

Can you make all the prime numbers less than 100 with 6 fours and these symbols?

4	4	4	4	4	4
+	-	×	÷	()	! √

2 =

17 =

41 =

67 =

3 =

19 =

43 =

71 =

5 =

23 =

47 =

73 =

7 =

29 =

53 =

79 =

11 =

31 =

59 =

83 =

13 =

37 =

61 =

89 =

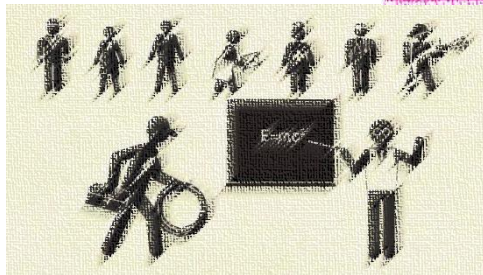
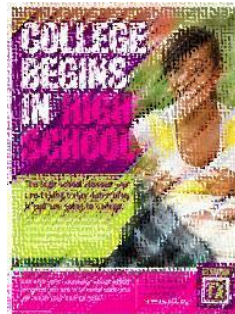
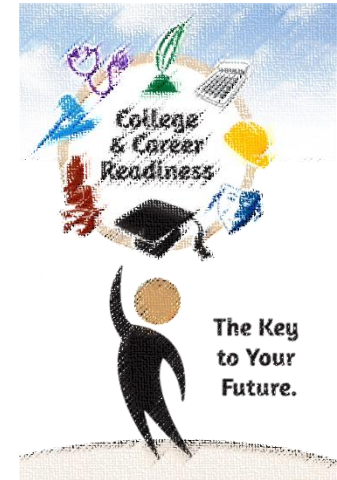
97 =

FOCUSING AND CONNECTING TO WHAT MATTERS MOST

Jan Scott, Ph.D.

Regional Director of Math Partnerships,
Scholastic Inc.

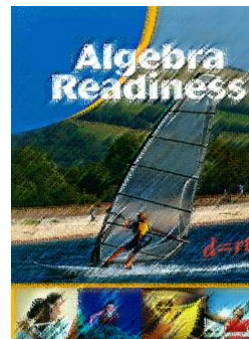




The Four Keys to College and Career READINESS

think:	<i>Key Cognitive Strategies</i>	know:	<i>Key Content Knowledge</i>
<i>Study Skills</i>	<i>Organization</i>	<i>Problem Solving</i>	<i>Research Skills</i>
act:	<i>Key Learning Skills and Techniques</i>	go:	<i>Key Transition Skills and Awareness</i>
<i>Planning</i>	<i>Time Use</i>	<i>Self-Management</i>	<i>Goal Setting</i>





Print Robert's Results

ALGEBRA READINESS FITNESS BREAKDOWN

100%

You're rock solid on the skills listed below. Crack & Pop a hot air balloon and push yourself even further by getting more practice with these video lessons:

