Engaging Real-World Investigations for Skill Development

Arjan Khalsa

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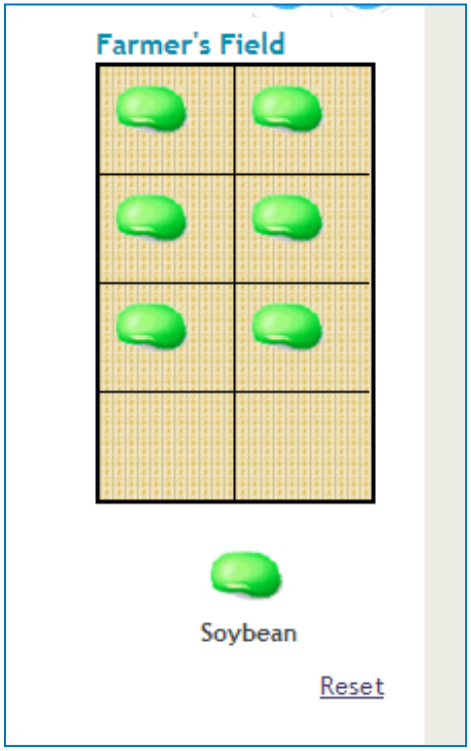
Agenda

- Real World Problems and Their Benefits
  - Common Core Expectations
  - Children's Learning Experiences
- Facilitating Classroom Dialogue
- The Three-Tiered Data Approach
- Looking at Sample Problems
- Student and Teacher Experience: Samples
- Experience a Real World Problem
- Create Your Own Real World Investigation

Arjan Khalsa

- CEO, Conceptua Math
- K-6 Teacher in early 1980's
- On team that first published NCTM standards in 1986
- FOSS Author
- Principle Investigator on three large scale, federally-funded math projects

Exploring What Real World Means



Three classes at Lakeview School are going on a field trip. The table shows the number of people in each class, including the teacher.

They can choose to use buses, vans, and cars.

	Total number of people
Mrs. Ruiz's Class	23
Mr. Yang's Class	25
Mrs. Evans' Class	24

Buses have 20 seats

Vans have 16 seats

Cars have 5 seats

Which three combinations can be used to take all three classes on the field trip?

- [ ] 1 bus and 4 vans
- [ ] 3 vans and 11 cars
- [ ] 1 bus and 1 van and 6 cars
- [ ] 1 bus and 8 cars
- [ ] 2 buses and 3 vans and 4 cars

A Child's World Has A Lot of Data

Authentic Data

- Intrinsic Motivation / Engagement
  - Daniel Pink, *Drive*
  - Creativity and perseverance
- Problem Solving Skills
  - Steve Leinwand, *Accessible Mathematics*
  - Abundant data presented in tables, charts, and graphs
  - Make conjectures and draw conclusions

"*Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace*."

