

1 **Exploring Proportional Relationships:**  
2 ***The Case of Mr. Donnelly***<sup>1</sup>  
3

4 Mr. Donnelly wanted his students to understand that quantities that are in a proportional  
5 (multiplicative) relationship grow at a constant rate and that there were three key strategies that  
6 could be used to solve problems of this type – scaling up, scale factor, and unit rate. He selected  
7 the Candy Jar task for the lesson since it was aligned with his goals, was cognitively challenging,  
8 and had multiple entry points.

9 

A candy jar contains 5 Jolly Ranchers (JRs) and 13 Jawbreakers (JBs). Suppose you had a new 10 candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly 11 Ranchers. How many Jawbreakers would you have? Explain how you know.
---

  
12

13 As students began working with their partners on the task, Mr. Donnelly walked around the room  
14 stopping at different groups to listen in on their conversations and to ask questions as needed  
15 (e.g., How did you get that? How do you know that the new ratio is equivalent to the initial ratio?).  
16 When students struggled to figure out what to do he encouraged them to look at the work they  
17 had done the previous day that included producing a table of ratios equivalent to 5 JRs: 13 JBs  
18 and a unit rate of 1 JR to 2.6 JBs. He also encouraged students to consider how much bigger the  
19 new candy jar must be when compared to the original jar.  
20

21 As he made his way around the room Mr. Donnelly also made note of the strategies students  
22 were using (see reverse side) so he could decide which groups he wanted to have present their  
23 work. After visiting each group, he decided that he would ask Groups 4, 5, and 2 to share their  
24 approaches (in this order) since each of these groups used one of the strategies he was targeting  
25 and the sequencing reflected the sophistication and frequency of strategies.  
26

27 During the discussion he asked the presenters (one student from each of the targeted groups) to  
28 explain what their group did and why and he invited other students to consider whether the  
29 approach made sense and to ask questions. He made a point of labeling each of the three  
30 strategies, asking students which strategy was most efficient in solving this particular task, and  
31 asking students questions that helped them make connections between the different strategies  
32 and to the key ideas he was targeting. Specifically he wanted students to see that that the scale  
33 factor identified by Group 5 was the same as the number of entries in the table created by Group  
34 4 (or the number of small candy jars that it would take to make the new candy jar) and that the  
35 unit rate identified by Group 2 was the factor that connected the JRs and JBs in each row of the  
36 table.  
37

38 Below is an excerpt from the discussion that took place around the unit rate solution that was  
39 presented by Jerry from Group 2.  
40

41 Jerry: We figured that there was 1 JR for 2.6 JBs so that a jar with 100 JRs would have 260  
42 JBs. So we got the same thing as the other groups.  
43 Mr. D.: Can you tell us how you figured out that there was 1 JR for 2.6 JBs?  
44 Jerry: We divided 13 by 5.  
45 Mr. D.: Does anyone have any questions for Jerry? Danielle?  
46 Danielle: How did you know to do  $13 \div 5$ ?  
47 Jerry: See we wanted to find out the number of JBs for 1 JR. So if we wanted JRs to be 1,  
48 we needed to divide it by 5. So now we needed to do the same thing to the JBs.  
49 Danielle: So how did you then get 260 JBs?  
50 Jerry: Well once we had 1 JR to 2.6 JBs it was easy to see that we needed to multiply each  
51 type of candy by 100 so we could get 100 JRs.

---

<sup>1</sup> This case is adapted from two sources: Smith, M. S., Silver, E. A., Stein, M. K., Boston, M., & Henningsen, M. A. (2005). *Improving instruction in rational numbers and proportionality: Using cases to transform mathematics teaching and learning, Volume 1*. New York: Teachers College Press; and Leinwand, S., Brahier, D., Huinker, D., Berry, R., Dillon, F., Larson, M., Leiva, M., Martin, G., & Smith, M.S. (2014). *Principles to Actions: Making Mathematics Work for All*. Reston, VA: National Council of Teachers of Mathematics.

