BOISE STATE UNIVERSITY

Transformations and Congruence in the CCSS-M 6-10

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Initiative for Developing Mathematical Thinking

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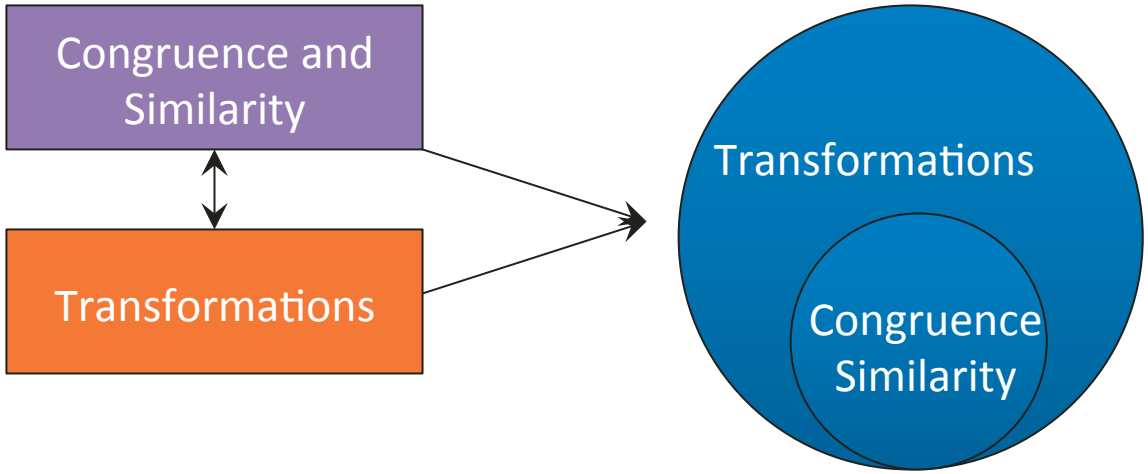
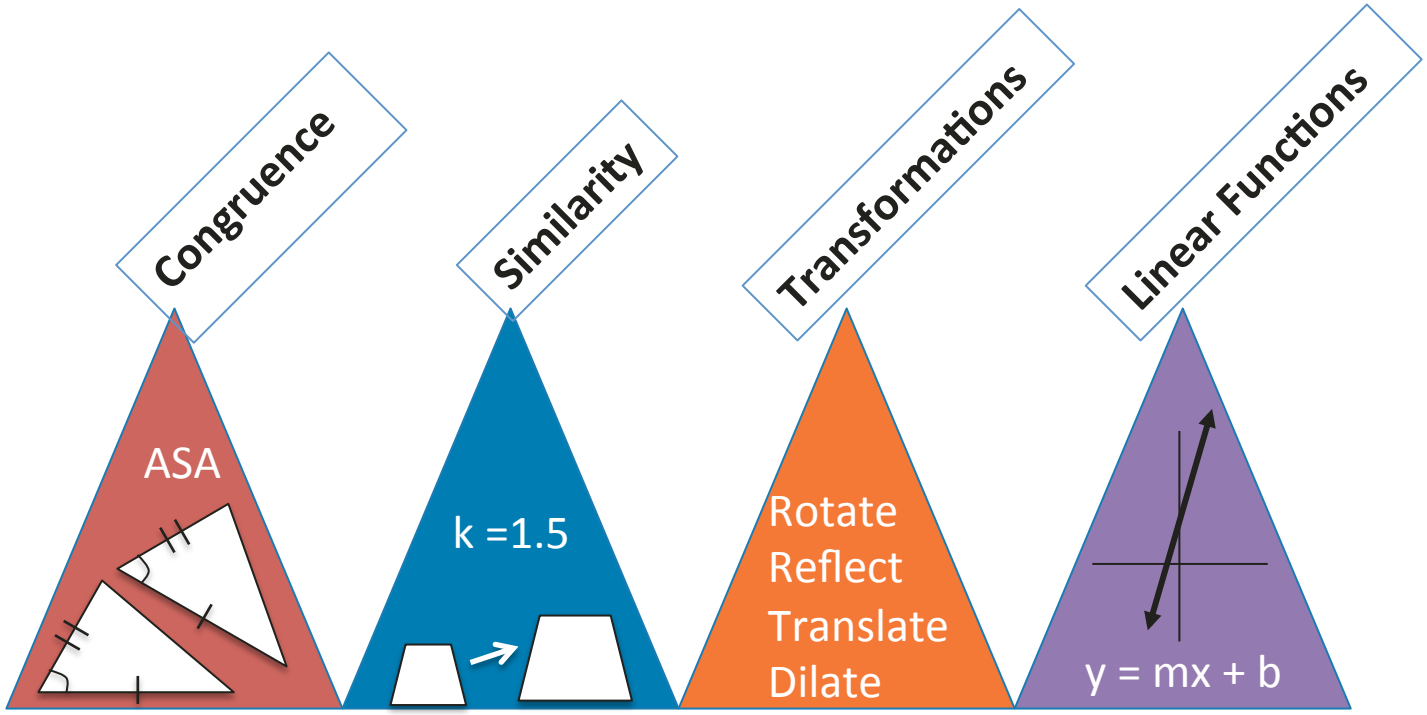
So…Why change?



