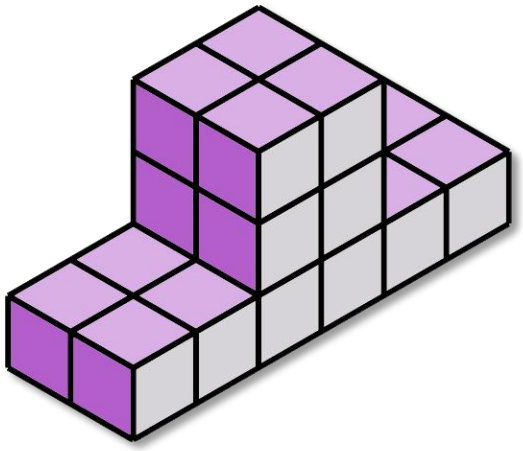
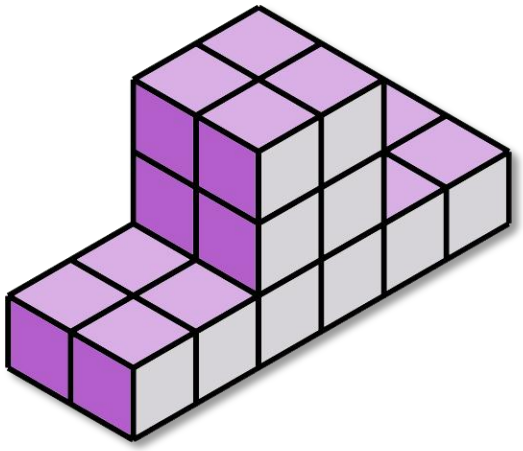
21964
Wares
Cubes, Cubes, and More Cubes

Volume is the total number of unit cubes it takes to fill a 3-dimensional object.



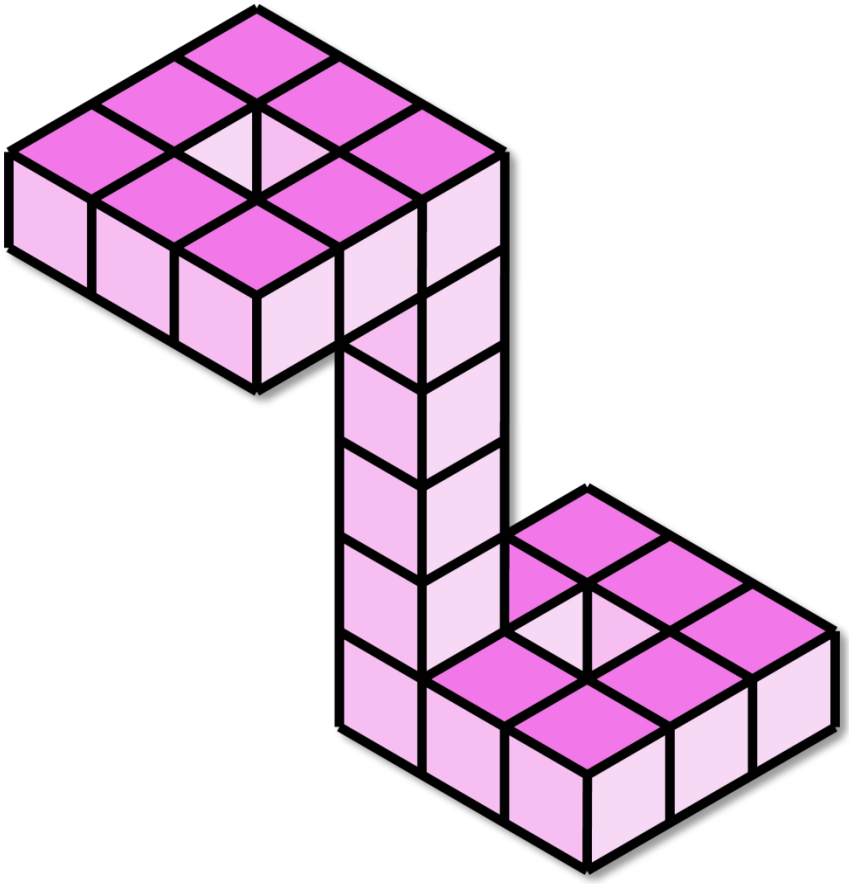
If we assume the solid shown above is made out of unit cubes, the **volume** of the solid is 20 unit^3.

Surface area is the total number of unit squares it takes to cover the outer surface of a 3-dimensional object.

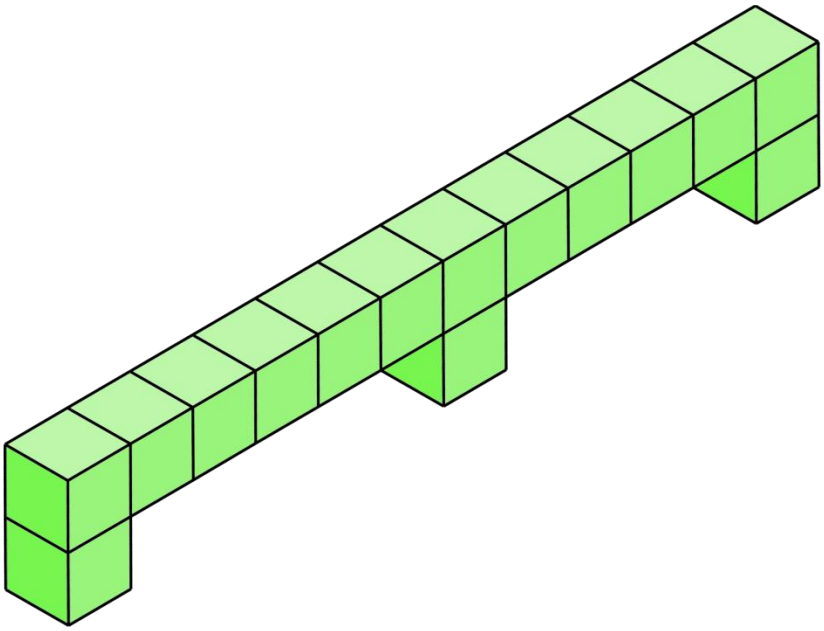


If we assume the solid shown above is made out of unit cubes, the **surface area** of the solid is 56 unit^2.

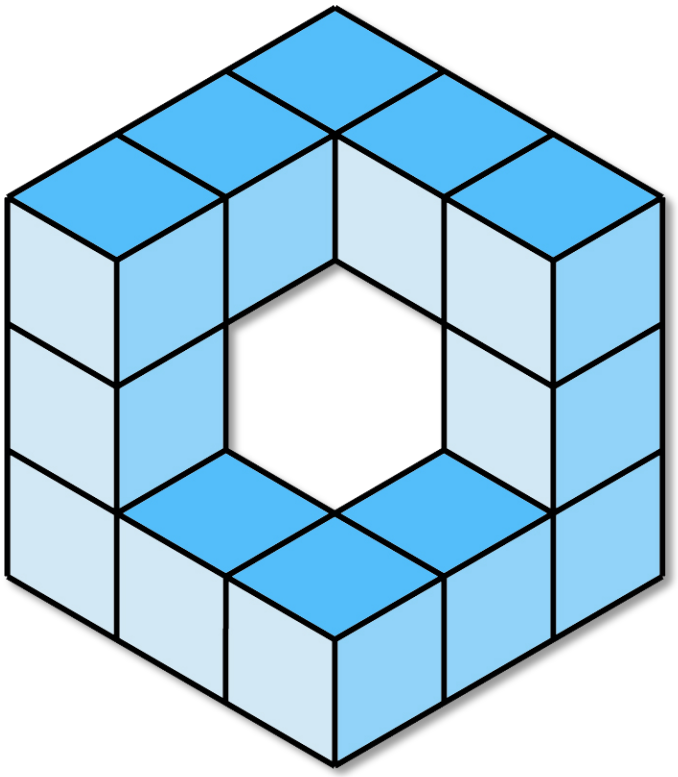
The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



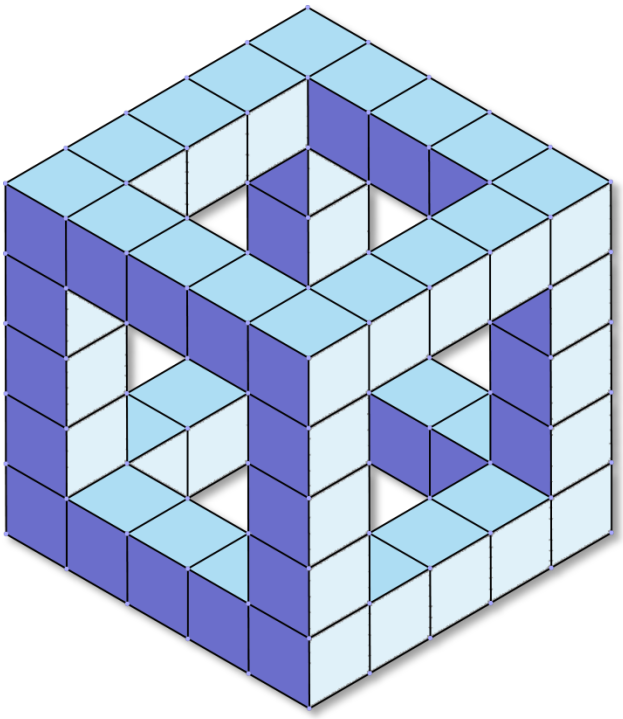
The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



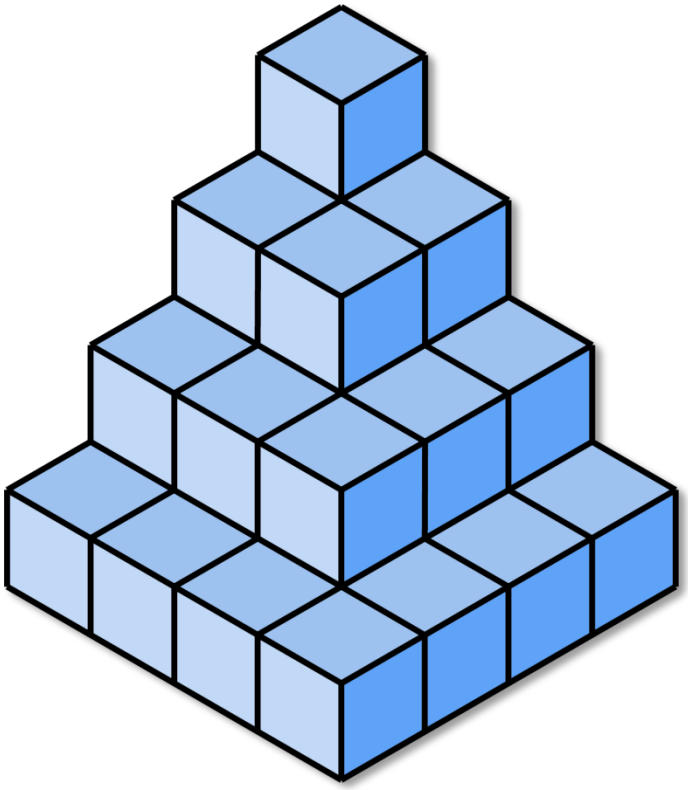
The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



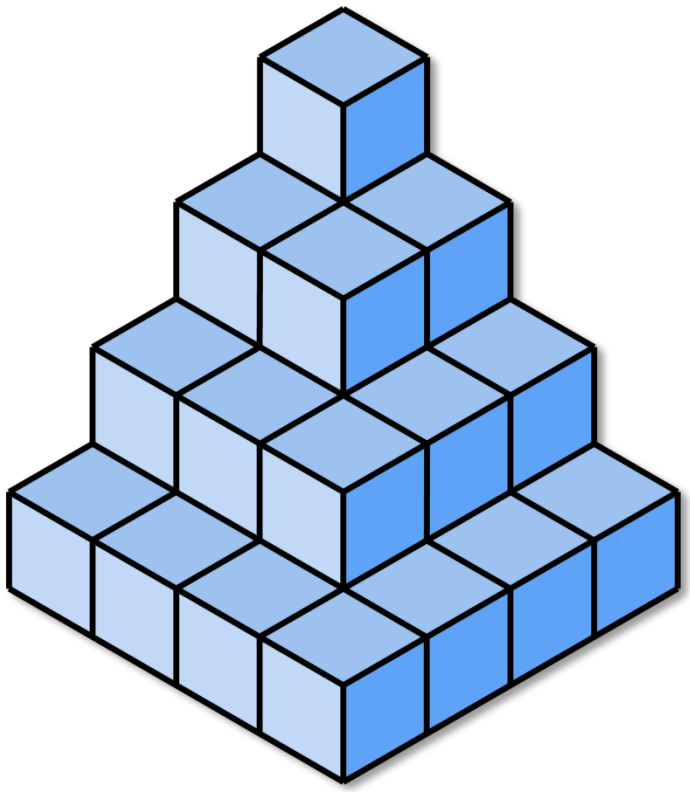
The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.