Standards for Mathematical Practice MP4

A Three-Tiered Approach to Data

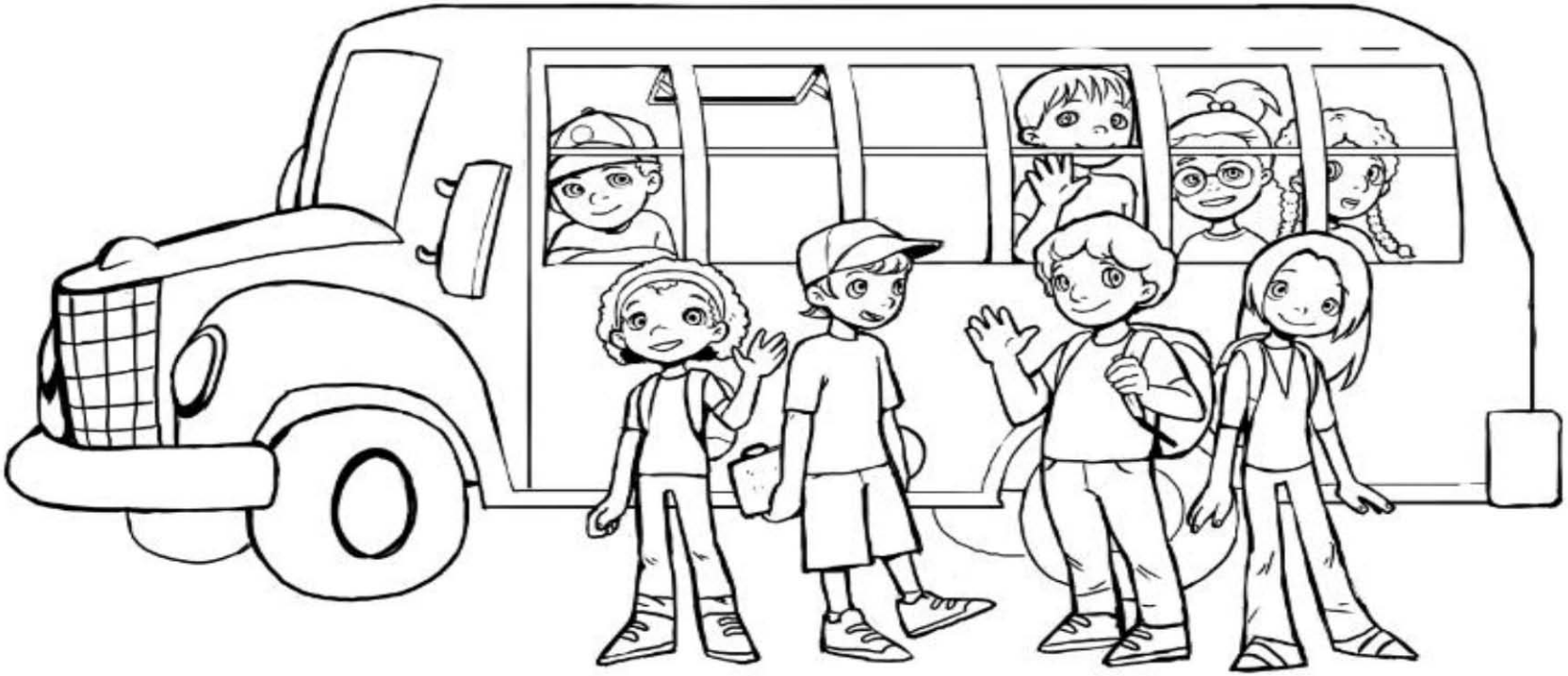
- Tier 1 – Data provided, student learns a heuristic model
- Tier 2 – Some data provided, student adds to the heuristic model
- Tier 3 – All data created by the student, student uses the heuristic model as an analytical tool

Let's Look at a Problem

Grade 3
Big Idea: Fractions are numbers that can represent quantities in different ways.

Alleah's Bus

Alleah's School—Alleah's Data



Alleah's School – TIER I

dents riding the bus that are at the back of the bus

Alleah's School - Create Some Data

Alleah's School—Create Some Data

Alleah's School – TIER II

Your School – TIER III

Student Work, Part 1

- Look at the picture of what Alleah saw on the next page and use it to write the fractions in the data chart below.
- Then, compare each fraction to 1/2 and explain whether the fractional part in the picture is less than, equal to, or greater than 1/2.

Find this in the Picture	**Write the Fraction**	**Compare the Fraction: Is it less than, equal to, or greater than 1/2?**
Fraction of the students who are on the school bus	4/8	equal to 1/2
Fraction of the bus windows that are closed	5/6	greater than 1/2
Fraction of the students who are wearing glasses	1/8	less than 1/2
Fraction of the students who are wearing baseball hats	2/8	less than 1/2
Fraction of the students who are waving	3/8	less than 1/2
Fraction of the students riding the bus that are at the back of the bus	3/4	greater than 1/2

Student Samples – Create Some Data

Alleah's School—Create Some Data

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Student Sample- Create Some Data

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Alleah's School—Create Some Data

Alleah's class had a class photo taken for the yearbook. Alleah described the picture to you with these sentences below.

Use the information below to complete the details in the picture on the next page and fill in the table below.

