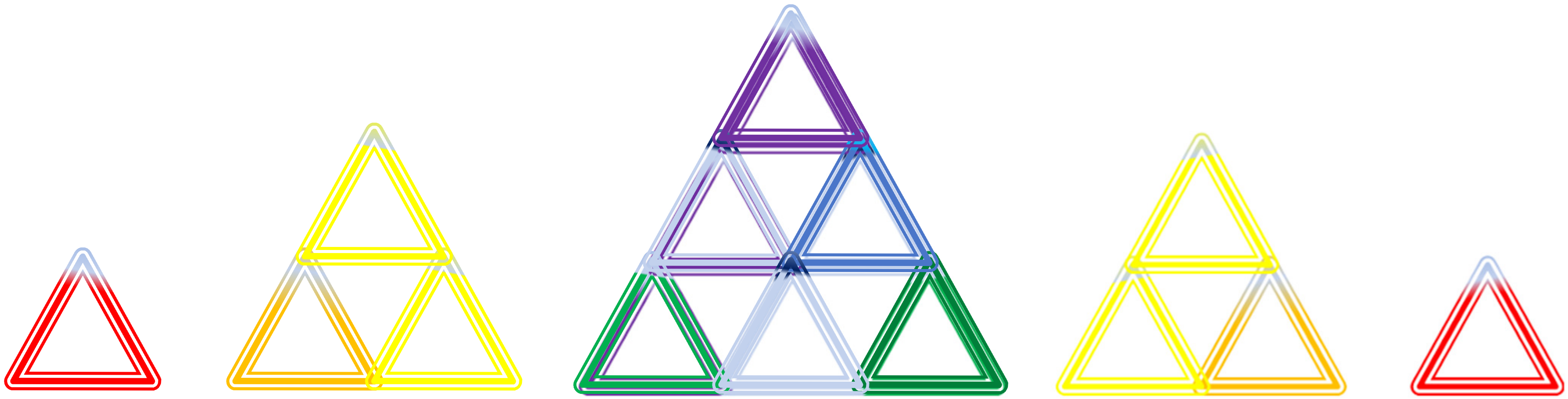
Algebra and the Common Core in Your Classroom



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What We Are Going To Do?

- **Analyze Patterns**
- **Write Equations to Describe Patterns**
- **Domain and Range**
- **Think About Questions to ask Students**
- **Think About Assessing Students**
- **Describe Patterns Recursively**
- **Make Connections to Content and Practice Standards**

How Are We Going To Do It?

- **Hands-on Models**
- **Mathematical Exploration**
- **Thinking About the Classroom**

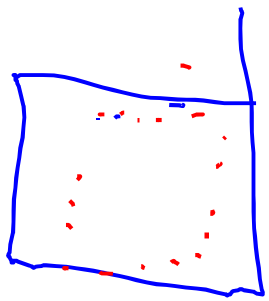
Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Principles to Action

- Clear Mathematical Goals for Student Learning
- Coherent Activities and Problems Aligned With Mathematical Goals
- Assess and Advance Student Understanding
- Allow Productive Struggle
- Facilitate Discourse To Foster Conceptual Understanding and Procedural Fluency
- Use Mathematical Representations to Support Learning
- Use Evidence of Student Thinking to Modify and Improve Instruction

Pattern A



Key

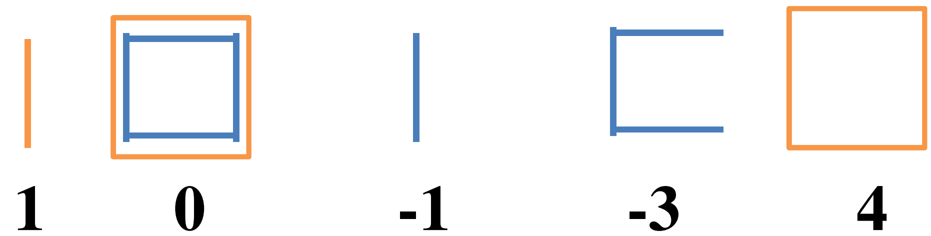
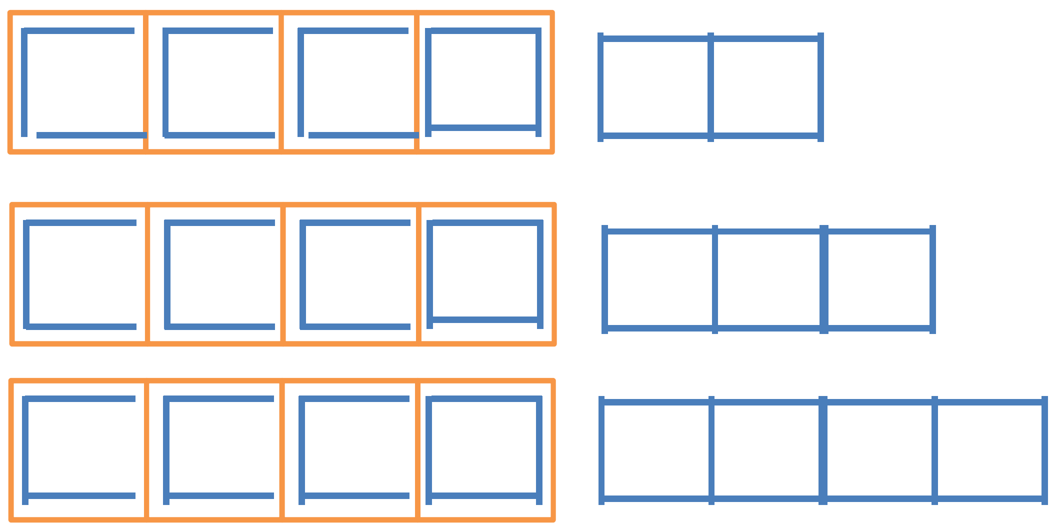


Figure 1

Figure 2

Figure 3



How many toothpicks will be in figure 5, figure n?

Pattern A

Key

1 0 -1 -3 4

Figure 1

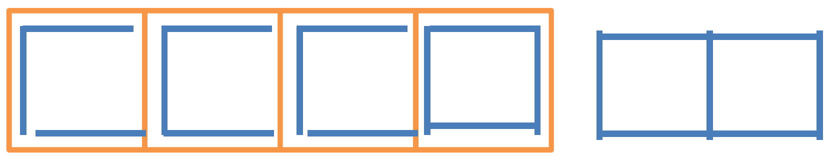


Figure 2

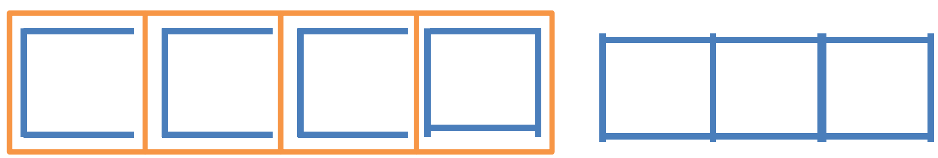
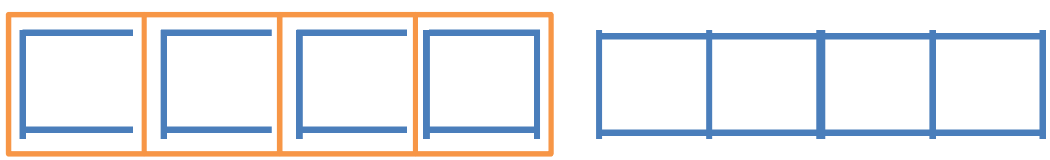


Figure 3



How many toothpicks will be in figure 5, figure n?

-7-3-3-3-3 or -4-3-3-3-3-3 or -7-3(4) or -4-3(5)

-7-3(n-1) -4 – 3n -7-3(n-1) -4 – 3n

Pattern A

Key

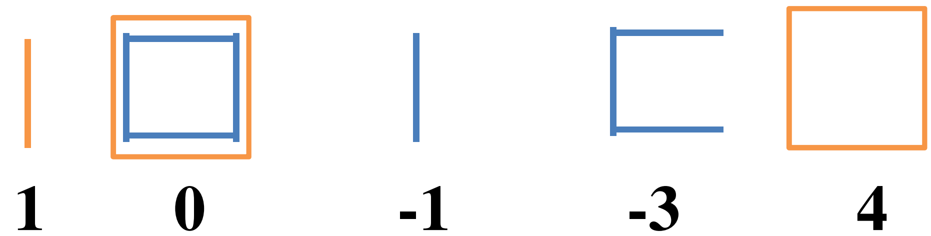
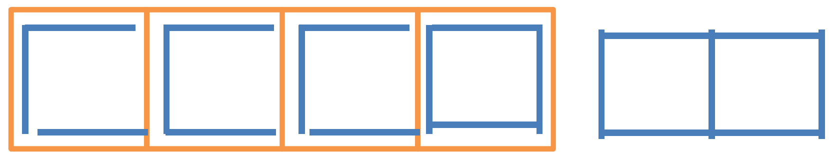


Figure 1



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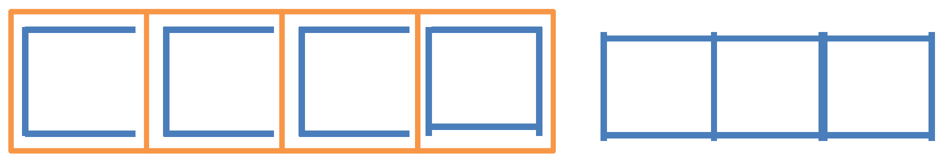
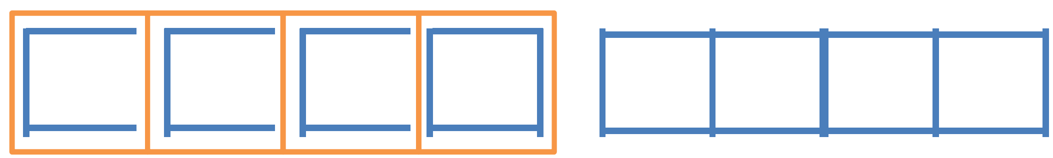


Figure 3



What questions can you ask students?

