


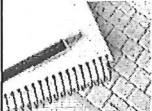
## WRITING: A TOOL TO ORGANIZE AND CLARIFY MATHEMATICS CONCEPTS

National Council of Teachers of Mathematics  
Regional Meeting and Exposition  
Las Vegas, NV October 25, 2013 11:00-12:00 Amazon N/O Rio  
Lynn Columbia, Lehigh University, [hlc0@lehigh.edu](mailto:hlc0@lehigh.edu)

## THE POWER OF WRITING AS A WAY OF LEARNING AND KNOWING




"Whatever we well understand we express clearly, and words flow with ease."  
Nicholas Boileau (1636-1711)



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**To raise achievement in mathematics for every student and effectively implement the Common Core State Standards in Mathematics in every classroom.**

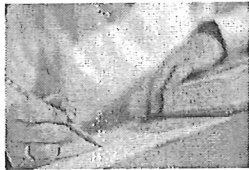


By adapting what we know works in our reading programs and applying it to mathematics instruction and creating language-rich classroom routines.

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### Why Write? Writing can: (1)

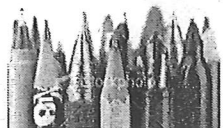
- Demand participation;
- Help to summarize, organize, relate, and associate ideas;
- Provide an opportunity to define, discuss, or describe an idea or concept;



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### Why Write? Writing can: (2)


- Encourage the personalization, assimilation, and accommodation of the concepts being taught;
- Provide an appropriate vehicle to express and focus on negative feelings and frustrations as well as to emote and rejoice in the beauty of learning mathematics concepts (Azzolino, 1990).



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### Why Write? Writing can: (3)

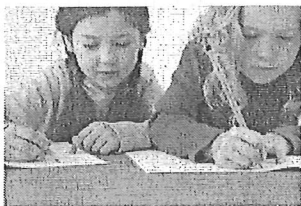
- Help students understand writing in the context of mathematics, and provide a context for learning to interpret word problems;
- Refine students' language regarding mathematics.



Why Write?

For teachers, writing can:

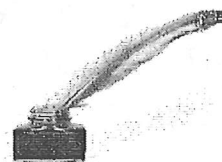
- Provide an open window on where the class stands;
- Help diagnose error patterns in students' understanding of mathematics;



Why Write?

For teachers, writing can:

- Provide evidence of where and why a student has failed to make connections;
- Show the beliefs and attitudes that students hold about mathematics. (Drake and Amspaugh, 1994)



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### Let's Write--Introduction

- Numeric autobiography—playful approach to how numbers play a role in our lives
- Select your favorite number—How does it appear in your life?
  - Columba-Three-3 degrees, 3 children, 3 dogs, lived in 3 states, worked long-term for 3 institutions, 1 out of 3 girls growing up, 3 in my street address, etc.

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### The Critical Role of Purposeful Writing in the Mathematics Classroom

- 1) Opportunity to organize and clarify concepts;
- 2) The benefits as a diagnostic tool; and
- 3) An opportunity to clear up misunderstandings or misconceptions.

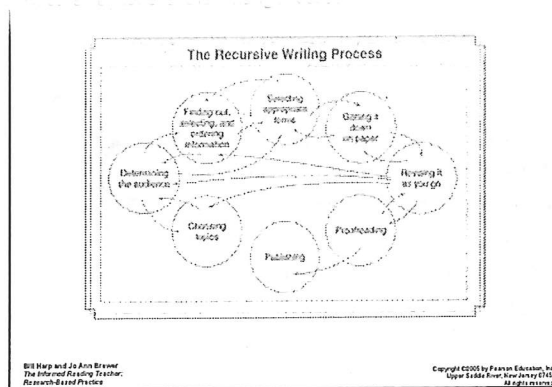
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### Graves Process Model: The Vocabulary of Writing

• Pre-Writing	• Getting Ready
• Drafting	• Getting it Down
• Revising	• Getting it Right
• Editing	• Checking it Out
• Publishing	• Going Public

