Two by Two

By now students have worked extensively with multiplication, and they have extended their knowledge of place value. With this background, they are ready to multiply two-digit numbers. This involves a more complex application of the distributive property than they have encountered so far. Students apply these skills in later work with the standard algorithm for multiplication.

Objective

Multiply 2 two-digit numbers.

Vocabulary

Draw a rectangle on the board, a little wider than tall. Label the height 3 and the width 4.

- **Ask:** *What is the area of the rectangle?* [12 square units] *How did you get that?* [Multiply height by width.]

- **Say:** *Any product of two factors can be modeled as a rectangular area. The model is called an area model.*

  Discuss this idea with students.

- The **area model** for multiplication is a model in which the side lengths of a rectangle represent the factors, and the area of the rectangle represents the product.

Arrays and area models can be used to illustrate why multiplication strategies work.

Set the Stage

Engage   WHOLE CLASS

Present the problem—

Mr. Amos bought 5 packages of fruit. Each package contains 4 apples and 2 bananas. How many apples does Mr. Amos have? How many bananas? How many pieces of fruit?

- **Ask:** *How do you find the number of apples?* [5 × 4] *The number of bananas?* [5 × 2] *The number of pieces of fruit?* [5 × (4 + 2) or (5 × 4) + (5 × 2)]

Have students consider the expression 5 × (4 + 2).

- **Ask:** *If I say the multiplication is distributed over the addition, what does that mean?* [The "5-multiplied-by" operates on both 4 and 2.]

- **Ask:** *What property of operations is this?* [distributive property]

- **Say:** *If Mr. Amos buys another package of fruit and now has (5 + 1) packages, the expression becomes (5 + 1) × (4 + 2). Tell me how the distributive property applies now.* [(5 + 1) is distributed over (4 + 2)]
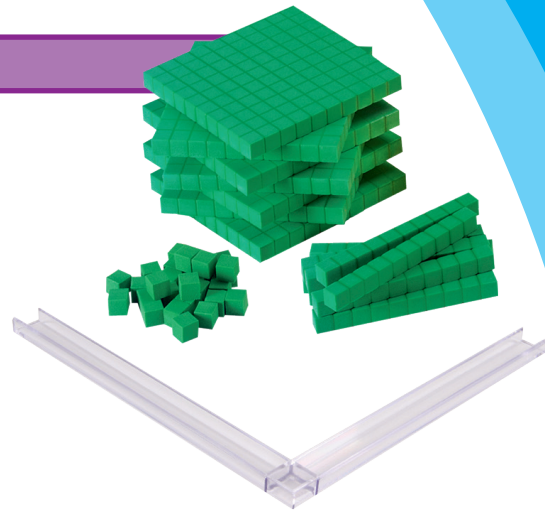
Warm-Up

Use this short thinking exercise to jump-start the instructional session.

Name __*Answer Key*__

2

3 ×	4 =	12
3 ×	44 =	132
3 ×	444 =	_____
3 × 4,444 =	_____	

Keep going, and then write three observations.

ANSWER: 1,332; 13,332. The products all begin with 1. The products all end with 2. The number 3 repeats in the middle 1 less time than the number of 4s.

COMMENTS & EXTENSIONS: Instead of multiplying by 3, try 4. What do you find? Now try 5 and 6.

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Foundation Skill Practice

Use this VersaTiles® activity to help students activate their prior knowledge.

10 by 10

Example

Find the product.

5 × 2 = 10, so 50 × 20 = ■

Think: **5 ×   2 =   10**
      50 × 2 =   100
So,   **50 × 20 = 1,000.**

Find the product.

1 3 × 4 = 12, so 30 × 40 = ■   **2** 3 × 7 = 21, so 30 × 70 = ■

3 5 × 6 = 30, so 50 × 60 = ■   **4** 7 × 4 = 28, so 70 × 40 = ■

5 2 × 9 = 18, so 20 × 90 = ■   **6** 5 × 8 = 40, so 50 × 80 = ■

7 80 × 40 = ■   **8** 60 × 40 = ■

9 90 × 30 = ■   **10** 40 × 50 = ■

11 A movie theater has 30 seats in each row. How many seats are there in 20 rows?

12 There are 40 rolls of theater tickets. There are 20 tickets in each roll. How many tickets are there?

Answer Box

| A	B	C	D	E	F
3,000	600	2,800	1,200	4,000	2,400
G	**H**	**I**	**J**	**K**	**L**
1,800	2,700	2,000	3,200	800	2,100

32   **Objective:** Find a product of tens multiplied by tens.