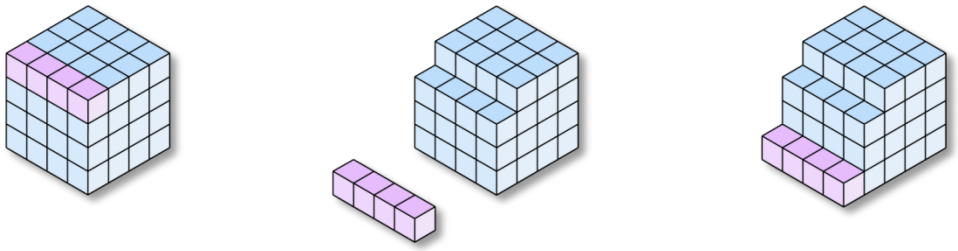


The solid shown below was made with unit cubes. How many cubes are not visible?



Activity 1

1. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.



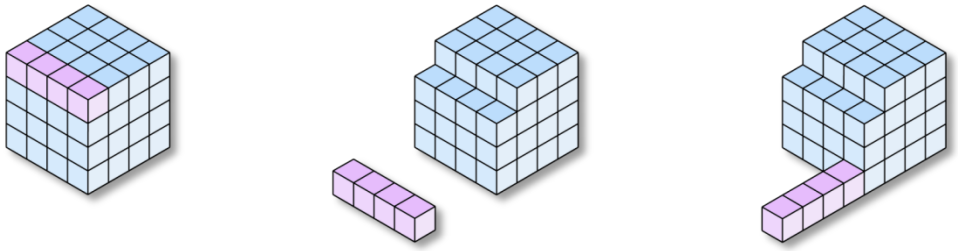
a b c

a. Surface area of the entire solid: 96 square units

b. Surface area of the larger portion:_____ sq units

c. Surface area of the entire solid: _____ sq units

2. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.



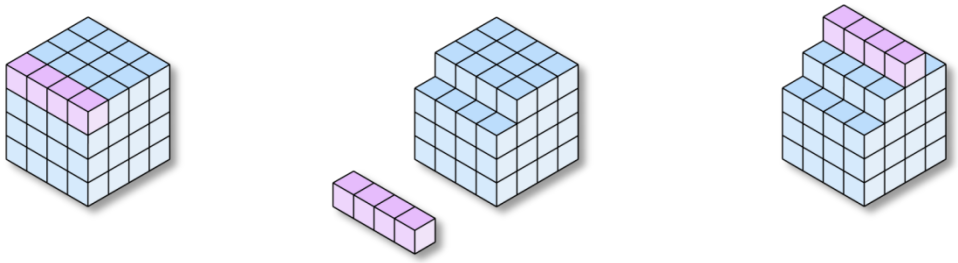
a b c

a. Surface area of the entire solid: 96 square units

b. Surface area of the larger portion:_____ sq units

c. Surface area of the entire solid: _____ sq units

3. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.



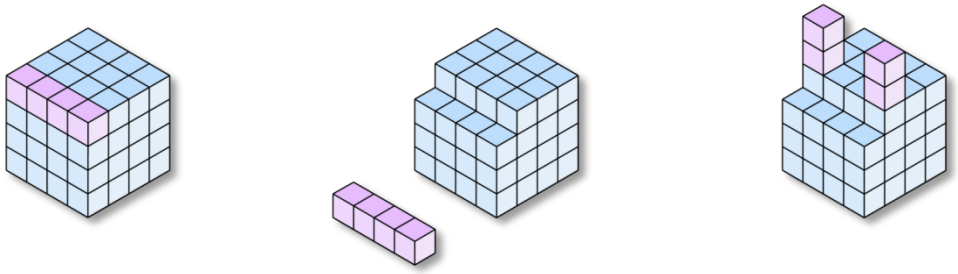
a b c

a. Surface area of the entire solid: 96 square units

b. Surface area of the larger portion:_____ sq units

c. Surface area of the entire solid: _____ sq units

4. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.



a b c

a. Surface area of the entire solid: 96 square units

b. Surface area of the larger portion:_____ sq units

c. Surface area of the entire solid: _____ sq units

Activity 2

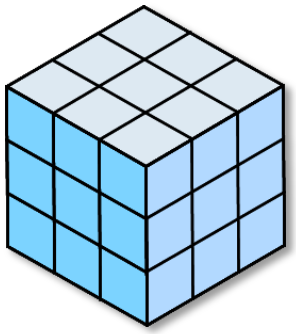
1. A large cube, made out of white material, was dipped into blue paint. It was then cut into 27 (= 3×3×3) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

b. How many of the smaller cubes have 2 blue faces?

c. How many of the smaller cubes have 1 blue face?

d. How many of the smaller cubes have 0 blue faces?



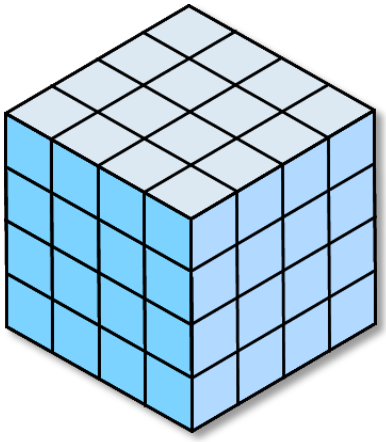
2. A large cube, made out of white material, was dipped into blue paint. It was then cut into 64 (= 4×4×4) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

b. How many of the smaller cubes have 2 blue faces?

c. How many of the smaller cubes have 1 blue face?

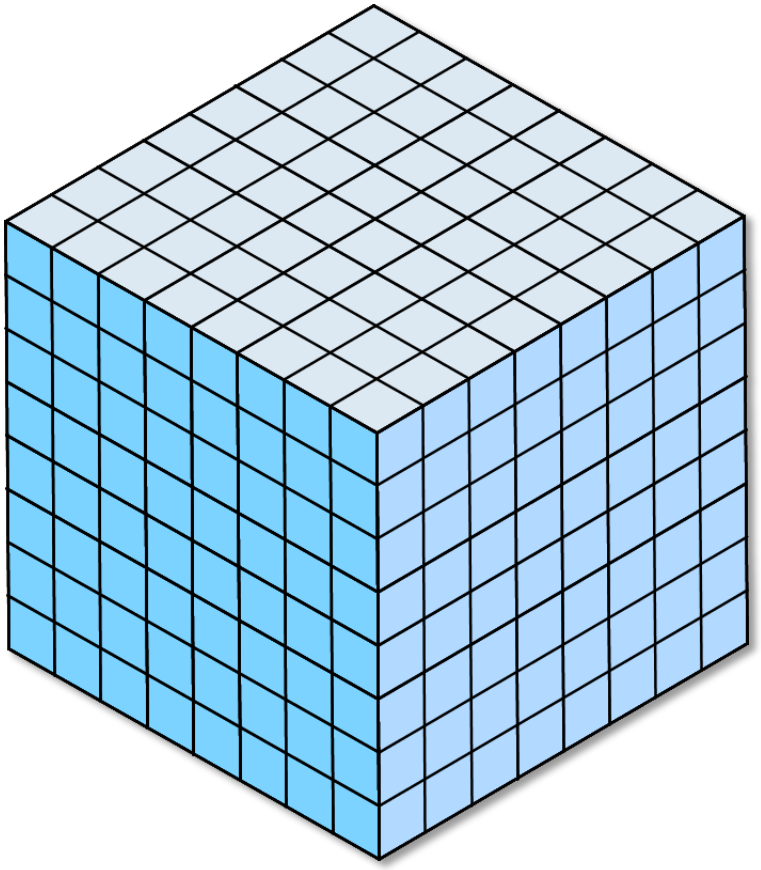
d. How many of the smaller cubes have 0 blue faces?



3. A large cube, made out of white material, was dipped into blue paint. It was then cut into 512 (= 8×8×8) smaller cubes.

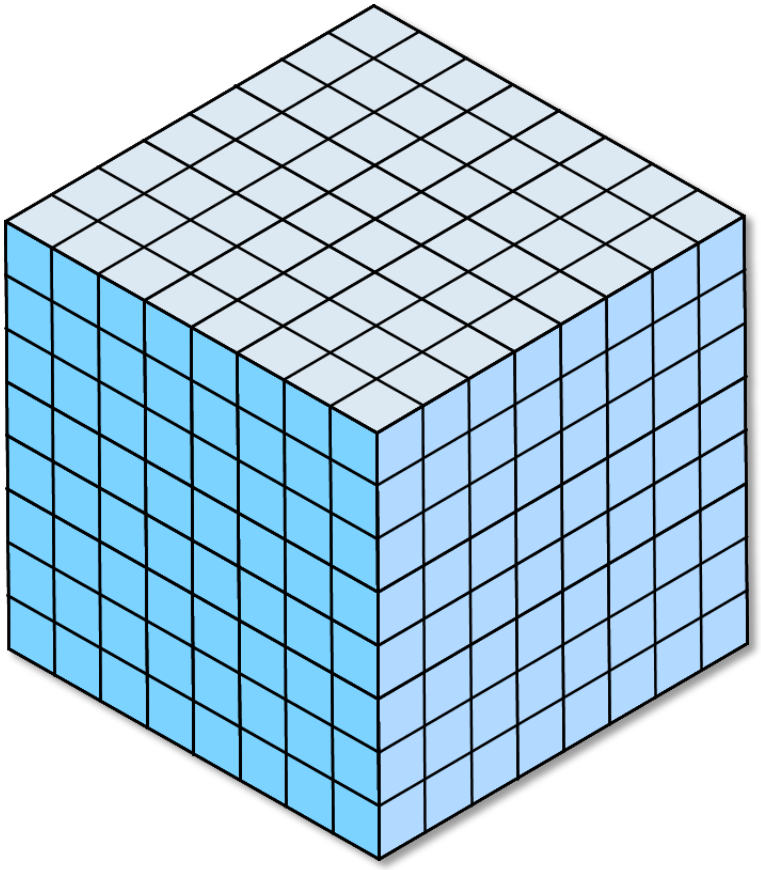
a. How many of the smaller cubes have 3 blue faces?

b. How many of the smaller cubes have 2 blue faces?



c. How many of the smaller cubes have 1 blue face?

d. How many of the smaller cubes have 0 blue faces?



4. A large cube, made out of white material, was dipped into blue paint. It was then cut into 1000 (= 10×10 × 10) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

b. How many of the smaller cubes have 2 blue faces?

c. How many of the smaller cubes have 1 blue face?

d. How many of the smaller cubes have 0 blue faces?

