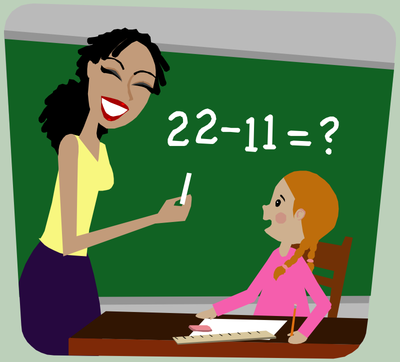
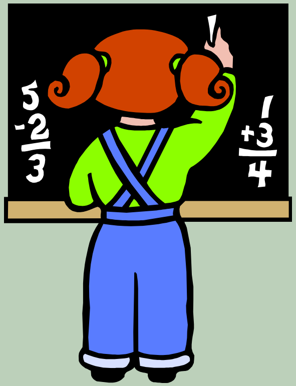
Aligning Assessments to the Common Core State Standards





Tracy Gruber, M.Ed, NBCT
Nevada Department of Education
K-12 Mathematics Specialist
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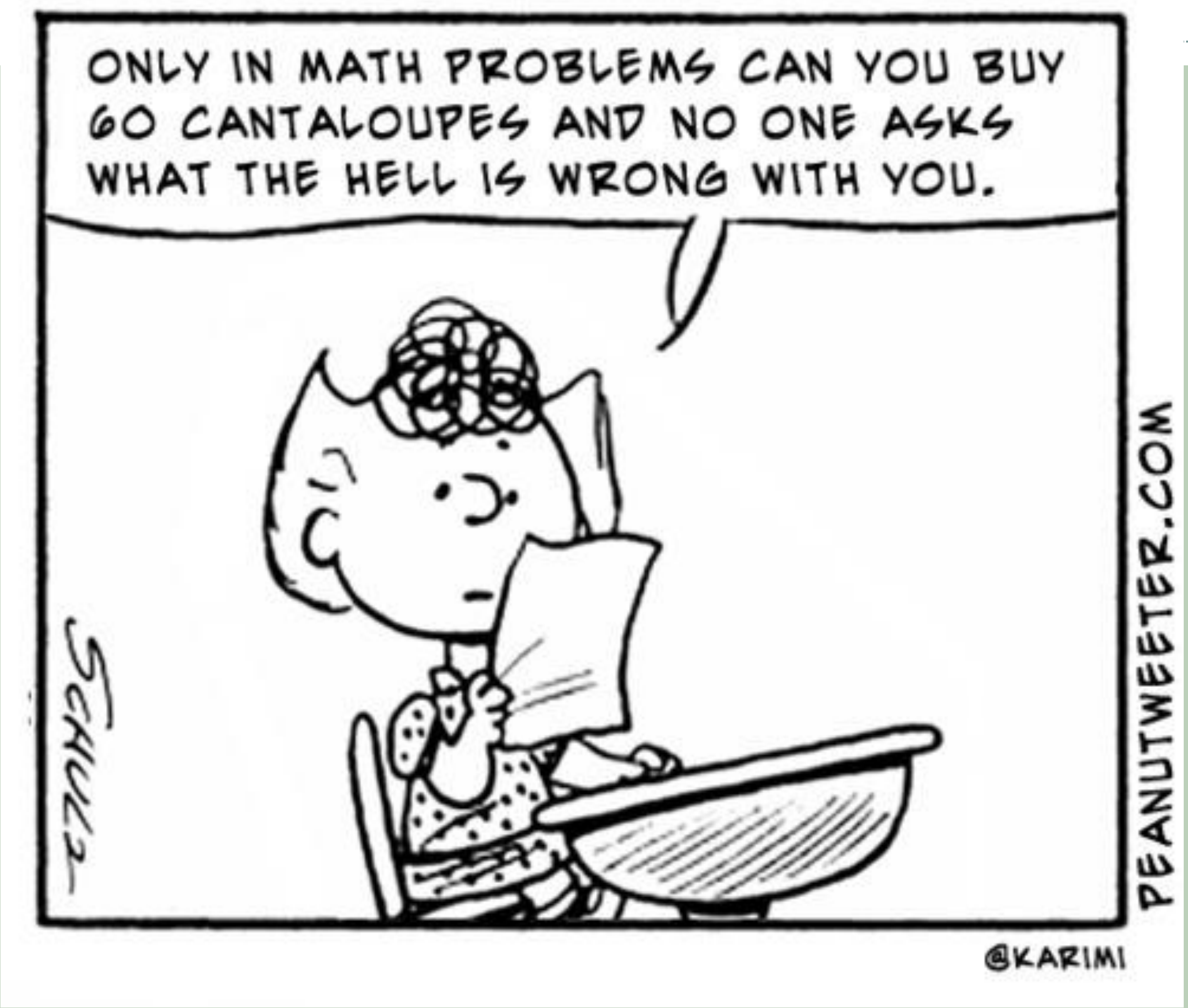
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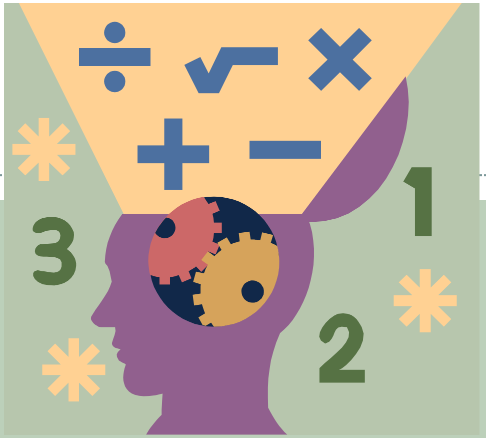
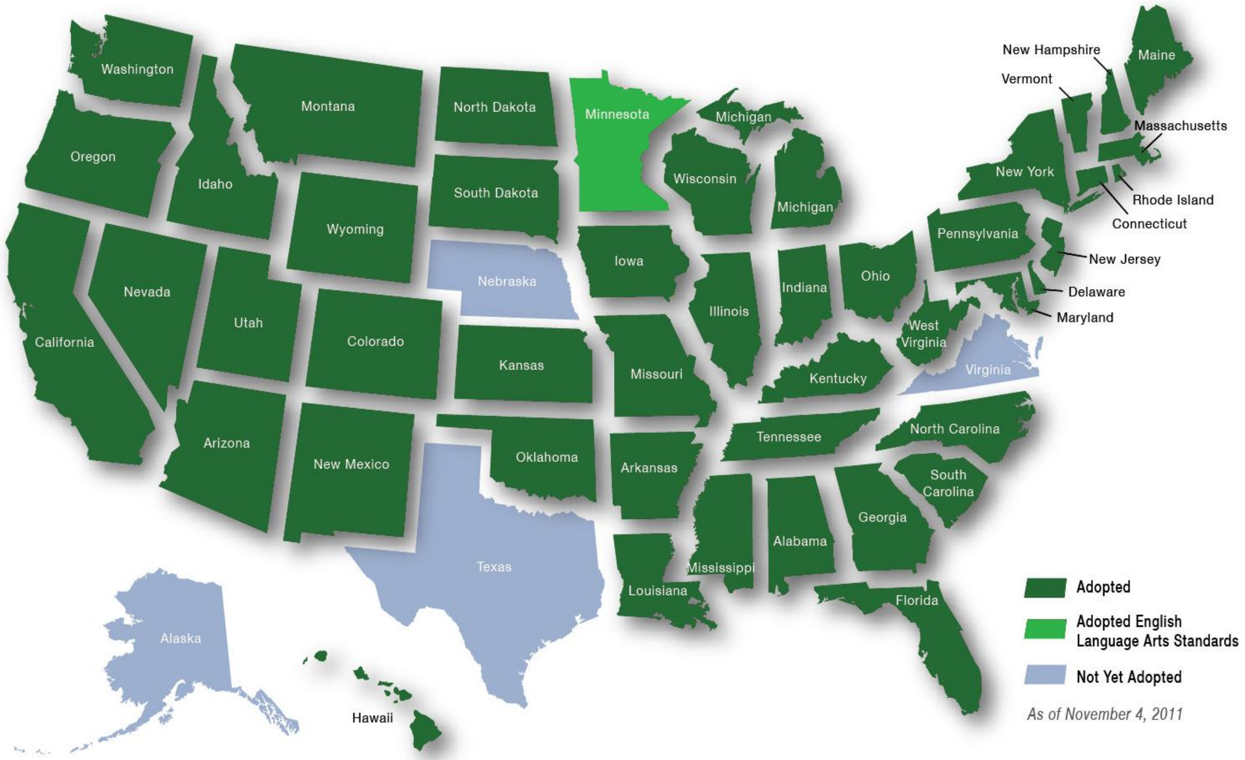


"The world is small now, and we're not just competing with students in our county or across the state. We are competing with the world," said Robert Kosicki, who graduated from a Georgia high school this year after transferring from Connecticut and having to repeat classes because the curriculum was so different. "This is a move away from the time when a student can be punished for the location of his home or the depth of his father's pockets."

Excerpt from Fox News, Associated Press. (June 2, 2010) States join to establish 'Common Core' standards for high school graduation.

Common Core State Standards

- Define the knowledge and skills students need for college and career
- Developed voluntarily and cooperatively by states; more than 40 states have adopted
- Provide clear, consistent standards in English language arts/literacy and mathematics

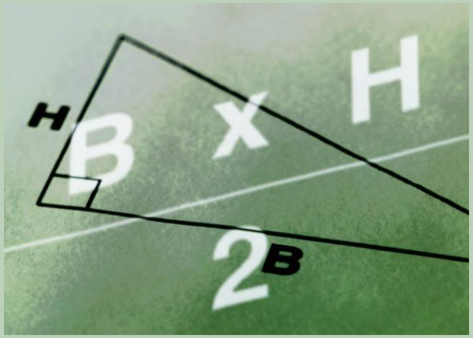


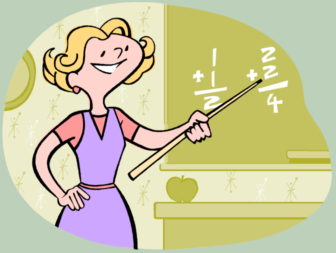
Source: www.corestandards.org

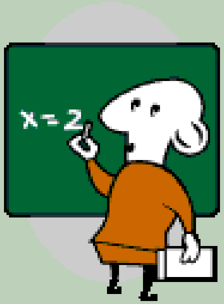
What questions will we try to answer today?