Pattern A

Key

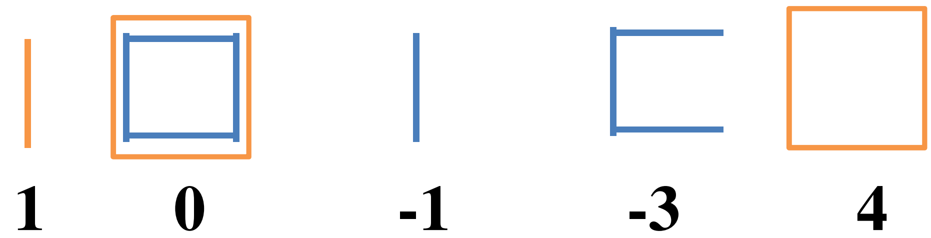


Figure 1

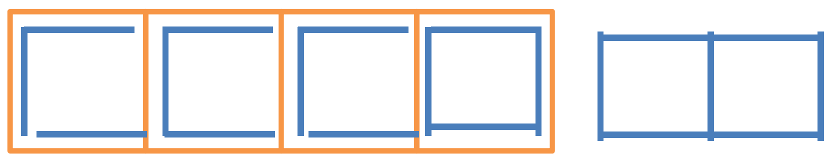


Figure 2

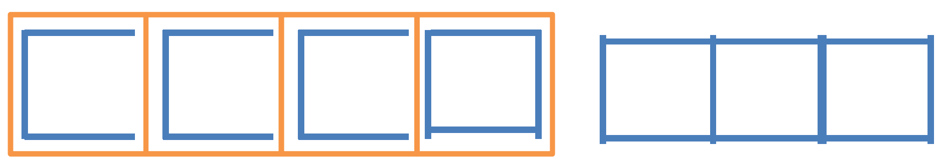
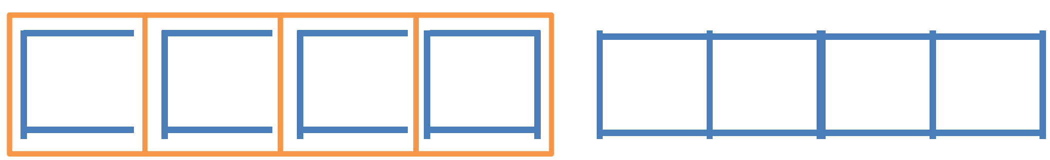


Figure 3



Explain how to get from Figure 3 to Figure 4.

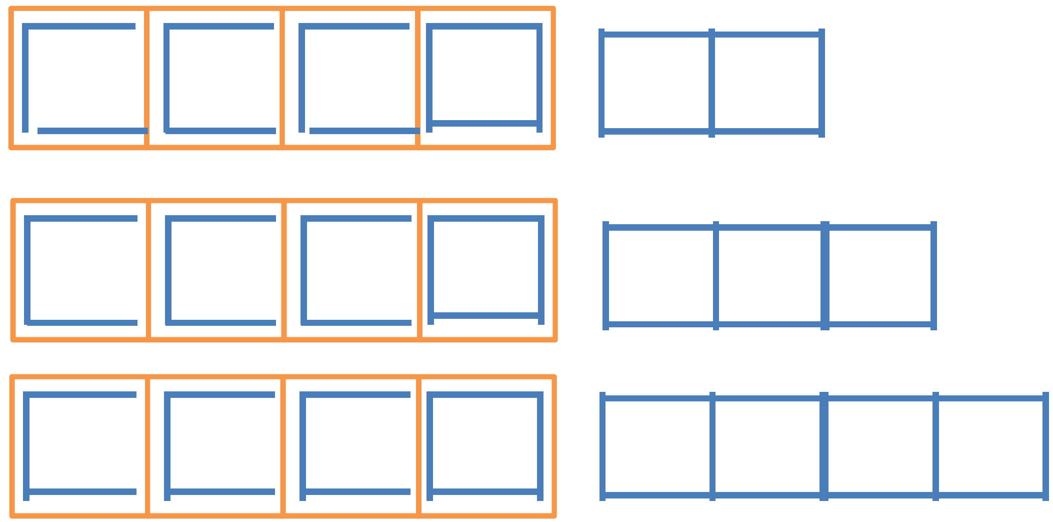
Explain how to get from Figure 1 to Figure 0.

Pattern A

Figure 1

Figure 2

Figure 3



Explain how to get from Figure 3 to Figure 4.
Explain how to get from Figure 1 to Figure 0.

$A_4 = A_3 - 3$ $\qquad$ $A_0 = A_1 + 3$

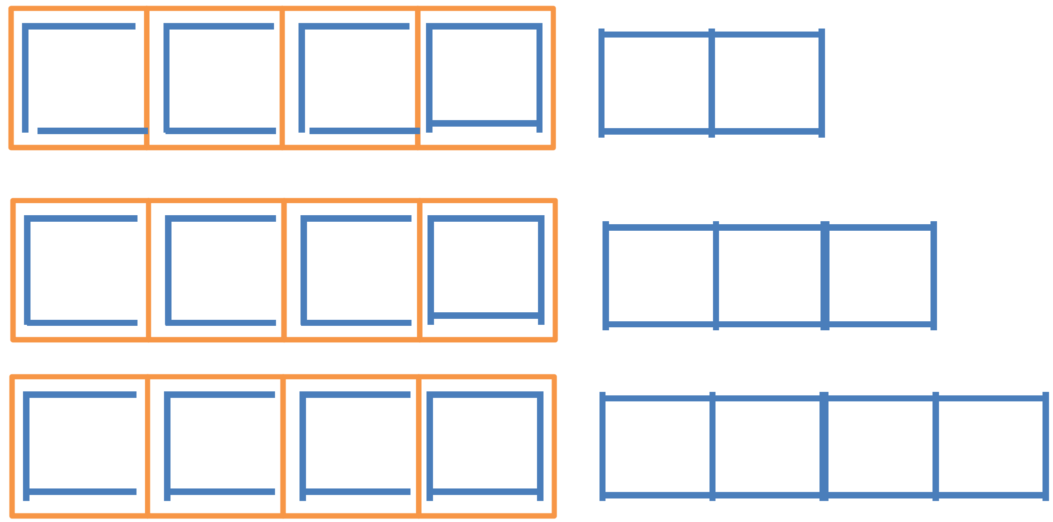
$A_n = A_{n-1} - 3$ $\qquad$ $A_{n-1} = A_n + 3$

Pattern A

Figure 1

Figure 2

Figure 3



Use words and mathematical symbols to show how to get from Figure 3 to Figure 4.
Use words and mathematical symbols to show how to get from Figure 1 to Figure 0.

Classroom Culture

– "questioning and deep thinking in which pupils will learn from shared discussions with teachers and from one another"

– "emphasis on the challenge to think for yourself (and not just work harder)"

– "teaching and learning have to be interactive."

– "what is needed is a culture of success, back by a belief that all can achieve."

-Black & Wiliam, *Inside the Black Box*, 2001

Planning

- What would your learning objective be that would allow you to use this activity?
- What previous knowledge do students need in order to complete this task?
- What questions would you ask students during the activity to help them make progress?
- What would you have students do after completing the task?

Strength In Numbers: Selecting And Setting Up A Task

- Mathematical Goals
- Prior Knowledge, Knowledge Needed, What Questions
- Ways To Solve (Student Eays)?

  What Misconceptions? What Errors?
- Expectations: Resources/Tools, Classroom Structure, Recording/Reporting
- Access to ***ALL***, Ensuring Understanding

Strength in Numbers, Ilana Seidel Horn, 2012

Strength In Numbers: Supporting Students' Exploration

Questions for:

- Getting Started/Making Progress
- Focus Thinking on Key Mathematical Ideas
- Access Student Understanding Mathematical Ideas
- Advance Understanding of Mathematical Ideas
- Encourage All Students to Share Thinking With Others or to Assess Their Understanding of Their Peers Ideas

***Strength in Numbers*, Ilana Seidel Horn, 2012**

Strength In Numbers: Supporting Students' Exploration

How will you ensure students remain engaged?

- What assistance will you give or what questions will you ask frustrated groups?
- What will you do if a group finished immediately? How will you extend the task?
- What will you do if a student/group focuses on nonmathematical aspects of the task?

***Strength in Numbers*, Ilana Seidel Horn, 2012**

Strength In Numbers: Sharing and Discussing the Task

How will you orchestrate classroom discussion?

- What solution paths will be shared? What order? Why?
- How will this help with the goals of the lesson?
- What specific questions will you ask so that students will:

1. make sense of the mathematical ideas?
2. expand on, debate and question the solutions being shared?

***Strength in Numbers*, Ilana Seidel Horn, 2012**

More Sharing and Discussing…

What specific questions will you ask so that students will:

3. make connections among the different strategies that are presented?
4. look for patterns?
5. begin to form generalizations?

How will you ensure all students have the opportunity to share their reasoning and thinking?