5. A large cube, made out of white material, was dipped into blue paint. It was then cut into n^3 ($= n \times n \times n$) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

b. How many of the smaller cubes have 2 blue faces?

c. How many of the smaller cubes have 1 blue face?

d. How many of the smaller cubes have 0 blue faces?

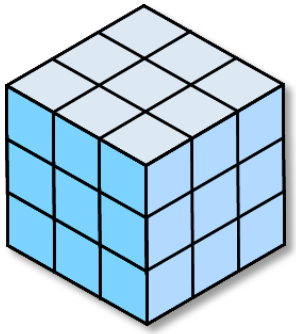
Activity 3

1. The large cube shown below is made with 27 (= 3 × 3 × 3) unit cubes.

a. What is the volume of the large cube? b. What is the surface area of the large cube?

c. How many of the unit cubes show 3 faces?
d. How many of the unit cubes show 2 faces?
e. How many of the unit cubes show 1 face?
f. How many of the unit cubes show 0 faces?

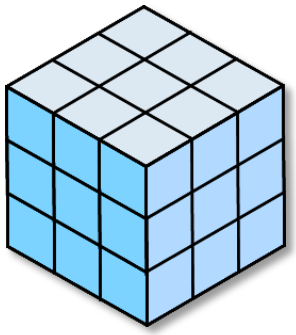
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.

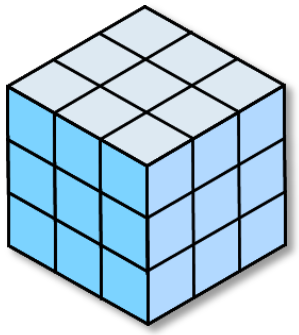
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

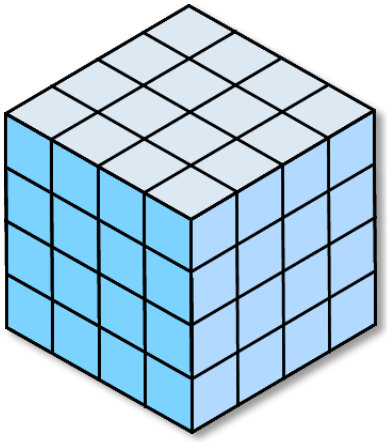
Recall a cube has 6 faces, 8 vertices, and 12 edges.



2. The large cube shown below is made with 64 (= 4 × 4 × 4) unit cubes.

a. What is the volume of the large cube? b. What is the surface area of the large cube?

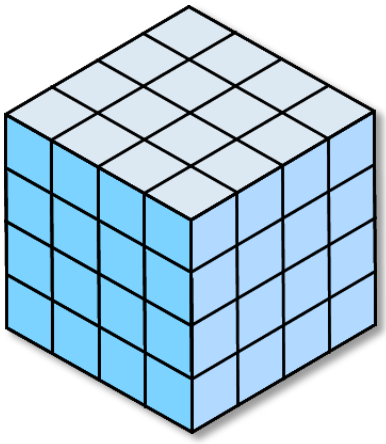
c. How many of the unit cubes show 3 faces?
d. How many of the unit cubes show 2 faces?
e. How many of the unit cubes show 1 face?
f. How many of the unit cubes show 0 faces? **Recall a cube has 6 faces, 8 vertices, and 12 edges.**



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.

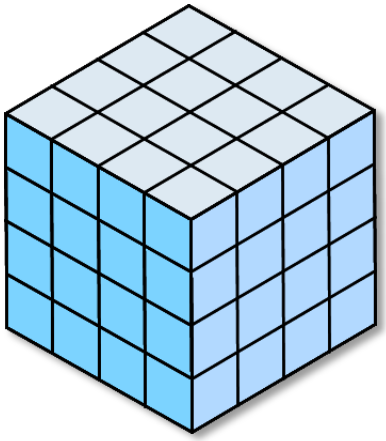
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.

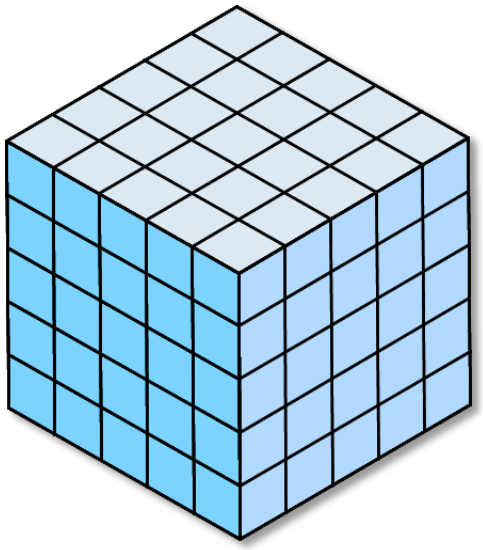


3. The large cube shown below is made with 125 (= 5 × 5 × 5) unit cubes.

a. What is the volume of the large cube? b. What is the surface area of the large cube?

c. How many of the unit cubes show 3 faces?
d. How many of the unit cubes show 2 faces?
e. How many of the unit cubes show 1 face?
f. How many of the unit cubes show 0 faces?

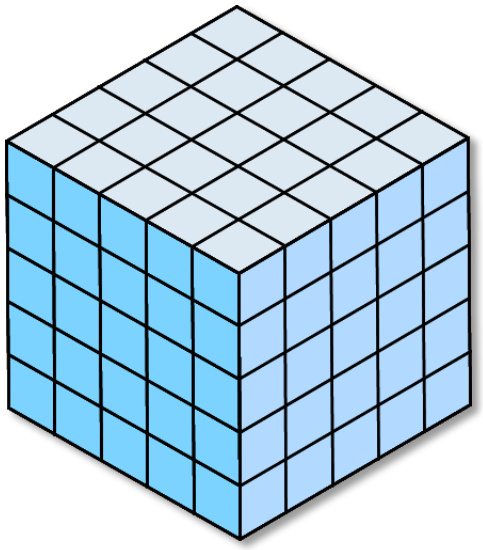
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.

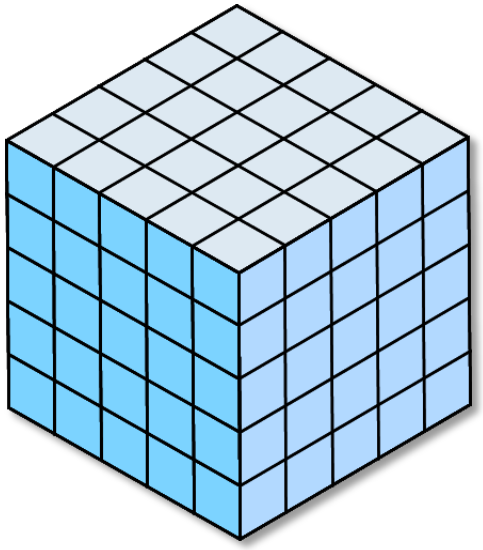
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.

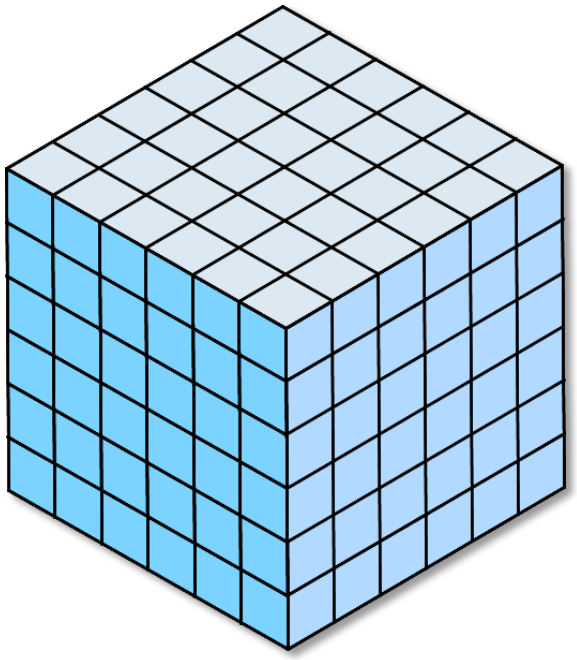


4. The large cube shown below is made with 216 (= 6 × 6 × 6) unit cubes.

a. What is the volume of the large cube? b. What is the surface area of the large cube?

c. How many of the unit cubes show 3 faces?
d. How many of the unit cubes show 2 faces?
e. How many of the unit cubes show 1 face?
f. How many of the unit cubes show 0 faces?

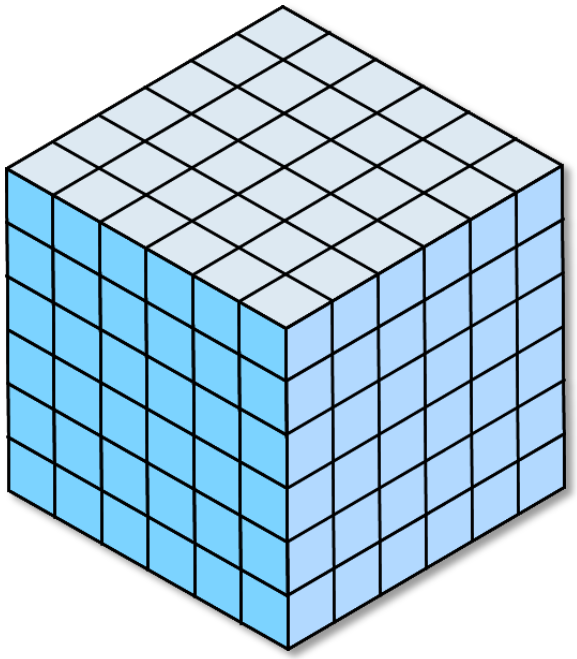
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.

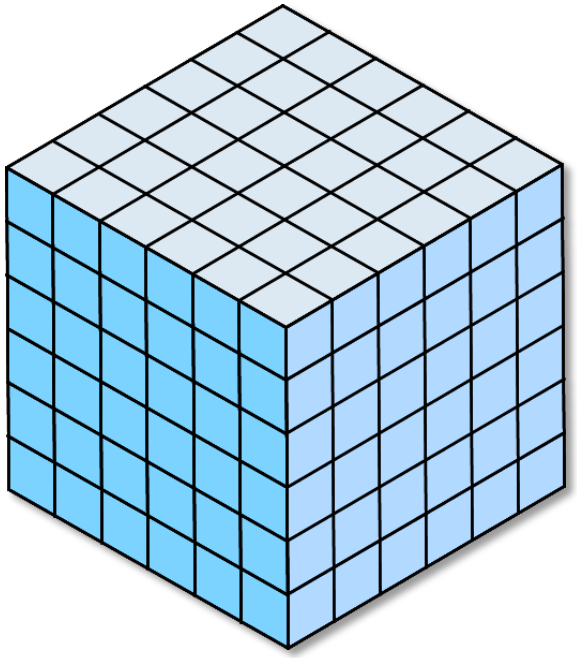
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



5. Visualize a large cube made out of 343 (= 7 × 7 × 7) unit cubes.

a. How many of the unit cubes show 3 faces? b. How many of the unit cubes show 2 faces?
c. How many of the unit cubes show 1 face?
d. How many of the unit cubes show 0 faces?

e. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
f. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.

g. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

h. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.

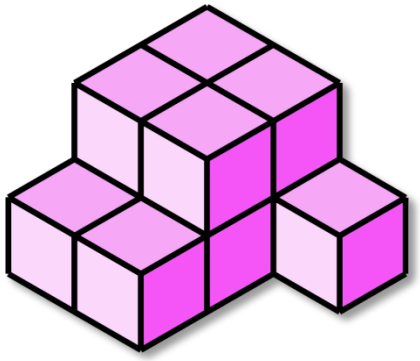
i. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

j. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.