- More students have brown eyes than have blue eyes.
- Fewer students have black hair than have brown hair.
- More students have blonde hair than have red hair.
- Less than 1/2 of the class wears glasses.
- More than 1/2 of the class is smiling in the class picture.
- More students wore blue shirts than wore red shirts, but more students wore red shirts than wore green shirts.

Total number of students: **Number of students in the class picture who...**	**Write the Fraction**	**Is the fraction less than, equal to, or greater than 1/2 of the class?**
Have brown eyes 7	$\frac{7}{12}$	more
Have blue eyes 4	$\frac{4}{12}$	less
Have eyes that are not brown or blue 1	$\frac{1}{12}$	less
Have black hair 2	$\frac{2}{12}$	less
Have brown hair 3	$\frac{3}{12}$	less
Have red or blonde hair 1 4	$\frac{1}{12}$ $\frac{4}{12}$	less
Wear glasses 3	$\frac{3}{12}$	Less than 1/2
Who smiled in the picture 10	$\frac{10}{12}$	More than 1/2
Who wore blue shirts 5	$\frac{5}{12}$	less
Who wore red shirts 4	$\frac{4}{12}$	less
Who wore green shirts 3	$\frac{3}{12}$	less

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Student Sample

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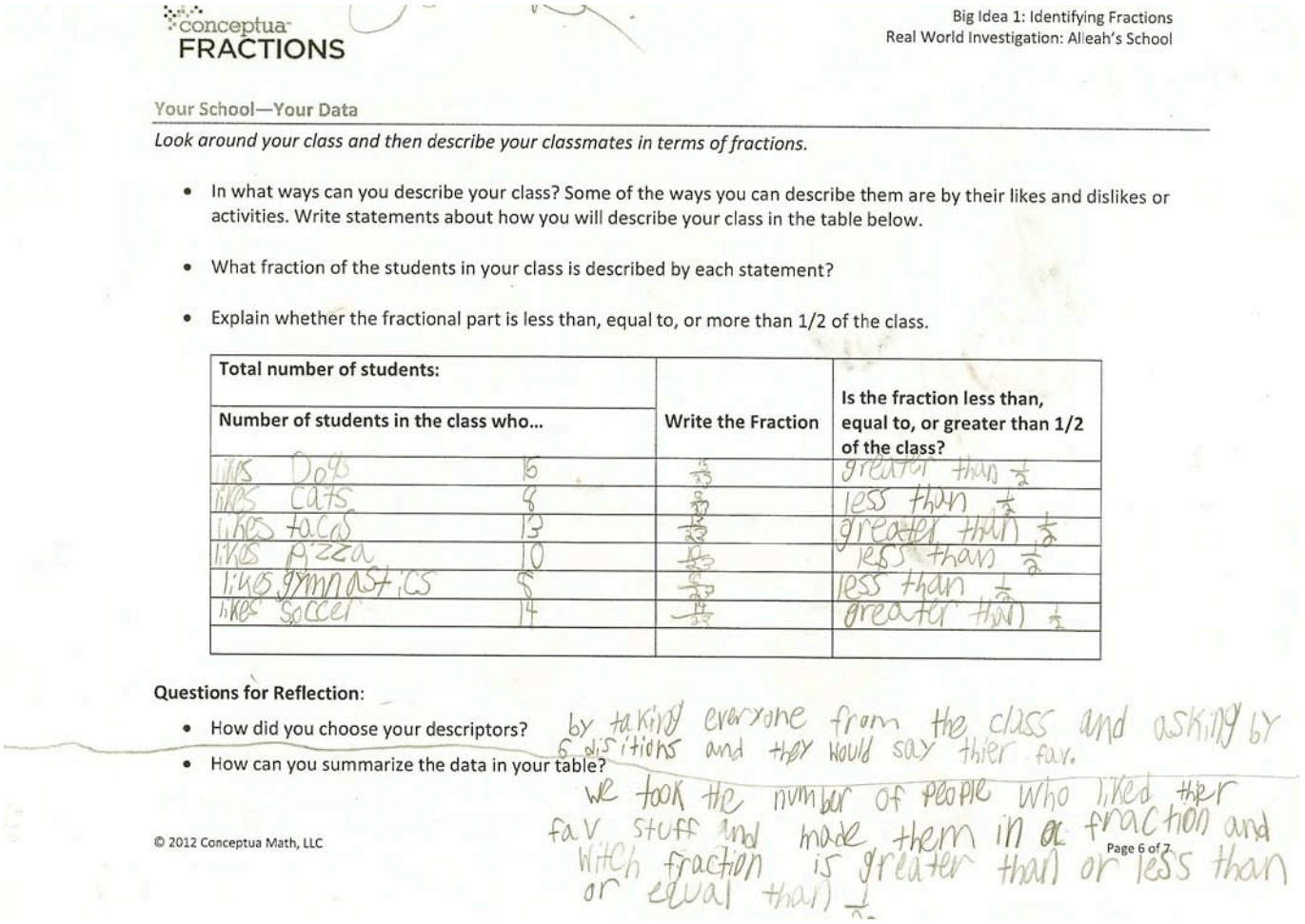
Total number of students: Number of students in the class picture who...	Write the Fraction	Is the fraction less than, equal to, or greater than 1/2 of the class?
Have brown eyes 7	7/12	greater than 1/2
Have blue eyes 4	4/12	less than 1/2
Have eyes that are not brown or blue 1	1/12	less than
Have black hair 3	3/12	less than
Have brown hair 4	4/12	less than
Have red or blonde hair 1 4	1/12 } 4/12	
Wear glasses 2	2/12	Less than 1/2
Who smiled in the picture 12	12/12	More than 1/2
Who wore blue shirts 5		
Who wore red shirts 3		
Who wore green shirts 2		