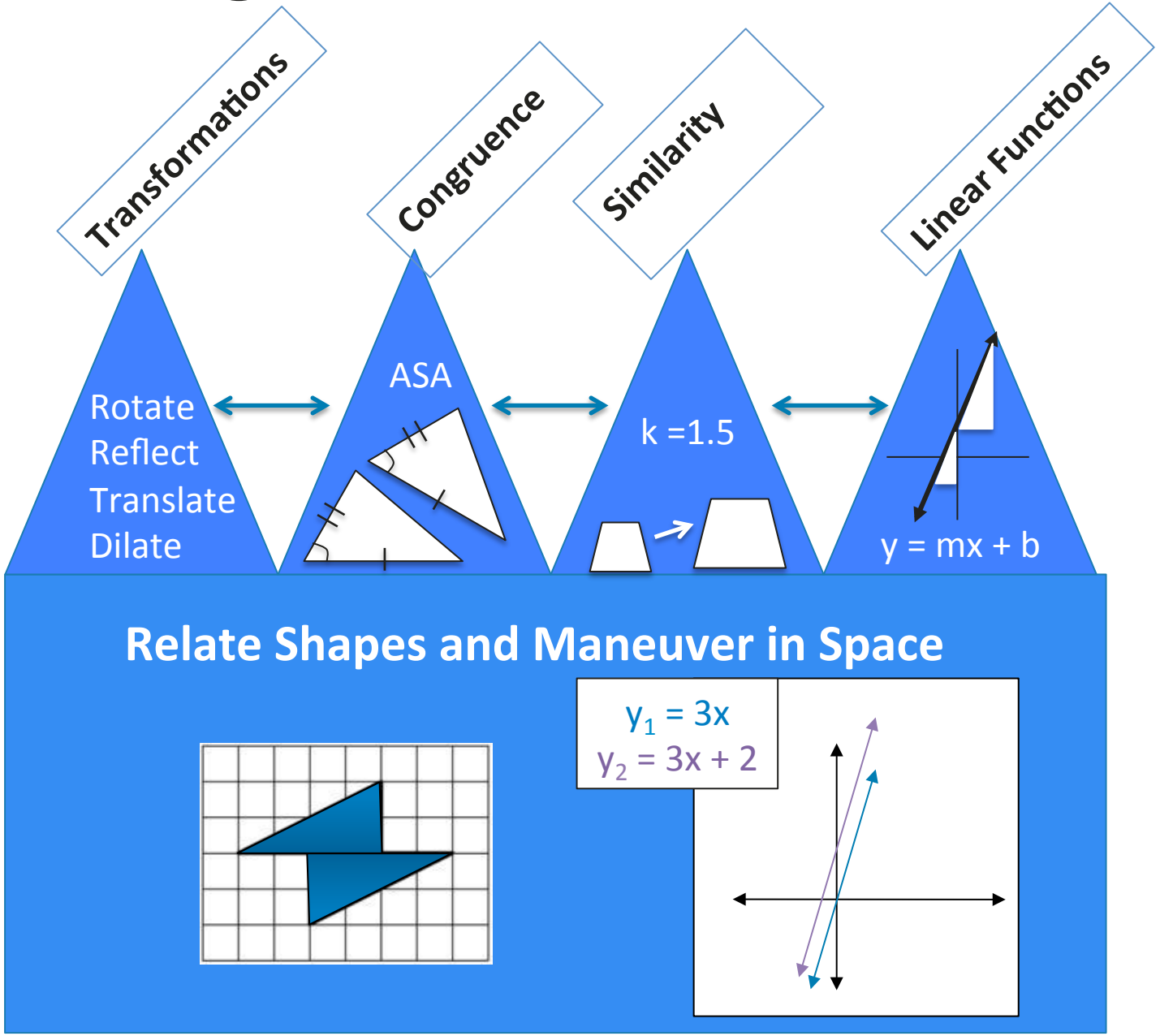
1. Finding coherence and structure across mathematics
2. Spatial reasoning in STEM fields