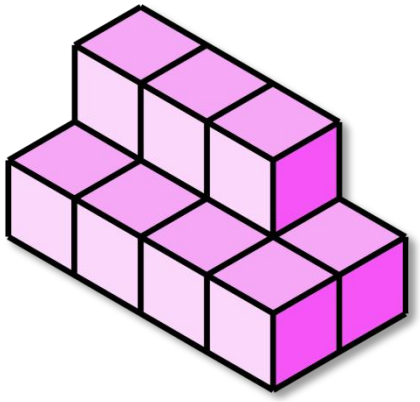
Recall a cube has 6 faces, 8 vertices, and 12 edges.

Activity 4

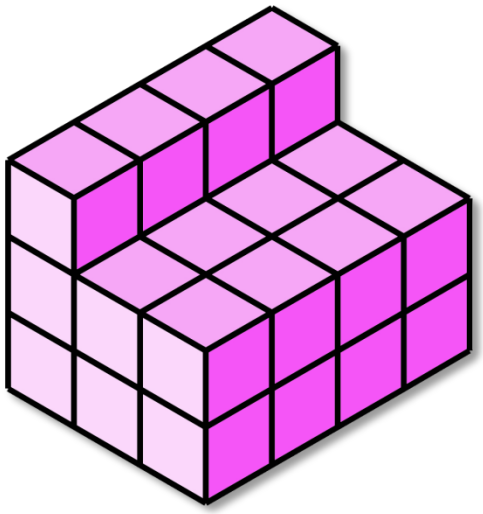
1. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.



2. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.

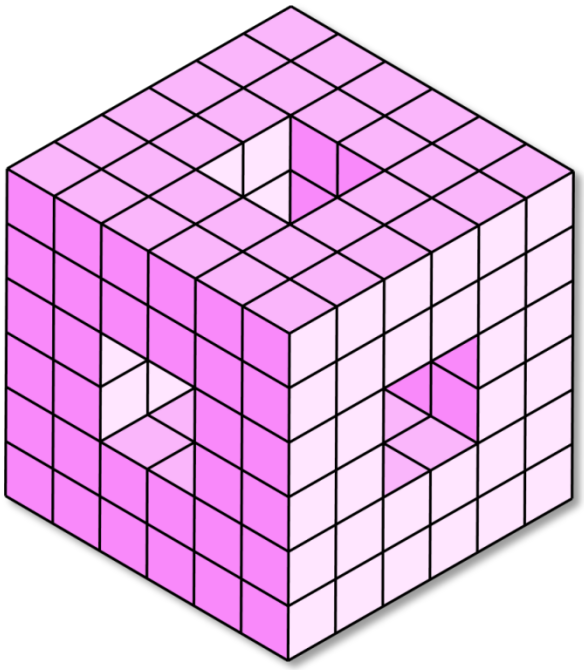


3. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.



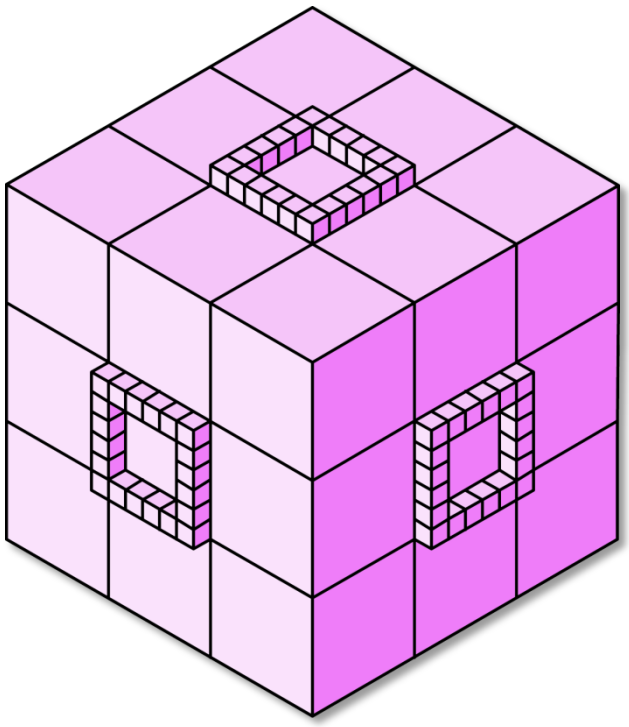
Activity 5

The following object was made from a 6 by 6 by 6 cube made out of unit cubes. Three holes were drilled through each pair of opposite faces. These holes went all the way through the opposite faces. Moreover, these holes removed the four unit cubes at the center of each of the six faces as shown. Of course, the process of drilling removed more cubes than just the ones on the surface. What is the volume and surface area of the object shown below?

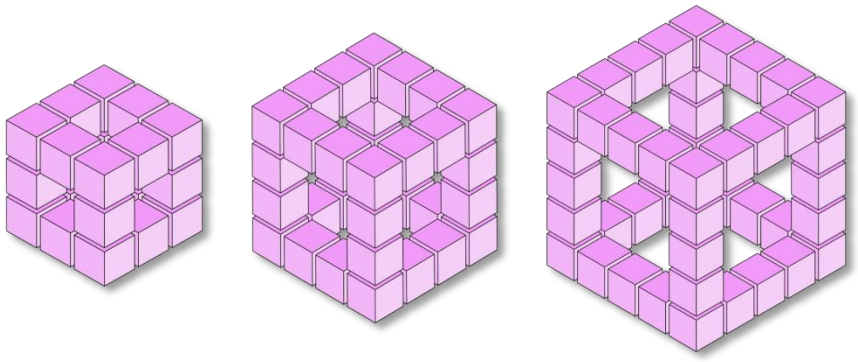


Activity 6

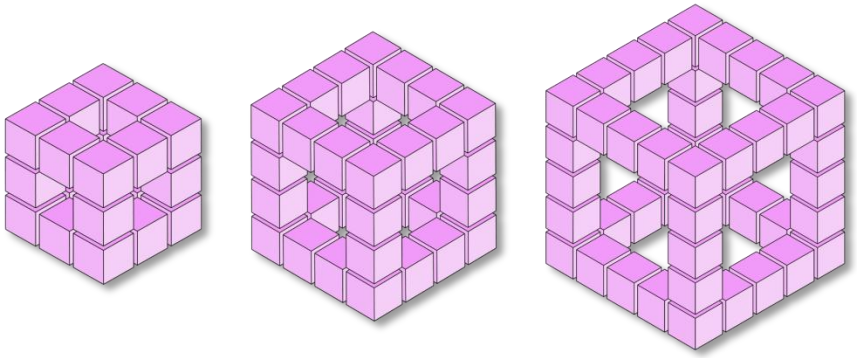
Elizabeth made the following sculpture with wooden cubes for her art project. The sculpture was made with cubes that are of two different sizes. The larger ones are unit cubes. The sculpture is solid (as opposed to hollow). The view from the opposite direction is identical to the view shown below. The views from these two directions show the entire sculpture. Find the volume and the surface area of Elizabeth's sculpture.



Activity 7



The first solid was constructed from a 3 units by 3 units by 3 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 1 square unit. The second solid was constructed from a 4 units by 4 units by 4 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 4 square units. The third solid was constructed from a 5 units by 5 units by 5 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 9 square units.

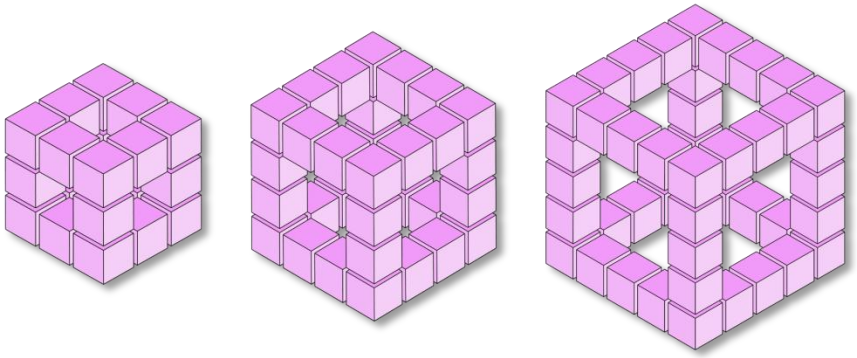


a. Find the volume and the surface area of the first solid.

b. Find the volume and the surface area of the second solid.

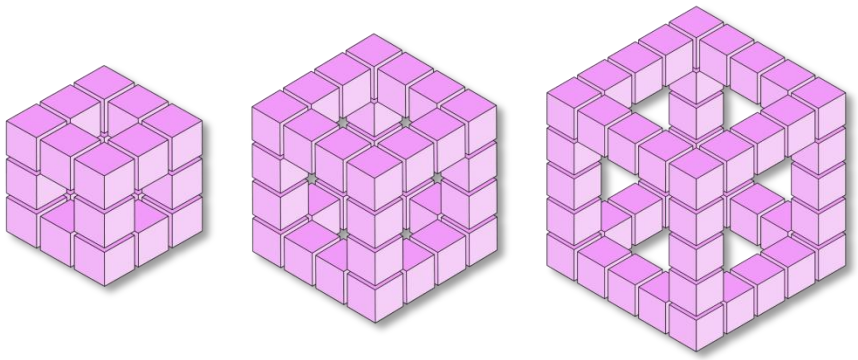
c. Find the volume and the surface area of the third solid.

Recall a cube has 6 faces, 8 vertices, and 12 edges.



d. Suppose now we start with a 6 units by 6 units by 6 units cube. We drill three intersecting holes from face-to-face of the cube. Each hole goes through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 16 square units. Use your mind’s eye to visualize the solid. Find the volume and the surface area of this solid.

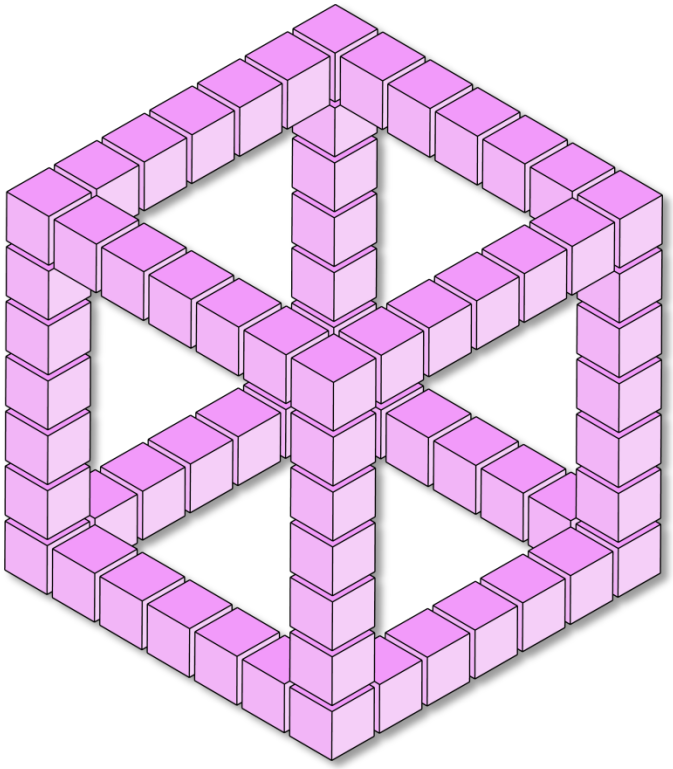
Recall a cube has 6 faces, 8 vertices, and 12 edges.



e. Consider an *n* units by *n* units by *n* units cube. Let us drill three intersecting holes from face-to-face of the cube. Each hole goes through the "centers" of the opposite faces of the original cube (as shown in the pictures above). The area of the cross-section of each of the three holes is $(n-2)^2$ square units. Use your mind's eye to visualize the solid. Find the volume and the surface area of this solid in terms of *n*.