## Stages of the Writing Process (Graves, 1983)

- Planning**
  - Listening
  - Brainstorming
  - Categorizing
  - Outlining
  - Charting, thinking about audience
  - Clarifying the task
- Drafting**
  - Composing sentences
  - Organizing ideas into paragraphs
  - Choosing words
  - Following conventions of form
- Sharing/Responding**
  - Share work to gain feedback
  - Peer editing
  - Reading/writing workshop
- Revising/Editing**
  - Re-writing for readers' benefit
  - Reorganizing for effect
  - Improving sentence structure and diction
- Publishing**
  - Sharing orally
  - Posting on bulletin board
  - Mailing or delivering
  - \*Publishing\* in class, school or other publication



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## Organize and Clarify Concepts for Understanding and Retention (OCCUR)

(Wellman, Columba, Kim & Moe, 2012)

Retain

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## Types of Writing: *Informal* [OCCUR]

In-Class	Focused; Free
Math Autobiographies	
Reading Logs	
Journals	
Letters	

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## Types of Writing: *Formal* [Writing Process]

Proofs
Process Papers
Summaries of Journal Articles
Solutions to Journal Problems
Research Papers
Lecture Notes

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• Let's examine some of these types of writing in more depth...

## Journals-One Minute Paper

One of the most effective ways students can use writing as an aid to learning is to keep a running account of what is going on each day in a particular class.



## Many names;

- Learning log
- Content journal
- Diary
- Jotter book
- Project journal
- Class journal
- Writer's notebook
- Process journal
- Double-entry notebook
- Applications journal
- Dialogue journal
- Response journal
- Math notes



Who keeps a journal?

## What is a journal?

- A record of your thoughts
- Others should be able to read it
- A reflection, an observation an experience you've had, a peeve, an insight, a significant memory, etc.



Dear Mr. Henshaw, Beverly Cleary (1983)



Originally published in 1947

## Why keep a journal?

1. Research has shown that writing a journal helps you to become a better writer. Writing is a skill and like any skill it needs to be practiced.
2. Keeping a journal helps you become more aware of yourself as a person.
3. It helps achieve detail in writing.
4. It becomes a resource file for essays you might want to write later.
5. Finally it shows you that writing can be fun.



*If you write with a pencil, I think words be all a stream gurgling. The reason I do is this: You, like, you, basic, with addition, subtraction, multiplication, and division, this, you, the math, right? It's complicated by the open stream. Now, other little streams feed into this which is like the more details math which starts off and builds upon your basis. On the math you hear and build with you, and more details that gets into the second of second grade, you build than basic dreams.*

*and if you, with feet, you bring further with hands, with sometimes, it gets so hard and so many steps that it might even be the steady in the eyes, coming against the beach, and off a base, you, round, with, it, what I think math is the sound of a stream gurgling.*

*7th grader*

What does mathematics mean to me? 7<sup>th</sup> grader

## Emoting the joy of mathematics...

*The math quiz was easy because that came real easy for me. I pay attention in class and knew what I was doing. It was on days, weeks, etc., temperatures. It was easy for me. I do my homework to know what I'm doing.*

## Developmental & ELL's



- When we position ourselves more as guides instead of gatekeepers, our adolescent learners gain writing skills and knowledge in mathematics.
- Examining our students' writing reveals emerging trends and master of concept or gaps in their learning and/or misconceptions.
- As teachers, we have all seen students who can give back a correct answer without understanding the content. If a student is unable to write an explanation, it does not mean they have a lack of understanding. Beautiful essay, but little in way of content.
- Early adolescents at a complex stage in their emotion and social development. Climate is significant in providing an optimal environment to meet the developmental needs of adolescents.
- Writing activities allow students to show they understand concepts by expressing them in another format and also provide practice in the correct use of standard English.

## A variation— The One Minute Paper




- What is it?
  - Students write answers to one or two questions for one minute
- What questions?
  - What was the main point of today's activity?
  - What are the most important questions remaining unanswered?
  - What was the muddiest point?

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### Thinking Breaks—Another Variation


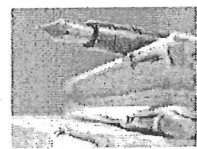
- **Thinking Breaks.** An open-ended format for a pause in discussion or problem solving for students to record their thinking.