Traditional Mathematics



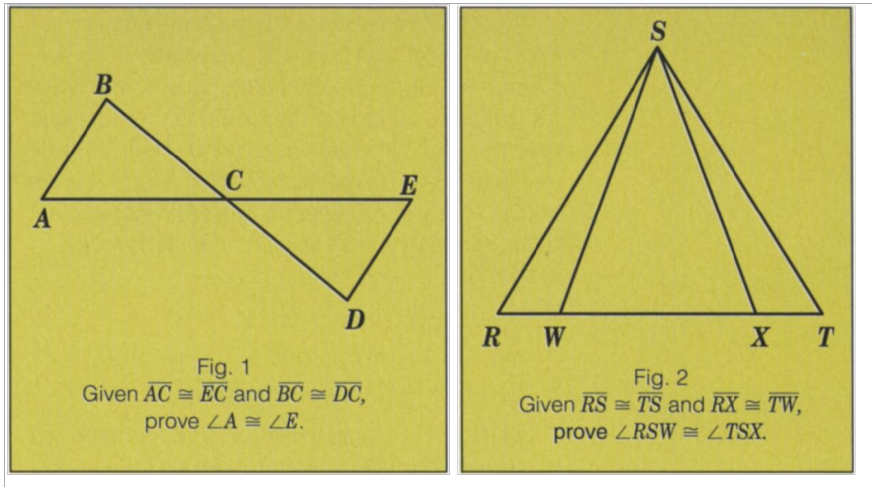
Iceberg analogy stolen shamelessly from Jeff Frykholm

Finding Structure in Mathematics



Connecting Transformations to CPCFC

- Okolica and Macrina (1992)
- Transformational geometry unit before deductive geometry
- Students prove congruence both by traditional methods and by transformations
- When students see transformation connection they more easily identify corresponding parts.



Holistic

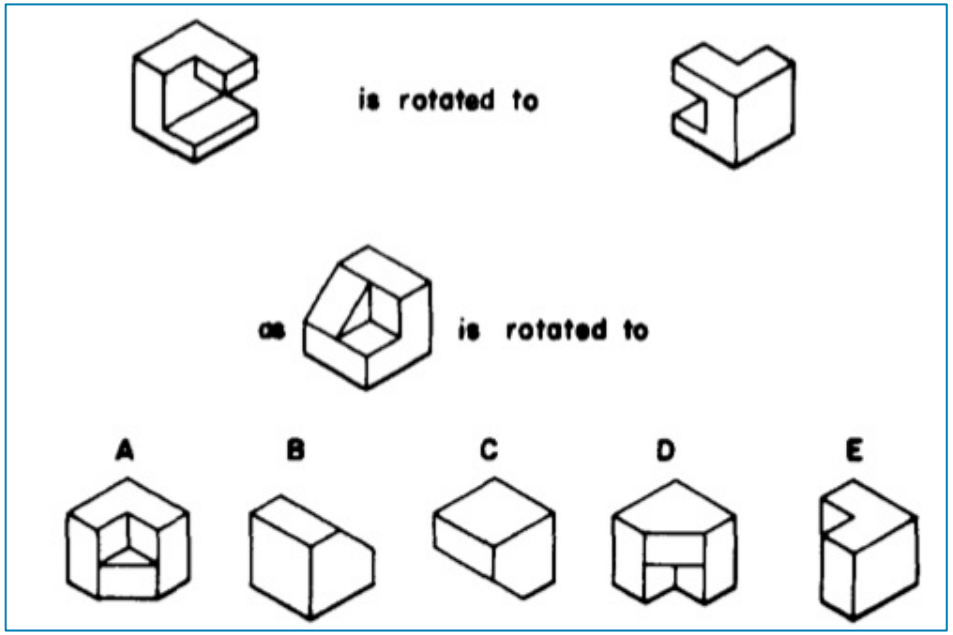
↓

Analytical

Spatial reasoning in chemistry

"A significant spatial ability main effect was found in this study when: (1) exam questions required students to mentally manipulate two-dimensional representations of molecules, and/or (2) exams focused on higher order cognitive skills such as problem solving."