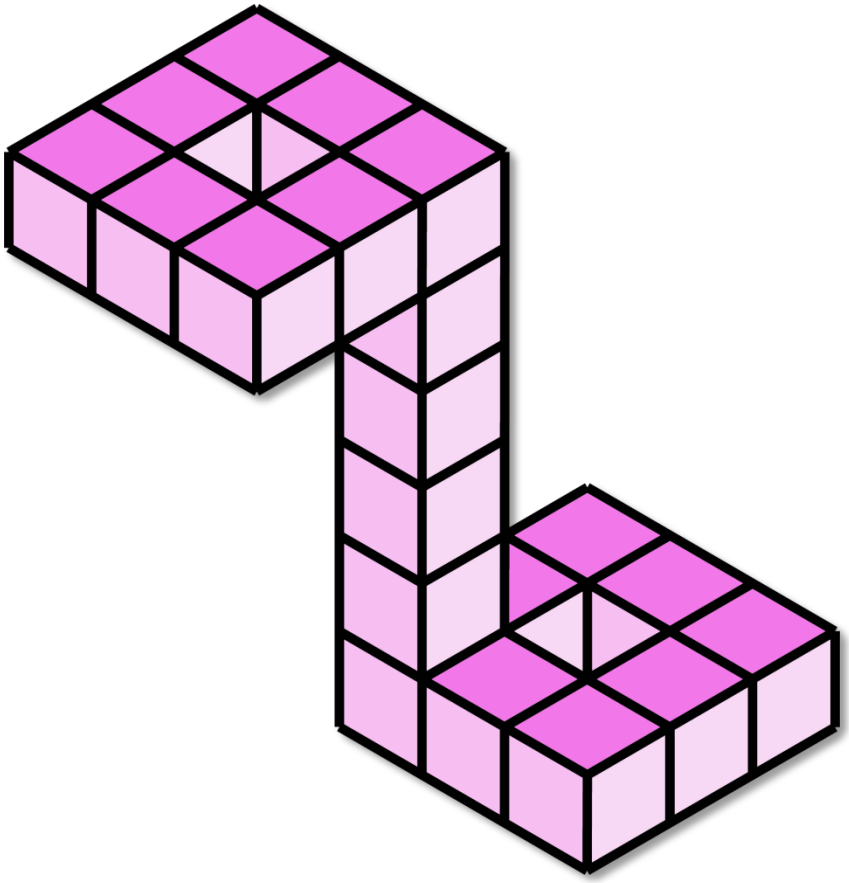
Recall a cube has 6 faces, 8 vertices, and 12 edges.

f. Use the formula that you derived in the previous part to find the volume and the surface area of the solid when $n = 7$. You may use the picture below to check your answers.



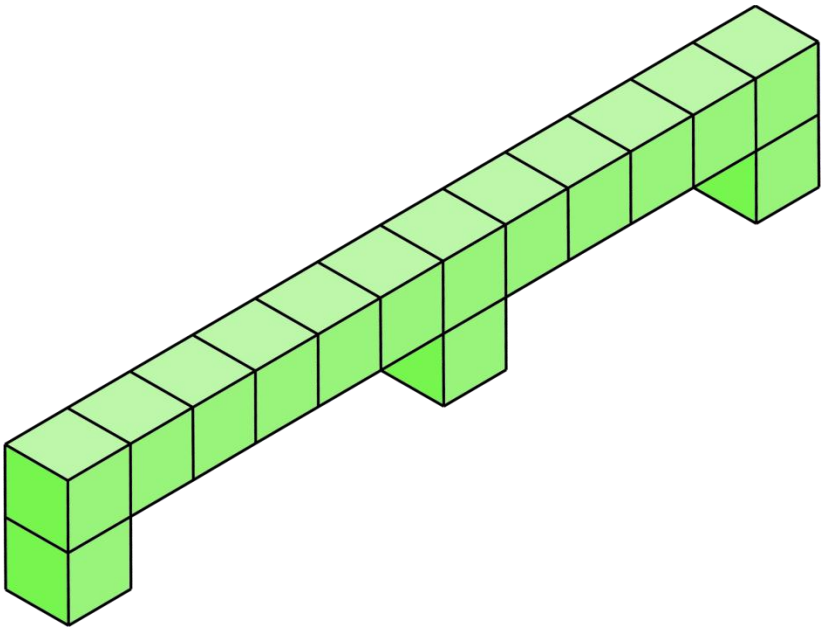
Answers

The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



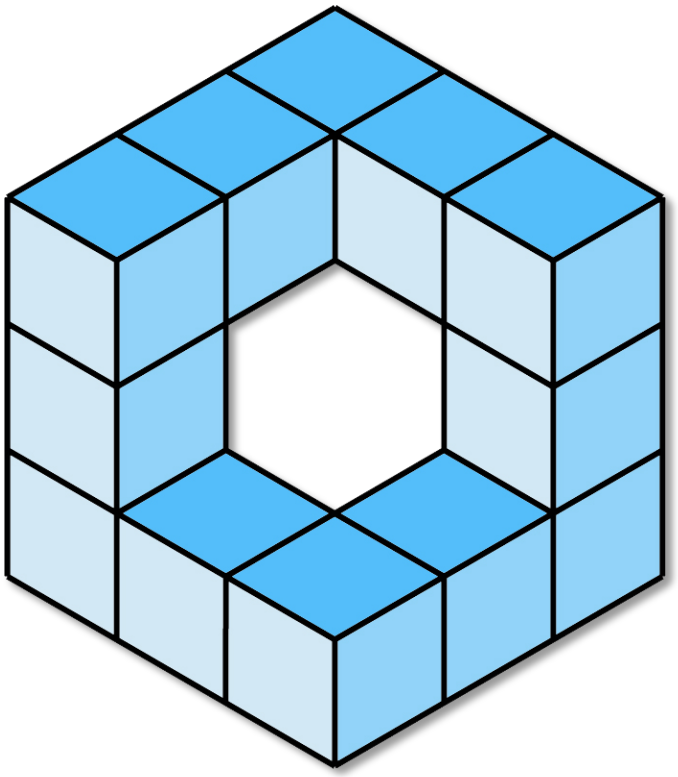
Ans:
Volume: 20 unit^3
Surface Area: 78 unit^2

The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



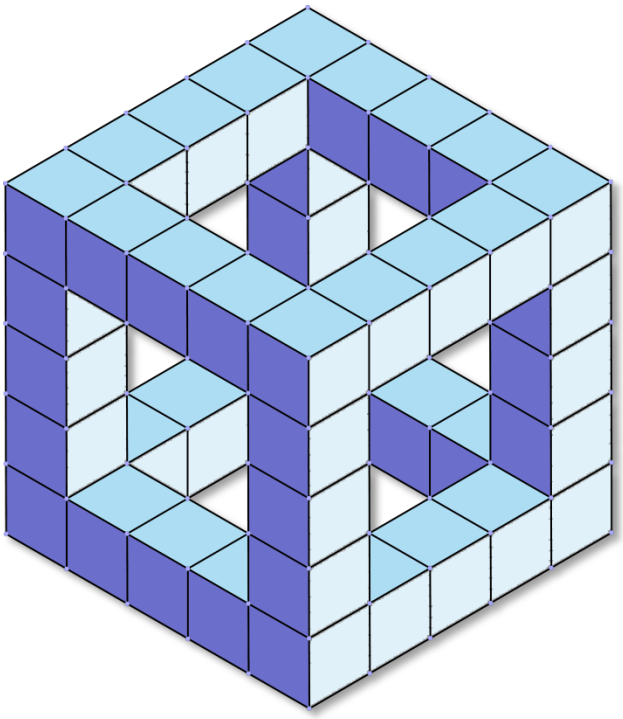
Ans:
Volume: 15 unit^3
Surface Area: 62 unit^2

The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



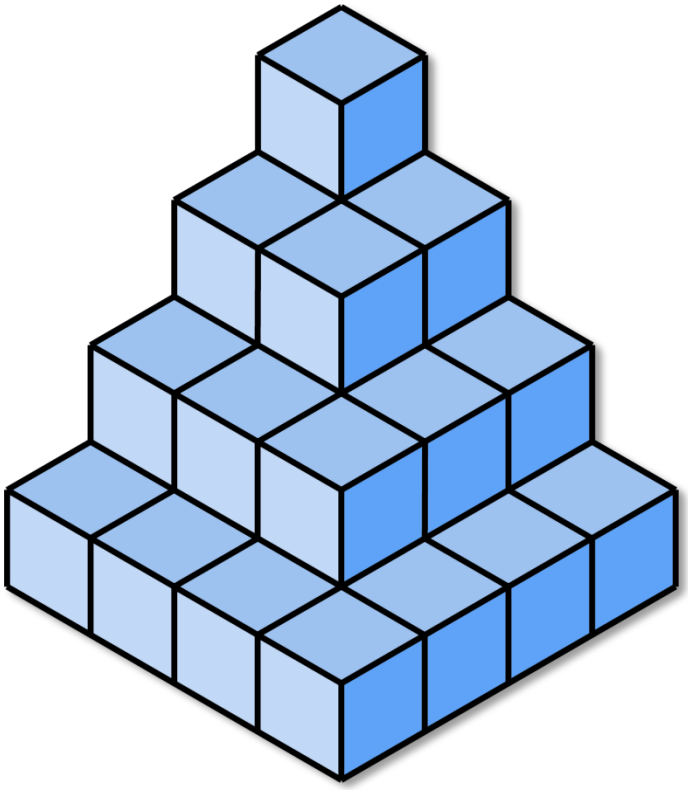
Ans:
Volume: 12 unit^3
Surface Area: 48 unit^2

The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



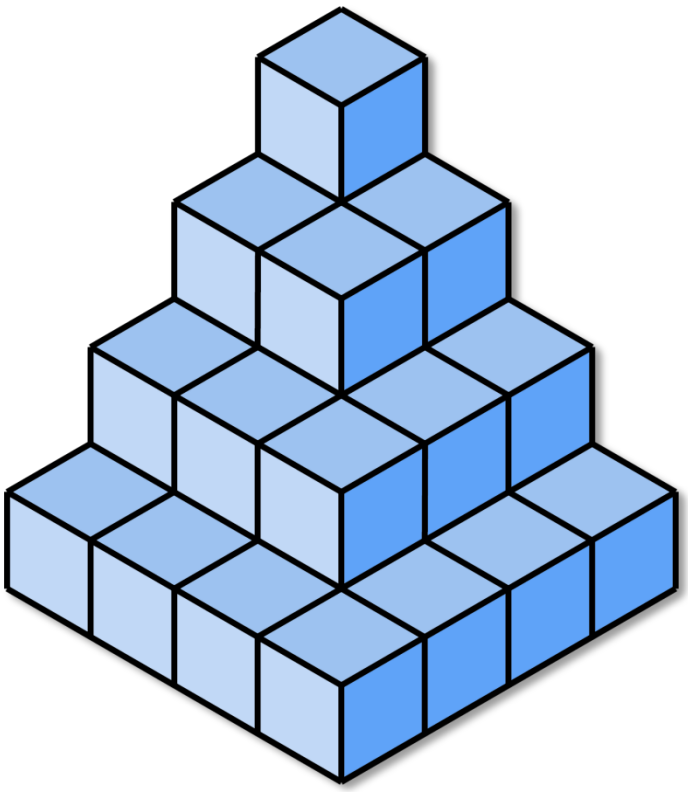
Ans:
Volume: 44 unit^3
Surface Area: 168 unit^2

The solid shown below was made with unit cubes. Find the volume and the surface area of the solid.