Thinking Breaks
Thinking Break: <i>In 1795 France adopted the metric system of measurement.</i>
Thinking Break: <i>In 1866 metric units were made legally acceptable in the US</i>
Thinking Break: <i>Dual systems continue to be used in the US.</i>



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### Advantages: One-Minute Paper/Thinking Break





- Only a few minutes of class time
- Immediate feedback: Have you really accomplished your objectives for this topic?
- Encourages active participation
- Easily read, tabulated and analyzed

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### When to use 1-Minute Paper/Thinking Break?

- After completing a topic
- During a topic—on-going assessment
- To help students identify patterns
- To help students articulate their thinking and understanding
- To help students identify interesting and important points






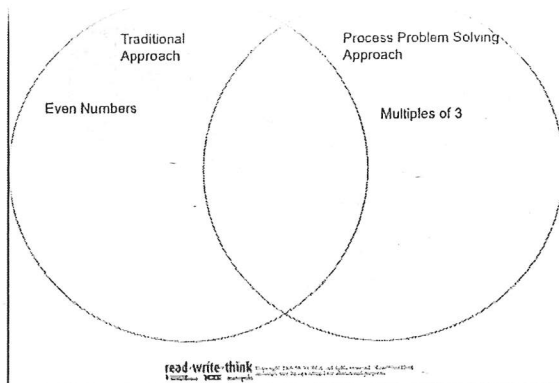
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### Cognitive Organizers



- Concept organizers, nonlinguistic representations, or concept maps are graphical representations of relationship between and among terms or concepts
- Help students to make sense integrating new ideas into their existing knowledge
- KWL chart, KWHL chart, Venn Diagrams, Inquiry Charts, Double-entry logs, Content Biographies/Autobiographies (numeric autobiographies), Brainstorming Webs



### Brainstorming Web

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### Brainstorming Web

5/10   .5   50/100

half note

50%   **1/2**   one half

1/2 hr.--30 minutes

Fifty cents

1/2 yr.--6 months

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### Webs Help Create Dialogue

- A frame of reference to connect concepts
- A rich medium of classroom dialogue
- Fosters thinking strategies
- Multiple representations
- Ability to make inferences
- Link prior knowledge to new situations
- Separate ideas into a new whole

8 cognitive organizers on NCTM website

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### Quick Writes

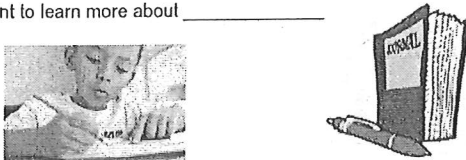
- Designed to give students the opportunity to reflect on their learning or to stimulate their thinking with short open-ended statements. Quick Writes can be used at the beginning, middle, or end of a lesson and are intended to take only a few moments.
- Entrance/Exit Slips. This learning to write technique eases the students into writing with just a few sentences at the beginning (Entrance) or at the end (Exit) of the class. Entrance slips help to focus students' attention on the topic for the day and to access prior knowledge. Exit slips help to assess the learning for the day and to determine if there were any misconceptions. Sticky notes, index cards, and half sheets of paper work well as entrance/exit slips.
- Entrance Slip
  - What did you learn from last night's homework assignment?
- Exit Slip
  - What are three things you learned in class today?

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59 Prompts NCTM website


### Writing Prompts

- I love math because....
- My best experience in math was when....
- One mathematics activity I really enjoy is \_\_\_\_\_ because \_\_\_\_\_.
- What questions are still unanswered at the end of this week?
- I want to learn more about \_\_\_\_\_



### Affective Considerations— Classroom Environment (Moynihan 2012)

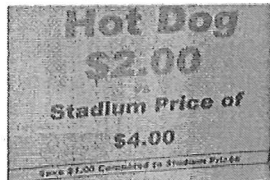
- Purposeful accessible learning
- Confidence building
- Spirit of collaboration/sense of pride
- Creativity/imagination



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### The important thing about math is...