Student Sample



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Big Idea 1: Identifying Fractions
Real World Investigation: Alleah's School

Your School—Your Data

Look around your class and then describe your classmates in terms of fractions.

- In what ways can you describe your class? Some of the ways you can describe them are by their likes and dislikes or activities. Write statements about how you will describe your class in the table below.
- What fraction of the students in your class is described by each statement?
- Explain whether the fractional part is less than, equal to, or more than 1/2 of the class.

Total number of students:		Write the Fraction	Is the fraction less than, equal to, or greater than 1/2 of the class?
Number of students in the class who...			
likes Dogs	16	$\frac{16}{25}$	greater than $\frac{1}{2}$
likes cats	9	$\frac{9}{25}$	less than $\frac{1}{2}$
likes tacos	13	$\frac{13}{25}$	greater than $\frac{1}{2}$
likes pizza	10	$\frac{10}{25}$	less than $\frac{1}{2}$
likes gymnastics	5	$\frac{5}{25}$	less than $\frac{1}{2}$
likes soccer	14	$\frac{14}{25}$	greater than $\frac{1}{2}$

Questions for Reflection:

- How did you choose your descriptors? by taking everyone from the class and asking by 5 questions and they would say thier fav.
- How can you summarize the data in your table? we took the number of people who liked thier fav stuff and made them in a fraction and with fraction is greater than or less than or equal than) $\frac{1}{2}$

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Student Sample

Questions for Reflection:

- How did you choose your descriptors? question #1: we chose our descriptors by asking everyone their favorites.
- How can you summarize the data in your table? question #2: more than 1/2 of the students like soccer in our class.

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Questions for Reflection:

- How did you choose your descriptors? My math class decided on two things for 7 topic to put down
- How can you summarize the data in your table? Out of 23 most students lik dogs out of 23 and tacos out of pizza, and soccer out of Gumnastics.

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Discussion About Student Behaviors

The students

- Liked the real-world school context.
- Were beginning to think with fractions.
- Understood many different fractions equivalent to ½ and were able to use ½ as a benchmark.
- Enjoyed examining their own work in fraction terms.
- See fractions in their world beyond cooking

Discussion About Teacher Behaviors

Teacher preparation

- It is a good idea to use the first problem as a way to introduce strategies that students can use to solve the problem.
- Reference the Conceptua Math Lesson Plan
- Need to anticipate where students will have difficulty in advance.
  - consider questions that can guide students
- Need to consider how the problem can be broken down if students struggle.
  - How to scaffold without sacrificing the challenge
- Need to plan for misconceptions

Intended Student Behaviors

Grouping the practice standards

1. Make sense of problems and persevere in solving them 6. Attend to precision	2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others	Reasoning and explaining
	4. Model with mathematics 5. Use appropriate tools strategically	Modeling and using tools
	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Seeing structure and generalizing

Habits of Mind of a Productive Mathematical Thinker

From *"Standards for Mathematical Practice"* document by Will McCallum of The University of Arizona.