52 Mr. D.: So Jerry's group multiplied by 100 but Danielle and her group (Group 5) multiplied by  
 53 20. Can they both be right? Amanda?  
 54 Amanda: Yes. Jerry's group multiplied 1 and 2.6 by 100 and Danielle and her group multiplied  
 55 5 and 13 by 20. Jerry's group multiplied by a number 5 times bigger than Danielle's  
 56 group because their ratio was 1/5 the size of the ratio Danielle's group used. So it is  
 57 the same thing.  
 58 Mr. D.: Do others agree with what Danielle is saying? (*Students are nodding their heads and*  
 59 *giving Danielle a thumbs up.*) So what is important here is that both groups kept the  
 60 ratio constant by multiplying both the JRs and JB's by the same amount. We call what  
 61 Jerry and his group found the **unit rate**. A unit rate describes how many units of one  
 62 quantity (in this case JB's) correspond to one unit of another quantity (in this case  
 63 JRs). (*Mr. Donnelly writes this definition on the board.*)  
 64 Mr. D.: I am wondering if we can relate the unit rate to the table that Group 4 shared. Take 2  
 65 minutes and talk to your partner about this. (*2 minutes pass*)  
 66 Mr. D.: Kamiko and Jerilyn (from Group 4), can you tell us what you were talking about?  
 67 Kamiko: We noticed that if we looked at any row in our table that the number of JB's in the row  
 68 was always 2.6 times the number of JRs in the same row.  
 69 Mike: Yeah we saw that too. So it looks like any number of JRs times 2.6 will give you the  
 70 number of JB's.  
 71 Mr. D.: So what if we were looking for the number of JB's in a jar that had 1000 JRs?  
 72 Mike: Well the new jar would be 1000 times bigger so you multiply by 1000.  
 73 Mr. D.: So take 2 minutes and see if you and your partner can write a rule that we could use  
 74 to find the number of JB's in a candy jar no matter how many JRs are in it.  
 75 (*After 2 minutes the discussion continues.*)  
 76  
 77 Towards the end of the lesson Mr. Donnelly placed the solution produced by Group 1 on the  
 78 document camera and asked students to decide whether or not this was a viable approach to  
 79 solving the task and to justify their answer. He told them they would have five minutes to write a  
 80 response that he would collect as they exited the room. He thought that this would give him some  
 81 insight as to whether or not individual students were coming to understand that ratios needed to  
 82 grow at constant rate that was multiplicative not additive.

Group 1 (incorrect additive)	Groups 3 and 5 (scale factor)	Groups 4 and 7 (scaling up)																	
<p><i>100 JRs is 95 more than the 5 we started with. So we will need 95 more JB's than the 13 I started with.</i></p> <p>5 JRs + 95 JRs = 100 JRs            13 JB's + 95 JB's = 108 JB's</p>	<p><i>You had to multiply the five JRs by 20 to get 100, so you'd also have to multiply the 13 JB's by 20 to get 260.</i></p> <p style="text-align: center;">(x20)</p> <p style="text-align: center;">5 JRs    <math>\longrightarrow</math>    100 JRs            13 JB's   <math>\longrightarrow</math>    260 JB's</p> <p style="text-align: center;">(x20)</p>	JR	JB	JR	JB														
		5	13	55	143														
		10	26	60	156														
		15	39	65	169														
		20	52	70	182														
		25	65	75	195														
		30	78	80	208														
		35	91	85	221														
		40	104	90	234														
		45	117	95	247														
		50	130	100	260														
Group 2 (unit rate)	Group 6 (scaling up)																		
<p><i>Since the ratio is 5 JRs for 13 JB's, we divided 13 by 5 and got 2.6. So that would mean that for every 1 JR there are 2.6 JB's. So then you just multiply 2.6 by 100.</i></p> <p style="text-align: center;">(x100)</p> <p style="text-align: center;">1 JR    <math>\longrightarrow</math>    100 JRs            2.6 JB's <math>\longrightarrow</math>    260 JB's</p> <p style="text-align: center;">(x100)</p>	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>JRs</td> <td>5</td> <td>10</td> <td>20</td> <td>40</td> <td>80</td> <td>100</td> </tr> <tr> <td>JBs</td> <td>13</td> <td>26</td> <td>52</td> <td>104</td> <td>208</td> <td>260</td> </tr> </table> <p><i>We started by doubling both the number of JRs and JB's. But then when we got to 80 JRs we didn't want to double it anymore because we wanted to end up at 100 JRs and doubling 80 would give me too many. So we noticed that if we added 20 JRs: 52 JB's and 80 JRs: 208 JB's we would get 100 JRs: 260 JB's.</i></p>					JRs	5	10	20	40	80	100	JBs	13	26	52	104	208	260
JRs	5	10	20	40	80	100													
JBs	13	26	52	104	208	260													
Group 8 (scaling up)																			
<p><i>We drew 100 JRs in groups of 5. Then we put 13 JB's with each group of 5 JRs. We then counted the number of JB's and found we had used 260 of them.</i></p>																			