Students as Mathematicians: A (CCSS) Modeling Approach

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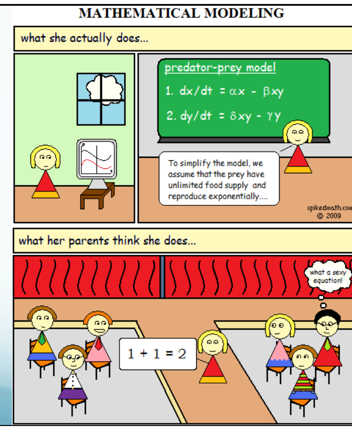


What Do Mathematicians Do?

1. Find an interesting or meaningful problem
2. Informally explore, experiment, collect data
3. From patterns in the data create conjectures, hypotheses, theories
4. Use toolbox of problem solving strategies to prove or disprove theories
5. Use known toolbox of basic skills
6. Extend or generalize – what else can I learn?
7. Publish / communicate
8. Go to Step 1

Modeling in Mathematics (CCSS)

- "Modeling is defined as both a conceptual category for high school mathematics and a mathematical practice and is an important avenue for motivating students to study mathematics, for building their understanding of mathematics, and for preparing them for future success."

Mathematical Practices (CCSS)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Classroom Goals: Setting the Tone

1. Enjoy math!
2. Embrace challenge.
3. Simplify and generalize.
4. Create models that help you understand your world.
5. Make connections.
6. Question your results, and reexamine your premises.
7. Articulate your thought process.
8. Work cooperatively.
9. Work independently.
10. Use technology.

Transforming the Classroom - BEFORE

- Precalculus Class – Traditionally had been taught using a textbook
- Students worked very hard but still were trying to "survive" the class
- Connections were artificial
- Students rightly questioned the purpose in learning the more difficult or abstract material
- It just was not working!

Transforming the Classroom - AFTER

- ELIMINATE THE TEXTBOOK (a LOT of work)
- Begin each unit with a realistic scenario
  - Extend scenarios to add more complexity and depth to the material
- Allow students to explore, discuss, "play"
- The development of the mathematical content becomes a necessary step in understanding and resolving the scenario
- Students are engaged, interested, willing to take chances with their thinking

Example: Apparent Size

- Application of radian measure
- Students used to focus on learning a formula
- Preview of apparent size:
  The Flagpole problem

Example: Apparent Size

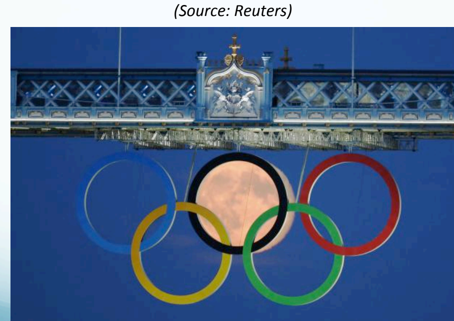
- Scenario: Standard flag sizing recommends that the length of the flag is at least 1/4 the height of the flagpole, and the width of the flag is approximately 2/3 the length of the flag. In addition, flag protocol requires that the flag should never touch the ground, even when flying at half-staff.
- Using only standard measuring tools, determine the height of the flagpole in front of the school. Use your results to calculate minimum and maximum dimensions of the replacement flag. Finally, research available flag sizes and their costs, and use the results to recommend the size of the replacement flag.

Example: Apparent Size

- Some students "discovered" a method that previewed apparent size:
  - From a distance, use a ruler to "measure" the flagpole and another object for reference.
  - Set up a proportion and solve!
- As the year progressed, there was further discussion of the strengths and weaknesses of this approach.

Scenario: The London Olympics

(Source: Reuters)



Scenario: The London Olympics

(Source: Reuters)

Question:

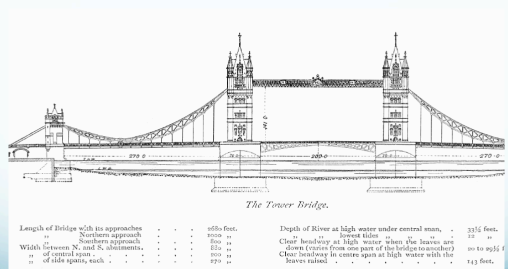
- How far was the photographer from the bridge?

Helpful Responses (only if they ask!)