Creating a Classroom Culture

- Students are encouraged to:
  - Express their point of view
  - Listen to others
  - Critique their own reasoning and the reasoning of others
- Methods
  - Provide the proper stimulus
  - Use productive talk moves
    - Waiting, reasoning, adding on
    - Repeating, re-voicing

Video

From *"Standards for Mathematical Practice"* document by Will McCallum of The University of Arizona.

Common Core Expectations

PARCC Partnership for Assessment of Readiness for College and Careers

TASK TYPES		
TYPE I: *TASKS ASSESSING CONCEPTS, SKILLS AND PROCEDURES*	TYPE II: *TASKS ASSESSING EXPRESSING MATHEMATICAL REASONING*	TYPE III: *TASKS ASSESSING MODELING / APPLICATIONS*
Type I tasks include a balance of conceptual understanding, fluency, and application. These tasks can	Type II tasks call for written arguments/justifications, critique of reasoning, or precision in mathematical	Type III tasks call for modeling/application in a real-world context or scenario (MP.4) and can also

A well-constructed real world problem can exercise all three types.

Common Core Expectations

- Claim 1: Concepts and Procedures
- Claim 2: Problem Solving
- Claim 3: Communicating Reasoning
- Claim 4: Modeling and Data Analysis

Smarter Balanced Assessment Consortium

A well-constructed real world problem can exercise all four claims.

Making Investigations Meaningful

- Relate to the lives of the students
- Involve problem articulation by the students
- Answers are likely to be judgments or decisions
- Involve reasoning and justifying, making good decisions

Let's Experience a Problem

Big Idea 8: Multiplication with Fractions

Let's Try a Real World Problem

- Multiplying Fractions
- 15 acres of land for sale
- Can Springfield build their sports complex?

Type of Sports Venue	Number They Want to Build	Size in Acres	Total Acreage
Soccer Field		2	
Football Field		1 1/2	
Baseball Field		3 3/4	
Basketball Court		1/10	
Set of Tennis Courts		1/4	
Outdoor Swimming Pool		2/10	
Skateboard Park		1/2	
Total			