***Strength in Numbers*, Ilana Seidel Horn, 2012**

More Sharing and Discussing…

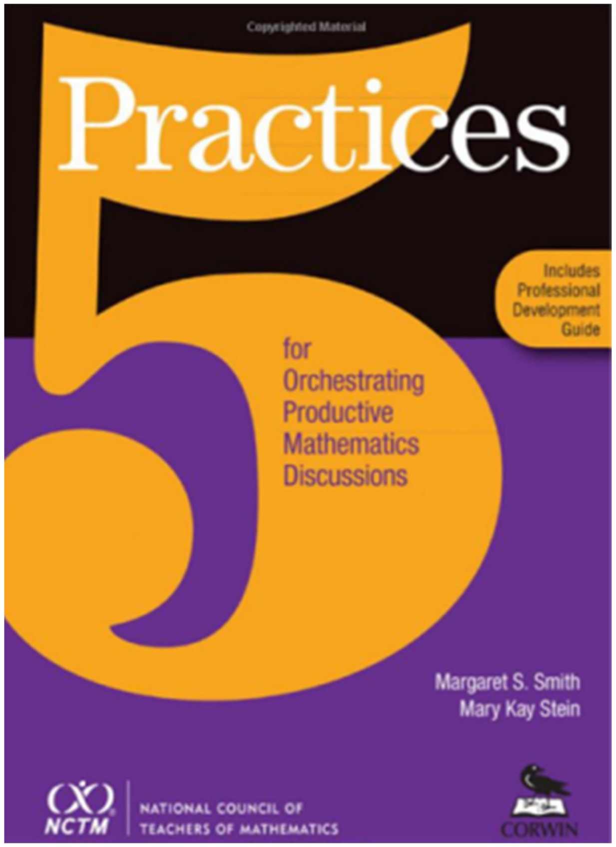
What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?

What will you do tomorrow that will build on this lesson?

Strength in Numbers*, *Ilana Seidel Horn, 2012

The 5 Practices

- **Anticipate**
- **Monitor**
- **Select**
- **Sequence**
- **Connect**

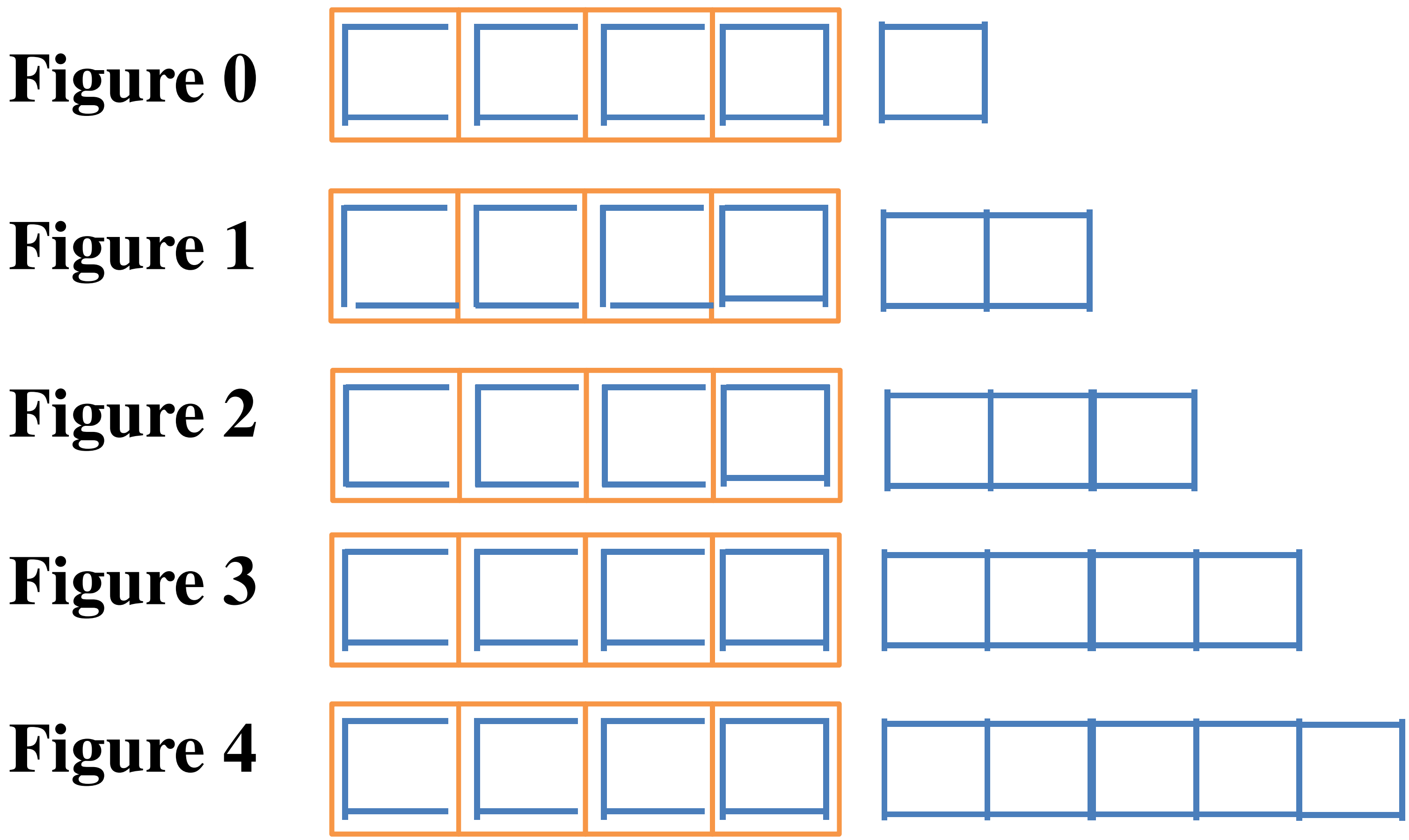


5 Practices for Orchestrating Productive Mathematics Discussions, by Margaret S. Smith and Mary Kay Stein (2011)

Principles to Action

- Clear Mathematical Goals for Student Learning
- Coherent Activities and Problems Aligned With Mathematical Goals
- Assess and Advance Student Understanding
- Allow Productive Struggle
- Facilitate Discourse To Foster Conceptual Understanding and Procedural Fluency
- Use Mathematical Representations to Support Learning
- Use Evidence of Student Thinking to Modify and Improve Instruction

What Would Figure 4 Look Like?
What would Figure 0 Look Like?



A Different Way To Look At It

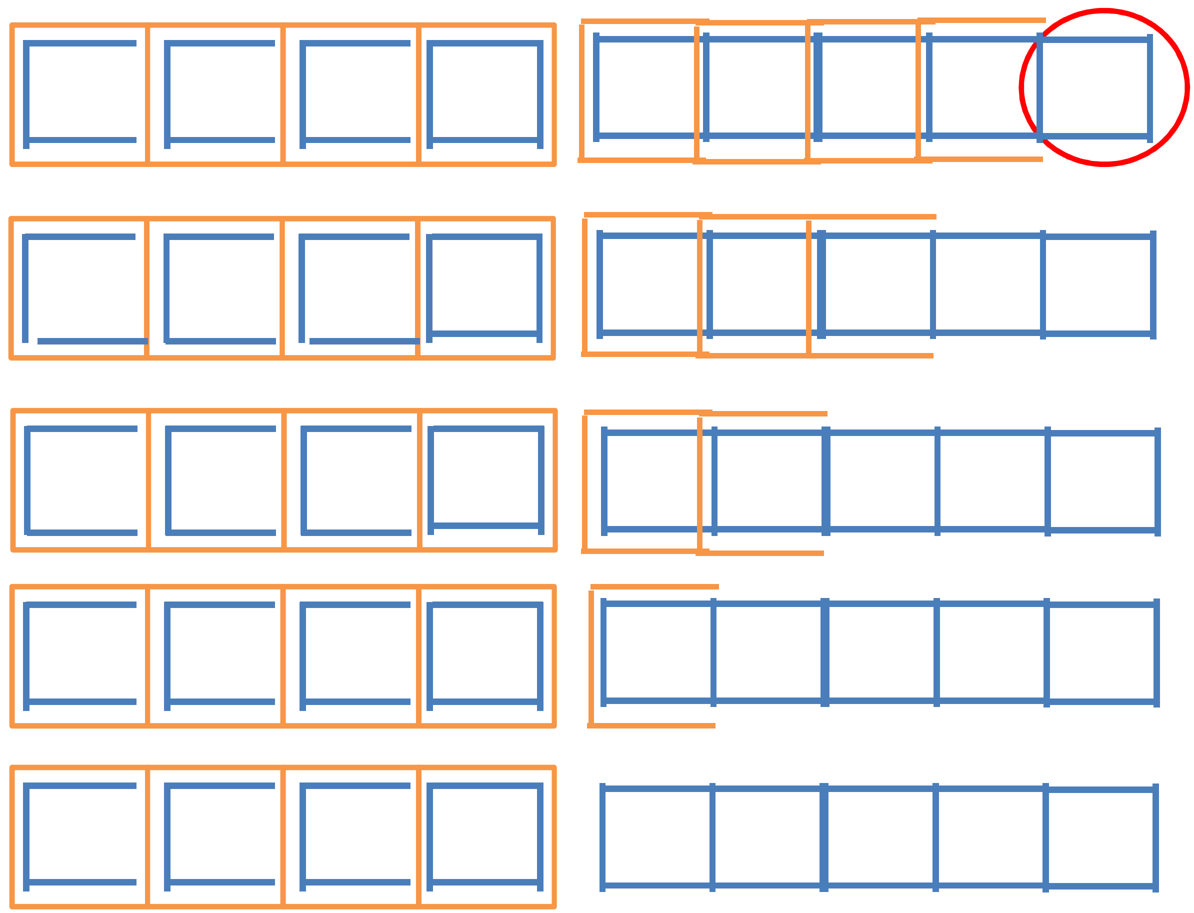
Figure 0

Figure 1

Figure 2

Figure 3

Figure 4



How many toothpicks are in each figure? Represent it using a table, graph and an equation (rule).