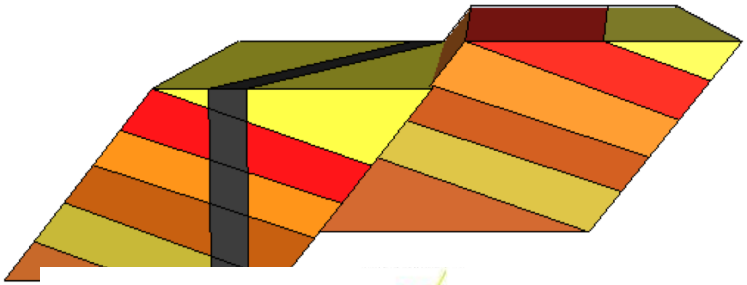
Pribyl and Bodner 1987, *Journal of Research in Science Teaching*

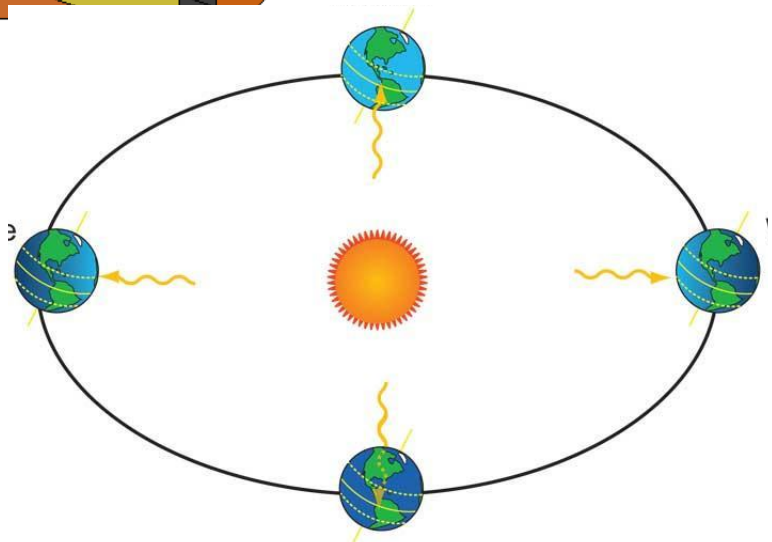


Example problem from Purdue Visualization of Rotations Test

Applications – Earth Science

Black, 2005, Journal of Geoscience Education





"…Between one fourth and one third of the total variation in ESC scores can be accounted for by spatial ability, an ability that has not been systematically fostered in traditional education."

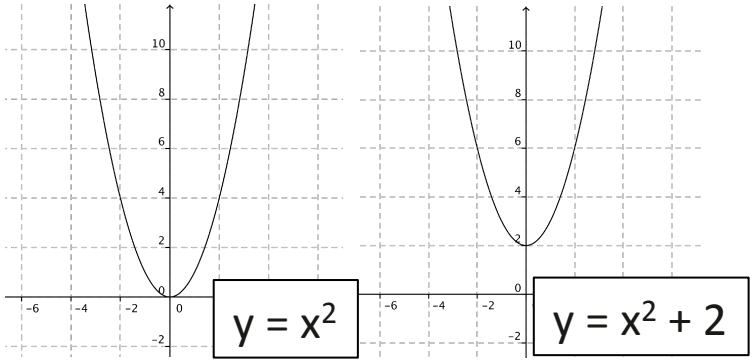
ESC = Earth Science Concept Test

Seeing Connections

Start with High School

Focus Area:
"The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation."

Focus Area:
"Geometric transformations of the graphs of equations correspond to algebraic changes in their equations."



Transformations across Geometry

High School

"The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation."

8th Grade

"use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze 2D figures and to solve problems."

7th Grade

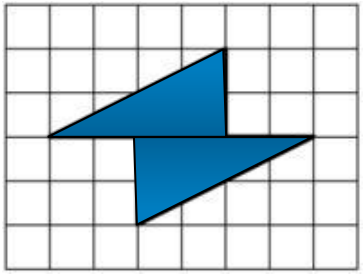
"reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions"

6th Grade

"find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles.

Transformations across Geometry

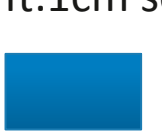
≤6th Grade



Find area that is shaded.