Ans:
Volume: 30 unit^3
Surface Area: 72 unit^2

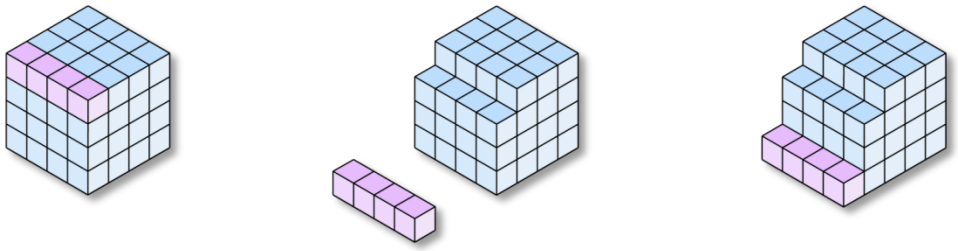
The solid shown below was made with unit cubes. How many cubes are not visible?



Ans: 14

Activity 1

1. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.

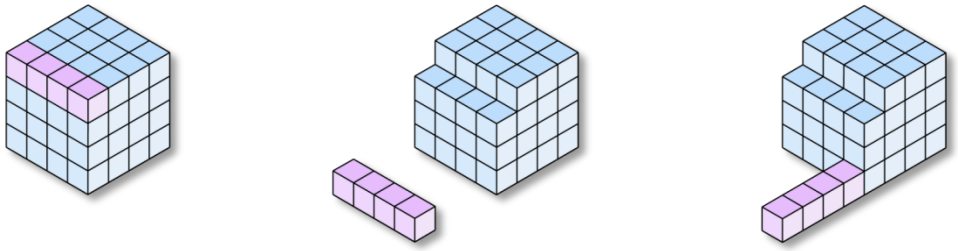


a b c

a. Surface area of the entire solid: 96 square units
b. Surface area of the larger portion:_____ sq units
c. Surface area of the entire solid: _____ sq units

Ans: 94 unit^2, 104 unit^2

2. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.

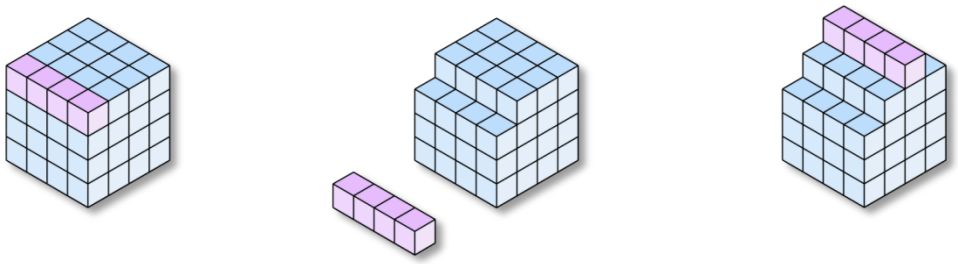


a b c

a. Surface area of the entire solid: 96 square units
b. Surface area of the larger portion:_____ sq units
c. Surface area of the entire solid: _____ sq units

Ans: 94 unit^2, 110 unit^2

3. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.

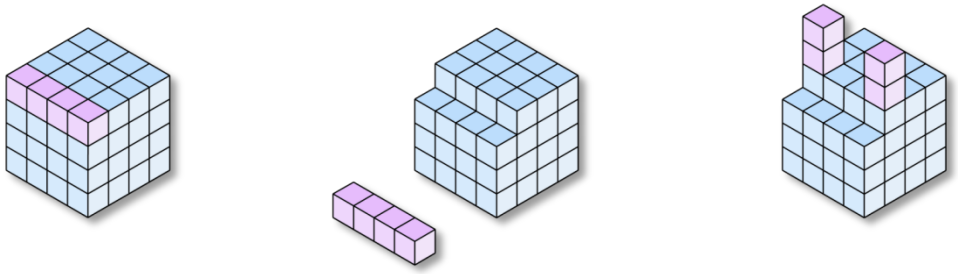


a b c

a. Surface area of the entire solid: 96 square units
b. Surface area of the larger portion:_____ sq units
c. Surface area of the entire solid: _____ sq units

Ans: 94 unit^2, 104 unit^2

4. The smaller cubes shown below are unit cubes. Without counting the unit squares on the surface fill in the blanks.



a b c

a. Surface area of the entire solid: 96 square units
b. Surface area of the larger portion:_____ sq units
c. Surface area of the entire solid: _____ sq units

Ans: 94 unit^2, 110 unit^2

Activity 2

1. A large cube, made out of white material, was dipped into blue paint. It was then cut into 27 (= 3×3×3) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

8

b. How many of the smaller cubes have 2 blue faces?

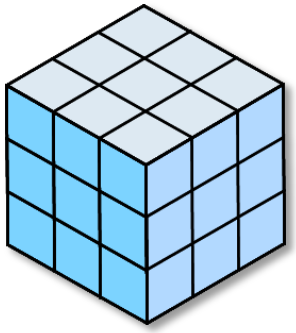
12

c. How many of the smaller cubes have 1 blue face?

6

d. How many of the smaller cubes have 0 blue faces?

1



2. A large cube, made out of white material, was dipped into blue paint. It was then cut into 64 (= 4×4×4) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

8

b. How many of the smaller cubes have 2 blue faces?

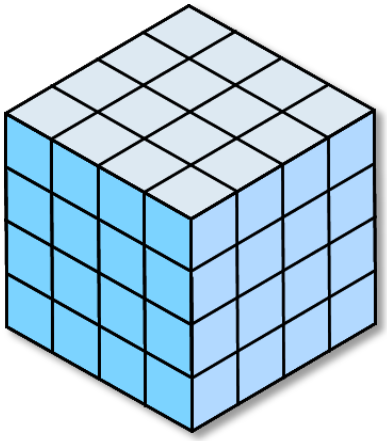
24

c. How many of the smaller cubes have 1 blue face?

24

d. How many of the smaller cubes have 0 blue faces?

8



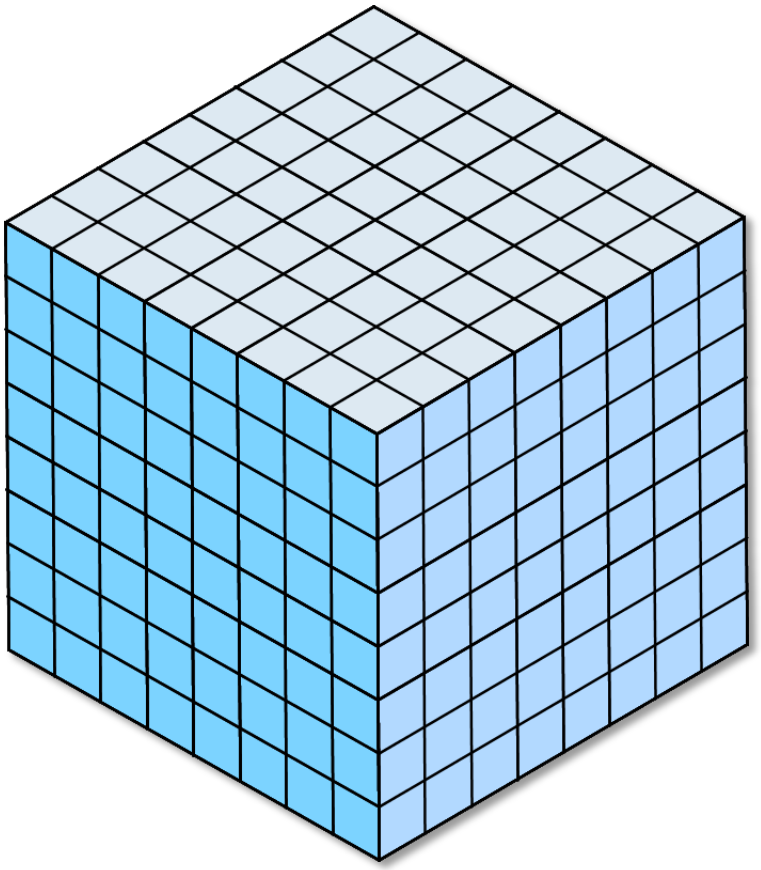
3. A large cube, made out of white material, was dipped into blue paint. It was then cut into 512 (= 8×8×8) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

8

b. How many of the smaller cubes have 2 blue faces?

72

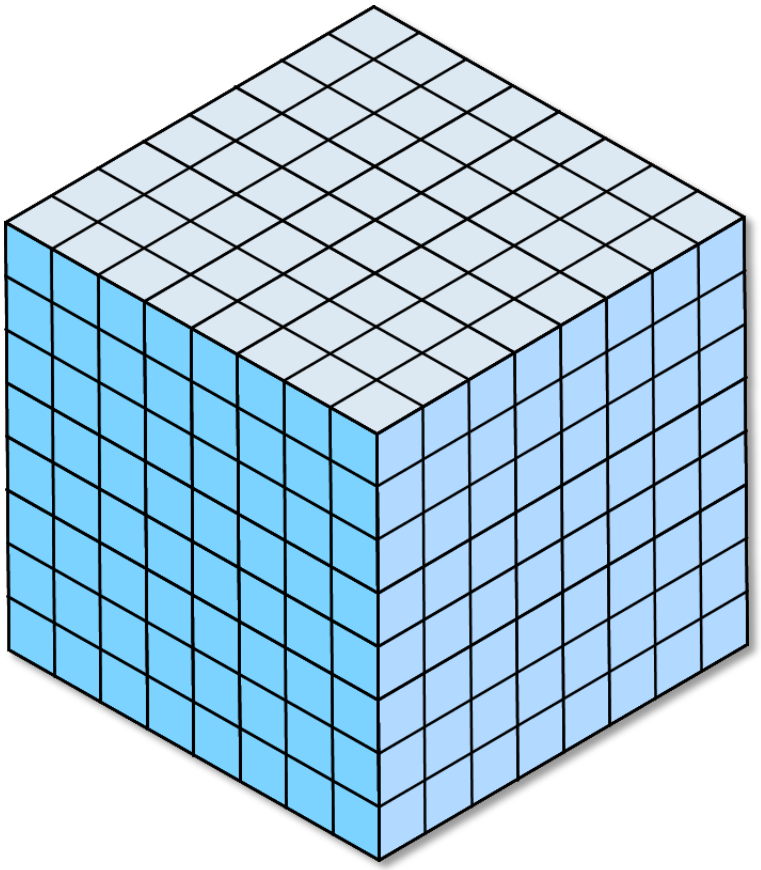


c. How many of the smaller cubes have 1 blue face?

216

d. How many of the smaller cubes have 0 blue faces?

216



4. A large cube, made out of white material, was dipped into blue paint. It was then cut into 1000 (= 10×10 × 10) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

8

b. How many of the smaller cubes have 2 blue faces?

96

c. How many of the smaller cubes have 1 blue face?

384

d. How many of the smaller cubes have 0 blue faces?

512

5. A large cube, made out of white material, was dipped into blue paint. It was then cut into n^3 (= $n \times n \times n$) smaller cubes.

a. How many of the smaller cubes have 3 blue faces?

8

b. How many of the smaller cubes have 2 blue faces?

$12(n-2)$

c. How many of the smaller cubes have 1 blue face?

$6(n-2)^2$

d. How many of the smaller cubes have 0 blue faces?