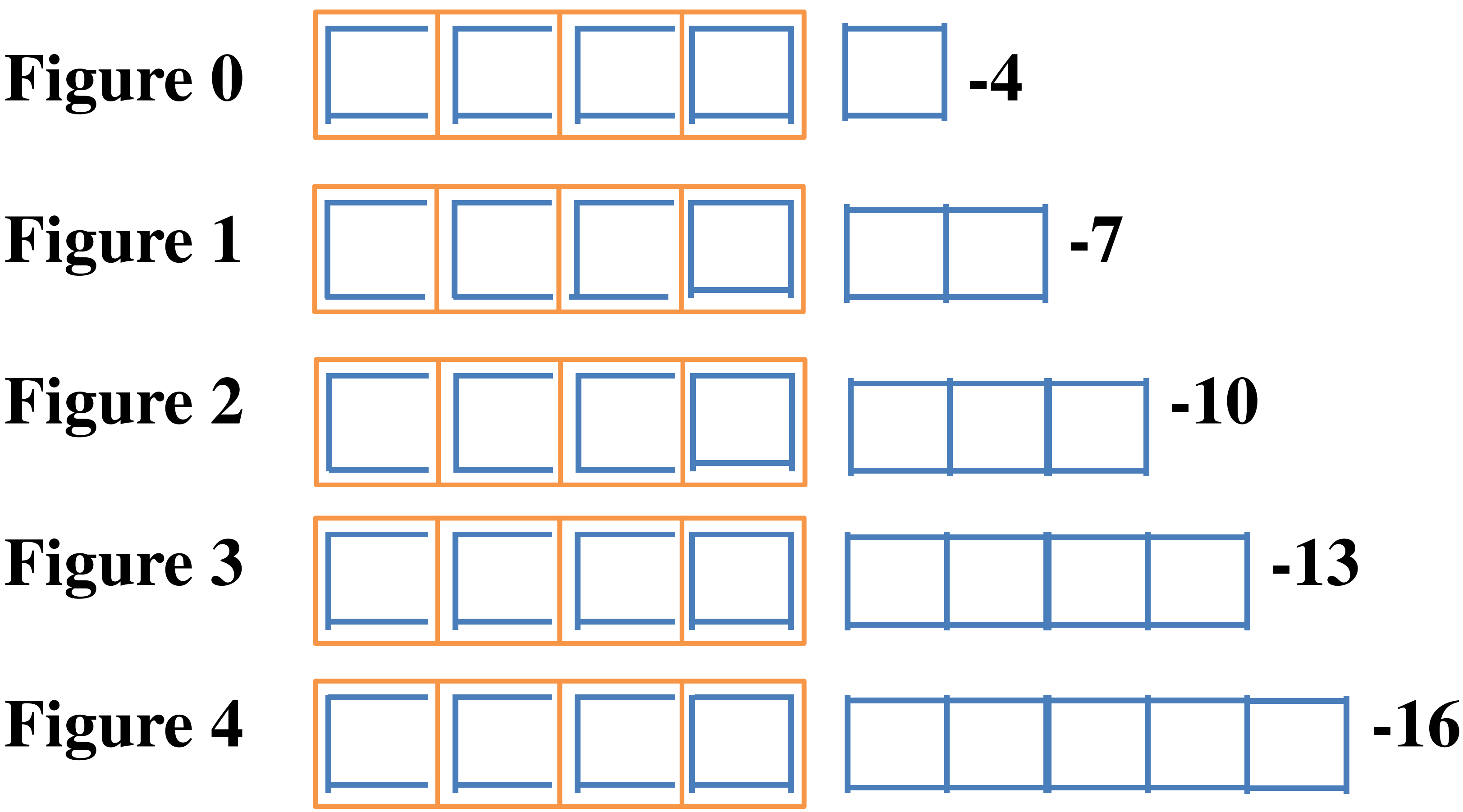


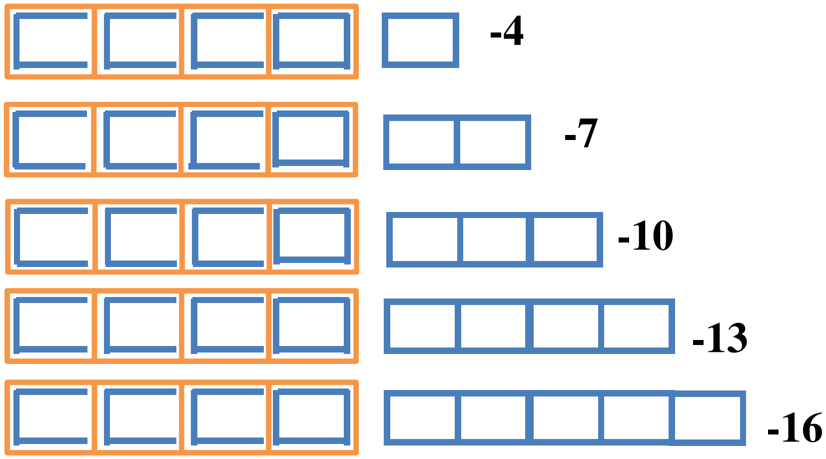
Figure 0

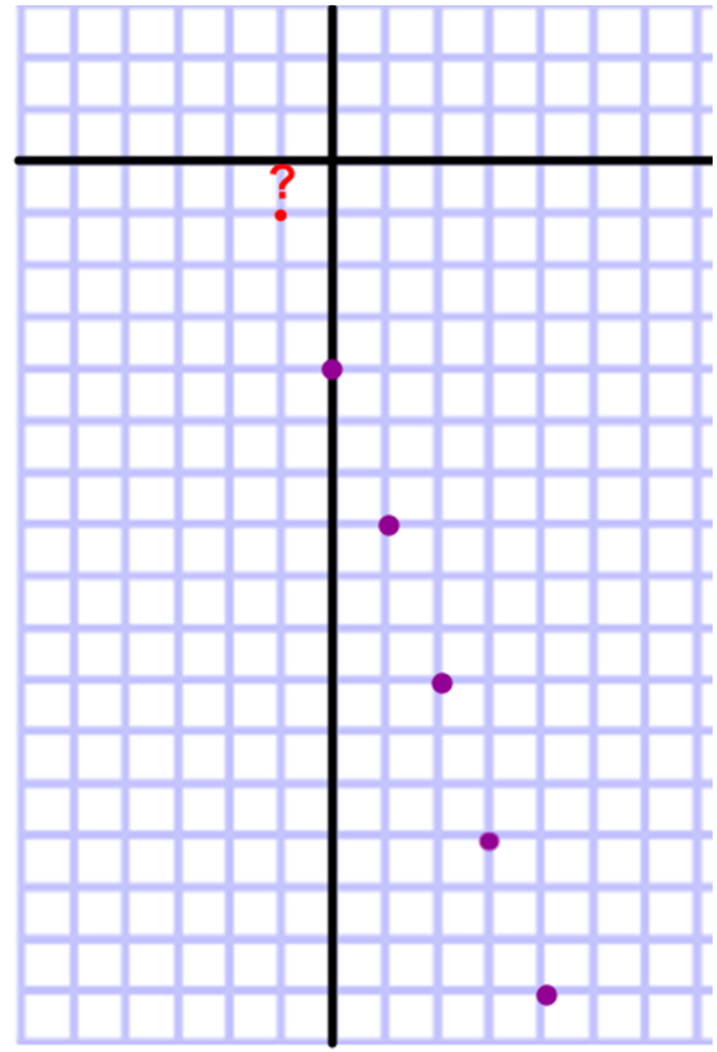
Figure 1

Figure 2

Figure 3

Figure 4



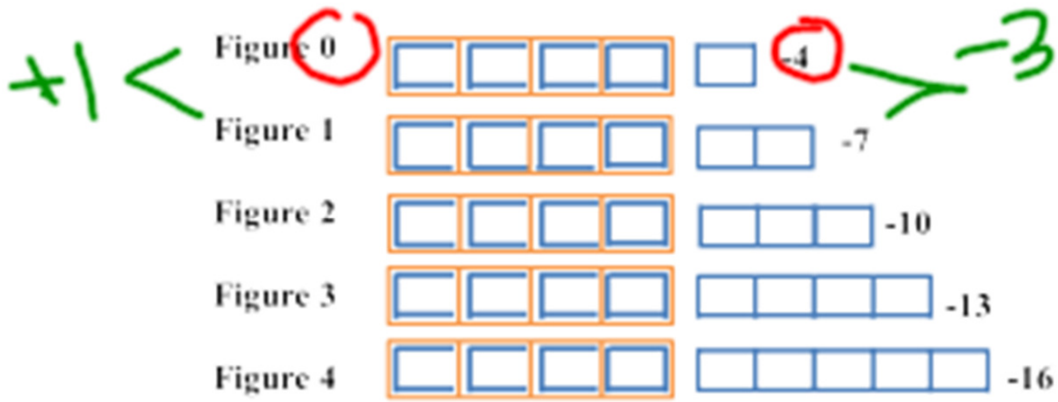


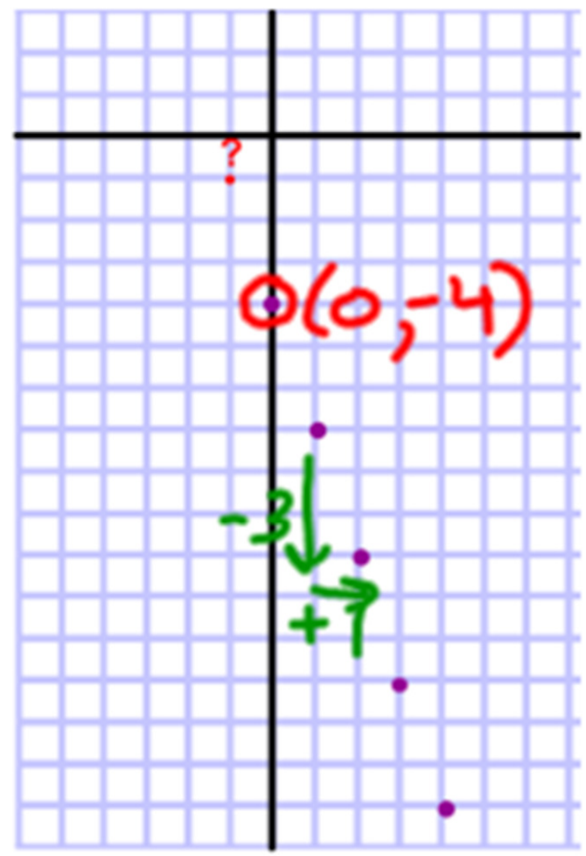
x or n (Figure #)	0	1	2	3	4
y (# of toothpicks)	-4	-7	-10	-13	-16

