Rethinking Probability in the Common Core and AP Statistics

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What would you do if…

Your doctor told you

“Well, I always just tell my patients to take an aspirin and get some sleep. It’s not very effective, but SOME people get better. I know that there have been some recent research studies looking at the effectiveness of this treatment, but frankly I just haven’t had the time to read them carefully or to think about how they might apply to my patients. So, why don’t you go home, take an aspirin and get some sleep.”

From *Developing Students’ Statistical Reasoning*, 2008. Garfield and Ben-Zvi, Foreword by Roxy Peck.

Would you

- Say thanks and head home to take an aspirin and a nap?
- Look for a new doctor?

Granted, scenario here is contrived to make a point, but it is an important point!

My wake up call…

- We expect professionals in other fields to stay current and to understand implications of research on practice.

- As a teaching professional, was I doing this???

- In mathematics and statistics education, much of what we have learned from research has not had an impact on classroom instruction. ☹
- We often ignore research in favor of anecdotal evidence. ☹ ☹

But when it come to probability…

My contention is

- We often ignore research

AND

- We often even ignore anecdotal evidence

IN FAVOR Of

- TRADITION!

The consequence

- Inconsistent understanding and application.

So why is probability so hard??

- My dog doesn't understand probability…
- But then neither does this guy…

Maybe we should rethink tradition

Humor me by doing a quick problem.

	Test Positive	Test Negative	Total
Has Virus	4	1	5
Does Not Have Virus	50	945	995
Total	54	946	1000

Suppose you are one of these 1000 people and you test positive. Should you be really worried?

If someone tests positive, what is the probability that they actually have the virus? That is, what proportion of people who test positive have the virus?

Easy, right?

Why is this hard?

Here is what this usually looks like…

The probability that a person has a certain virus is 0.005. A test used to detect the virus in a person is positive 80% of the time if the person has the virus and 5% of the time if the person does not have the virus. Let A be the event that "the person is infected" and B be the event "the person tests positive." If a person tests positive, what is the probability that the person has the virus.

And here is how we expect students to solve this problem…

Let A be the event that "the person is infected" and B be the event "the person tests positive."

P(A) = 1/200 = 0.005 $P(not\ A)$ = 0.995

P$(B|A)$ = 0.80 $P(B|\ not\ A)$ = 0.05

Then use

$$P(A|B)=\frac{P(A)\cdot P(B|A)}{P(A)\cdot P(B|A)+P(not\ A)\cdot P(B|not\ A)}$$

This is hard for students! But this is the same problem that people solve intuitively if they have a data table. We need to connect the two!

Let’s teach them to do this…

	Test Positive	**Test Negative**	**Total**
Has Virus	4	1	5
DoesNot Have Virus	50	945	995
Total	54	946	1000

Same problem, same answer, but easy!

<u>NOT</u> “dumbing” down—students can still do all of the same types of problems. Just <u>takes</u> <u>advantage of how people naturally reason.</u>

The disconnect

- The research—students are much better able to reason in terms of natural frequencies than in terms of probabilities or proportions.
- The anecdotal evidence—students can solve complex probability problems when given a data table.
- The disconnect—we continue to teach classical probability and probability rules and formulas rather than teaching how to translate probability information into long-run or large group information.

For a great short discussion, see

"Chances Are" article in the New York Times online by Steve Strogatz

http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/

From this article…

“The trick is to think in terms of ‘natural frequencies’—in simple counts of events rather than the more abstract notions of percentages, odds, or probabilities. As soon as you make this mental shift, the fog lifts.”

Interesting study…

From “Calculated Risks” by Gert Gigerenzer.

Told doctors in Germany and the U.S. the following: a woman in a low risk group (age 40 to 50 with no family history of breast cancer) has a positive mammogram. What is the probability that she actually has cancer?