$(n-2)^3$

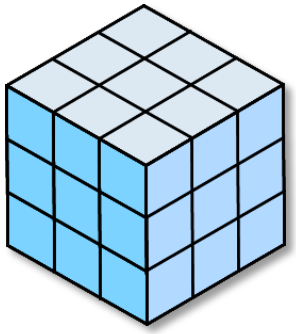
Activity 3

1. The large cube shown below is made with 27 (= 3 × 3 × 3) unit cubes.

a. What is the volume of the large cube? Ans: 27 u^3
b. What is the surface area of the large cube? Ans: 54 u^2

c. How many of the unit cubes show 3 faces? Ans: 8
d. How many of the unit cubes show 2 faces? Ans: 12
e. How many of the unit cubes show 1 face? Ans: 6
f. How many of the unit cubes show 0 faces? Ans: 1

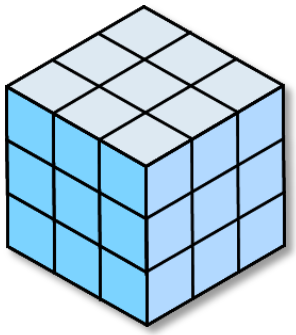
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 19 u^3
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
Ans: 54 u^2

i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 15 u^3
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
Ans: 78 u^2

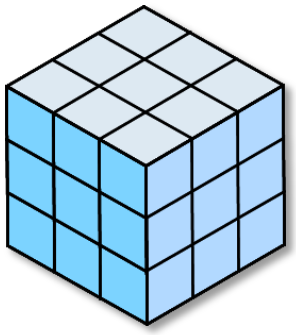
Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 21 u^3

l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
Ans: 78 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.

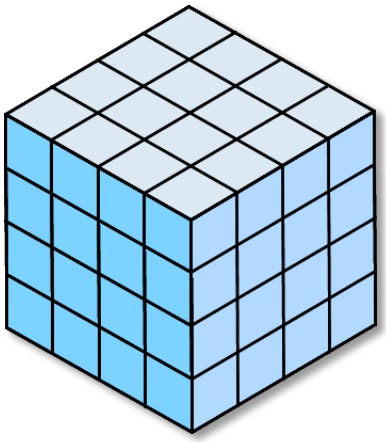


2. The large cube shown below is made with 64 (= 4 × 4 × 4) unit cubes.

a. What is the volume of the large cube? Ans: 64 u^3
b. What is the surface area of the large cube? Ans: 96 u^2

c. How many of the unit cubes show 3 faces? Ans: 8
d. How many of the unit cubes show 2 faces? Ans: 24
e. How many of the unit cubes show 1 face? Ans: 24
f. How many of the unit cubes show 0 faces? Ans: 8

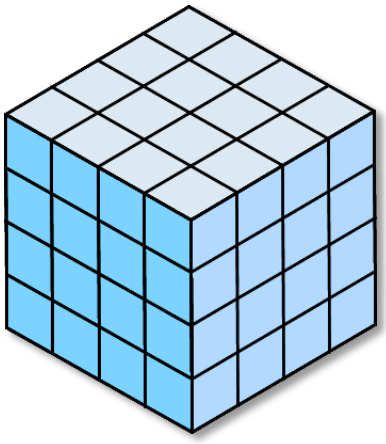
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 56 u^3
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 96 u^2

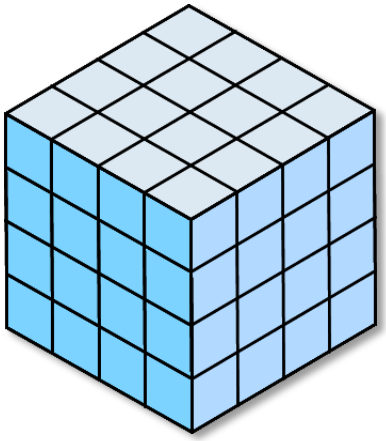
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 40 u^3
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 120 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 40 u^3
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 144 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.

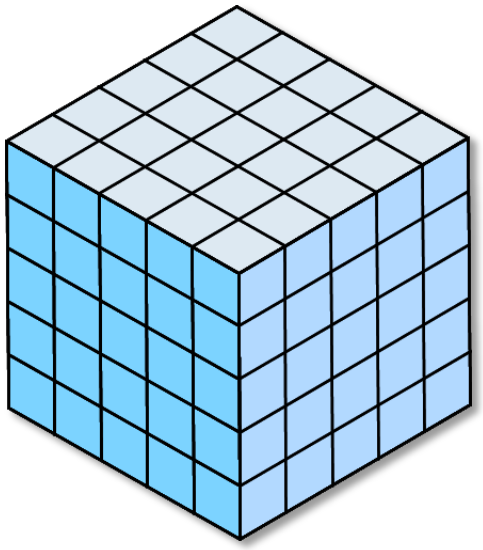


3. The large cube shown below is made with 125 (= 5 × 5 × 5) unit cubes.

a. What is the volume of the large cube? Ans: 125 u^3
b. What is the surface area of the large cube? Ans: 150 u^2

c. How many of the unit cubes show 3 faces? Ans: 8
d. How many of the unit cubes show 2 faces? Ans: 36
e. How many of the unit cubes show 1 face? Ans: 54
f. How many of the unit cubes show 0 faces? Ans: 27

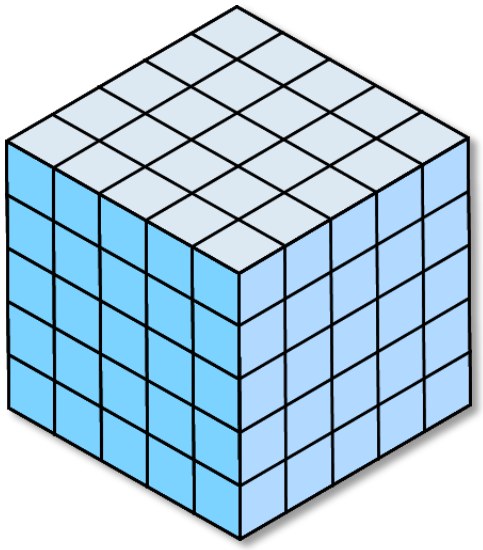
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 117 u^3
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 150 u^2

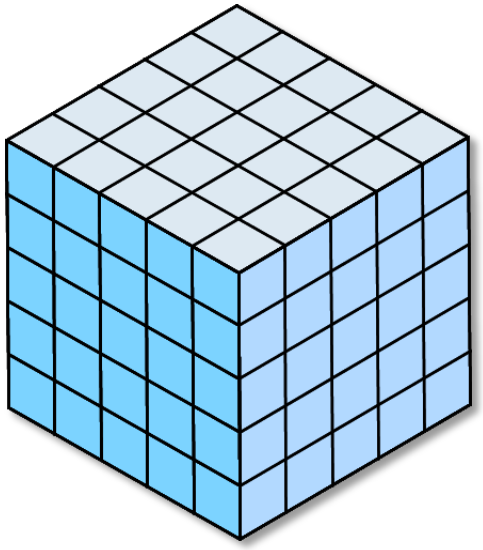
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 89 u^3
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 174 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 71 u^3
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 222 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.

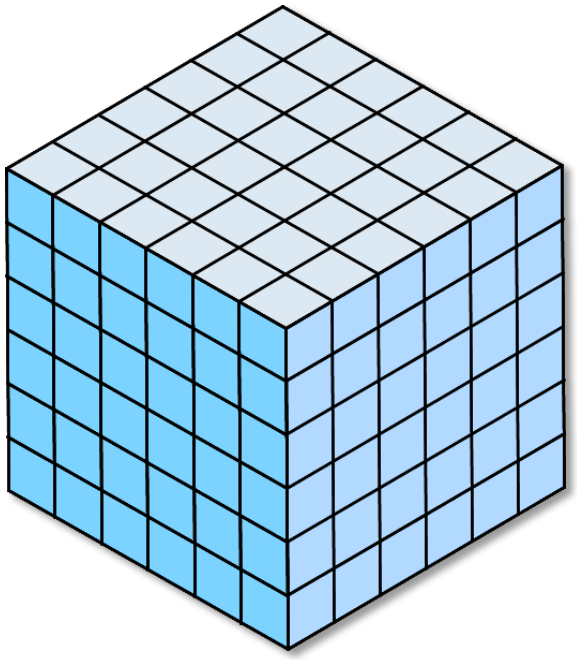


4. The large cube shown below is made with 216 (= 6 × 6 × 6) unit cubes.

a. What is the volume of the large cube? Ans: 216 u^3
b. What is the surface area of the large cube? Ans: 216 u^2

c. How many of the unit cubes show 3 faces? Ans: 8
d. How many of the unit cubes show 2 faces? Ans: 48
e. How many of the unit cubes show 1 face? Ans: 96
f. How many of the unit cubes show 0 faces? Ans: 64

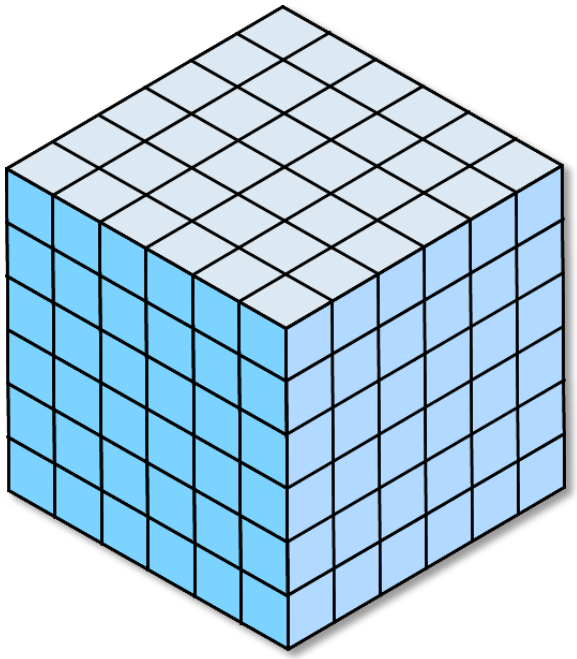
Recall a cube has 6 faces, 8 vertices, and 12 edges.



g. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 208 u^3
h. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact. Ans: 216 u^2

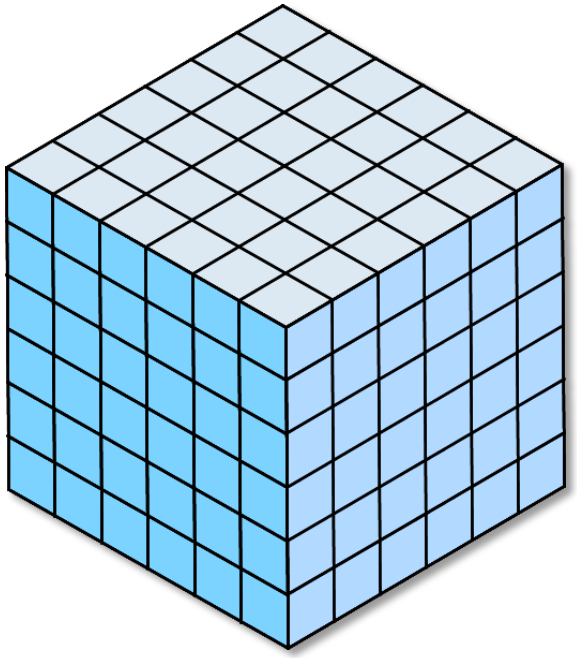
i. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 168 u^3
j. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact. Ans: 240 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.



k. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 120 u^3
l. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact. Ans: 312 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.