$y = -3x - 4$

What questions can we ask…

- to give all students access?
- to create productive classroom discourse?
- to engage students in the mathematical practice standards?
- to connect to prior knowledge?
- to lead into future knowledge?
- to understand what students know and don't know?
- to address student misconceptions?
- to address content standards?
- to see different solution paths?
- to inform future instruction?





+1

x or n (Figure #)	0	1	2	3	4
y (# of toothpicks)	-4	-7	-10	-13	-16

-3

$y = -3x - 4$

How do we address mathematical content and practice standards?

- What do the slope and the y-intercept represent in the context of this problem?
- What is the domain and range?
- How would you build “ Figure -1 ” ?
- How do you get from one figure to the next?
- How do you go backwards from one figure to the next?
- What is a recursive rule?
- Is the number of toothpicks an arithmetic sequence?
- Explain how you came up with your equations or rule.
- Explain how the explicit and recursive rules are related.

Pattern A.1

Figure 3

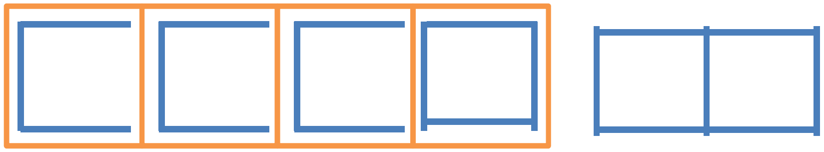


Figure 4

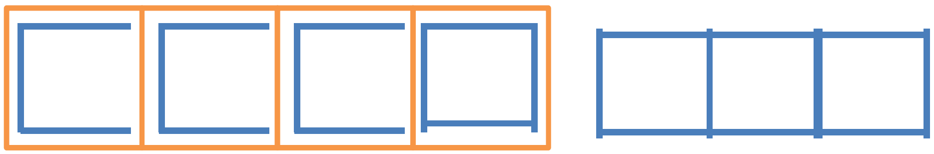
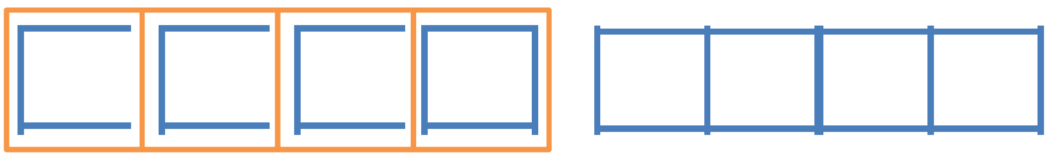


Figure 5



What questions would you ask students?

Pattern A.2

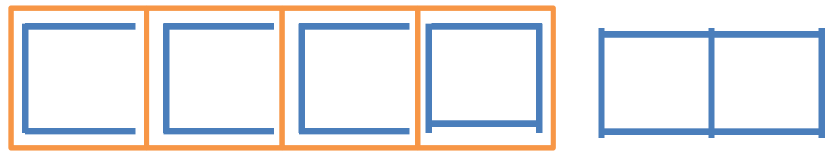
Figure 4 

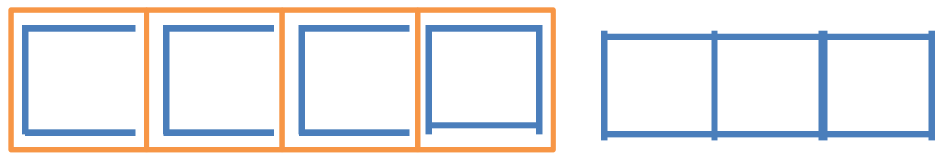
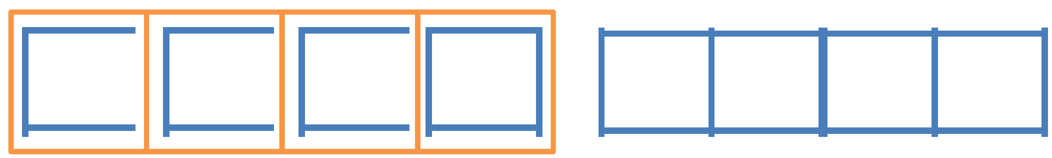
Figure 6 

Figure 8 

What questions would you ask students?

Pattern B

Figure 1

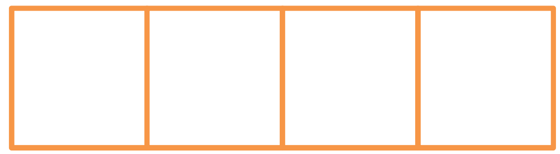


Figure 2

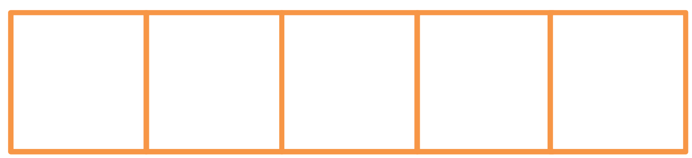
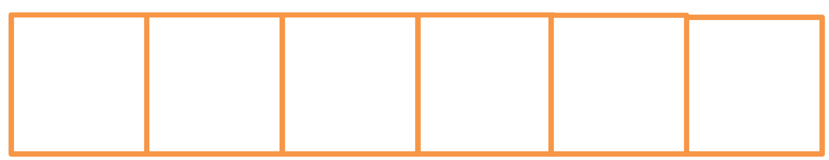


Figure 3



What will Figure -2 look like if you only have negative toothpicks to build with?

Pattern B

Figure 1

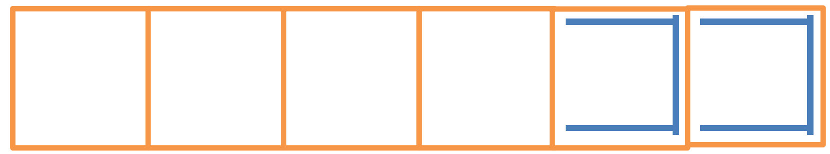


Figure 2

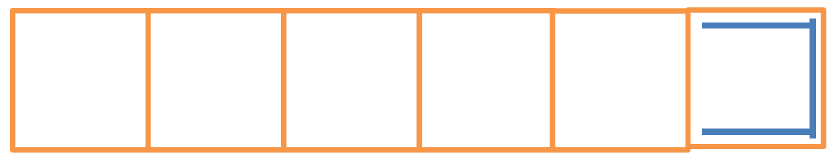
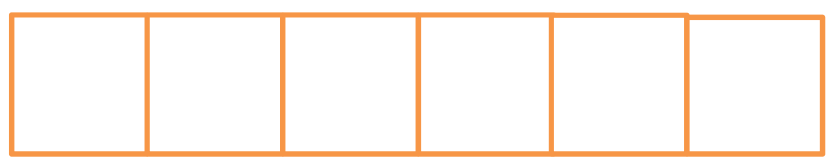
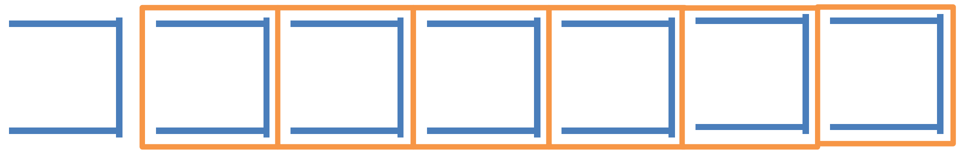


Figure 3



What will Figure -2 look like if you only have negative toothpicks?

What figure is this?