Free Access to Real Word Problems

http://www.nctm.org/publications/worlds/

www.conceptuamath.com

Big Idea 5, Real World Problem

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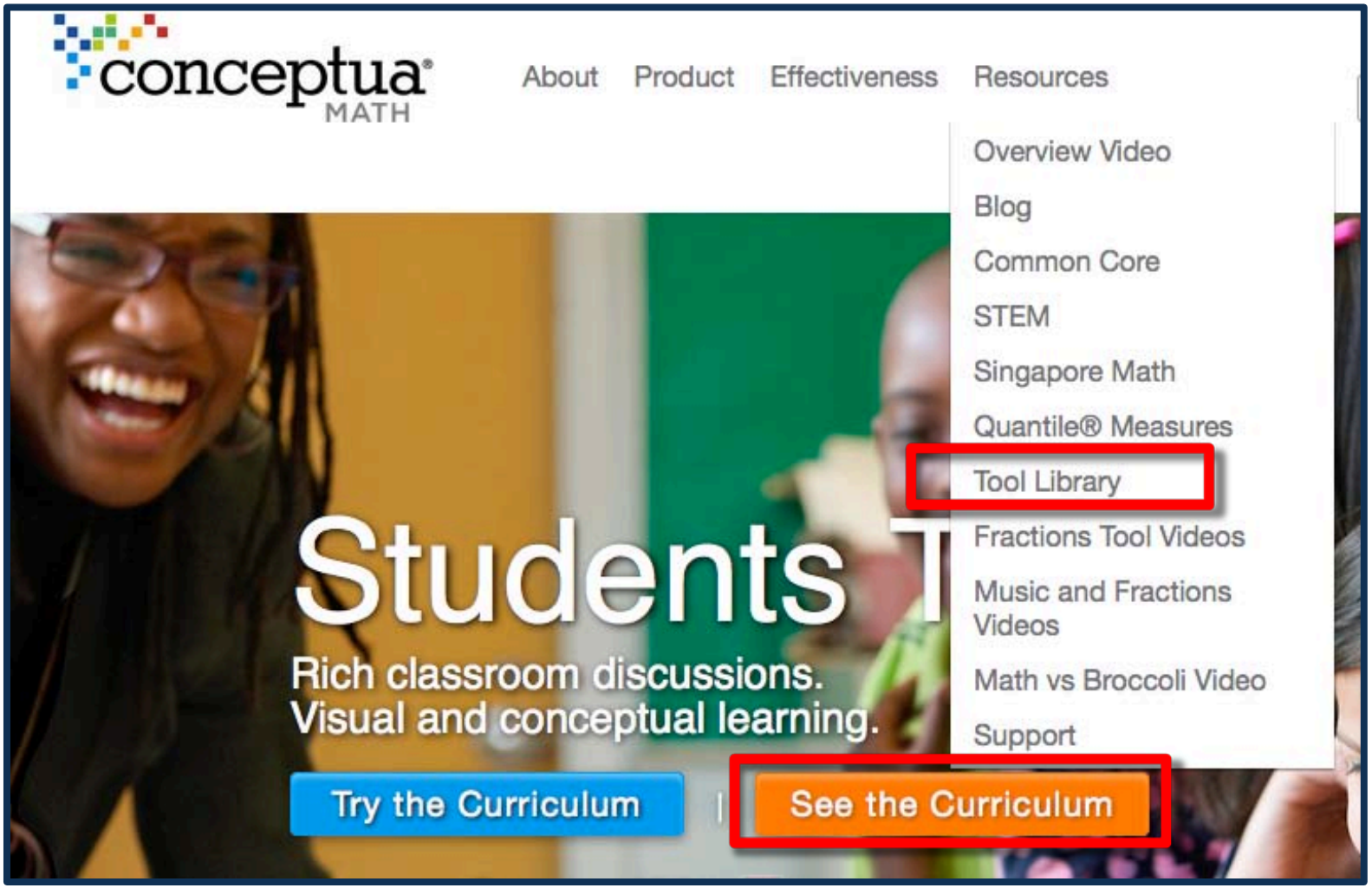
Many ways to Access: Tools