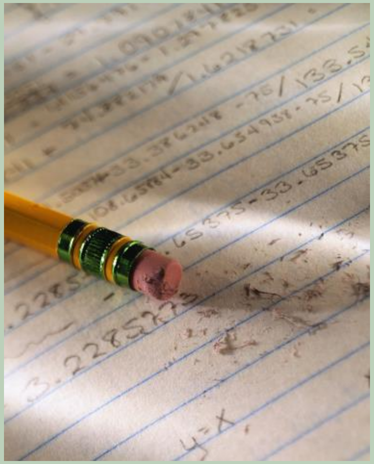
- How can teachers plan instruction that takes into account the shifts in the CCSS-M and meets the needs of all learners?
- How will the new assessment system help educators understand what students have learned and how to support future student learning?

Mathematics Instruction:
How much really needs to change?

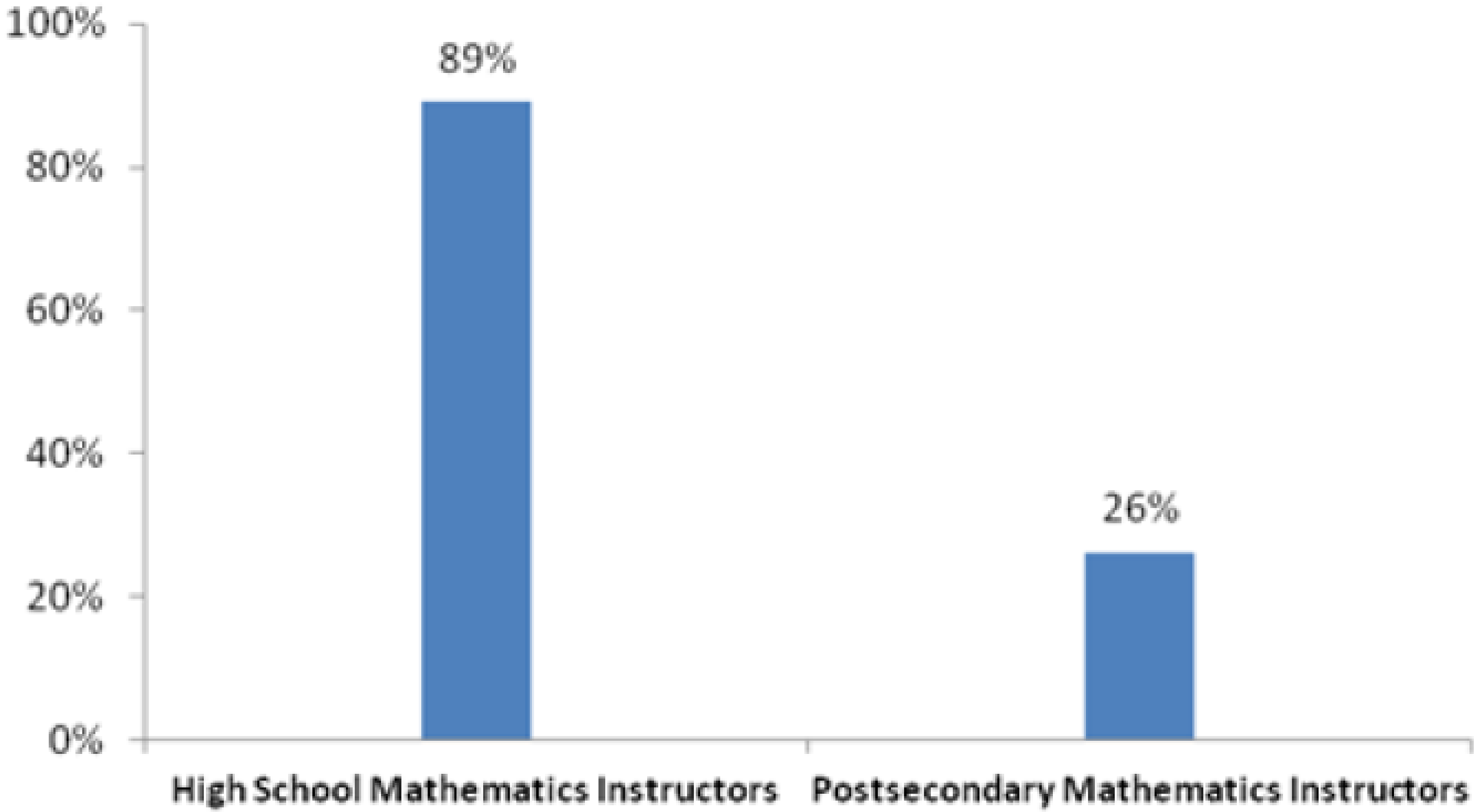








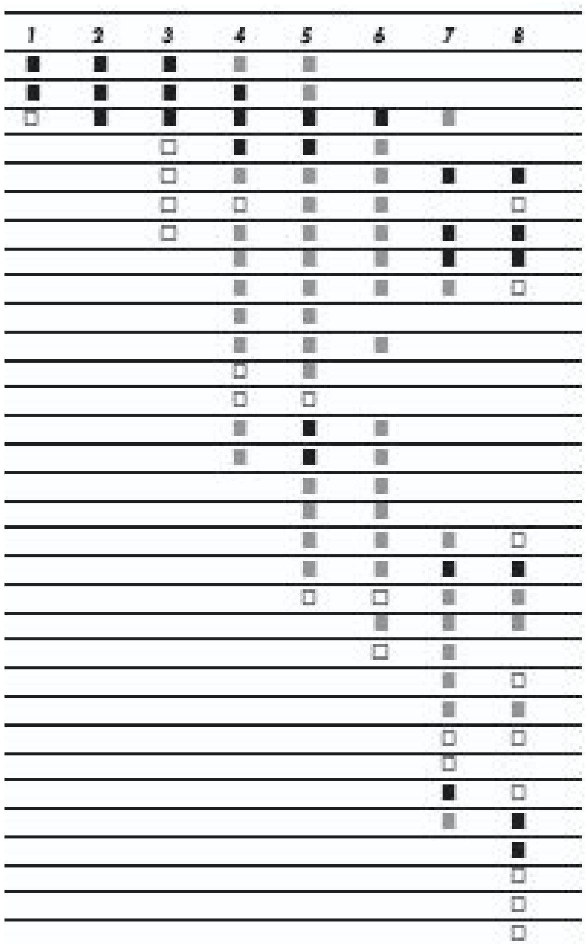
What percentage of mathematics educators reported that their students are prepared for college-level work in mathematics?

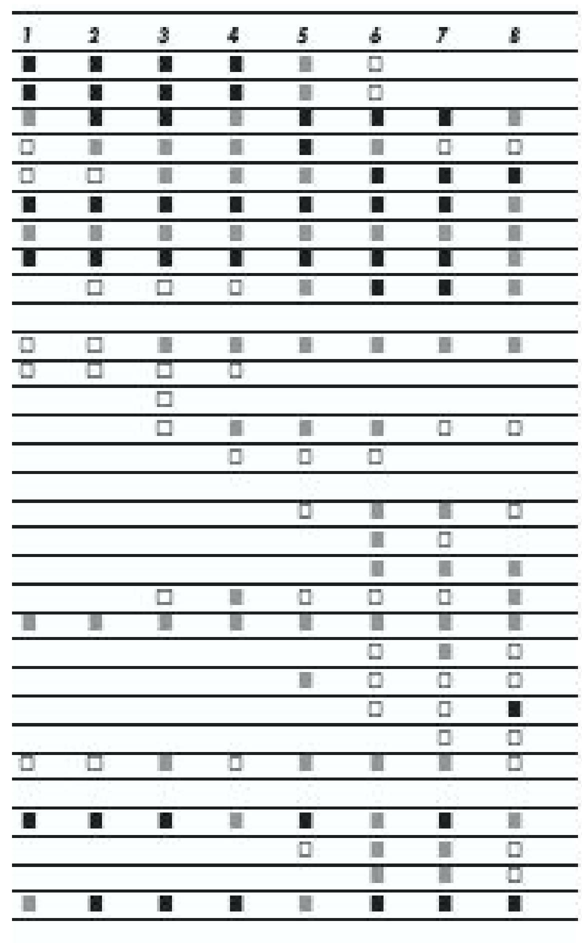


Source: *ACT National Curriculum Survey 2009*, Appendix B, Tables B.8 and B.9, page 43