5. Visualize a large cube made out of 343 (= 7 × 7 × 7) unit cubes.

a. How many of the unit cubes show 3 faces? Ans: 8
b. How many of the unit cubes show 2 faces? Ans: 60
c. How many of the unit cubes show 1 face? Ans: 150
d. How many of the unit cubes show 0 faces? Ans: 125

e. What is the volume of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
Ans: 335 $unit^3$
f. What is the surface area of the remaining solid, if all the cubes that are showing 3 faces are removed? Assume all the other cubes remain intact.
Ans: 294 $unit^2$

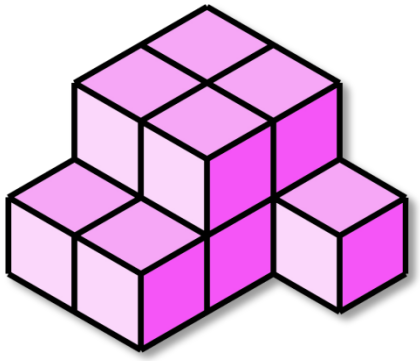
g. What is the volume of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
Ans: 283 $unit^3$
h. What is the surface area of the remaining solid, if all the cubes that are showing 2 faces are removed? Assume all the other cubes remain intact.
Ans: 318 $unit^2$

i. What is the volume of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
Ans: 193 $unit^3$
j. What is the surface area of the remaining solid, if all the cubes that are showing 1 face are removed? Assume all the other cubes remain intact.
Ans: 414 $unit^2$

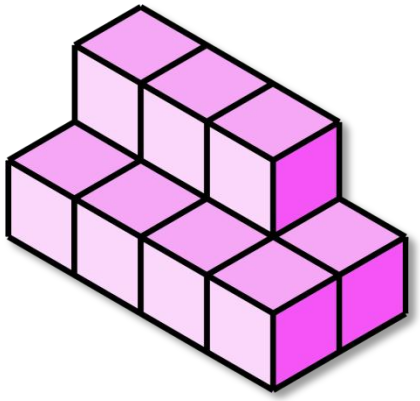
Recall a cube has 6 faces, 8 vertices, and 12 edges.

Activity 4

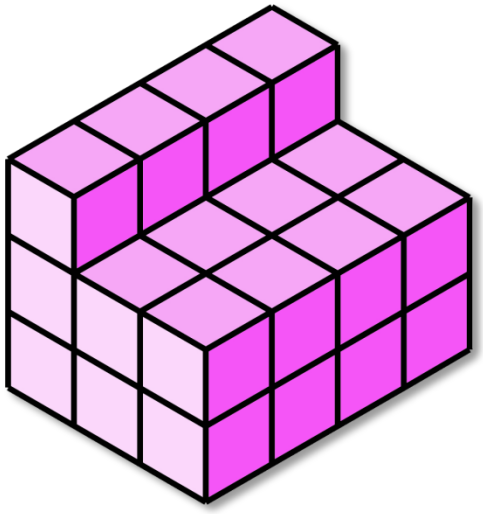
1. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.
Ans: 32



2. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.
Ans: 30



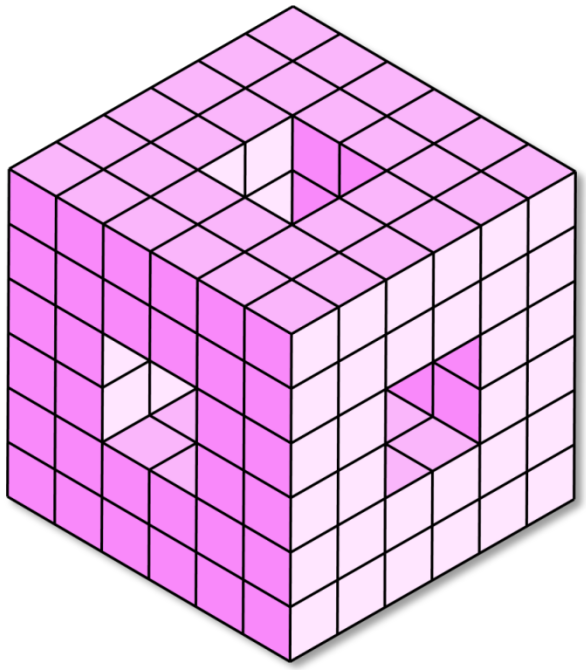
3. The following is a picture of stacks of solid cubes. Determine the number of faces that are glued together.
Ans: 106



Activity 5

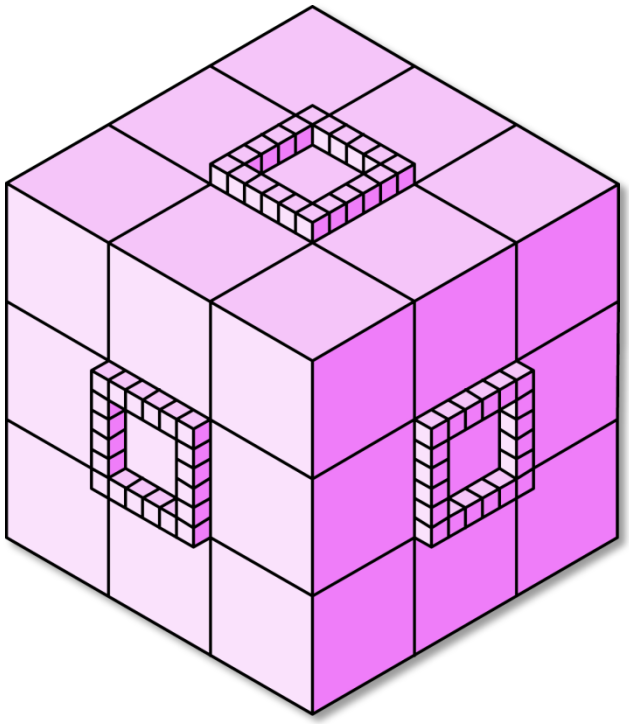
The following object was made from a 6 by 6 by 6 cube made out of unit cubes. Three holes were drilled through each pair of opposite faces. These holes went all the way through the opposite faces. Moreover, these holes removed the four unit cubes at the center of each of the six faces as shown. Of course, the process of drilling removed more cubes than just the ones on the surface. What is the volume and surface area of the object shown below?

Ans: Vol: 160 u^3, SA: 288 u^2

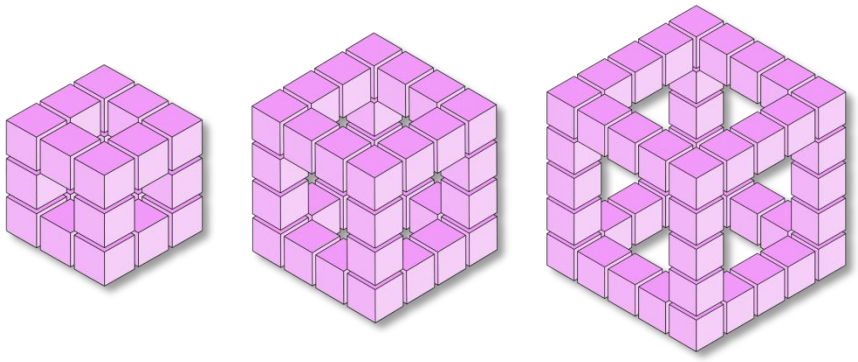


Activity 6

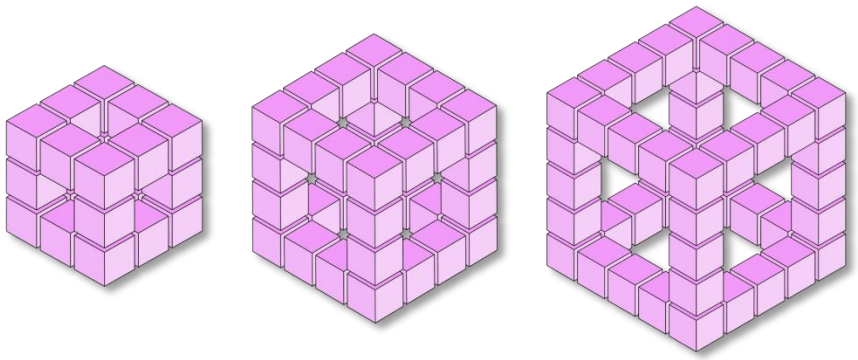
Elizabeth made the following sculpture with wooden cubes for her art project. The sculpture was made with cubes that are of two different sizes. The larger ones are unit cubes. The sculpture is solid (as opposed to hollow). The view from the opposite direction is identical to the view shown below. The views from these two directions show the entire sculpture. Find the volume and the surface area of Elizabeth's sculpture.
Ans: Vol: 27 & 15/27 u^3, SA: 60 & 2/3 u^2



Activity 7



The first solid was constructed from a 3 units by 3 units by 3 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 1 square unit. The second solid was constructed from a 4 units by 4 units by 4 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 4 square units. The third solid was constructed from a 5 units by 5 units by 5 units cube (not shown in the picture). Three intersecting holes were drilled from face-to-face of the cube. Each hole went through the “centers” of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 9 square units.



a. Find the volume and the surface area of the first solid.

Vol: 20 u^3, SA: 72 u^2

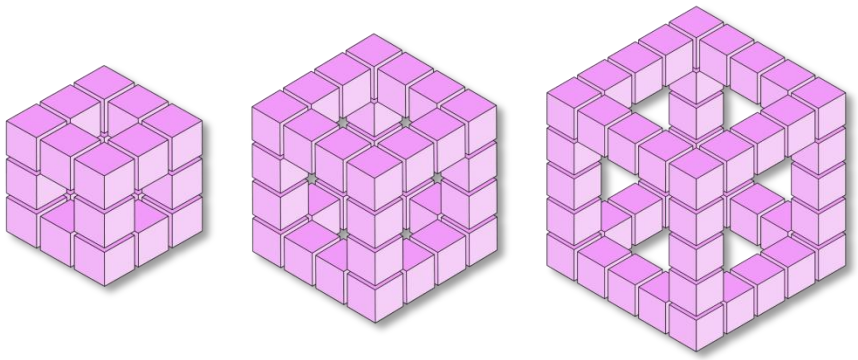
b. Find the volume and the surface area of the second solid.

Vol: 32 u^3, SA: 120 u^2

c. Find the volume and the surface area of the third solid.

Vol: 44 u^3, SA: 168 u^2

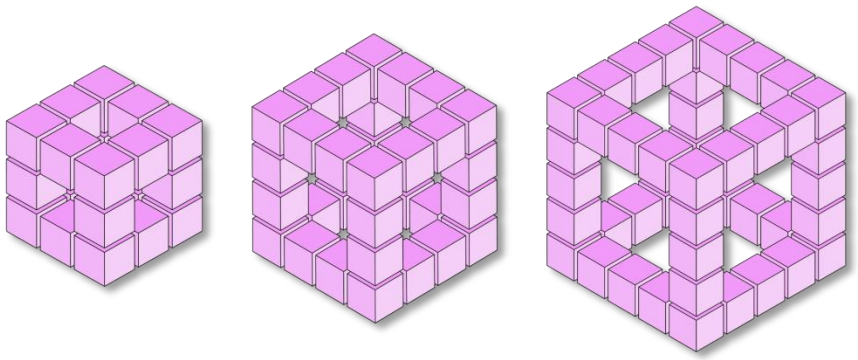
Recall a cube has 6 faces, 8 vertices, and 12 edges.



d. Suppose now we start with a 6 units by 6 units by 6 units cube. We drill three intersecting holes from face-to-face of the cube. Each hole goes through the "centers" of the opposite faces of the original cube. The area of the cross-section of each of the three holes is 16 square units. Use your mind's eye to visualize the solid. Find the volume and the surface area of this solid.

Vol: 56 u^3, SA: 216 u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.



e. Consider an n units by n units by n units cube. Let us drill three intersecting holes from face-to-face of the cube. Each hole goes through the "centers" of the opposite faces of the original cube (as shown in the pictures above). The area of the cross-section of each of the three holes is $(n-2)2$ square units. Use your mind's eye to visualize the solid. Find the volume and the surface area of this solid in terms of n.

Vol: $12n-16$ u^3, SA: $48n-72$ u^2

Recall a cube has 6 faces, 8 vertices, and 12 edges.

f. Use the formula that you derived in the previous part to find the volume and the surface area of the solid when $n = 7$. You may use the picture below to check your answers.

Vol: 68 u^3, SA: 264 u^2