Using Math Drawings and Math Talk for Understanding Algorithms

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This paper is based on the CCSS-M standards, *The NBT Progression for the Common Core State Standards* by The Common Core Writing Team (7 April, 2011), commoncoretools.wordpress.com, and on Fuson, K. C. & Beckmann, S. (Fall/Winter, 2012-2013). Standard algorithms in the Common Core State Standards. *National Council of Supervisors of Mathematics Journal of Mathematics Education Leadership, 14 (2)*, 14-30 (which is posted at http://www.math.uga.edu/~sybilla/ as is this talk file). For this handout or if you have difficulties finding either of the above, send me an email karenfuson@mac.com with NCTM 2014 in the subject line.

For more details about the CCSS-M and visual supports please see the series of flexible webcasts I have made. There are 9 ½ hours so far and will be 13 hours in all. The links to these webcasts are listed under Projects on my Northwestern University webpage: http://www.sesp.northwestern.edu/profile/?p=61 I will update my Northwestern University webpage as new webcasts are completed, so you can find the new links there.

Overview

Taken together, the NBT Progression document summarizes that *the standard algorithm* for an operation implements the following mathematical approach with minor variations in how the algorithm is written:

a. decomposing numbers into base-ten units and then carrying out single-digit computations with those units using the place values to direct the place value of the resulting number;
b. using the one-to-ten uniformity of the base ten structure of the number system to generalize to large whole numbers and to decimals.

General methods that will generalize to and become standard algorithms can and should be developed, discussed, and explained initially using a visual model. Helping step methods that clarify the meaning or use of place value, relate easily to parts of visual models, or prevent common errors can be used initially to support student understanding and accuracy. These lead to variations of standard algorithms by dropping steps or writing them in a more efficient way.

Criteria for better variations of how the algorithm is written are:

Given the CCSS emphasis on meaning-making, variations in ways to record the standard algorithm that support and use place value correctly should be emphasized.

Given the centrality of single-digit computations in algorithms, variations that make such single-digit computations easier should be emphasized.

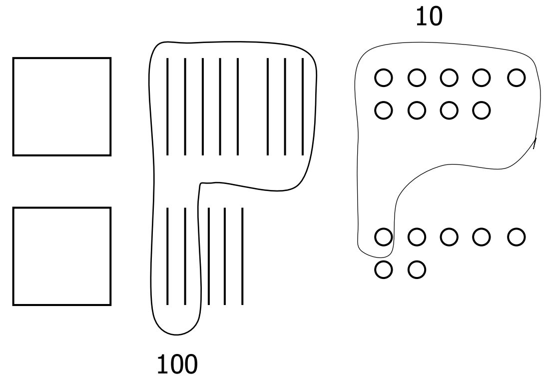
Written methods may involve different kinds of steps, e.g., ungrouping (borrowing) to be able to subtract and the actual subtracting. These kinds of steps can alternate or can be completed all at once. Variations in which the kinds of steps alternate can introduce errors and be more difficult, so methods without such alternations should be emphasized.

Written variations can keep the initial multidigit numbers unchanged, or single-digit numbers can be written so as to change (or seem to change) the original numbers. The former variations are conceptually clearer and so should be emphasized.

Many students prefer to calculate from left to right, consistent with how they read numbers and words. Variations that can be undertaken left to right are helpful to many students, especially initially, so they should be emphasized.

Written Variations of Standard Algorithms

Quantity Model ↔ Good Variations



New Groups Below

$$\begin{array}{cccc} & 1 & 8 & 9 \\ + & 1 & 5 & 7 \\ \hline & \overset{1}{3} & \overset{1}{4} & 6 \end{array}$$