VersaTiles® student book, page 32

Explore   WHOLE CLASS

Distribute Base Ten Blocks and Factor Tracks. Present the problem $12 \times 14 =$ _____

Have students work on a model for 12×14 using the Factor Track. Elicit that the goal is to model the problem as a rectangle with sides 12 and 14. Students build the side lengths in the tracks and they build the product rectangle by filling in the area defined by the sides.
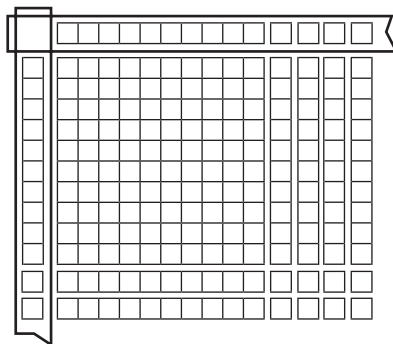
product rectangle built with 1 flat, 6 rods, 8 units
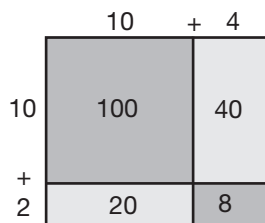
sketch of area model

Have students identify the 4 partial-product rectangles, sketch the area model, and then write the equation suggested by the model. Ask students to explain how these different models represent the same number.

$$12 \times 14 = (10 + 2) \times (10 + 4)$$
$$= (10 \times 10) + (2 \times 10) + (10 \times 4) + (2 \times 4)$$
$$= 100 + 20 + 40 + 8 = 168$$

Explore & Explain   SMALL GROUPS

Prepare ahead Students will need Base Ten Blocks and Factor Tracks.

The activity helps students visualize the multiplication of 2 two-digit numbers. It helps them see the role of place value and illustrates the inner workings of the distributive property. Students build and sketch manipulative models, sketch area models, and write equations to find products.
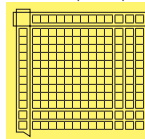
Materials

- Base Ten Blocks
- Factor Track™

Explore & Explain

Lesson
2 Two by Two   Name ___*Answer Key*___

Try This
■ Use Base Ten Blocks and a Factor Track to model each problem.
■ In problems 1–2, sketch your model.
■ In problems 3–4, sketch an area model.
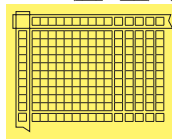■ In each problem, complete the number sentences.

1. $12 \times 13 = (10 + 2) \times (10 + 3)$

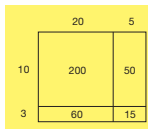Students may use shorthand notation for flats, rods, and units

$= (10 \times 10) + (\boxed{2} \times 10)$
$+ (10 \times 3) + (\boxed{2} \times 3)$
$= \boxed{100} + \boxed{20} + \boxed{30} + \boxed{6} = \boxed{156}$

2. $11 \times 15 = (\boxed{10} + \boxed{1}) \times (\boxed{10} + \boxed{5})$

$= (10 \times 10) + (\boxed{1} \times \boxed{10})$
$+ (\boxed{10} \times \boxed{5}) + (\boxed{1} \times \boxed{5})$
$= \boxed{100} + \boxed{10} + \boxed{50} + \boxed{5} = \boxed{165}$

3. $13 \times 25 = (\boxed{10} + \boxed{3}) \times (\boxed{20} + \boxed{5})$

	20	5
10	200	50
3	60	15

$= (\boxed{10} \times \boxed{20}) + (\boxed{3} \times \boxed{20})$
$+ (\boxed{10} \times \boxed{5}) + (\boxed{3} \times \boxed{5})$
$= \boxed{200} + \boxed{60} + \boxed{50} + \boxed{15} = \boxed{325}$

4. $24 \times 32 = (\boxed{20} + \boxed{4}) \times (\boxed{30} + \boxed{2})$

	30	2
20	600	40
4	120	8
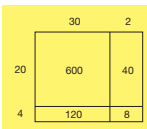
$= (\boxed{20} \times \boxed{30}) + (\boxed{4} \times \boxed{30})$
$+ (\boxed{20} \times \boxed{2}) + (\boxed{4} \times \boxed{2})$
$= \boxed{600} + \boxed{120} + \boxed{40} + \boxed{8} = \boxed{768}$

Find the product. Show your work on the back.