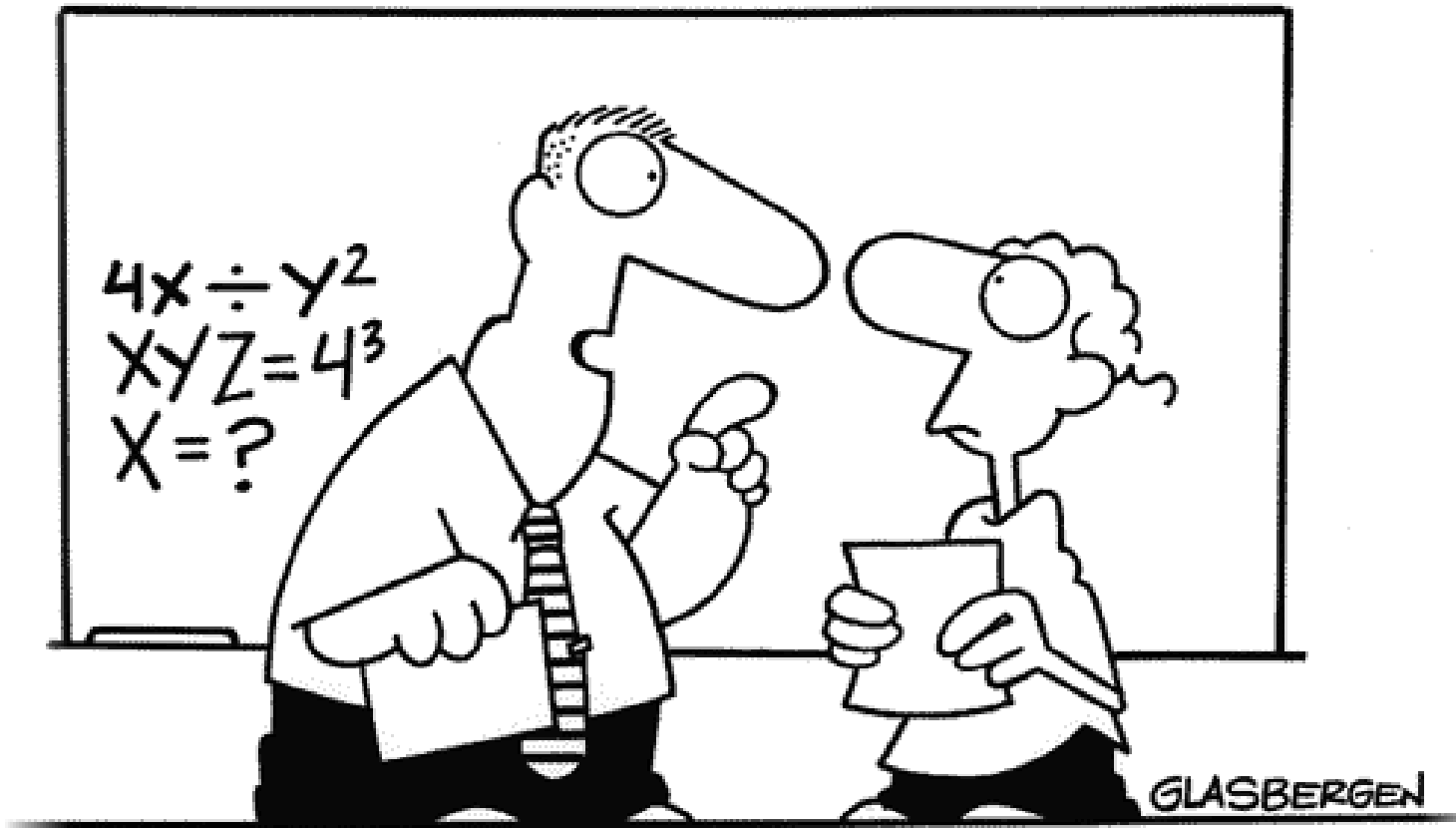
- Decimal Operations
- Fractions



Math Curriculum Guide
Algebra Readiness
2012-13

FIT 4 ALGEBRA





“Algebra class will be important to you later in life because there’s going to be a test six weeks from now.”



“Why is it important for today’s kids to learn algebra? Because *I* had to learn this junk in school and now it’s *your* turn, that’s why!”

The importance of algebra . . .

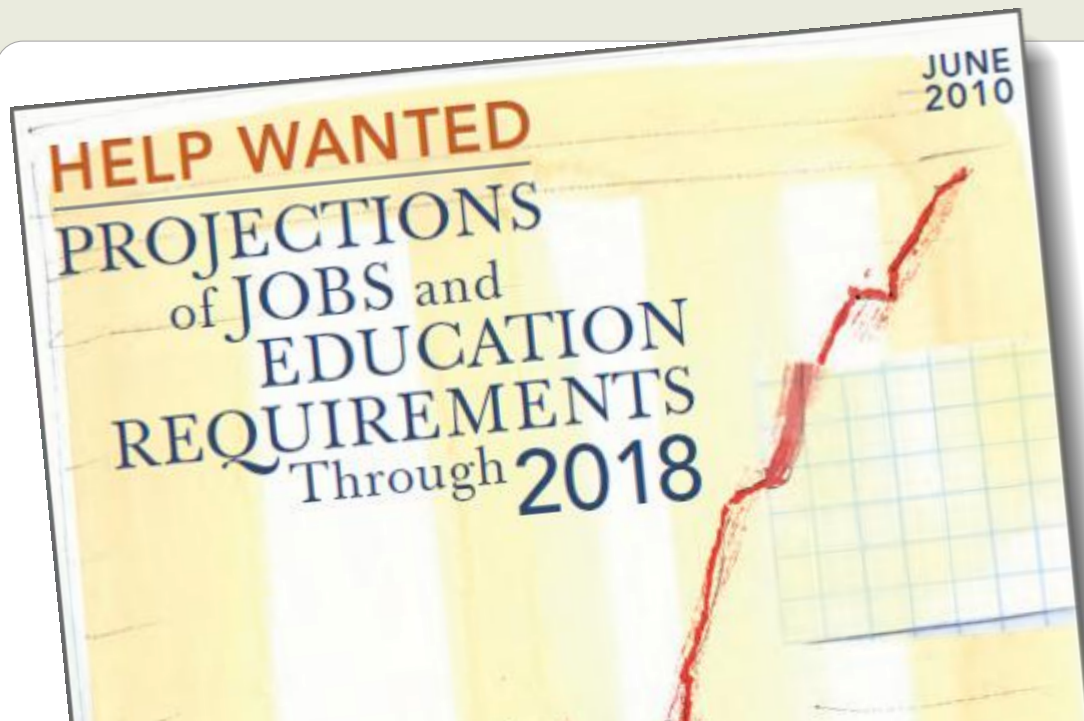
A white question mark icon inside a light green diamond shape.