The shape of math in A+ countries

Mathematics topics intended at each grade by at least two-thirds of A+ countries



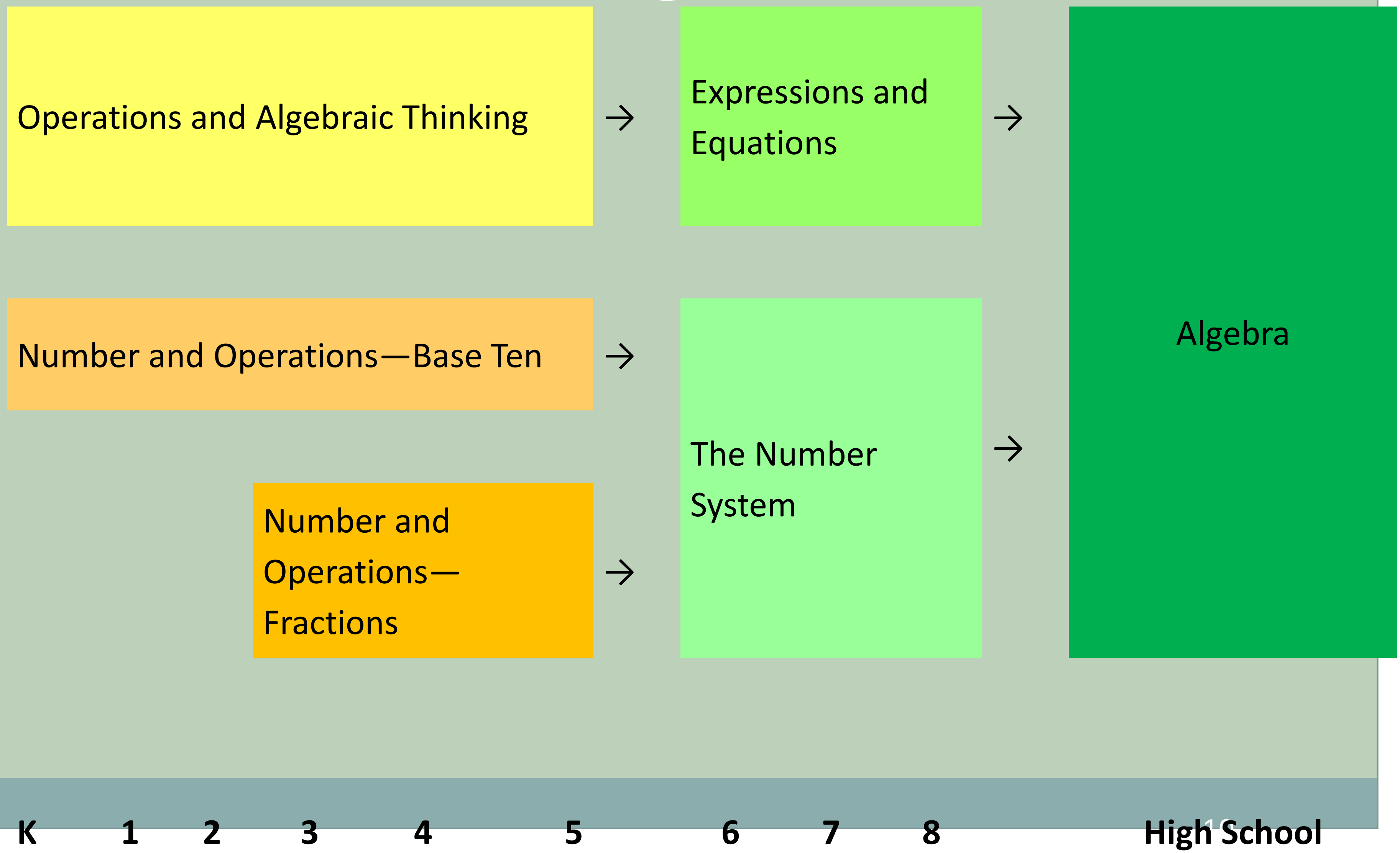


Mathematics topics intended at each grade by at least two-thirds of 21 U.S. states

[1] *Schmidt, Houang, & Cogan, "A Coherent Curriculum: The Case of Mathematics." (2002).*

Traditional U.S. Approach

Focusing Attention Within Number and Operations



What The Disconnect Means for Students:

- Nationwide, many students in two-year and four-year colleges need remediation in math.
- Remedial classes lower the odds of finishing the degree or program.
- Need to set the agenda in high school math to prepare more students for postsecondary education and training. *(I would add K-12)*

Standards for Mathematical Practices

- 8 Practices for K-12 that are the heart of what students should be doing with mathematics (application and using mathematics).
- http://www.youtube.com/watch?v=m1rxkW8ucAI&list=UUF0pa3nE3aZAfBMT8pqM5PA

Features of CCSSM and Implications for Assessment

Assessing through authentic connections of content and practices

*"Designers of curricula, assessments, and professional development should all **attend to the need to connect the mathematical practices to mathematical content in mathematics instruction**." (CCSSM, pg. 8)*

Make Sense of Problems and Persevere in Solving Them

"Does this make sense?"

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

http://www.illustrativemathematics.org/standards/practice

Reason Abstractly and Quantitatively

"The ability to contextualize and decontextualize.“

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

http://www.illustrativemathematics.org/standards/practice

Construct Viable Arguments and Critique the Reasoning of Others

"Distinguish correct logic or reasoning from that which is flawed."

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

http://www.illustrativemathematics.org/standards/practice

Model With Mathematics

"Analyze relationships mathematically.“

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

http://www.illustrativemathematics.org/standards/practice

Use Appropriate Tools Strategically

"Explore and deepen understanding of concepts using tools."

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.

http://www.illustrativemathematics.org/standards/practice

Attend to Precision

"Communicate precisely.“

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

http://www.illustrativemathematics.org/standards/practice

Look For and Make Use of Structure

"Shift perspectives to discern a pattern or structure.“

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7\times5+7\times3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.

http://www.illustrativemathematics.org/standards/practice

Look For and Express Regularity in Repeated Reasoning

"Maintain oversight of the process, while attending to the details.“

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

http://www.illustrativemathematics.org/standards/practice

Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others	Reasoning and explaining
4. Model with mathematics 5. Use appropriate tools strategically	Modeling and using tools
7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Seeing structure and generalizing

General Discussion

- **With a partner or neighbor please discuss:**

  "1. Which Mathematical Practices do you currently use in your classroom?

  2. Which Mathematical Practice will you having your students engage in next week?"

- **Share any major insights with our group.**

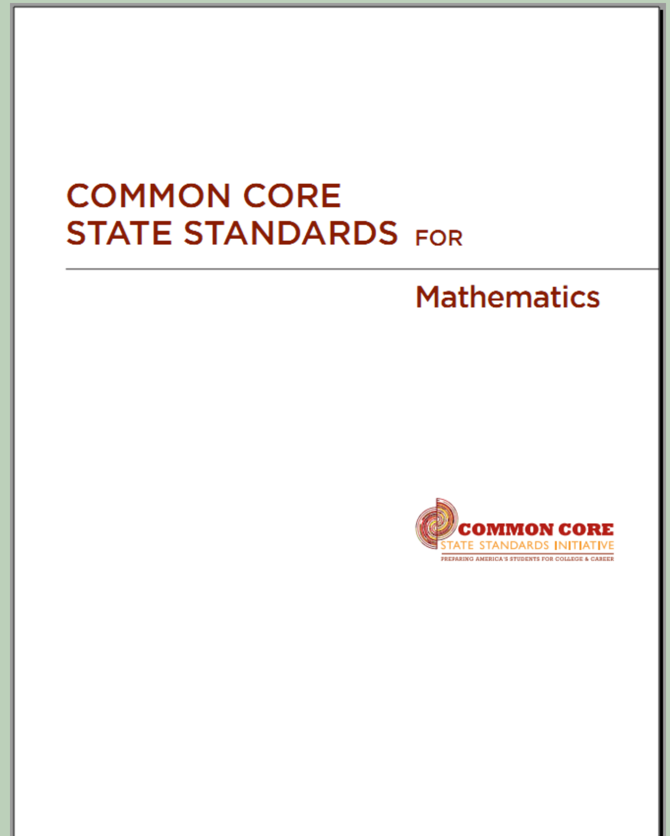
Five Basic Characteristics to Support Quality Mathematics Teaching

- Precision: Mathematical statements are clear and unambiguous. At any moment, it is clear what is known and what is not known.
- Definitions: They are the bedrock of the mathematical structure. They are the platform that supports reasoning. No definitions, no mathematics.
- Reasoning: The lifeblood of mathematics. The engine that drives problem solving. Its absence is the root cause of teaching and learning by rote.
- Coherence: Mathematics is a tapestry in which all the concepts and skills are interwoven.
- Purposefulness: Mathematics is goal-oriented, and every concept or skill is therefore a purpose. Mathematics is not just fun and games.

(Wu, 2011)

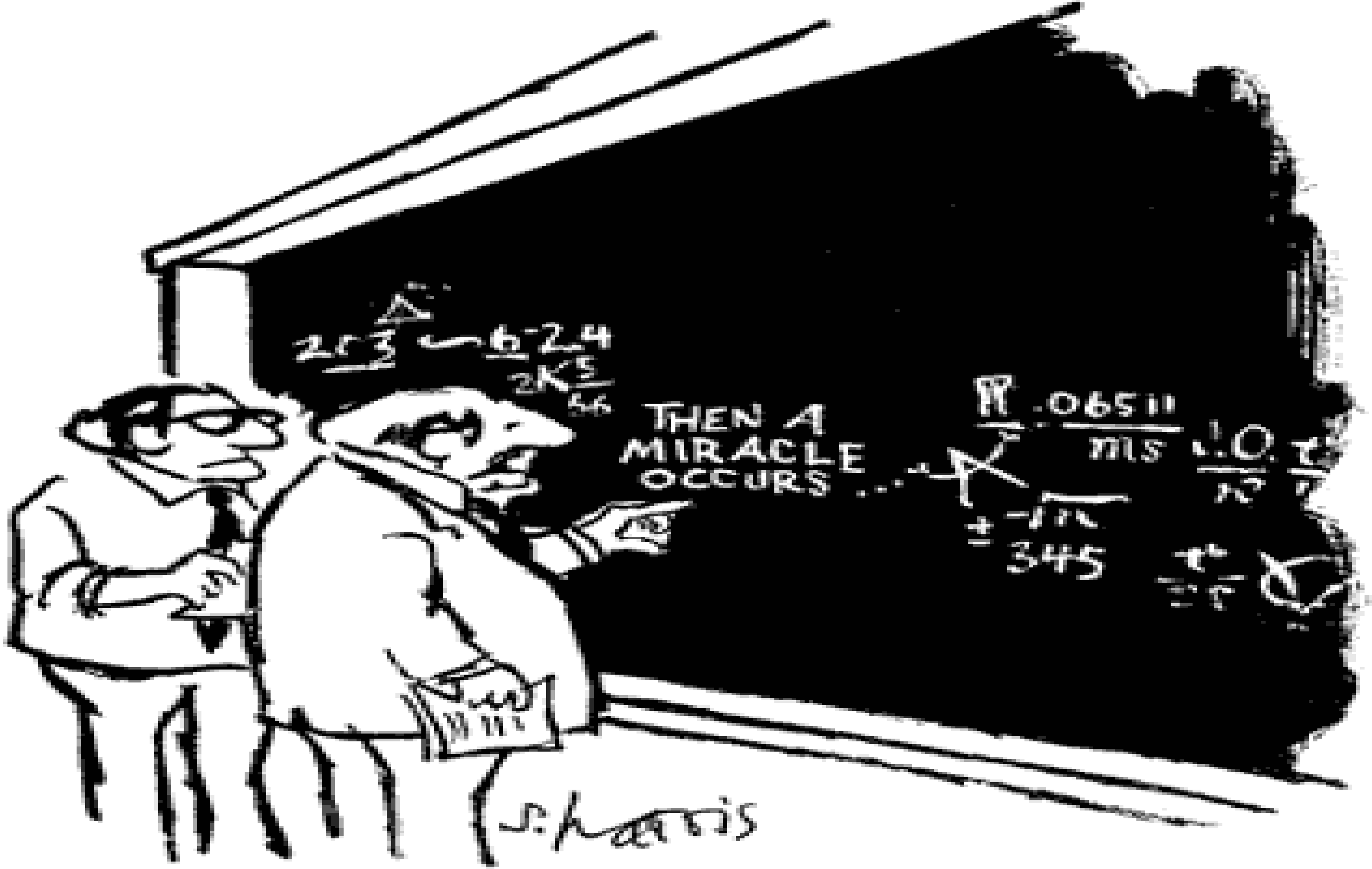
The CCSSM Requires Three Shifts in Mathematics