- ...you learn all the time with it. (Naomi, Grade 2)
- ...it makes you think. (Corey, Grade 2)
- ...you need it to succeed in life. (Caleb, Grade 5)
- ...you can use it in any job, at any time, on any day. (Soshana, Grade 5) (Moynihan, 2012)



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### It's like...


- ...a ride because sometimes it's bumpy and sometimes it's smooth (Nuni, Grade 2)
- ...a slide because when you get on it, you can think and learn really fast. (Caroline, Grade 3)
- ...getting a gift because you can always use it. (Sari, Grade 4)
- ...a hole in the ground—it's endless and you will need it everywhere you go. (Matt, Grade 5) (Moynihan, 2012)



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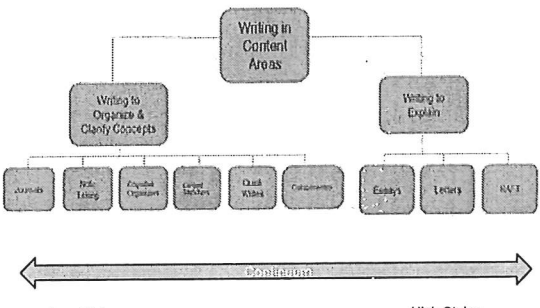
### Writing to Explain

- Typically more formal or extended selections of writing
- Often this type of writing is "published"
  - Ready to deliver to the public or the teacher
  - Producing a public version of text
- Usually includes the following: substantial, planned, authoritative, conventional, audience centered, revised, edited, and assessed for a grade
- Such as letters, essays, RAFT model (role, audience, format, and topic)

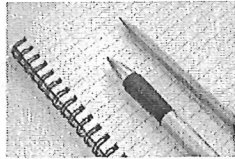


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### Writing in the Mathematics Classroom (and other classrooms)



A word of caution: The emphasis of the writing in the mathematics classroom is to promote concept development and to provide feedback—not to grade writing skills...



"I am never as clear about any matter as when I have just finished writing about it."

James Van Allen (1991)  
Physicist and Author



This is our goal as mathematics teachers.



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Thank you for your attention and participation.

Wishing you a terrific NCTM Regional conference in Las Vegas, great writing, and good teaching!



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## References

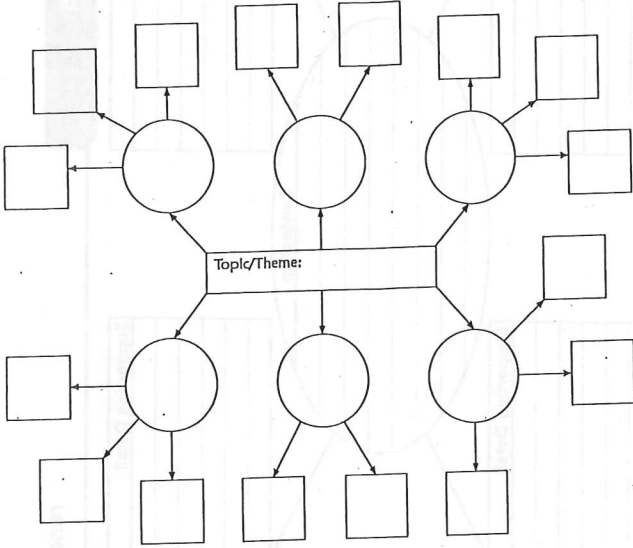
- Azzolino, A. (1990). Writing as a tool for teaching mathematics: The silent revolution. In T. J. Cooney, & C. R. Hirsh (Eds.), *Teaching and learning mathematics in the 1990's* (pp. 92-100). Reston, VA: NCTM.
- Columba, L., Kim, C., & Moe, A. J. (in press, 2014). *Content at the core: Integrating literacy in the middle grades*. AZ: Holcomb Hathaway.
- Drake, R. M., & Amspaugh, L. B. (1994). What writing reveals in mathematics. *Focus on learning problems in mathematics*, 16(3), 43-50.
- Moynihan, C. (2012). *Math sense: The look, sound, and feel of effective instruction*. Portland, ME: Stenhouse.
- Wellman, D., Graves, D. H. (1983). *Writing: Teachers & children at work*. Portsmouth, NH: Heinemann.