To make the question specific, also given the following information:

- The probability that a woman in this low risk group has breast cancer is 0.8 percent.
- If a woman has breast cancer, the probability that she will have a positive mammogram is 90 percent.
- If a woman does not have breast cancer, the probability that she will have a positive mammogram is 7 percent.

What would the doctors tell the woman with a positive mammogram?

- 25 German doctors estimated the probability that the woman has breast cancer given that she had a positive mammogram. Estimates ranged from...
- 1 percent to 90 percent!!
- 8 of 25 doctors said 90%.
- 9 of 25 doctors said between 50 and 80 percent.
- 8 of 25 said 10 percent or less.
- American doctors did worse! About 95% said somewhere around 75%

So what is the right answer???

- The probability that a woman in this low risk group has breast cancer is 0.8 percent.
- If a woman has breast cancer, the probability that she will have a positive mammogram is 90 percent.
- If a woman does not have breast cancer, the probability that she will have a positive mammogram is 7 percent.

Asked another way…

- 8 out of 1000 women in this group have breast cancer.
- Of these 8 women with breast cancer, 90%, which is about 7, will have a positive mammogram.
- Of the 992 women who do not have breast cancer, 7%, which is about 70, will have a positive mammogram.
- Imagine the group of women who would have a positive mammogram. How many of these women actually have breast cancer?

The Probability Literacy Deficit

- Stubborn misconceptions
- Poor understanding of chance behavior
- Lack of understanding of the ideas of conditional probability

All of these contribute to a lack of literacy and make it difficult to develop inferential reasoning.

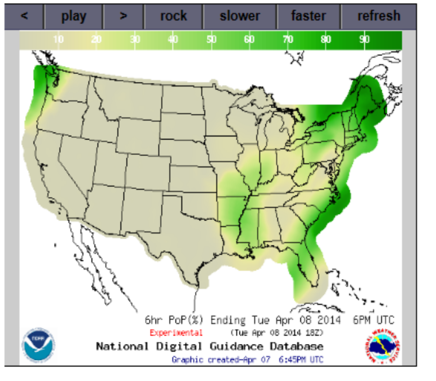
Deficit is linked to how we teach probability

Heavy early emphasis on “classical” probability based on equally likely outcomes and games of chance does not prepare students to understand chance behavior or to understand and interpret probability statements often encountered in the media.

From this week’s headlines…

- “Underwater Search in Area of Highest Probability” (4/4/14)
- “Low probability, high impact: Tsunamis join list of Gulf Coast threats” (4/4/14)
- Probability of precipitation

Probability of Precip



This image is a computer generated probability forecast for precipitation during each 6-hr period given. The imagery is based on the GFS MOS product.

From this week's headlines…

- "Scientists Predict The Probability of Photos Going Viral on Facebook" (4/7/14)
- Panel Urges Low-Dose Aspirin to Reduce Pre-eclampsia Risk" (4/7/14)

From this week's headlines…

Could Dads' Obesity Raise Autism Risk for Kids?

Study finds slight increase linked to weight of fathers, not mothers

MONDAY, April 7, 2014 (HealthDay News) -- Children born to obese fathers, but not obese mothers, may have a slightly higher risk of autism than kids with thinner dads, a large new study suggests.

Researchers found that of nearly 93,000 Norwegian children they followed, those born to obese dads had double the risk of developing autism. But the odds were still small: just under 0.3 percent were diagnosed with autism, versus 0.14 percent of kids with normal-weight fathers.

The findings, published online April 7 in *Pediatrics*, are the first to link fathers' obesity to autism risk. And experts stressed that it's not clear whether dads' extra pounds, per se, cause the increase.

From this week's headlines…

Stress Hormone May Drive Risk-Taking by Teen Motorists

MONDAY, April 7, 2014 (HealthDay News) -- Teens whose brain chemistry is less affected by stressful situations could be at increased risk for car crashes, a small Canadian study suggests.