Tool Videos for Students and Teachers

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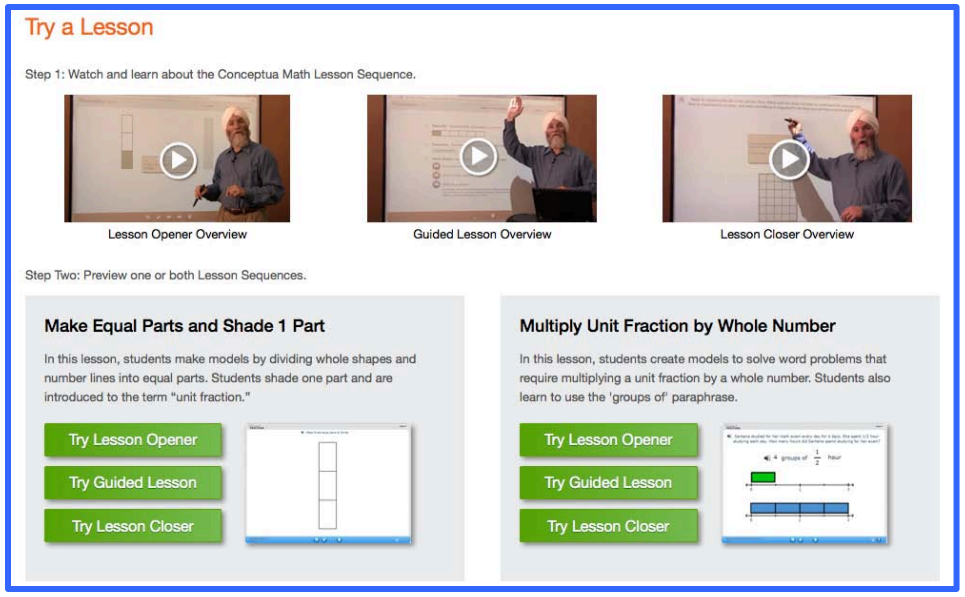
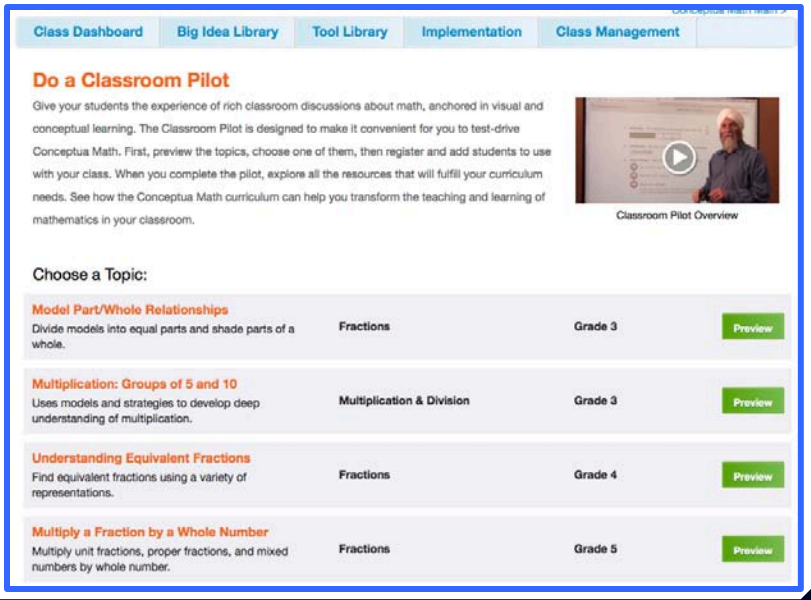
About Product Effectiveness Resources

- Overview Video
- Blog
- Common Core
- STEM
- Singapore Math
- Quantile® Measures
- Tool Library
- **Fractions Tool Videos**
- Music and Fractions Videos
- Math vs Broccoli Video
- Support

Conceptua Fractions Tool Videos. The Conceptua Fr
built for teaching fractions. The first set of videos demo
teachers can use the Fractions Tools to lead classroom
about fractions concepts. The second set of videos is f
learning fractions using our interactive, visual models.

Many ways to access: Curriculum





Map Out Your Own Real World Problems

- Choose a grade range
- Decide upon a context
- Tier 1 - What are some examples of data that you would provide?
- Tier 2 - Withhold some data. Has the investigation become more interesting?
- Tier 3 – How can the student make this truly their own investigation? Will their answers be important to them?

Springville's Sports Complex

Big Idea: Multiplication with fractions is similar to multiplication with whole numbers in that you are often finding the product of groups of items. They are different in that you can multiply with fractions to find parts of an amount.

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Springville's Sports Complex — Springville's Data

The Springville Parks Department is planning to build a sports complex. They want to build a soccer field, 2 football fields, 3 baseball fields, 4 basketball courts, and 2 sets of tennis courts.

Use this information to make models and complete the table below.

- The Springville Parks Department found 15 acres of land for sale. If they purchased this land, would there be enough acres to build all of the venues that they wanted to include in the sports complex? How do you know?

- **If there is enough land**, are there enough acres for an additional venue? If so, what could be added?

  If there isn't enough land, what would have to be eliminated from the plan in order to build the sports complex on that land?

Type of Sports Venue	Number They Want to Build	Size in Acres	Total Acreage
Soccer Field		2	
Football Field		1 1/2	
Baseball Field		3 3/4	
Basketball Court		1/10	
Set of Tennis Courts		1/4	
Outdoor Swimming Pool		2/10	
Skateboard Park		1/2	
Total			

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Springville's Sports Complex — Create Some Data

The Springville Parks Department just purchased 20 acres of land on which they want to build a new sports complex. Local residents have suggested types of sports venues they would like included in the complex. You have been hired by the Parks Department, and your boss wants you to create a plan for the complex.