5. $12 \times 19 =$ ___ $(10 + 2) \times (10 + 9) = (10 \times 10)$
$+ (2 \times 10) + (10 \times 9) + (2 \times 9)$
$= 100 + 20 + 90 + 18 = 228$

6. $35 \times 22 =$ ___ $(30 + 5) \times (20 + 2) = (30 \times 20)$
$+ (5 \times 20) + (30 \times 2) + (5 \times 2)$
$= 600 + 100 + 60 + 10 = 770$

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Reinforce the Concept

Explain & Elaborate   WHOLE CLASS

Have students discuss the activity. Note whether they developed a routine.

- **Ask:** *Did you notice that you used the distributive property? Refer to problem 5. Present and discuss.*

$$12 \times 19 = (10 + 2) \times (10 + 9)$$
$$= (10 + 2) \times 10 + (10 + 2) \times 9$$
$$= (10 \times 10) + (2 \times 10) + (10 \times 9) + (2 \times 9)$$

Elicit that $(10 + 2)$ is first distributed over $(10 + 9)$, and then 10 and 9 are each distributed over $(10 + 2)$. Have students draw the area model and explain how these different models show the same number.

Evaluate   WHOLE CLASS

Have students find the product 13×18 using drawings and/or equations. Ask students to show you how they used place value to generate the 4 partial products.

Independent Practice

Use this VersaTiles® activity to give students more practice with the skills they learned in the lesson.

Two by Two

Example

Find the product.

$15 \times 23 = \blacksquare$
- Decompose the factors.
  $15 \times 23 = (10 + 5) \times (20 + 3)$
- Use the Distributive Property.
  $(10 + 5) \times (20 + 3) = (10 + 5) \times 20 + (10 + 5) \times 3$
  $= (10 \times 20) + (5 \times 20) + (10 \times 3) + (5 \times 3)$
  $= 200 + 100 + 30 + 15$
  $= \mathbf{345}$

Find the product.

1. $14 \times 16 = \blacksquare$
2. $13 \times 28 = \blacksquare$
3. $21 \times 25 = \blacksquare$
4. $32 \times 24 = \blacksquare$
5. $28 \times 16 = \blacksquare$
6. $22 \times 23 = \blacksquare$
7. $35 \times 12 = \blacksquare$
8. $18 \times 22 = \blacksquare$
9. $15 \times 33 = \blacksquare$
10. $44 \times 21 = \blacksquare$
11. $18 \times 52 = \blacksquare$
12. $15 \times 34 = \blacksquare$

Answer Box

A	B	C	D	E	F
525	936	364	510	768	924
G	**H**	**I**	**J**	**K**	**L**
448	495	396	506	420	224

Objective: Multiply two 2-digit numbers.   33

Re-Engage

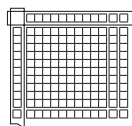Use this page to give students additional concrete-to-representational-to-abstract practice.

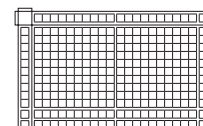Lesson **2**   Two by Two   Name ___**Answer Key**___

Use a Factor Track and Base Ten Blocks to build the model. Complete the number sentence.

1. $11 \times 12 = (10 + 1) \times (10 + 2)$

   $= (10 \times 10) + (1 \times 10)$
   $+ (10 \times 2) + (1 \times 2)$
   $= \boxed{100} + \boxed{10} + \boxed{20} + \boxed{2} = \boxed{132}$

2. $12 \times 21 = (10 + \boxed{2}) \times (20 + \boxed{1})$

   $= (10 \times 20) + (2 \times 20)$
   $+ (\boxed{10} \times 1) + (\boxed{2} \times 1)$
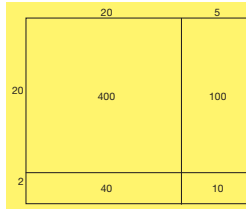 $= \boxed{200} + \boxed{40} + \boxed{10} + \boxed{2} = \boxed{252}$

Use a Factor Track and Base Ten Blocks to model the problem. Complete the area sketch. Complete the number sentence.

3. $13 \times 15 = (10 + \boxed{3}) \times (10 + \boxed{5})$

	10	5
10	100	50
3	30	15
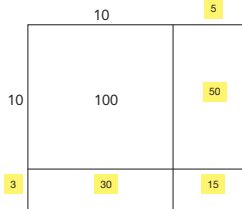
   $= (10 \times 10) + (\boxed{3} \times 10)$
   $+ (10 \times \boxed{5}) + (\boxed{3} \times \boxed{5})$
   $= 100 + \boxed{30} + \boxed{50} + \boxed{15} = \boxed{195}$

4. $22 \times 25 = (\boxed{20} + \boxed{2}) \times (\boxed{20} + \boxed{5})$

	20	5
20	400	100
2	40	10

   $= (20 \times 20) + (\boxed{2} \times \boxed{20})$
   $+ (\boxed{20} \times \boxed{5}) + (\boxed{2} \times \boxed{5})$
   $= \boxed{400} + \boxed{40} + \boxed{100} + \boxed{10} = \boxed{550}$

Find the product. Show your work on the back.