Name: \_\_\_\_\_ Date: \_\_\_\_\_

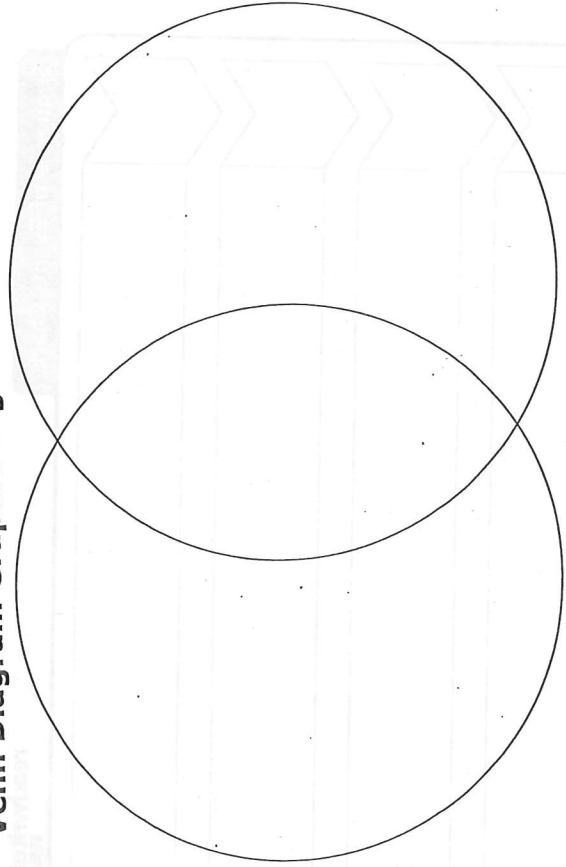
### Concept Map

The concept map below is a way for you to show relationships between words and concepts. An arrow connecting two words shows that those words are related in some way. You can add more arrows and more bubbles to the map as needed. When you're done, you can group words that go together with a circle or box.



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### Venn Diagram Graphic Organizer

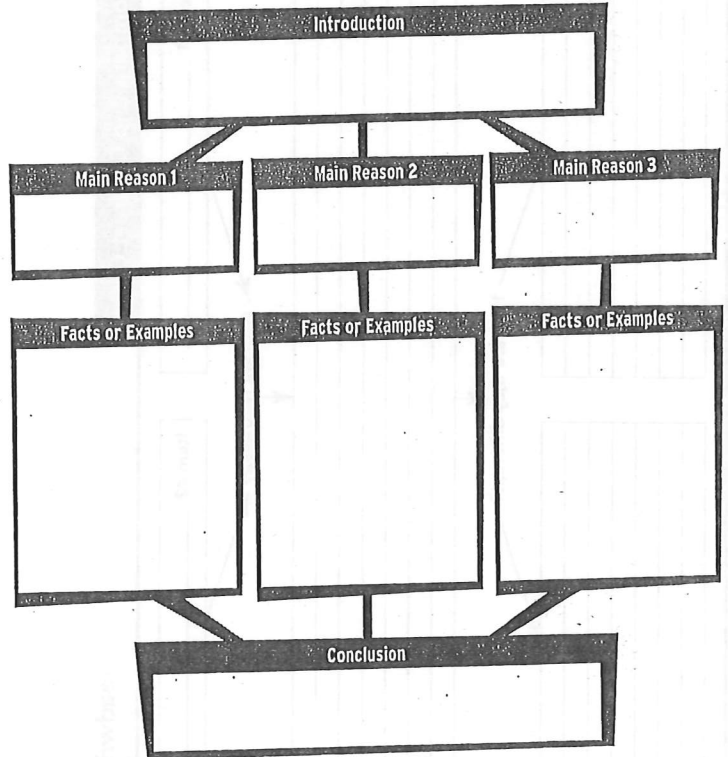


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## PERSUASION MAP

by: \_\_\_\_\_

topic: \_\_\_\_\_






## Thinking Breaks

Thinking Break
Thinking Break
Thinking Break

## Brainstorming Web

1/2

### 59 writing prompts for math teachers

#### Prompts for math journals (assessing disposition)

- My best experience with math was when \_\_\_\_\_.
- My worst experience with math was when \_\_\_\_\_.
- I love math because...
- I hate math because \_\_\_\_\_.
- I want to become better at math so that I \_\_\_\_\_.
- Is math your favorite subject? Why or why not?
- What did you like most about your math class last year? What did you like the least?
- People who are good at math \_\_\_\_\_.
- One mathematics activity I really enjoy is \_\_\_\_\_ because \_\_\_\_\_.