- **Focus** strongly where the standards focus
- **Coherence**: **Think** across grades and **link** to major topics within grades
- **Rigor**: In major topics, pursue **conceptual understanding**, procedural skill and **fluency**, and **application** with equal intensity



The Three Mathematical Shifts:

What do they really mean?



"I think you should be more explicit here in step two."

Shift #1: Focus

Key Areas of Focus in Mathematics

Grade	Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction - concepts, skills, and problem solving and place value
3–5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra and linear functions

Shift #1: Focus

Content Emphases by Cluster

The Smarter Balanced Content Specifications help support focus by identifying the content emphasis by cluster. The notation [m] indicates content that is major and [a/s] indicates content that is additional or supporting.

Grade 4 Cluster-Level Emphases

m = major clusters; a/s = additional and supporting clusters

Operations and Algebraic Thinking

[m] Use the four operations with whole numbers to solve problems.

[a/s] Gain familiarity with factors and multiples.

[a/s] Generate and analyze patterns.

Number and Operations in Base Ten

[m] Generalize place value understanding for multi-digit whole numbers.

[m] Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

[m] Extend understanding of fraction equivalence and ordering.

[m] Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

[m] Understand decimal notation for fractions, and compare decimal fractions.

Shift #2: Coherence

Think Across Grades, and Link to Major Topics Within Grades

- Carefully connect the learning within and across grades so that students can build new understanding on foundations built in previous years.
- Begin to count on solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

Shift #2: Coherence

Think Across Grades

Example: Fractions

"The **coherence** and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percents, and negative fractions). **The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected**."

Source: Final Report of the National Mathematics Advisory Panel (2008, p. 18)

Shift #2: Coherence

Think Across Grades

Grade 4	4.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
Grade 5	5.NF.4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. 5.NF.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
Grade 6	6.NS. Apply and extend previous understandings of multiplication and division to divide fractions by fractions. 6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent

Shift #2: Coherence

Link to major work within grade

Example: Data Representation

Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Standard 3.MD.3

Coherence: Some Standards from Early Grades are Critical Through Grade 12

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

What it Looks Like in Grade 3

True or False:

3 x 8 = 20 + 4	T	F
50 ÷ 10 = 5 x 1	T	F
9 x 9 = 8 x 10	T	F

What it Looks Like in Grade 5

True or False:

$$\frac{1}{2} \times \frac{1}{3} = \frac{3}{6} \times \frac{1}{3}$$

$$\frac{2}{2} \times \frac{1}{3} = \frac{3}{6} \times \frac{1}{3}$$

What it Looks Like in Grade 8

Tell how many solutions:

$3x + 17 = 3x + 12$

What it Looks Like in High School

$X^4 - 5x^3 + x^2 + 2x + 1 =$

Drag the correct expression to make a true equation.

$x^3 + (x + 1)^2 + X^4 - 6x^3$

$X^4 - 3x^3 + 2x^3 + x^2 + 2x + 1$

$X^4 - 5x^3 + x + x + 2x + 1$

…

Shift #3: Rigor

In Major Topics, Pursue Conceptual Understanding, Procedural Skill and Fluency, and Application

- The CCSSM require a balance of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations
- Pursuit of all threes requires equal intensity in time, activities, and resources.

Shift #3: Rigor
Solid Conceptual Understanding

- Teach more than "how to get the answer" and instead support students' ability to access concepts from a number of perspectives
- Students are able to see math as more than a set of mnemonics or discrete procedures
- Conceptual understanding supports the other aspects of rigor (fluency and application)

Shift #3: Rigor

Application

- Students can use appropriate concepts and procedures for application even when not prompted to do so.
- Teachers provide opportunities at all grade levels for students to apply math concepts in "real world" situations, recognizing this means different things in K-5, 6-8, and HS.
- Teachers in content areas outside of math, particularly science, ensure that students are using grade-level-appropriate math to make meaning of and access science content.

Shift #3: Rigor

Procedural Skill and Fluency

- The standards require speed and accuracy in calculation.
- Teachers structure class time and/or homework time for students to practice core functions such as single-digit multiplication so that they are more able to understand and manipulate more complex concepts.

Shift #3: Rigor

Required Fluencies for Grades K-6

Grade	Standard	Required Fluency
K	K.OA.5	Add/subtract within 5
1	1.OA.6	Add/subtract within 10
2	2.OA.2 2.NBT.5	Add/subtract within 20 (know single-digit sums from memory) Add/subtract within 100
3	3.OA.7 3.NBT.2	Multiply/divide within 100 (know single-digit products from memory) Add/subtract within 1000
4	4.NBT.4	Add/subtract within 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2,3	Multi-digit division Multi-digit decimal operations

How Can Assessments Deliver on the Promise of Focus, Coherence and Rigor?

- ***FOCUS: Assessments focus where the standards focus.***

  Major content represents the majority of points and problems on assessments.

- ***COHERENCE: Assessments honor the coherence in the standards.***

  Balance of tasks assessing individual standards and related standards within the context of the grade and, as relevant, the progressions.

- ***RIGOR: Assessments reflect the rigor of the standards.***

  Balance of tasks assessing conceptual understanding, procedural skill and fluency, and application of mathematics to solve problems.

General Discussion

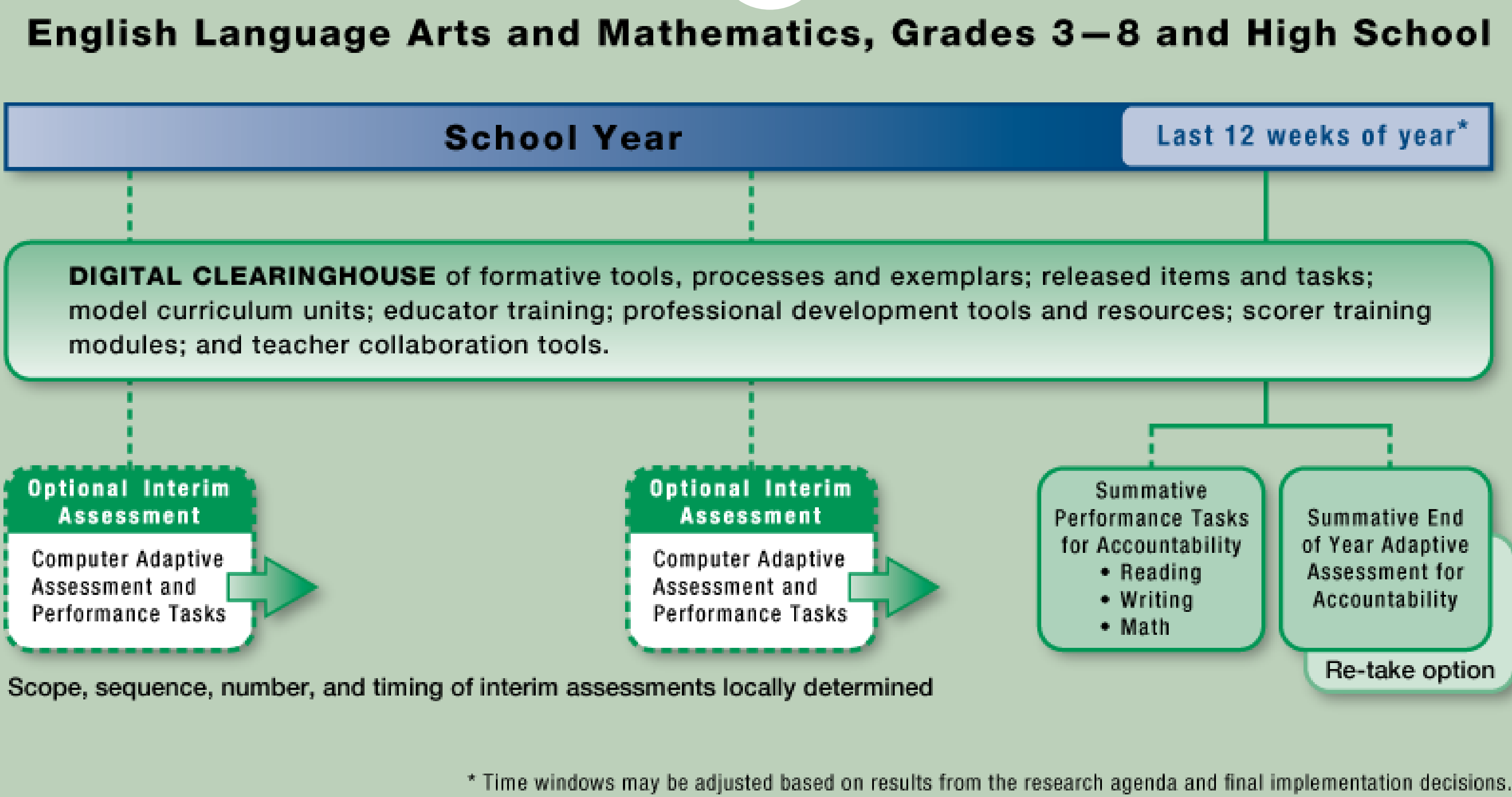
- **With a partner or neighbor please discuss:**

  "Discuss the importance of the shifts and how it will impact your classroom instruction."

