A Study of Middle School and College Students' Misconceptions About Solving Multi-Step Linear Equations

Amber Powell

SUNY FREDONIA

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Question

What mistake(s) do you think students will make the most when solving this equation?

$$-(x + 4) = 3(8 + 2x)$$

And this one?

$$4x + 3 = 5x - 11$$

Question

What error was made the most when solving this equation?

$$a = 19a + 18$$

And this one?

$$x/3 = 8$$

Purpose for Research

- Previous teaching experience at a high-needs, low income city school district
  - Taught AIS Math grades 6 – 8
  - Co-taught 7th grade math (special education and ESL students)
- Observations when teaching multi-step linear equations, students:
  - Were able to recall rules to solve the problem
  - Could not recall basic math facts without using strategies
  - Struggled to perform integer operations
  - Could not remember mathematical properties (i.e. distributive property)

Prior Research

Four categories of teaching/learning (Bright, 1981):

- Instrumental - students follow rules without understanding them
- Relational - students learn specific rules from more general ones
- Intuitive - students solve problems based on prior knowledge
- Formal - students have complete understanding of symbols, ideas and terminology in the problem solving process

Prior Research

Two groups of math students (Kieran, 1992):

- Arithmetic thinkers - tend to use trial-and-error substitution to solve equations because the answer is more intuitive to them
- Algebraic thinkers – use inverses and "undoing" strategies to solve equations because they recognize they are trying to find the value of the unknown

Prior Research

- Sign Confusion (Herscovics & Linchevski, 1994)
  - $6x – 7 = 11$
  - Students forget the subtraction sign places a negative value on 7
- Equation balance (Bright, 1981)
  - "left hand side" equals "right hand side"
- Misunderstanding of "equal" sign (Knuth, Alibali, Hattikudur, McNeil & Stephens, 2008)

Textbook Teaching Methods

- Direct instruction
- Instruction through manipulatives
- Self-discovery

Hypothesis

- 8th grade students will make the most mistakes with the distributive property and integer operations when solving multi-step linear equations
- College students in a non-majors mathematics course will make the same mistakes as 8th graders
- There will be no significant difference in scores between the two different groups in this study

Demographics

MATH 110 Students (77)	8th Grade Students (36)
44 Females	19 females
33 Males	17 males
59 Freshmen	2 IEPs
8 Sophomores	0 504 plans
6 Juniors	0 ELLs
4 Seniors	

Design

- Instrument contained 15 multi-step linear equations
- Participants given 15 minutes at beginning of one class period to complete the assessment
- Calculators were not permitted
- Students were also asked to answer a three question, free response survey

Data Collection

- Administered 113 assessments
- Two versions of the instrument were used – same questions, different order
- Nine criteria evaluated:
  - combining like terms
  - moving terms
  - distributing terms
  - using negative numbers in operations
  - using variables other than x
  - using the four basic operations (addition, subtraction, multiplication, and division)

Data Collection

- Three question survey
  - Favorite problem
  - Easiest problem
  - Most difficult problem
- Comments

Instrument

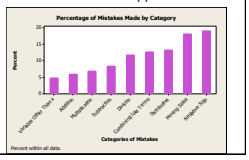
1. $3x+7=16$	2. $6-7x=13$	3. $2x-5=8$
4. $9x+4=8x$	5. $4x+3=5x-11$	6. $5(x-4)=30$
7. $(x+2)4=28$	8. $7x-6+9=4x$	9. $8x-2-3x=-x+4$
10. $-(x+4)=3(8+2x)$	11. $\frac{x}{3}=8$	12. $q+13=29$
13. $-3+4+10x=11x$	14. $r+2r=6$	15. $a=19a+18$

Data Analysis

- Data comparison:
  - variables measured
  - grade level
  - gender
- Minitab used for analysis
  - Analysis of Variance
  - Tukey Test for Significance

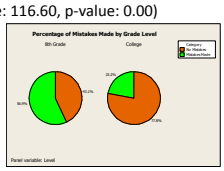
Results

- Mistakes that occurred frequently
  - Operations using negative numbers
  - Moving numbers and variables to the opposite side of the equal sign