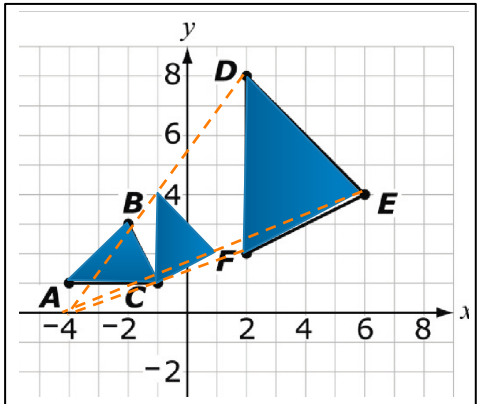
7th Grade

40 ft:1cm scale



20 ft:1cm scale

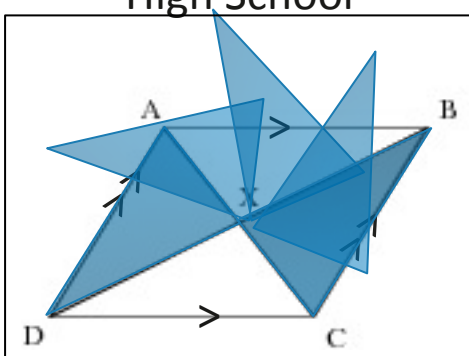
8th Grade



Describe transformations that take ΔABC to ΔDEF

Adapted from sample SBAC item

High School



Can we prove that the shaded triangles are congruent?

Adapted from Illustrative Mathematics

Connecting Symbols with Geometry

High School

"Geometric transformations of the graphs of equations correspond to algebraic changes in their equations."

8th Grade

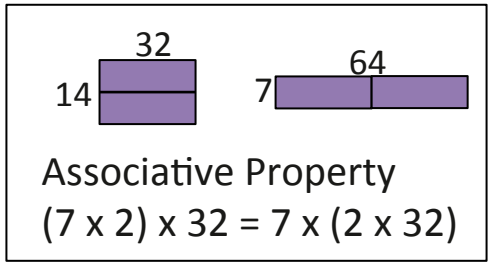
"Understand the connections between proportional relationships, lines, and linear equations."

1-7th Grade

Write equivalent mathematical expressions based on properties of operations.

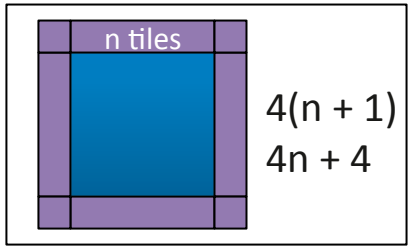
Connecting Symbols with Geometry

1-7th Grade



Associative Property

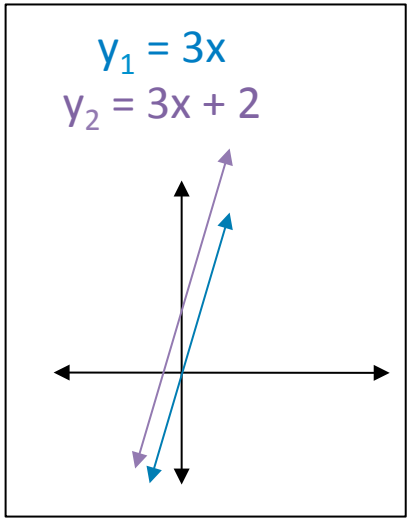
$(7 \times 2) \times 32 = 7 \times (2 \times 32)$



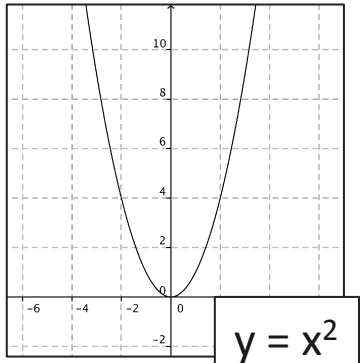
$4(n + 1)$

$4n + 4$

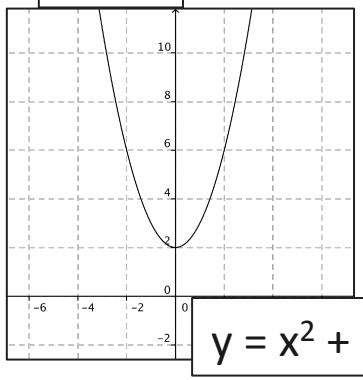
8th Grade



High School



$y = x^2$



$y = x^2 + 2$

"The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation."

K-5: Decompose / Compose / Create / Compare / Partition / Reason with shapes*, Lines of symmetry (fold and draw), Coordinate plane movement

* Shaping gets overlooked
* Visual ↔ Verbal (a lot)
* "Questions surrounding identification and definition can get in the way of developing imagery."

(*NCTM Developing Essential Understanding of Geometry Grades 6-8, page 25, 27)

Practical Considerations

- Spatial reasoning warm-ups
- Connect spatial reasoning across curriculum (e.g. area formulas, graphing, parallel lines)
- Move or combine congruence and transformation chapters
- Encourage high school students to defend congruence or similarity via transformations AND standard criteria, relating the two.

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Webinars available at dmt.boisestate.edu

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