- **Share any major insights with our group.**

Features of the CCSSM and Implications for Assessment

A Balanced Assessment System



Key Structural Features of the CCSSM Have Implications for Assessment

- The Standards are not flat - not all content is emphasized equally.
- The Standards are not a sum of parts - all levels of the hierarchy have been designed to function as content.
- The Standards are not a grab-bag of topics – the Standards define specific learning progressions.
- The content standards aren’t the only standards - Mathematical practices must be connected to content.

Features of CCSSM and Implications for Assessment

Assessing Individual Content Standards or Parts of Standards

Alignment in Context: The Cluster, Domain, and Grade

3.NF.2

Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Can tasks for part (b) include fractions like 6/3 that are equal to whole numbers?

Features of CCSSM and Implications for Assessment

Assessing Individual Content Standards or Parts of Standards

Alignment in Context: Neighboring Grades and Progressions

What are progressions?

Many or most of the content standards in K-8 represent steps or stages along a progression of learning and performance.

Why are progressions important for item writers?

They are context for alignment questions. Progression-sensitive tasks will help teachers implement the standards with fidelity.

Where can I find more information?

Progressions documents are narratives of the standards across grade levels, informed by research on children's cognitive development and by the logical structure of mathematics.

http://math.arizona.edu/~ime/progressions/#products

Features of CCSSM and Implications for Assessment

Assessing Individual Content Standards or Parts of Standards

Alignment in Context: Neighboring Grades and Progressions

Certain cluster headings use language with a sense of motion from grade to grade. Some examples:

Grade 2

Work with equal groups of objects to **gain foundations for** multiplication.

Grade 4

Generalize place value understanding for multi-digit whole numbers.

Extend understanding of fraction equivalence and ordering.

Build fractions from unit fractions by **applying and extending previous understandings** of operations on whole numbers.

Grade 5

Apply and extend previous understandings of multiplication and division **to** multiply and divide fractions.

Grade 6

Apply and extend previous understandings of multiplication and division **to** divide fractions by fractions.

Apply and extend previous understandings of numbers **to** the system of rational numbers.

Apply and extend previous understandings of arithmetic **to** algebraic expressions.

Apply and extend previous understandings of operations with fractions **to** add, subtract, multiply, and divide rational numbers.

Features of CCSSM and Implications for Assessment

Assessing at All Levels of the Content Hierarchy

Smarter Balanced's Claim 1 assessment targets are designed to take the cluster context into consideration.

Cluster headings as assessment targets: In the CCSSM the cluster headings usually serve to communicate the larger intent of a group of standards. For example, a cluster heading in Grade 4 reads: "Generalize understanding of place value for multi-digit numbers." Individual standards in this cluster pinpoint some signs of success in the endeavor, but the important endeavor itself is stated directly in the cluster heading. In addition, the word "generalize" signals that there is a multi-grade progression in grades K-3 leading up to this group of standards. With this in mind, the cluster headings can be viewed as the most effective means of communicating the focus and coherence of the standards. Therefore, this content specifications document *uses the cluster headings as the targets of assessment* for generating evidence for Claim #1. For each cluster, guidance is provided that gives item developers important information about item/task considerations for the cluster. Sample items are also be provided that

Source: Smarter Balanced Content Specifications, pg. 28.

Features of CCSSM and Implications for Assessment

Minimizing pitfalls of traditional multiple choice questions

- High-quality machine-scoreable tasks are critical.
- Some examples of how to do this in CBT that are machine scored:
  - Gridded response
  - True/False with multiple options
  - Drag and drop

Features of CCSSM and Implications for Assessment

Fluency as a special case of assessing individual content standards.

- ***Fluent*** in the standards means ***fast*** and ***accurate***.
- The word *fluency* was used judiciously in the Standards to mark the endpoints of progressions of learning that begin with solid underpinnings and then pass upward through stages of growing maturity.
- Assessing the full range of the standards means assessing fluency where it is called for in the standards.
  - Some of these fluency expectations are meant to be mental and others with “pencil and paper.” But for each of them, there should be no hesitation in getting the answer with accuracy.

Features of CCSSM and Implications for Assessment

Measuring fluency

Smarter Balanced Claim #1: Concepts & Procedures. Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

"…throughout the K-6 standards in CCSSM there are also individual content standards that set expectations for fluency in computation (e.g., fluent multiplication and division within the times tables in Grade 3). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. Technology may offer the promise of assessing fluency more thoughtfully than has been done in the past. This, too, is part of 'measuring the full range of the standards."

Source: Smarter Balanced Content Specifications, pg. 26.

Features of CCSSM and Implications for Assessment

Measuring fluency

Item Stem

For items 1a – 1e, determine whether each equation is True or False.

1a.	$\sqrt{32} = 2^{\frac{5}{2}}$	True	False
1b.	$16^{\frac{3}{2}} = 8^2$	True	False
1c.	$4^{\frac{1}{2}} = \sqrt[4]{64}$	True	False
1d.	$2^8 = \left(\sqrt[3]{16}\right)^6$	True	False
1e.	$\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$	True	False

Fluency and students' ability to see structure are both valuable in this Claim 1 item

Source: Smarter Balanced Assessment Consortium.

Features of CCSSM and Implications for Assessment

Measuring fluency

For items 1a–1c, choose Yes or No to show whether putting the number 7 in the box would make the equation true.

1a.	$10 \times \square = 70$	◯ Yes	◯ No
1b.	$48 \div \square = 6$	◯ Yes	◯ No
1c.	$63 \div \square = 9$	◯ Yes	◯ No

Source: Sample item MAT.03.SR.1.000OA.C.237 http://www.smarterbalanced.org/smarter-balanced-assessments/#item

Features of CCSSM and Implications for Assessment

Smarter Balanced's Claims Embody Specific Mathematical Practices in the Presence of Content Standards

Claim #2: Problem Solving. Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #3: Communicating Reasoning. Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim #4: Modeling and Data Analysis. Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Features of CCSSM and Implications for Assessment

Connecting Content Standards and Mathematical Practices in Assessment

- Practices combine easily and a single student behavior could be thought of as exhibiting multiple practices at once.
- Mathematical practices change through the grades as students grow in mathematical maturity and in the sophistication with which they apply mathematics.
  - Need to ensure grade-level appropriate expectations

Julie's Shirts

In a sale, all the prices are reduced by 25%.

1. Julie sees a jacket that cost $32 before the sale. How much does it cost in the sale? Show your calculations.

In the second week of the sale, the prices are reduced by 25% of the previous week's price. In the third week of the sale, the prices are again reduced by 25% of the previous week's price. In the fourth week of the sale, the prices are again reduced by 25% of the previous week's price.

2. Julie thinks this will mean that the prices will be reduced to $0 after the four reductions because 4 x 25% = 100%. Explain why Julie is wrong.

3. If Julie is able to buy her jacket after the four reductions, how much will she have to pay? Show your calculation.

4. Julie buys her jacket after the four reductions. What percentage of the original price does she save? Show your calculations.

Features of CCSSM and Implications for Assessment

Connecting Content Standards and Mathematical Practices in Assessment

"To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #4. Tasks generating evidence for Claim #4 in a given grade will draw upon knowledge and skills articulated in the progression of standards up to that grade, with strong emphasis on the major work of the grades."

Source: Smarter Balanced Content Specifications, pg. 71.

Features of CCSSM and Implications for Assessment

Connecting Content Standards and Mathematical Practices in Assessment

A Closer Look at Claim #3: Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

(Connection to MP.3: Construct viable arguments and critique the reasoning of others)

Features of CCSSM and Implications for Assessment

Connecting Content Standards and Mathematical Practices in Assessment

Reasoning is a refrain in the content standards

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Note generally such words as *justify* a conclusion, *prove* a statement, *explain* the mathematics; also *derive*, *assess*, *illustrate*, and *analyze*.

Claim #3 tasks have fine "grain size."

The Standards ask students not just to Reason, but to "reason about X," where X is key grade-level mathematics such as properties of operations, relationships between addition and subtraction or between multiplication and division, fractions as numbers, variable expressions, linear/nonlinear functions, etc.

A Closer Look at Claims #2 and #4

Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Modeling & Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

General Discussion

- **With a partner or neighbor please discuss:**

  "Discuss the assessment implications and how it impacts your classroom, school, and district."

- **Share any major insights with our group.**

The Assessment *System*

More to it than a Summative Test

Assessment System Components

Formative Assessment Practices

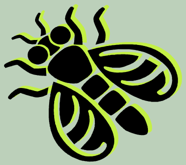
- Research-based, **on-demand tools and resources for teachers**
- Aligned to **Common Core**, focused on increasing student learning and enabling **differentiation of instruction**
- **Professional development** materials include model units of instruction and publicly released assessment items, formative strategies

> **"Few initiatives are backed by evidence that they raise achievement. Formative assessment is one of the few approaches proven to make a difference."**
>
> **- Stephanie Hirsh, Learning Forward**

Tasks for Discussion
Formative, Interim or Summative?

Three frogs sit on a log and 18 flies in the air,
How many flies should each frog get if each frog gets a fair share?

Show your work or explain how you found your answer.



Sixteen frogs sit on a log and 139 flies in the air,
How many flies should each frog get if each frog gets a fair share?