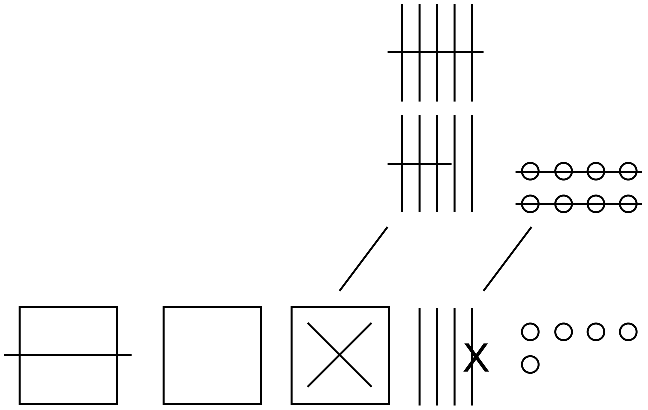
Show All Totals

$$\begin{array}{cccc} & 1 & 8 & 9 \\ + & 1 & 5 & 7 \\ \hline & 2 & 0 & 0 \\ & 1 & 3 & 0 \\ & & 1 & 6 \\ \hline & 3 & 4 & 6 \end{array}$$

Current Common

New Groups Above

$$\begin{array}{cccc} & \overset{1}{1} & \overset{1}{8} & 9 \\ + & 1 & 5 & 7 \\ \hline & 3 & 4 & 6 \end{array}$$



Ungroup Everywhere First, Then Subtract Everywhere

Left → Right

$$\begin{array}{cccc} & & 13 & \\ & 2 & \cancel{14} & 16 \\ & \cancel{3} & \cancel{4} & \cancel{6} \\ - & 1 & 8 & 9 \\ \hline & 1 & 5 & 7 \end{array}$$

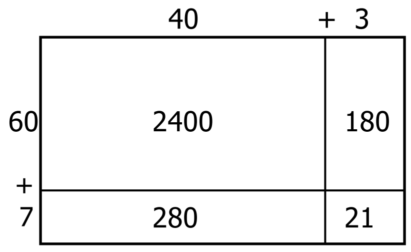
Right → Left

$$\begin{array}{cccc} & & 13 & \\ & 2 & \cancel{3} & 16 \\ & \cancel{3} & \cancel{4} & \cancel{6} \\ - & 1 & 8 & 9 \\ \hline & 1 & 5 & 7 \end{array}$$

R → L Ungroup, Then Subtract, Ungroup, Then Subtract

$$\begin{array}{cccc} & & 13 & \\ & 2 & \cancel{3} & 16 \\ & \cancel{3} & \cancel{4} & \cancel{6} \\ - & 1 & 8 & 9 \\ \hline & 1 & 5 & 7 \end{array}$$

Area Model



Place Value Sections

$$\begin{array}{r} 2400 \\ 180 \\ 280 \\ +\quad 21 \\ \hline 2\overset{1}{8}81 \end{array}$$

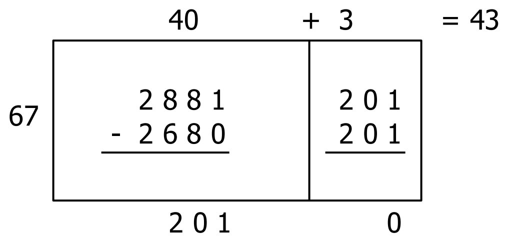
Expanded Notation

$$\begin{array}{rcr} 43 & = & 40 + 3 \\ \times\ \ 67 & = & 60 + 7 \\ \hline 60 \times 40 & = & 2400 \\ 60 \times \ \ 3 & = & 180 \\ 7 \times 40 & = & 280 \\ 7 \times \ \ 3 & = & 21 \\ \cline{3-3} & & 2\overset{1}{8}81 \end{array}$$

1-Row

$$\begin{array}{r} \overset{\overset{1}{2}}{4}3 \\ \times\quad 67 \\ \hline 301 \\ 258 \\ \hline 2881 \end{array}$$

Rectangle Sections



Expanded Notation

$$\begin{array}{r} \left.\begin{array}{r} 3 \\ 40 \end{array}\right] 43 \\ 67\,\overline{)\,2881} \\ -\ 2680 \\ \hline 201 \\ -\ 201 \\ \hline \end{array}$$

Digit by Digit

$$\begin{array}{r} 43 \\ 67\,\overline{)\,2881} \\ -\ 268 \\ \hline 201 \\ -\ 201 \\ \hline \end{array}$$