5. $12 \times 17 = \underline{}$ $(10 + 2) \times (10 + 7) = (10 \times 10)$
   $+ (2 \times 10) + (10 \times 7) + (2 \times 7)$
   $= 100 + 20 + 70 + 14 = 204$

6. $15 \times 21 = \underline{}$ $(10 + 5) \times (20 + 1) = (10 \times 20)$
   $+ (5 \times 20) + (10 \times 1) + (5 \times 1)$
   $= 200 + 100 + 10 + 5 = 315$

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Daily Routine

Anchor Poster

Create an anchor poster with your class to summarize the concept of the area model and distributive property. It might include one-digit and two-digit examples along with an area model and symbolic representations.
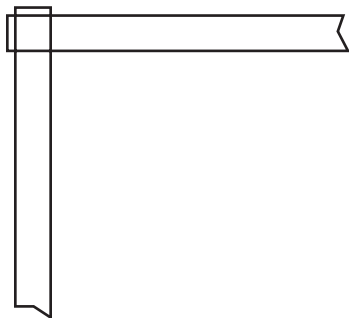
Place this anchor chart in the writing center.

WRITING ASSIGNMENT: What does the word "distribute" mean? Explain using pictures, numbers, and words.

Try This

- Use Base Ten Blocks and a Factor Track to model each problem.
- In problems 1–2, sketch your model.
- In problems 3–4, sketch an area model.
- In each problem, complete the number sentences.
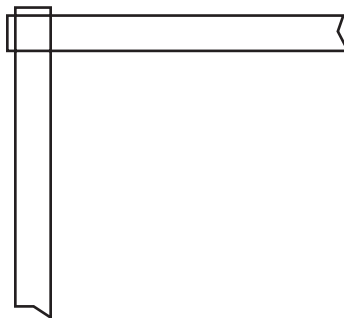
1. $12 \times 13 = (10 + 2) \times (10 + 3)$

= $(10 \times 10) + (\_\_\_ \times 10)$
  + $(10 \times 3) + (\_\_\_ \times 3)$
= $\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_$

2. $11 \times 15 = (\_\_\_ + \_\_\_) \times (\_\_\_ + \_\_\_)$

= $(10 \times 10) + (\_\_\_ \times \_\_\_)$
  + $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$
= $\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_$

3. $13 \times 25 = (\_\_\_ + \_\_\_) \times (\_\_\_ + \_\_\_)$

= $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$
  + $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$
= $\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_$

4. $24 \times 32 = (\_\_\_ + \_\_\_) \times (\_\_\_ + \_\_\_)$

= $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$
  + $(\_\_\_ \times \_\_\_) + (\_\_\_ \times \_\_\_)$
= $\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_$

Find the product. Show your work on the back.

5. $12 \times 19 = $ _____

6. $35 \times 22 = $ _____

Two by Two

Find the product.

$15 \times 23 = \blacksquare$
- Decompose the factors.
  $15 \times 23 = (10 + 5) \times (20 + 3)$
- Use the Distributive Property.
  $(10 + 5) \times (20 + 3) = (10 + 5) \times 20 + (10 + 5) \times 3$
  $= (10 \times 20) + (5 \times 20) + (10 \times 3) + (5 \times 3)$
  $= 200 + 100 + 30 + 15$
  $= \mathbf{345}$

Find the product.

1 $14 \times 16 = \blacksquare$

2 $13 \times 28 = \blacksquare$

3 $21 \times 25 = \blacksquare$

4 $32 \times 24 = \blacksquare$

5 $28 \times 16 = \blacksquare$

6 $22 \times 23 = \blacksquare$

7 $35 \times 12 = \blacksquare$

8 $18 \times 22 = \blacksquare$

9 $15 \times 33 = \blacksquare$

10 $44 \times 21 = \blacksquare$

11 $18 \times 52 = \blacksquare$
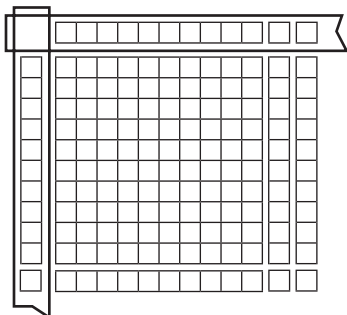
12 $15 \times 34 = \blacksquare$