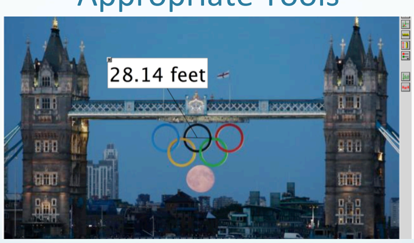
- Although the photographer's distance is unknown, there are established measurements for many of the objects in the picture. Think about what information might be helpful!
  - For example, there is information on the dimensions of the Tower Bridge.
- The angular diameter of the moon varies between 29.3' and 34.1'
  - This could be given to students in radians as the apparent size. Here, the apparent size is approximately 0.01 radians

Helpful Responses (only if they ask!)

(Source: Wikipedia)

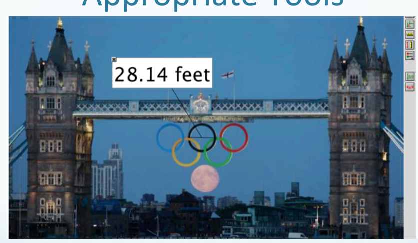


Appropriate Tools



$$\text{Apparent Size} = \frac{\text{diameter}}{\text{distance}}$$

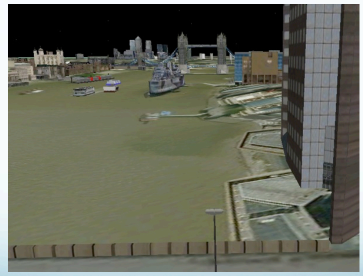
Appropriate Tools



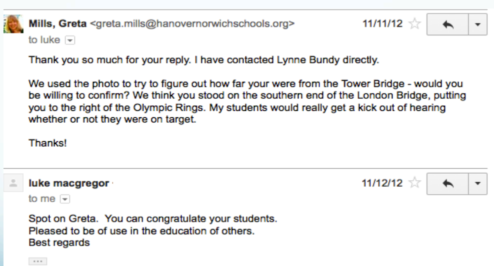
$$0.01 = \frac{28.14\ (\text{ft})}{\text{distance (ft)}}$$

Appropriate Tools

Appropriate Tools



Reality Check!



Mills, Greta <greta.mills@hanovernorwichschools.org> 11/11/12
to luke

Thank you so much for your reply. I have contacted Lynne Bundy directly.

We used the photo to try to figure out how far your were from the Tower Bridge - would you be willing to confirm? We think you stood on the southern end of the London Bridge, putting you to the right of the Olympic Rings. My students would really get a kick out of hearing whether or not they were on target.

Thanks!

luke macgregor 11/12/12
to me

Spot on Greta. You can congratulate your students.
Pleased to be of use in the education of others.
Best regards

Scenario: The Mathematics of Sound

- Technology allows us to explore the relationship between frequency and pitch in music
- Geometer's Sketchpad, Geogebra, and Audacity will play pure sine waves
- With a basic understanding of Pythagorean tuning, we can create an octave scale

The Fundamental and the Octave

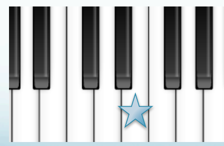
Pythagorean Tuning

- A_3 = 220 Hertz
- A_4 = 440 Hertz
- Octaves are in the ratio of 2:1
- Other intervals create other ratios
- A "perfect fifth" is considered pleasing to the ear
- Build the notes of a scale using "perfect fifth" tuning (aka Pythagorean Tuning)

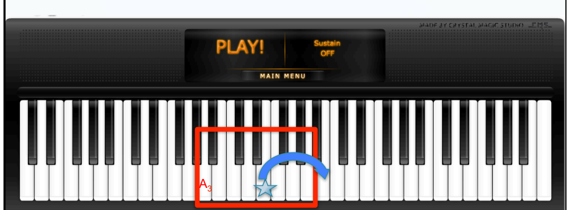
The perfect fifth is 7 semitones above the fundamental

Calculating Frequencies

$$E = \frac{3}{2}A_3 = \frac{3}{2}(220\text{ Hz}) = 330\text{ Hz}$$



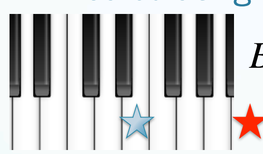
Staying in the octave



The next perfect fifth above E is B

... outside of the octave.

