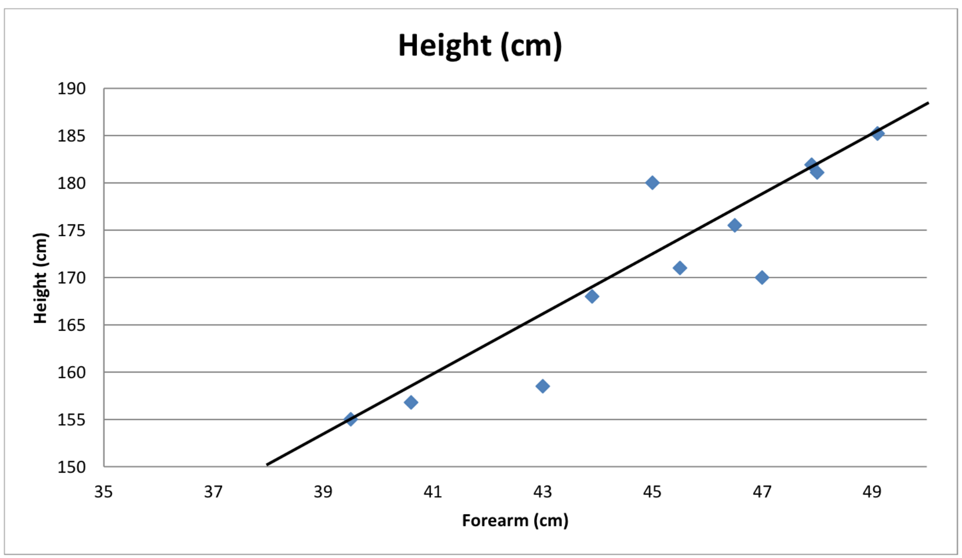
47	170
49.1	185.2
48	181.1
47.9	181.9
40.6	156.8
45.5	171
46.5	175.5
43	158.5

Estimated line of best fit:

Interpret…



$$y - 155 = 3.15(x - 39.5)$$

$$y - 155 = 3.15x - 124.43$$

$$y = 3.15x + 30.57$$

Unpacking the CCSS-M: Descriptor Elements

1. **Underlying <u>Cognitive or Conceptual Principles</u>:** components of cognitive framework for making meaning; "big ideas"
2. **Student <u>Strategies, Inscriptions (Representations), and Misconceptions</u>:** how students make their reasoning and intermediate understandings visible
3. **<u>Mathematical Distinctions and Multiple Models</u>:** emerging distinctions, and models for reasoning, that support increasingly sophisticated and nuanced building of the big ideas
4. **<u>Coherent Structure</u>**: recurring themes or frameworks for reasoning, which can be fostered deliberately in instruction to support student investigation and reflection.
5. **<u>Bridging Standards</u>:** identify intermediate understandings, address CCSS-M grain size variations, and signal major instructional gaps that might not otherwise be addressed, for student progress and transitions.

VDM LT: A sample of descriptor elements

1. **Cognitive or Conceptual Principles:**

   --Distributions have shape, center, and spread

   --Central tendency involves varied notions of balance, typicality

2. **Student Strategies, Inscriptions (Representations), and Misconceptions:**

   --Pictographs, bar graphs, line (dot) plots, histograms, box plots

3. **Mathematical Distinctions and Multiple Models:**

   --Different representations, and different measures, show and hide different features of a distribution

   --The mean: as balance point, as fair share

4. **Coherent Structure**:

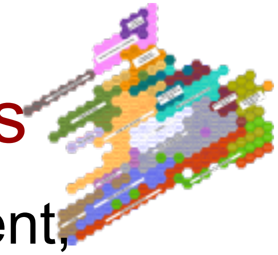
   --Statistical investigation cycle

5. **Bridging standards**

   --Adding circle graphs (K-5)

   --Boxplots separated from histograms

Value of Learning Trajectories to Teachers



- Support understanding of student conceptual development, intermediate understandings and misconceptions
- Support managing the range of preparation and needs of students—teachers can identify tasks and discourse that support improved proficiency
- Identify clusters of related concepts at grade level
- Provide opportunities to strengthen mathematical content knowledge
- Suggest rich uses of classroom assessment
- Clarify what to expect of students' preparation from <u>last</u> year, and what will be expected of students <u>next</u> year
- Support cross-grade (vertical) instructional collaboration and coordination

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