Answer Box

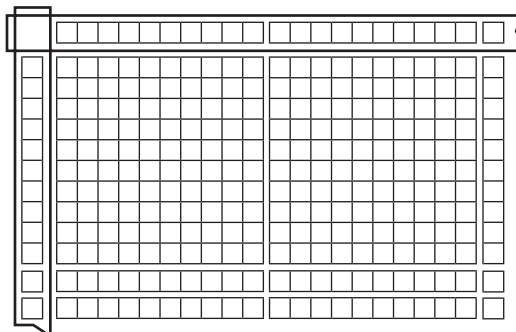| A	B	C	D	E	F
525	936	364	510	768	924
G	**H**	**I**	**J**	**K**	**L**
448	495	396	506	420	224

Name _____

Use a Factor Track and Base Ten Blocks to build the model.
Complete the number sentence.

1. $11 \times 12 = (10 + 1) \times (10 + 2)$

```
= (10 × 10) + (1 × 10)
  + (10 × 2) + (1 × 2)
= ____ + ____ + ____ + ____ = ____
```

2. $12 \times 21 = (10 + \underline{}) \times (20 + \underline{})$

```
= (10 × 20) + (2 × 20)
  + (____ × 1) + (____ × 1)
= ____ + ____ + ____ + ____ = ____
```
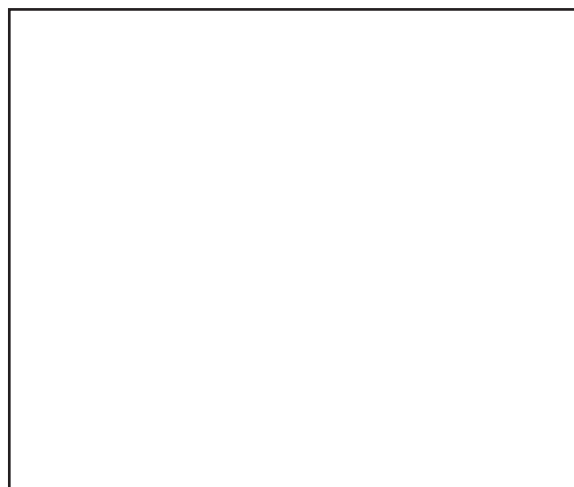
Use a Factor Track and Base Ten Blocks to model the problem.
Complete the area sketch. Complete the number sentence.

3. $13 \times 15 = (10 + \underline{}) \times (10 + \underline{})$

```
        10
   ┌──────────┬────┐
   │          │    │
10 │   100    │    │
   │          │    │
   ├──────────┼────┤
   │          │    │
   └──────────┴────┘
```
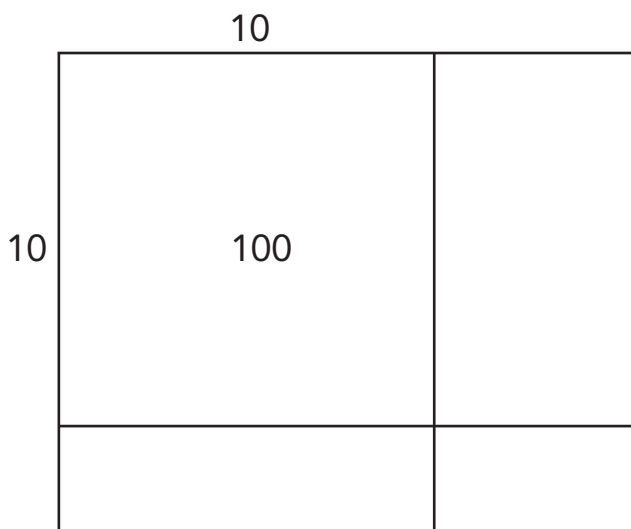
```
= (10 × 10) + (____ × 10)
  + (10 × ____) + (____ × ____)
= 100 + ____ + ____ + ____ = ____
```

4. $22 \times 25 = (\underline{} + \underline{}) \times (\underline{} + \underline{})$

```
   ┌───────────────────┐
   │                   │
   │                   │
   │                   │
   │                   │
   │                   │
   │                   │
   └───────────────────┘
```

```
= (20 × 20) + (____ × ____)
  + (____ × ____) + (____ × ____)
= ____ + ____ + ____ + ____ = ____
```

Find the product. Show your work on the back.

5. $15 \times 17 =$ _____

6. $15 \times 21 =$ _____