Pattern B

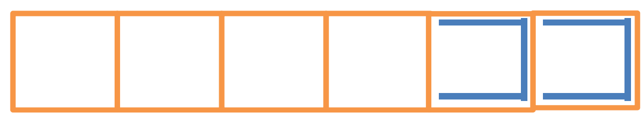
Figure 1 

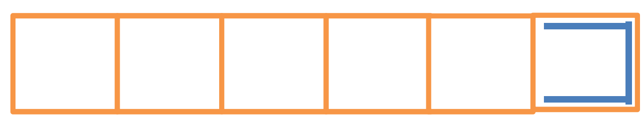
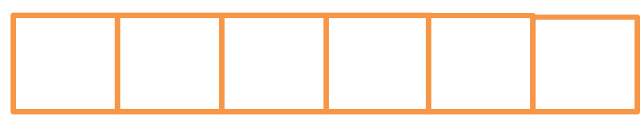
Figure 2 

Figure 3 

The Content

- Expressions
- Equations
- Systems of Equations
- Graphical Representation
- Construct and Compare Linear, Quadratic and Exponential Models
- Modeling
- Use Functions to Model Relationships
- Understanding Integers

The Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Pattern C

Figure 1

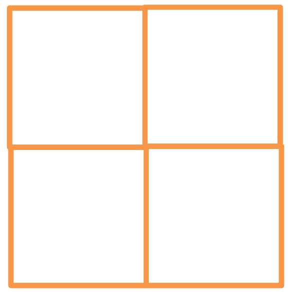


Figure 2

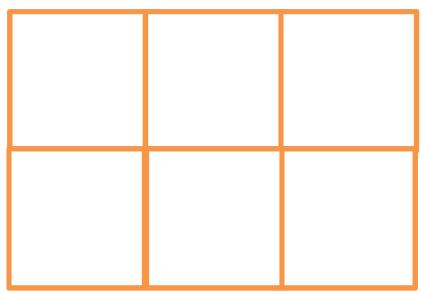
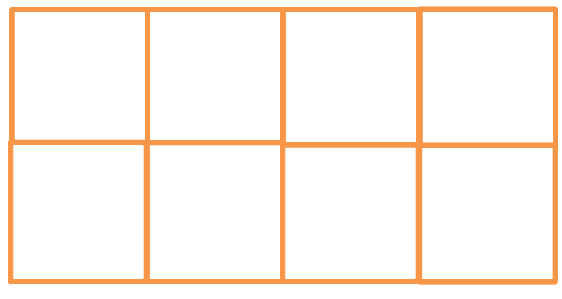


Figure 3



Pattern D

Figure 1

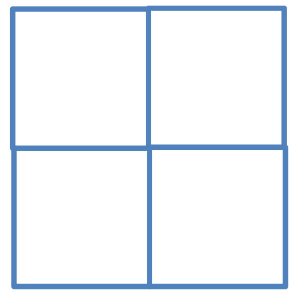


Figure 2

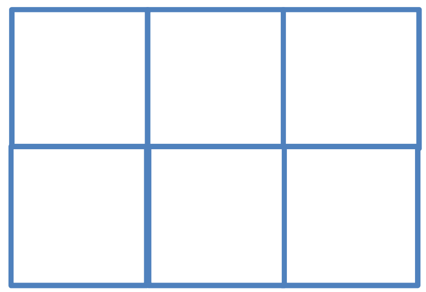


Figure 3

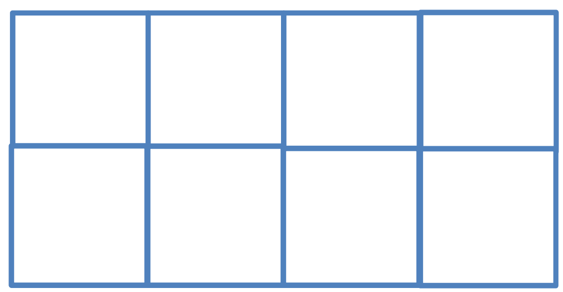


Fig. 1 Fig. 2 Fig. 3

Fig #	#tpicks

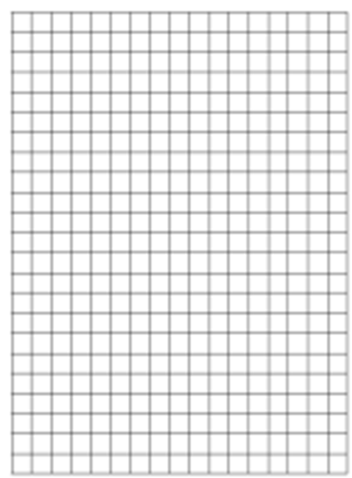
1. How many toothpicks in Figure 1? Figure 2? Figure 3? Find and record the number of toothpicks needed to build Figure 4?

2. How many toothpicks are there in Figure 6?

3. Record your data in a table. What is the slope? Find an equation to model the number of toothpicks in the n^{th} figure (or x^{th} figure).

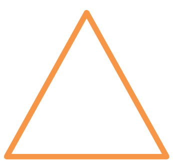
4. What does the slope represent in this problem? What does the y-intercept represent in this problem?

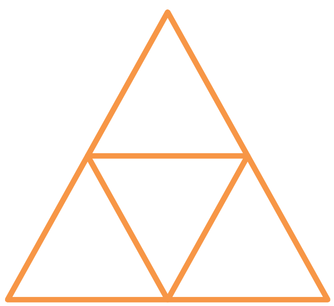
5. How is this problem the same as the other patterns? How is it different than the other patterns?
(use complete sentences...)



6. What would Figure -3 look like?

How Many Toothpicks?





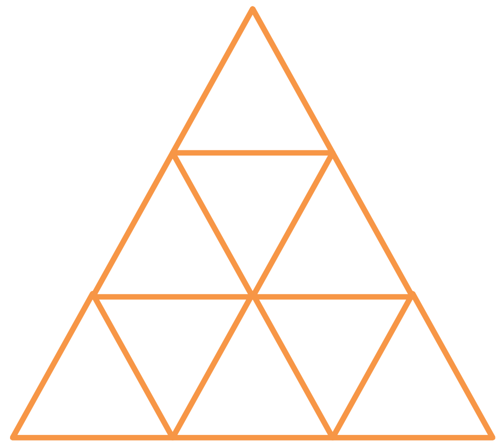


Fig. 1 **Fig. 2** **Fig. 3**

What questions can we ask?

A Difference Table

Figure # (x)	1	2	3	4	5
# Toothpicks (y)	3	9	18	30	45
Δy	6	9	12	15	
$\Delta^2 y$	3	3	3		

To get figure 5 you take figure 4 and add 5 triangles. Each triangle has 3 toothpicks. 5*3=15.

To get figure 5, you take the toothpicks you added to figure 3 to get figure 4 an add 1 more triangle. A triangle has 3 toothpicks.

To get figure “n” you add 3n toothpicks to figure “ n-1”

To get figure n you take what you added to “n-2” to get “n-1” and add 3 toothpicks.

A Difference Table

$1.5x^2 + 1.5x$

Figure # (x)	1	2	3	4	5
# Toothpicks (y)	3	9	18	30	45
Δy	6	9	12	15	
$\Delta^2 y$	3	3	3		

1.5

$3x + 1.5$

Linear Regression?

Systems of Equations?

The y-intercept = 0 so c = 0 in $ax^2 + bx + c$

A Difference Table

Figure # (x)	1	2	3	4	5	
# Toothpicks (y)	3	9	18	30	45	$f(x)=1.5x^2+1.5x$
Δy	6	9	12	15		$f'(x)=3x + 1.5$
$\Delta^2 y$	3	3	3			$f''(x) = 3$

Linear Regression?

Systems of Equations?

The y-intercept = 0 so c = 0 in $ax^2 + bx + c$

Going Backwards?

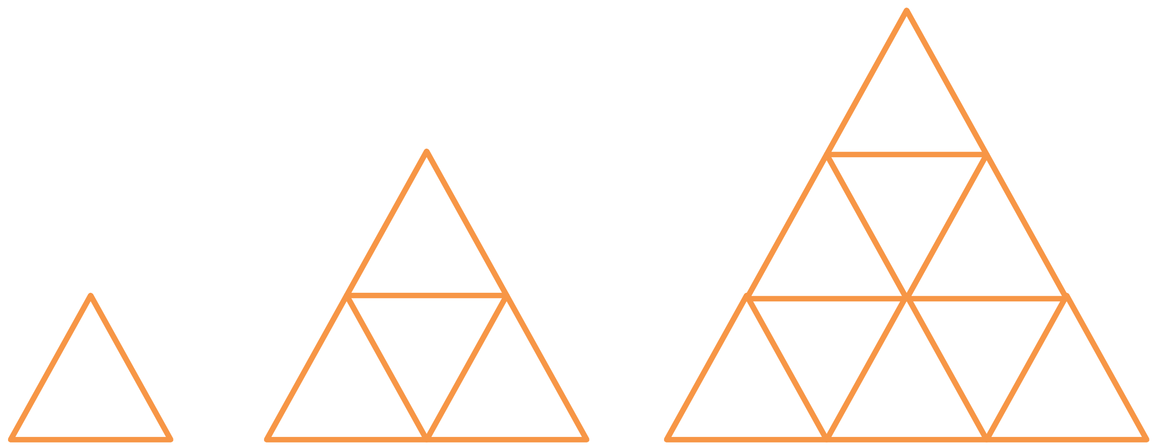
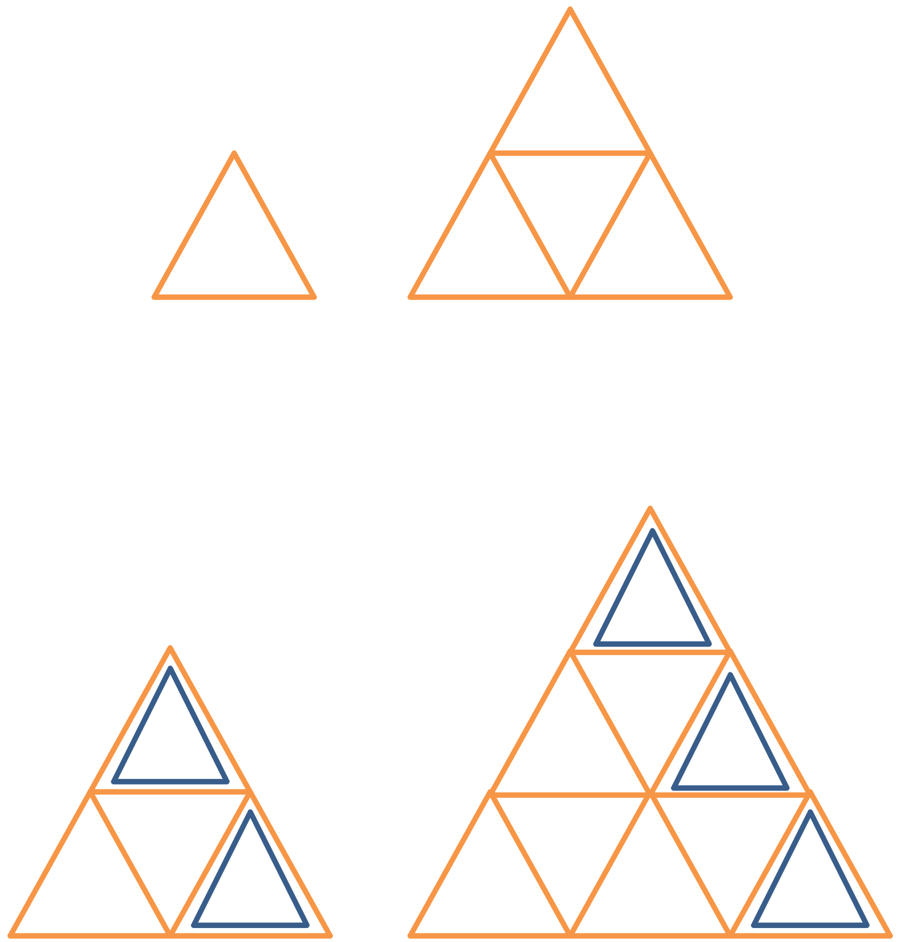