Calculating Frequencies



$$B_4 = \frac{3}{2}E = 495\text{ Hz}$$

Drop the Octave (B_4 to B_3):

$$B_3 = \frac{1}{2}\left(\frac{3}{2}E\right) = \frac{3}{4}E = 247.5\text{ Hz}$$

Remaining Frequencies

PITCH	FREQUENCY
A_3	220
A# / Bb	234.9
B	247.5
C	264.3
C# / Db	278.4
D	297.3

PITCH	FREQUENCY
D# / Eb	313.2
E	330
F	352.4
F# / Gb	371.3
G	396.4
G# / Ab	417.7
A_4	440

GEOMETER'S SKETCHPAD

Extensions

- Compare Pythagorean tuning to Equal Temperament Tuning
- Explore the aural illusion of the missing fundamental
- Use harmonics to replicate real instruments
- Audacity software: free program where you can sample and analyze real sounds

Scenario: Replicating Sound

- Using appropriate technology (such as Audacity), record and analyze the sound generated by playing a single note on an instrument of your choice, allowing the sound to diminish over time. Develop a mathematical model of the sound, and use appropriate technology to test your model's ability to replicate the sound.

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Sample: Guitar String

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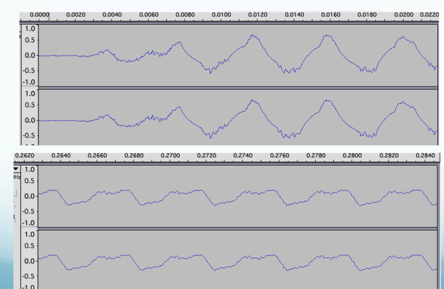
Audacity Software



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Period and Frequency

- The period can be measured with a ruler and is approximately 5.6 cm
- The horizontal axis gives us different time values; 0.002 seconds measures approximately 2.8 cm
- Therefore, the period in seconds is approximately 0.004 seconds

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Period and Frequency

- The *frequency* of a note is its pitch.
- Frequency is the reciprocal of the period and is measured in Hertz (cycles per second)
- Therefore, the frequency is approximately 250 Hz

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Mathematical Model, part I

- The formula for a pure sine wave given a fundamental frequency is

$$y = A\sin(2\pi f \cdot t)$$

- Therefore, our first model is

$$y = \sin(2\pi \cdot 250 \cdot t)$$

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What about Amplitude? Finding the Decay Rate

- It takes approximately 0.25 seconds for the *graph* of the wave to reach an amplitude that is 55% of its original amplitude
- This gives a decay function for the amplitude:

$$(0.55)^{\frac{t}{0.25}}$$

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Mathematical Model, part II

$$y = (0.55)^{\frac{t}{0.25}} \sin(2\pi \cdot 250t)$$

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Refining the Model: Overtones

- Each overtone has a frequency that is a multiple of the fundamental
- Each overtone has a smaller amplitude than the fundamental
- Playing with the overtones, one possible model is …

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Refining the Model: Overtones

$$y = (0.55)^{\frac{t}{0.25}} \begin{bmatrix} \sin(2\pi \cdot 250t) \\ +0.55\sin(2 \cdot 2\pi \cdot 250t) \\ +0.2\sin(3 \cdot 2\pi \cdot 250t) \end{bmatrix}$$

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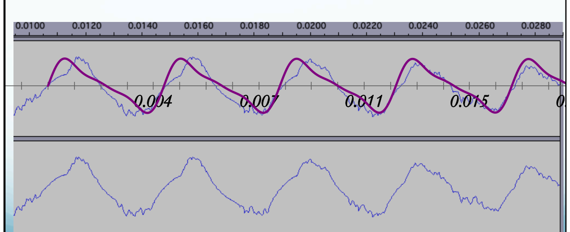
Testing the Model



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Resources for Modeling

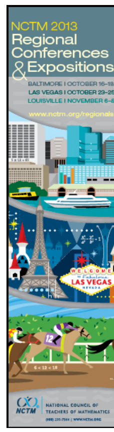
- Math Modeling Handbook
  - Published by CoMAP
  - Seeks to support the implementation of the Common Core State Standards for Mathematics in the high school mathematical modeling conceptual category.
  - Provides modules and guides for twenty topics together with references to specific CCSSM modeling standards for which the topics may be appropriate.
  - Helps students to develop a *modeling disposition*

High School Contests in Modeling

- November: High School Mathematical Contest in Modeling (HiMCM) sponsored by the Consortium for Mathematics and its Applications (CoMAP; www.comap.com)
- March: Moody's Mega Math Challenge(M3 Challenge) sponsored by the Society for Industrial and Applied Mathematics (SIAM; m3challenge.siam.org)
  - Information on the M3 Challenge is available in the back of the Hall

Thank you!

GMills@OAPB.org



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