Safe-driving teens appear to have higher levels of the stress hormone cortisol, said study author Marie Claude Ouimet, an associate professor of medicine and health sciences at the University of Sherbrooke, in Quebec.

At the start of the study, the researchers measured each teen's stress response by asking them to solve a series of math problems, telling them that $60 would be awarded to the person with the highest score.

Doctors took saliva samples from teens to measure their cortisol levels before and after the math problems, and used those samples to estimate each kid's response to stress.

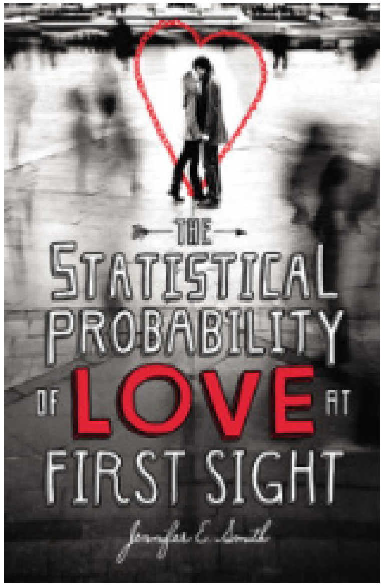
The teens were then let loose on the roads, in cars that measured their driving behavior using a series of sensors, cameras and GPS.

Researchers found that the teens with a higher cortisol response to stress were less likely to crash or experience a near crash, according to the results published online April 7 in the journal *JAMA Pediatrics*.

From this week's headlines…

Jennifer E. Smith – The Statistical Probability of Love at First Sight

BY MIENEKE | PUBLISHED 2 APRIL, 2014



Who would have guessed that four minutes could change everything?

Imagine if she hadn't forgotten the book. Or if there hadn't been traffic on the expressway. Or if she hadn't fumbled the coins for the toll. What if she'd run just that little bit faster and caught the flight she was supposed to be on. Would it have been something else – the weather over the Atlantic or a fault with the plane?

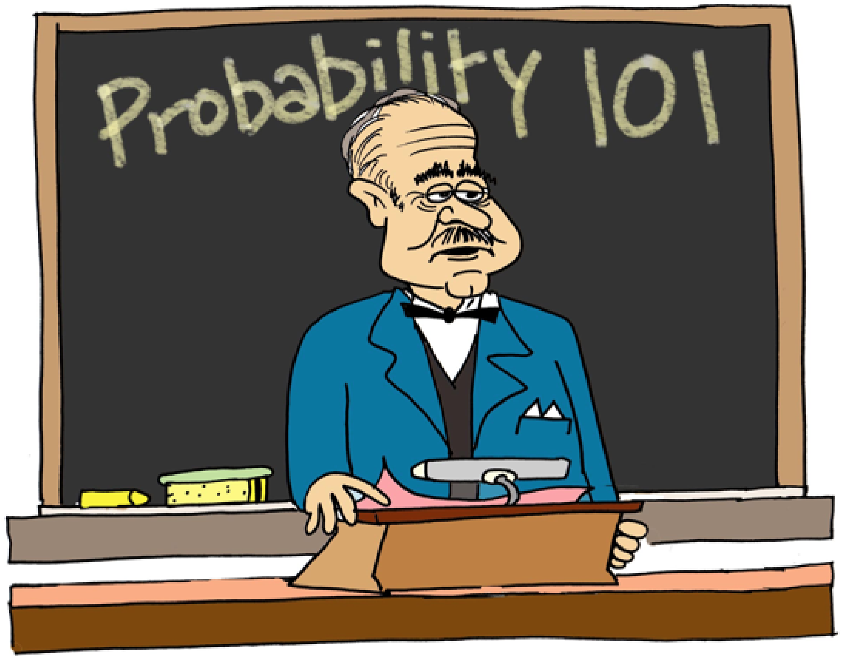
Hadley isn't sure if she believes in destiny or fate but, on what is potentially the worst day of each of their lives, it's the quirks of timing and chance events that mean Hadley meets Oliver…

Set over a 24-hour-period, Hadley and Oliver's story will make you believe that true love finds you when you're least expecting it.