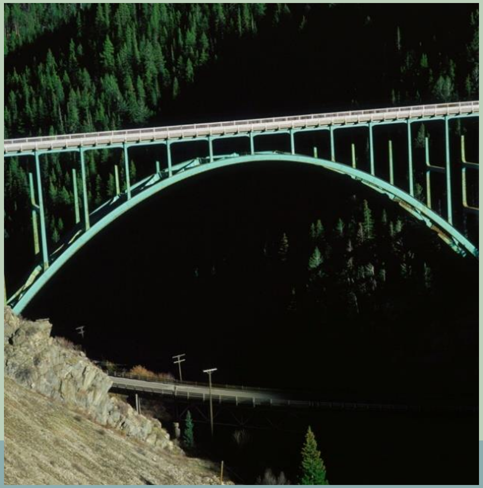
How many flies are still in the air after each frog receives an equal number?

Show your work or explain how you found your answer.

An Overview of Smarter Balanced's Approach

Content Specifications …

- Create a bridge between standards and assessment and, ultimately, instruction
- Organize the standards around major constructs & big ideas
- Express what students should learn and be able to do



Assessment Claims for Mathematics

Concepts and Procedures	"Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency."
Problem Solving	"Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies."
Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Data Analysis and Modeling	"Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

Assessing Fluency (Grade 3)

Mark each equation True or False.

_____ 3 x 8 = 10 + 10 + 4

_____ 6 x 2 = 15 – 3

_____ 42 ÷ 7 = 24 ÷ 6

Standards Addressed:

3.OA.7 Multiply/divide within 100

2.OA.2 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

1.OA.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

Using Technology to Assess Understanding Fractions as Numbers

The numbers 0 and 3/5 are shown on the number line. Put a point on the line to represent the number 1.

Selected Response

Multiple Correct Options

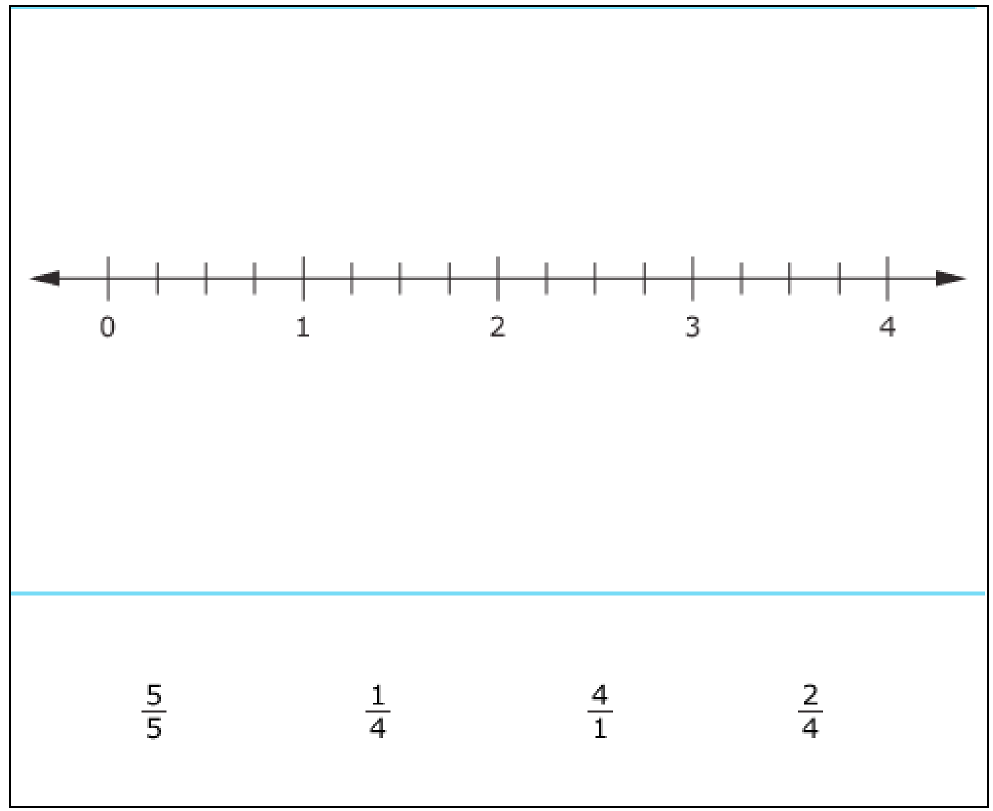
Which of the following statements is a property of a rectangle? Select all that apply.

- ☐ Contains three sides
- ☐ Contains four sides
- ☐ Contains eight sides
- ☐ Contains two sets of parallel lines
- ☐ Contains at least one interior angle that is acute
- ☐ Contains at least one interior angle that is obtuse
- ☐ All interior angles are right angles
- ☐ All sides have the same length
- ☐ All sides are of different length

Grade 3

15

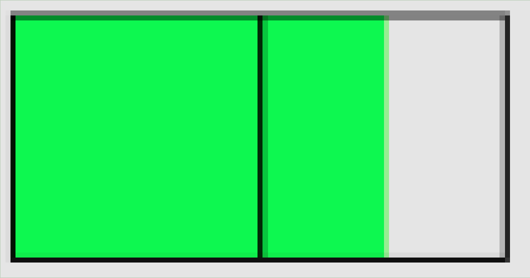
Drag each fraction to the correct location on the number line.



3.NF.A.3b Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram*.

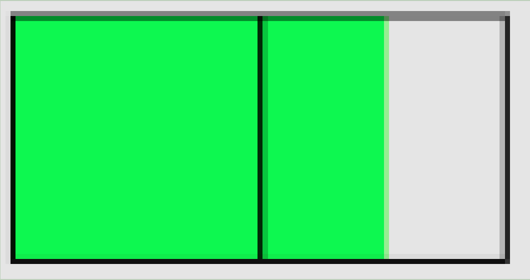
Beyond the Number Line: Other Representations that Support Student Understanding of Fractions

- What fraction is represented by the shaded area?



Notice the use of "represented." The shaded area is not "equal" to a fraction.

- The fraction represented by the shaded area is ¾. Based on this:
  - Draw an area that represents ¼.
  - Draw an area that represents 1.
- The fraction represented by the shaded area is equal to 3/2. Based on this:
  - Draw an area that represents ½.
  - Draw an area that represents 1.

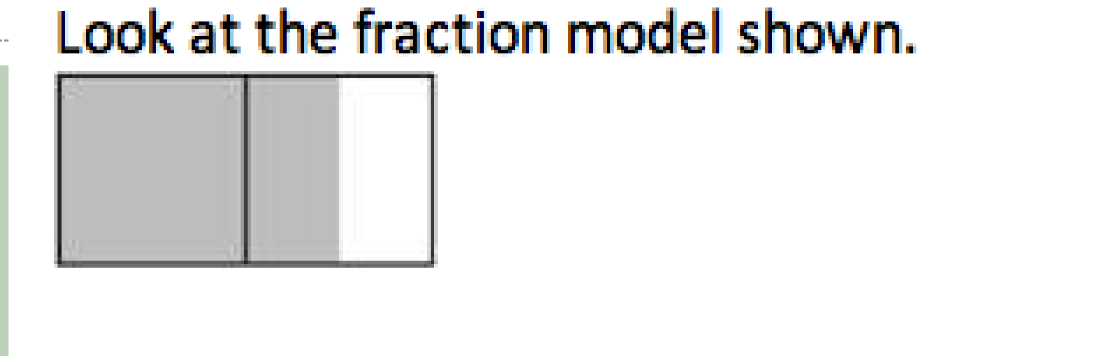


Linking Operations with Fractions to Operations with Whole Numbers

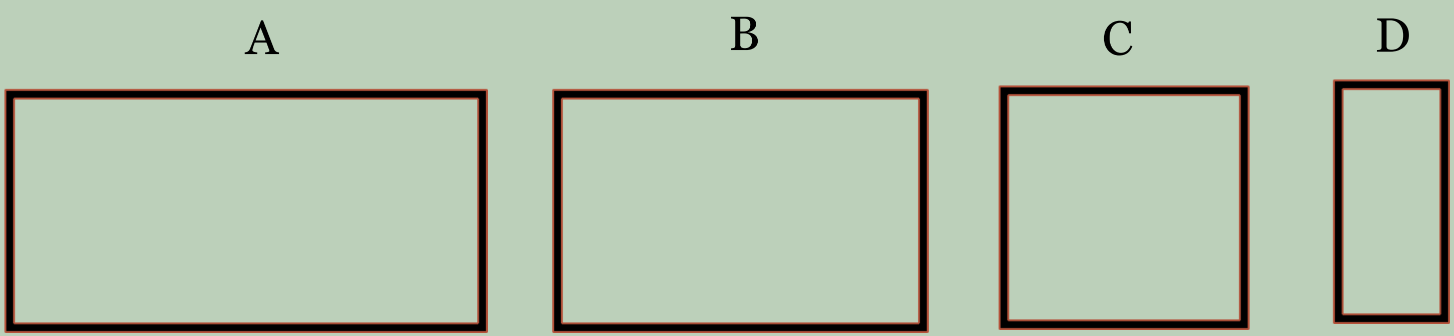
"Children must adopt new rules for fractions that often conflict with well-established ideas about whole number" (p.156)

Bezuk & Cramer, 1989

Fractions Example

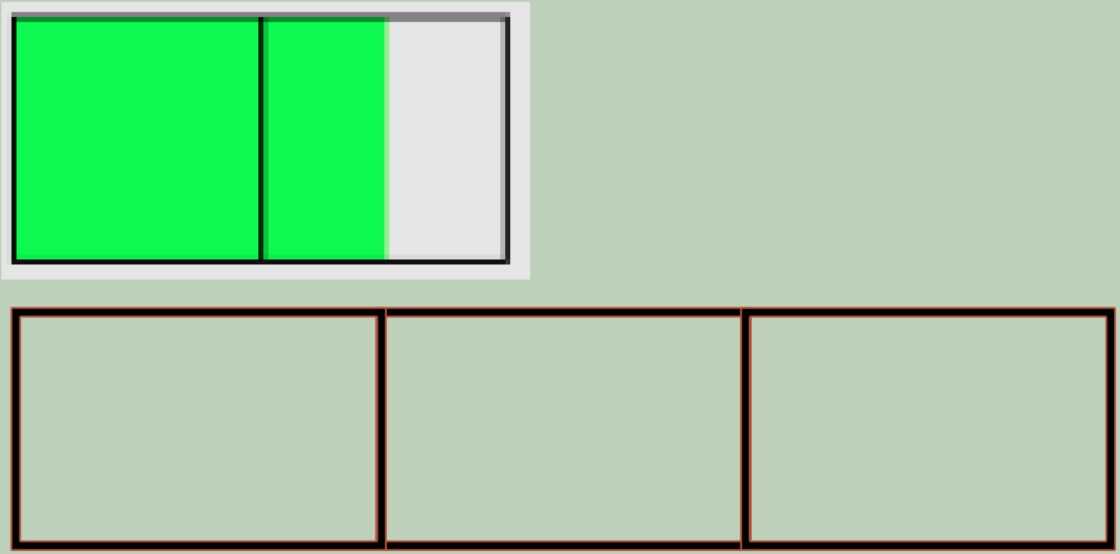


The shaded area represents 3/2. Drag the figures below to make a model that represents 3 x 3/2.