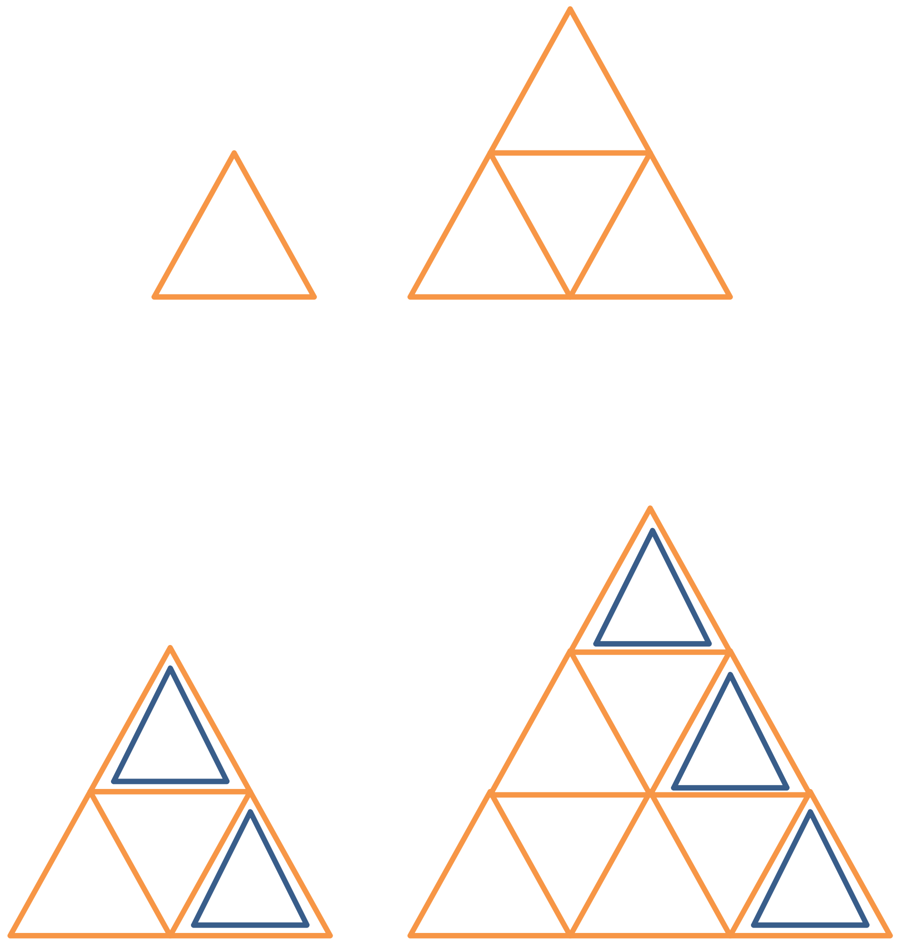
Fig. 1 Fig. 2 Fig. 3

How can you use the negative toothpicks to get from figure 3 to figure 2, from 2 to 1, from 1 to 0, from n to n-1, 0 to -1, from -1 to -2?

Compare the to two figures to the bottom two figures.



3 positive **9 positive**



9 positive
6 negative

18 positive
9 negative

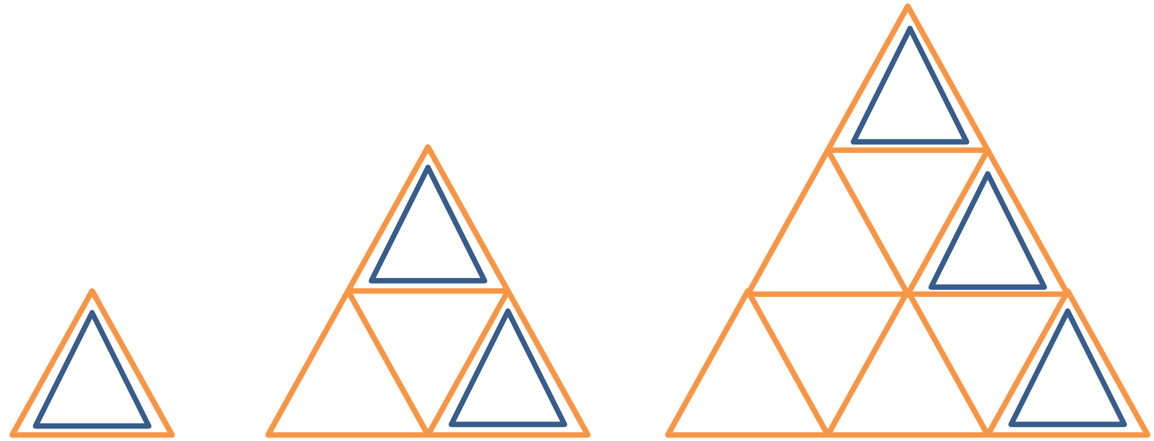


Fig.	0	1	2
#TPs	0	3	9

To get figure n, take figure n+1 and add 3(n+1) negative toothpicks:

$$A_n = A_{n+1} + (-3)(n+1)$$

$$A_{n+1} = A_n + 3(n+1)$$

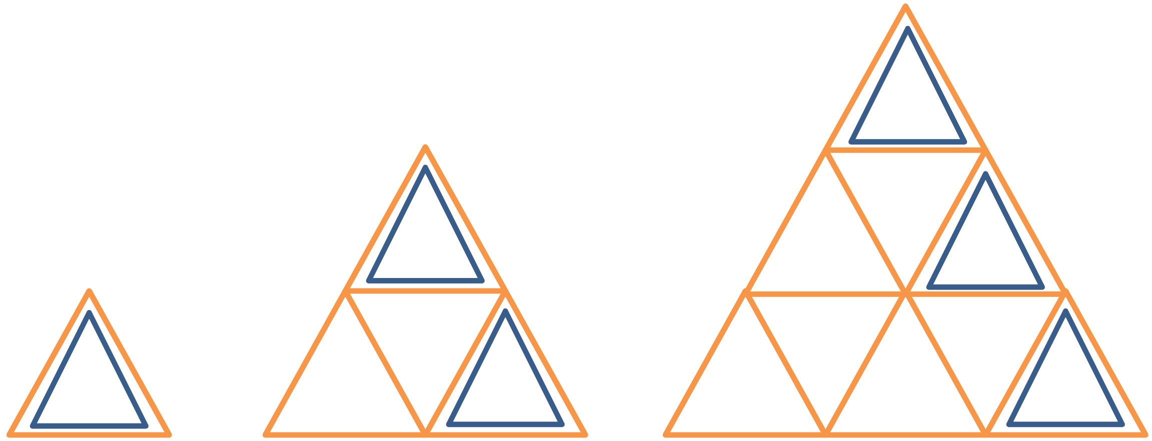


Fig.	0	1	2
#TPs	0	3	9

How many toothpicks will be in figure -1? Figure -2?
Convince yourself. Record your reasoning.

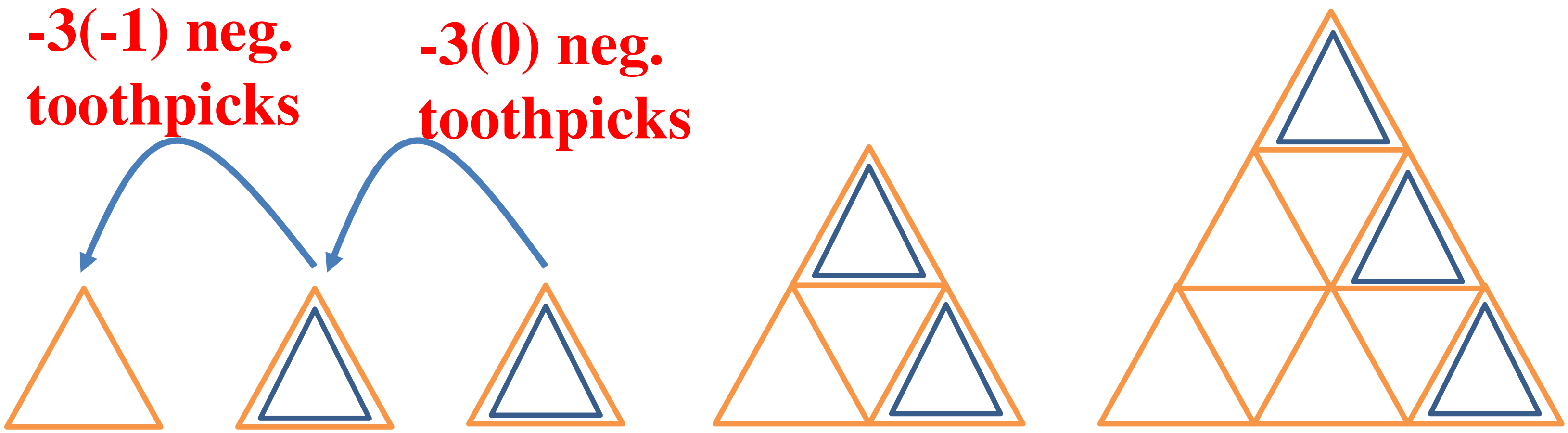


Fig.	-2	-1	0	1	2
#TPs			0	3	9

To get figure -1, I go to figure zero and 3(0) negative toothpicks. So zero.