Probability Literacy

None of these headlines lend themselves to traditional classical probability thinking or interpretations.

Did not see one example that involved drawing balls from urns or selecting socks at random from a drawer that contains 6 blue socks and 5 black socks!



We teach probability the old fashioned way. We urn it.

What to do???

- Rethink and refocus
  - Emphasize natural frequencies
  - Interpretation of probabilities as “long-run” or “large group”.
  - Teach how to translate probability information by thinking long-run or large group.
    - Long-run: What would happen if I did this 1000 times?
    - Large group: What if I had a group of 1000

EARLY AND OFTEN!

Implications for the Classroom and Classroom Activities

- Develop an understanding of chance behavior.
- Interpret probabilities as long-run or large group.
- Translate probability information into long-run or large group information.
- Challenge “predictions” using simulation.
- Develop conditional probability thinking.

Develop Understanding of Chance Behavior

- Most people don't have a good sense of what random "looks like".
- Tendency to over-interpret short term patterns and clusters.
- Think random implies uniform or evenly spread.

Coin Toss Activity

- Half the class makes up a sequence of 100 H's and T's that they think would be consistent with what might happen if you tossed a coin 100 times.
- Half the class actually tosses a coin 100 times.
- Look at "streaks". Distribution of longest run of H's looks different for real coin tossing than for sequences that students make up.

Counterintuitive

Bart Holland, *What Are the Chances* (2002)

People find it hard to understand that randomness comes in “bunches”.

Things that occur at random will create some areas where there are no occurrences and some clusters.

Can’t focus on the bunches without asking if what you are seeing is consistent with chance behavior.

Interpreting Probabilities

- Move quickly from classical (3 chances out of 6) to interpretations that start with
  - If I did this 1000 times, I would expect to see…
  - If I had a group of 1000 people, I would expect to see…

This leads naturally to…

Teach Translating Probability Information

Teach students how to translate probability information into long-run or large group information, which then let's students reason naturally to answer questions.

Revisiting Gigerenzer Example

Probability Statement	Translation
The probability that a woman in this low risk group has breast cancer is 0.8 percent.	In a group of 1000 low-risk women, about 8 will have breast cancer and about 992 will not have breast cancer.
If a woman has breast cancer, the probability that she will have a positive mammogram is 90 percent.	In a group of 1000 low risk women, about 8 will have breast cancer and 7 of them will have a positive mammogram.
If a woman does not have breast cancer, the probability that she will have a positive mammogram is 7 percent.	In a group of 1000 low risk women, about 992 will not have breast cancer and 70 of them will have a positive mammogram.

Challenge Misconceptions, but…

- It is not enough to demonstrate that misconception is wrong.