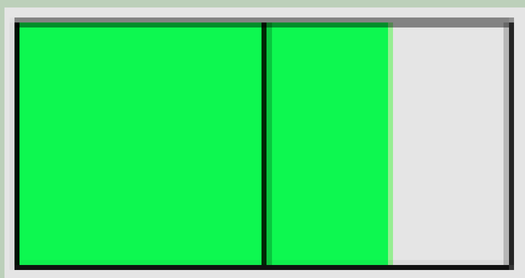
Student A drags three of shape B, which is equal in area to the shaded region. This student probably has good understanding of cluster 5.NF.B he knows that 3 x 3/2 is equal to 3 iterations of 3/2. Calculation of the product is not necessary because of the sophisticated understanding of multiplication.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

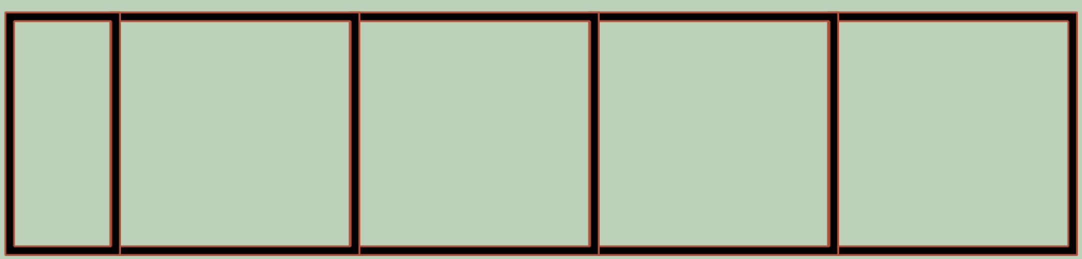


Student B reasons that 3 x 3/2 = 9/2 = 4 ½. She correctly reasons that since the shaded area is equal to 3/2, the square is equal to one whole, and drags 4 wholes plus half of one whole to represent the mixed number.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

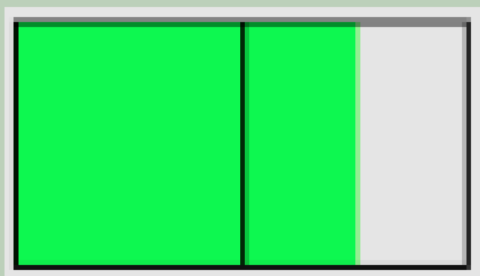


Note that unlike the previous chain of reasoning, this requires that the student determines how much of the shaded area is equal to 1.

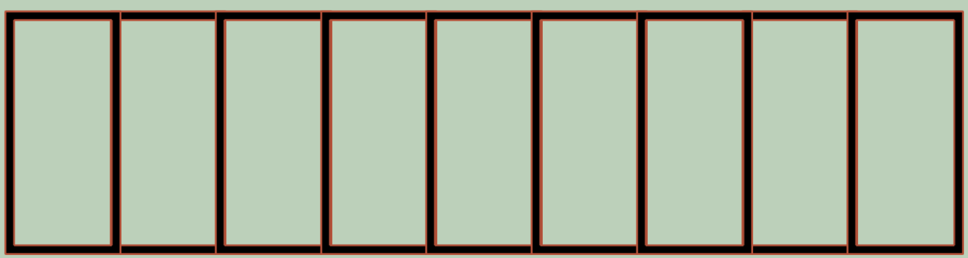


Student C multiplies 3 x 3/2 = 9/2. She reasons that since the shaded area is 3/2, this is equal to 3 pieces of size ½. Since 9/2 is 9 pieces of size ½, she drags nine of the smallest figure to create her model.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.



This chain of reasoning links nicely back to the initial development of 3/2 in 3.NF.1 “understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*, illustrating the coherence in the standards across grades 3-5.



How can K-2 work on operations with whole numbers and work on fractions in grade 3 support students' thinking about these problems in grades 4 & 5?

$$4 + \frac{5}{2} = ?$$

Even before learning the exact sum, can students tell you between which two whole numbers the answer lies?

$$4 \times \frac{5}{2} = ?$$

Even before learning the exact product, what can students tell you about the value of the product?

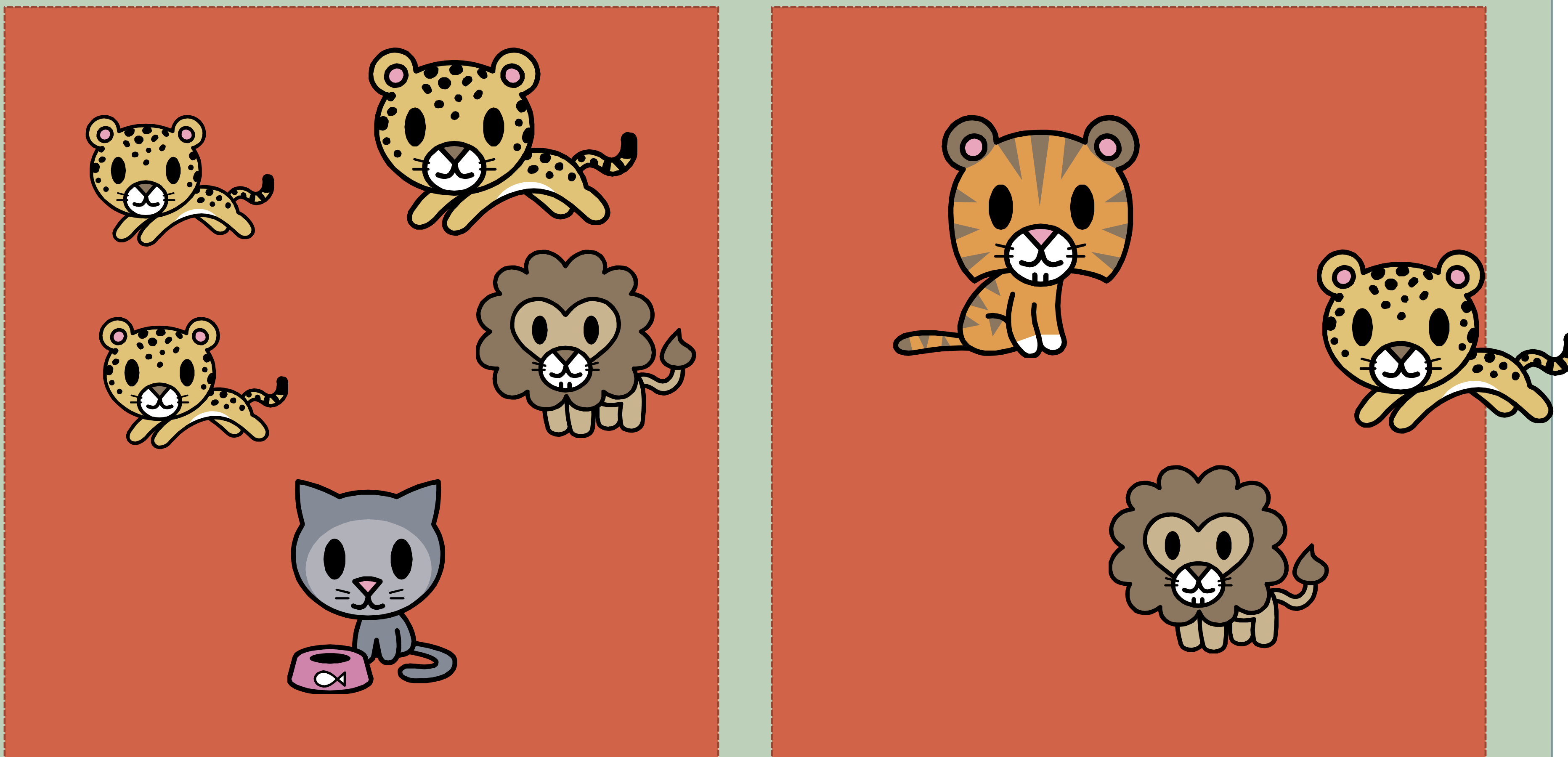
At what grade should students be able to solve these problems?

$$\frac{2}{5}+\frac{2}{3}=\frac{1}{10}+\frac{4}{6}+\frac{?}{?}$$

$\frac{1}{2}+\frac{2}{3}=\frac{1}{?}+\frac{2}{3}$	$\frac{3}{6}+\frac{2}{3}=\frac{1}{?}+\frac{2}{3}$	$\frac{3}{6}+\frac{2}{3}=\frac{1}{?}+\frac{4}{6}$	$\frac{1}{2}+\frac{2}{3}=\frac{4}{6}+\frac{3}{?}$

$$\frac{1}{3}+\frac{1}{3}=\frac{1}{?}+\frac{1}{3}$$

Explain the flaw in the chain of reasoning. In the first group, 3/5 of the cats have spots. In the second group, 1/3 of the cats have spots. All together, 4/8 of the cats have spots. Therefore, 3/5 + 1/3 = 4/8.