Use this information to create models and complete the table below.

- Your boss wants at least 16 -- but no more than 18 -- of the 20 acres to be used for sports venues. Your boss wants at least 4 different types of sports venues. What types of sports venues and how many of each will you include?
- How many acres are occupied by each type of venue? How many of the 20 acres of land are occupied by sports venues?
- The land that is not used for sports venues can be used for picnic areas, parking lots, playgrounds, or other purposes. How much space in your plan is not occupied by sports venues, and how would you suggest that space be used? Be prepared to explain your decisions and your calculations.

Type of Sports Venue	Number You Want to Build	Size in Acres	Total Acreage
Soccer Field		2	
Football Field		1 1/2	
Baseball Field		3 3/4	
Basketball Court		1/10	
Set of Tennis Courts		1/4	
Outdoor Swimming Pool		2/10	
Skateboard Park		1/2	
Total			

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Your Sports Complex – Your Data

The Springville Parks Department is having a competition for students to design the town's new sports complex. You need to determine what sports venues to include and how many of each type to include in your complex. You may use the suggested sports venues or include some of your own.

Type of Sports Venue	Number You Want to Build	Size in Acres	Total Acreage

- What types of venues would you include in your sports complex?
- How many of each venue would you include?
- What is the size of each venue?
- How many acres would your complex occupy in total?
- Use grid paper to create a visual representation of your sports complex. Be prepared to explain your decisions and calculations.

Questions for Reflection:

- Are the sizes of your venues represented to scale?
- Why do you think your design would win the competition?

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Student Name: _____

Big Idea 8: Multiplying with Fractions

Use this rubric with *Real World Investigation: Your Sports Complex—Your Data*

	3- Advanced Problem Solver	2- Accomplished Problem Solver	1- Emerging Problem Solver	0-Beginning Problem Solver	Student score	Teacher score
Demonstrates understanding of the Big Idea	My solution demonstrates my complete understanding that multiplication with fractions is similar to multiplication with whole numbers, but is different in that I can multiply with fractions to find parts of an amount, in addition to more advanced concepts.	My solution demonstrates my complete understanding that multiplication with fractions is similar to multiplication with whole numbers, but is different in that I can multiply with fractions to find parts of an amount.	My solution demonstrates that I have some understanding that multiplication with fractions is similar to multiplication with whole numbers, but is different in that I can multiply with fractions to find parts of an amount.	My solution does not demonstrate an understanding that multiplication with fractions is similar to multiplication with whole numbers, but is different in that I can multiply with fractions to find parts of an amount, or I did not try to solve the problem.		
Makes sense of problems and perseveres in solving them	I read and understood all parts of the problem and stayed focused on finding a solution. If I encountered difficulty, I was able to work through it.	I read and understood most parts of the problem and stayed focused on finding a solution. If I encountered difficulty, I was able to work through it.	I had some difficulty understanding the problem, but sought assistance from at least two people. I may have been off task at times.	I did not understand the problem and did not seek assistance. I did not persevere and did not finish the problem.		
Attends to precision	My computations were at least 95% accurate and were reasonable. I completed all parts of the problem and did more than was required.	My computations were at least 90% accurate and were reasonable. I completed all parts of the problem.	My computations were at least 80% accurate, but were unreasonable. I may have left part of the problem unanswered.	My computations contained many errors and/or are incomplete.		
Utilizes appropriate strategies, models and tools	I independently determined appropriate strategies, models and tools and I applied them appropriately to solve the problem.	I asked for help in determining the best strategies, models and tool, and I used appropriate ones for the problem.	The strategies, models and tools I used were not appropriate for the problem.	I did not attempt to use strategies, models or tools to solve the problem.		
Constructs viable arguments	I explained and justified my solution and answered all questions from peers and/or adults clearly and with well thought out reasons. My answers were accurate.	I explained and justified parts of my solution and answered some questions from peers and/or adults. I made two or fewer errors.	I attempted to explain my solution, but my explanation was unclear or contained more than two inaccuracies. I was not able to answer questions.	I did not or could not explain my solution.		

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