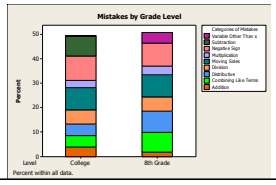
Results

- Eighth grade students made twice the number of mistakes that MATH 110 students did (F-value: 116.60, p-value: 0.00)



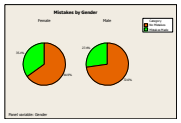
Results

- 8th graders and MATH 110 students made different types of mistakes



Results

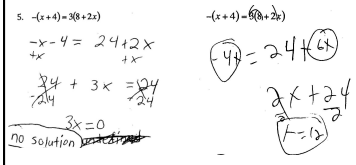
- Males made fewer mistakes than females (F-value: 6.60, p-value: 0.01)
- There was no difference in the types of mistakes made based on gender



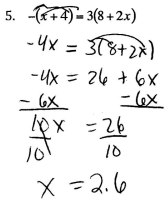
Results

Problem #5: $-(x+4)=3(8+2x)$		
Skill Tested	College Students	8th graders
Distribution Property	5 (6%)	19 (53%)
Moving Terms to Other Side of Equal Sign	6 (8%)	8 (22%)
Addition/Subtraction Mistakes	6 (8%)	2 (6%)
Multiplication/Division Mistakes	4 (5%)	4 (11%)
Mistakes Involving Negative Numbers	7 (9%)	12 (33%)
Left Problem Blank	1 (1%)	6 (17%)

Student Work



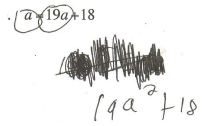
Student Work



Results

Problem #4: $a=19a+18$		
Skill Tested	College Students	8th graders
Moving Terms to Other Side of Equal Sign	13 (17%)	11 (31%)
Addition/Subtraction Mistakes	5 (6%)	3 (8%)
Multiplication/Division Mistakes	0 (0%)	4 (11%)
Mistakes Involving Negative Numbers	12 (16%)	9 (25%)
Mistake Due to Variable Other than x	1 (1%)	8 (22%)
Left Problem Blank	5 (6%)	7 (19%)

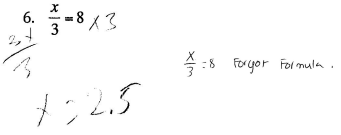
Student Work



Results

Problem #14: $\frac{x}{3}=8$		
Skill Tested	College Students	8th graders
Moving Terms to Other Side of Equal Sign	9 (12%)	11 (31%)
Multiplication/Division Mistakes	8 (10%)	10 (28%)
Left Problem Blank	5 (6%)	7 (19%)

Student Work



Survey Results

- Mixed results
- Would reportedly perform better with calculator

Implications for Teaching

- Teachers need to continuously reinforce integer operations as a part of instruction
- Students need to understand fractions
- Students should use a calculator more effectively, and not depend on it to calculate

Application of Results

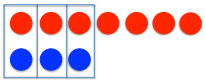
- Spiral education
  - Basic facts, integer operations and fractions incorporated into lessons and homework
- Manipulatives
  - Student generated fraction tiles
  - Bingo chips (two colors)
- Interactive White Boards
  - National Library of Virtual Manipulatives

Bingo Chips

Example: -7 + 3 = x

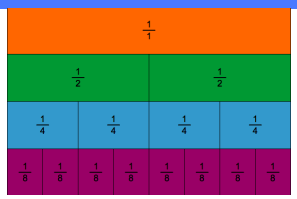
Let red = negative

Let blue = positive



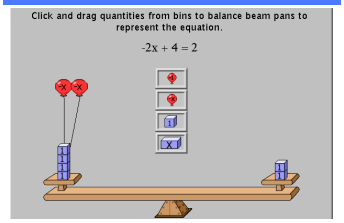
How many are left?

Fraction Bars



What is 1/4 + 3/8?

Balance



Future Research

- Administer instrument to each grade level at one district
- Administer two instruments, and allow the use of a calculator for one
- Focus on manipulatives as a part of teaching

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Questions?

Amber Powell
Email: powell@fredonia.edu

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