General Discussion

- **With a partner or neighbor please discuss:**

  "Discuss a concept/skill that you teach and how the progression of this concept/skill is taught in the grade prior and after."

- **Share any major insights with our group.**

Topic 1: Fractions

The Progression Across Grades 3-5

Definition of “fraction”

Definition. Let k, l be whole numbers with $l > 0$. Divide each of the line segments $[0,1]$, $[1,2]$, $[2,3]$, $[3,4]$, $\ldots$ into l segments of equal length. These division points together with the whole numbers now form an infinite sequence of equally spaced markers on the number line (in the sense that the lengths of the segments between consecutive markers are equal to each other). The first marker to the right of 0 is by definition $\frac{1}{l}$. The second marker to the right of 0 is by definition $\frac{2}{l}$, the third $\frac{3}{l}$, etc., and the k-th is $\frac{k}{l}$. The collection of these $\frac{k}{l}$'s for all whole numbers k and l, with $l > 0$, is called the *fractions*. The number k is called the *numerator* of the fraction $\frac{k}{l}$, and the number l its *denominator*.

For typographical reasons, a fraction $\frac{k}{l}$ is sometimes written as k/l. We adopt for convenience the **convention** that *the fraction notation* $\frac{k}{l}$ *or* k/l *automatically assumes that* $l > 0$. It is common to call $\frac{k}{l}$ a *proper* fraction if $k < l$, and *improper* if $k \geq l$. Note that by the way we define a fraction,

> *we make no distinction between proper fractions and improper fractions.*

3.NF.A Develop understanding of fractions as numbers.

- On a piece of paper, draw a line segment that is 6 inches long.
- Label the left endpoint 0 and the right endpoint 3.
- Locate and label the numbers 1 and 2 on the number line with respect to locations of 0 and 3.
- Locate and label the numbers 1/3, 5/3, and 9/3.

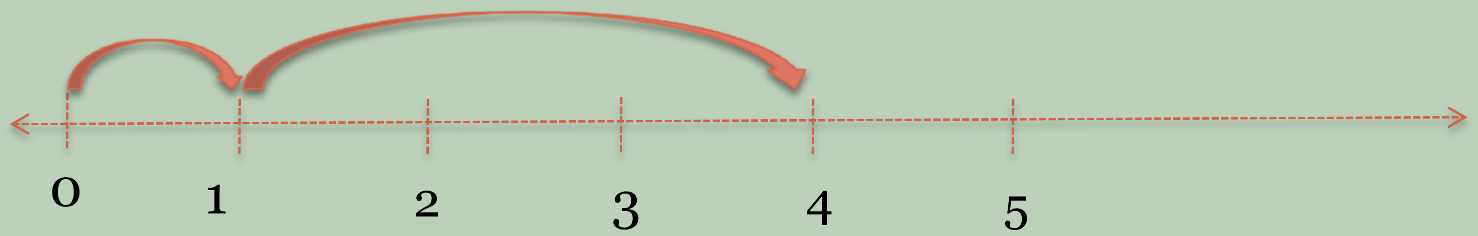
If you were doing this with Grade 3 students, would you change any direction above? What additional support may be needed? Why?

3.NF.A Develop understanding of fractions as numbers.

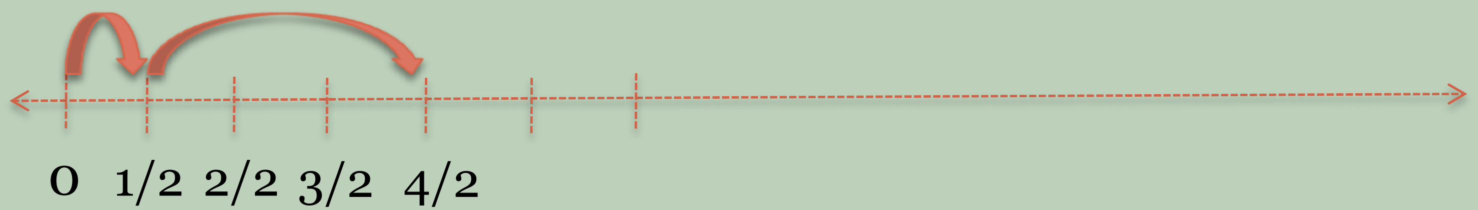
- Next, label all fractions with denominator 3 along the part of the number line you have drawn.
  - How many fractions with denominator 3 do you have labeled?
- Discuss with your table: What parallels are there between how whole numbers are introduced in kindergarten and this introduction to fractions?

Operations with fractions should be a natural extension of operations with whole numbers

1 + 3 = ?



$$\frac{1}{2} + \frac{3}{2} = ?$$



Performance Tasks

Performance Tasks

- Extended projects demonstrate real-world writing and analytical skills
- May include online research, group projects, presentations
- Require 1-2 class periods to complete
- Included in both interim and summative assessments
- Applicable in all grades being assessed
- Evaluated by teachers using consistent scoring rubrics

> **"The use of performance measures has been found to increase the intellectual challenge in classrooms and to support higher-quality teaching."**
>
> **- Linda Darling-Hammond and Frank Adamson, Stanford University**

Discuss in your group the impact of what is happening in this picture in regards to classroom instruction.

Viewing Smarter Balanced Sample Items & Tasks

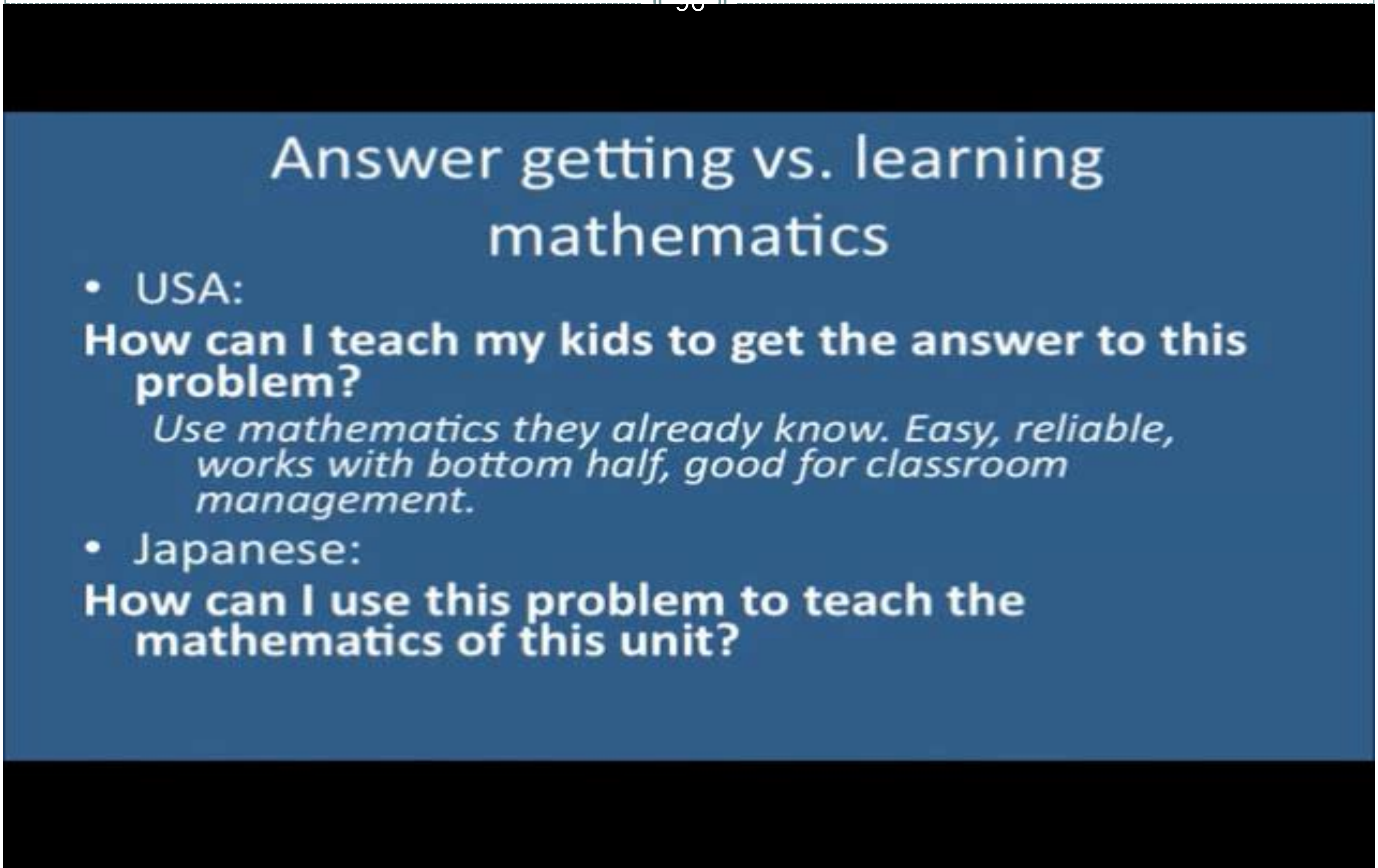
- Sample Items
  - http://sampleitems.smarterbalanced.org/itempreview/sbac/

- Full Practice Tests Available
  - http://sbac.portal.airast.org/practice-test/

Instructional Resources

- www.achievethecore.org
- www.illustrativemathematics.org
- http://insidemathematics.org/index.php/mathematical-content-standards
- http://map.mathshell.org/materials/tasks.php
- http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm
- http://commoncoretools.me/illustrative-mathematics/
- http://illuminations.nctm.org/Activities.aspx?grade=all&srchstr=problem%20solving
- www.pta.org/4446.htm
- http://math.arizona.edu/~ime/progressions/#products
- http://www.smarterbalanced.org/k-12-education/common-core-state-standards-tools-resources/

We need to work on changing our focus: http://vimeo.com/3092498



- “There's a lot of value to getting it just about right, and a lot of cost to getting it wrong, and what we discovered is that mathematics does not break down into lesson-sized pieces.”

- Phil Daro

Questions

Thank You

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