- When I study for a test, I \_\_\_\_\_.
- When it comes to math, I find it difficult to \_\_\_\_\_.
- When I hear someone say math is fun, I \_\_\_\_\_.
- When I see a word problem, the first thing I do is \_\_\_\_\_.

- Then I \_\_\_\_\_.
- Does mathematics or math class scare you in any way?
  - Draw a picture of a mathematician and describe what a mathematician does.
  - Explain how you feel about mathematics now as compared to before you took this class.

#### Prompts for learning logs (assessing learning)

- My three personal goals for this term are \_\_\_\_\_.
- Explain everything you know about \_\_\_\_\_.
- What is a \_\_\_\_\_? Write all you can about \_\_\_\_\_.
- Find something that you learned today that is similar to something you already knew. Write about these similarities.
- How would you use what you learned today in your life?
- The main idea today's lesson was \_\_\_\_\_.
- Write a definition in your own words of a \_\_\_\_\_.
- What is the most significant thing you learned this week?
- What questions are still unanswered at the end of this week?
- Last week in math I learned \_\_\_\_\_.
- My favorite part of math last week was \_\_\_\_\_.
- The hardest part of math last week was \_\_\_\_\_.
- This is how I used math this week (outside of school) \_\_\_\_\_.
- Describe any discoveries you made about mathematics today, this week/month/year.
- Give two examples of situations in which you have used, seen or can find the concept of \_\_\_\_\_, OUTSIDE of this classroom.
- I want to learn more about \_\_\_\_\_.
- I wish I knew more about \_\_\_\_\_.
- I need more help understanding \_\_\_\_\_.

## Entrance Slip.

Entrance Slip
What did you learn from last night's homework assignment?

## Exit Slip.

Exit Slip
What are the three most important things you learned in class today?

- My math grade now is \_\_\_\_\_ because \_\_\_\_\_.
  - What was your grade on the exam/quiz/project? If you were not satisfied with your score, what can you do to improve? If you were happy with your score, what did you do well?
  - Write instructions for a (insert grade level here) grader to follow when (adding fractions, finding percentages, calculating averages, etc.)
  - How would you describe a \_\_\_\_\_?
  - The difference between \_\_\_\_\_ and \_\_\_\_\_ is \_\_\_\_\_.
  - Compare and contrast the terms \_\_\_\_\_.
  - What patterns do you notice in \_\_\_\_\_.
  - You know several ways to \_\_\_\_\_ (solve an equation, add fractions, etc.) Which method is your favorite? Why?
  - Make a list of objects or figures in the room which have \_\_\_\_\_.
  - How can you tell?
  - Write a letter to your teacher explaining what you understand about \_\_\_\_\_, and what is still giving you trouble.
  - Write and solve a word problem whose solution involves \_\_\_\_\_.
  - Describe practical uses for \_\_\_\_\_.
  - Write \_\_\_\_\_ possible test questions for this unit.
  - Write a letter to a student who will be taking this class next year, giving some advice about this class.
- Prompts to use for problem-solving (assessing process)
- The most important part of solving a problem is \_\_\_\_\_.
  - Describe the process you undertook to solve this problem. (Problem needs to be provided.)
  - I knew I was right when \_\_\_\_\_.
  - Tips I would give a friend to solve this problem are \_\_\_\_\_.
  - Could you have found the answer by doing something different? What?
  - What strategy did you use to solve this problem and why?
  - Was this problem difficult or easy? Why?
  - Were you frustrated with this problem? Why or why not?
  - Where else could you use this type of problem solving?
  - What would happen if you missed a step? Why?
  - What other strategies could you use to solve this problem?