To get to figure -2, I go to figure -1 and add 3(-1) negative toothpick. So -3 negative toothpicks. Remove 3 negative toothpicks. So positive 3 toothpicks are left.

Principles to Action

- Clear Mathematical Goals for Student Learning
- Coherent Activities and Problems Aligned With Mathematical Goals
- Assess and Advance Student Understanding
- Allow Productive Struggle
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- Use Mathematical Representations to Support Learning
- Use Evidence of Student Thinking to Modify and Improve Instruction

Exponential Functions

- $y = 2^x$

x	0	1	2	3	4	5
y	1	2	4	8	16	32
Δy	1	2	4	8	16	

$f(x) = 2^x$

The y values are a geometric sequence given by the recursive formula:

$A_x = 2 * A_{x-1}$

Motivation:

http://illuminations.nctm.org/Lesson.aspx?id=2630

- $y = 5^x$

x	0	1	2	3	4	5
y	1	5	25	125	625	3125

Δy 4 20 100 500 2500

$f(x) = 5^x$

The y values are a geometric sequence given by the recursive formula:

$A_x = 5 * A_{x-1}$ where $A_0 = 1$

The difference also form a geometric sequence

$A_x = 5 * A_{x-1}$ where $A_0 = 4$

The difference is the differences are 16, 80, 400…

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What about exponential decay?

$y = (0.5)^x$

x	0	1	2	3	4	5
y	1	1/2	1/4	1/8	1/16	1/32

Δy 1/2 1/4 1/8 1/16 1/32

$f(x) = (0.5)^x$

The y values are a geometric sequence given by the recursive formula:

$A_x = 0.5 * A_{x-1}$ where $A_0 = 1$

The differences follow the same recursion with $A_0 = 0.5$

Motivation: Halving Paper Of Different Dimensions

Generalizing…

5   5·5   5·5·5

5·5-5   5·5·5-5·5

$y = b^x$

x	0	1	2	3	4	5	…	n
y	1	b	b^2	b^3	b^4	b^5	…	
Δy		$b-1$	$b^2 - b$	b^3-b^2	b^4-b^3	b^5-b^4	…	$b^{n-1}(b-1)$

$b(b-1)$ $b^2(b-1)$ $b^4(b-4)$

$f(x) = b^x$

The y values are a geometric sequence given by the recursive formula:

$A_x = b * A_{x-1}$ where $A_0 = 1$

The differences use the same recursion.

Let's look at b^{n-1} and b^n

The difference is $b^n - b^{n-1}$ the previous difference is $b^{n-1} - b^{n-2}$

Multiplying this by b, we get $b^n - b^{n-1}$

The differences of an exponential function of the form $f(x) = b^x$ will be a geometric sequence that has the common ratio.

What about $g(x) = a*b^x$ or $h(x) = a*b^x + c$?

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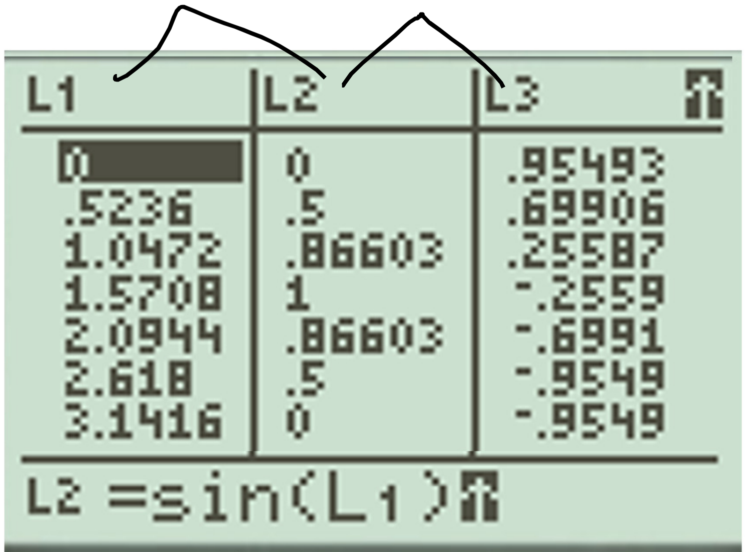
Trigonometric

$y = \sin(x)$ Let's use radians!!!

x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	$6\pi/6$
y	0	½	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	½	0

Δy ½ … that got ugly fast

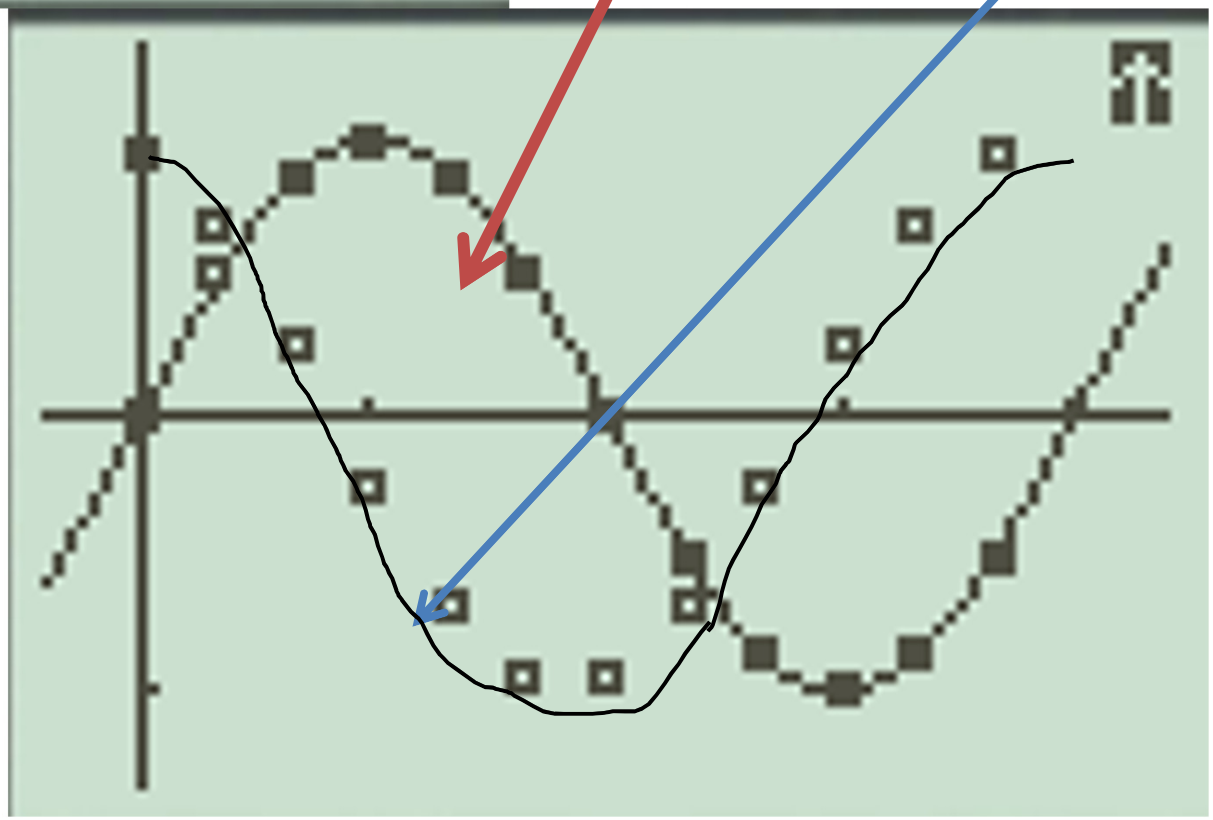


L1	L2	L3
0	0	.95493
.5236	.5	.69906
1.0472	.86603	.25587
1.5708	1	⁻.2559
2.0944	.86603	⁻.6991
2.618	.5	⁻.9549
3.1416	0	⁻.9549

L2 =sin(L1)

L3 = ΔList(L3)/(π/6)

L1 vs. L2 L2 vs. L3



The Content

- Exponential/Trig/
- Expressions
- Equations
- Systems of Equations
- Graphical Representation
- Construct and Compare Linear, Quadratic and Exponential Models
- Modeling
- Use Functions to Model Relationships

The Practices

1. Make sense of problems and persevere in solving them.
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References

- Toothpick activity adapted from the *Seeing Math Secondary* materials, developed by the Concord Consortium and distributed by PBSTeacherLine.
- CCSS Math Standards and Appendix A for Mathematics.

Questions?

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I WILL POST THIS AND THE HANDOUTS TO THE NCTM WEBSITE LATER TODAY!