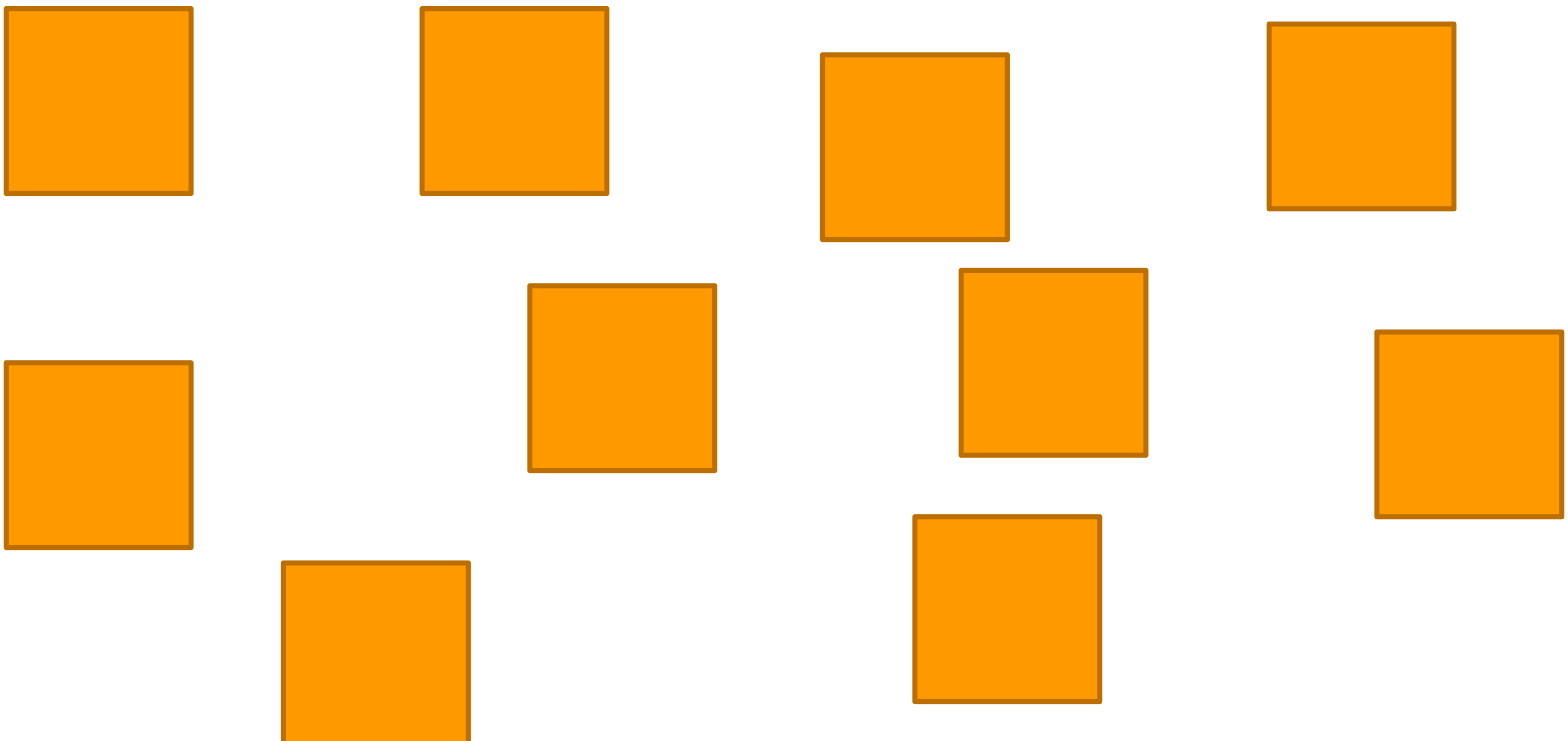
- Students must do the testing (usually via simulation or observation), then reconcile with prior belief and explain why prior belief was incorrect.

Examples

- Middle school
  - 6's are harder to roll
- High school
  - Conjunction fallacy
    - P(elementary school teacher) = .3
    - P(elementary teacher and female) greater than .3

Continued

Can you construct a population with P(T) = .3 and P(T and F) is greater than .3?



Continued

	Female	Male	Total
Teacher			300
Not Teacher			700
Total			1000

Is there any way to fill in this table that would result in P(F and T) > .3?

Conditional Probability Thinking

- Are the following two probabilities the same? If not, which do you think is greater?

P(over 6 feet tall given professional basketball player)

P(professional basketball player given over 6 feet tall)

First probability is interpreted as proportion of professional basketball players that are over 6 ft. tall.

Second probability is interpreted as proportion of people who are over 6 ft. tall who are professional basketball players.

Conditional Probability Thinking

P(positive test given disease)

P(disease given positive test)

Can be very different!

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Thanks!

If you have questions or would like a copy of this presentation, email

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