How important is algebra to a student's chance of attending college?

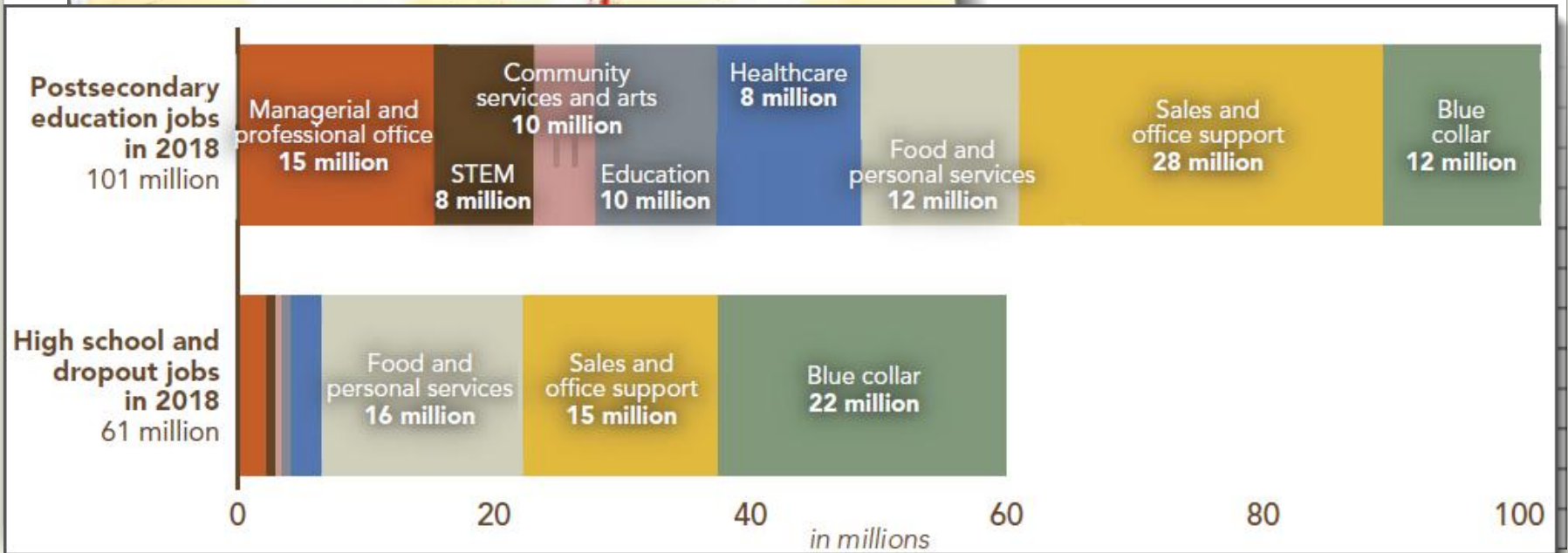
A white answer icon (the letter 'A' with a colon) inside a light green diamond shape.

Students who take a **year of algebra** and follow with a **year of geometry** nearly **DOUBLE** their chances of going to college -- by doing that alone!

Students who complete Algebra II are more than twice as likely to **GRADUATE** from college.



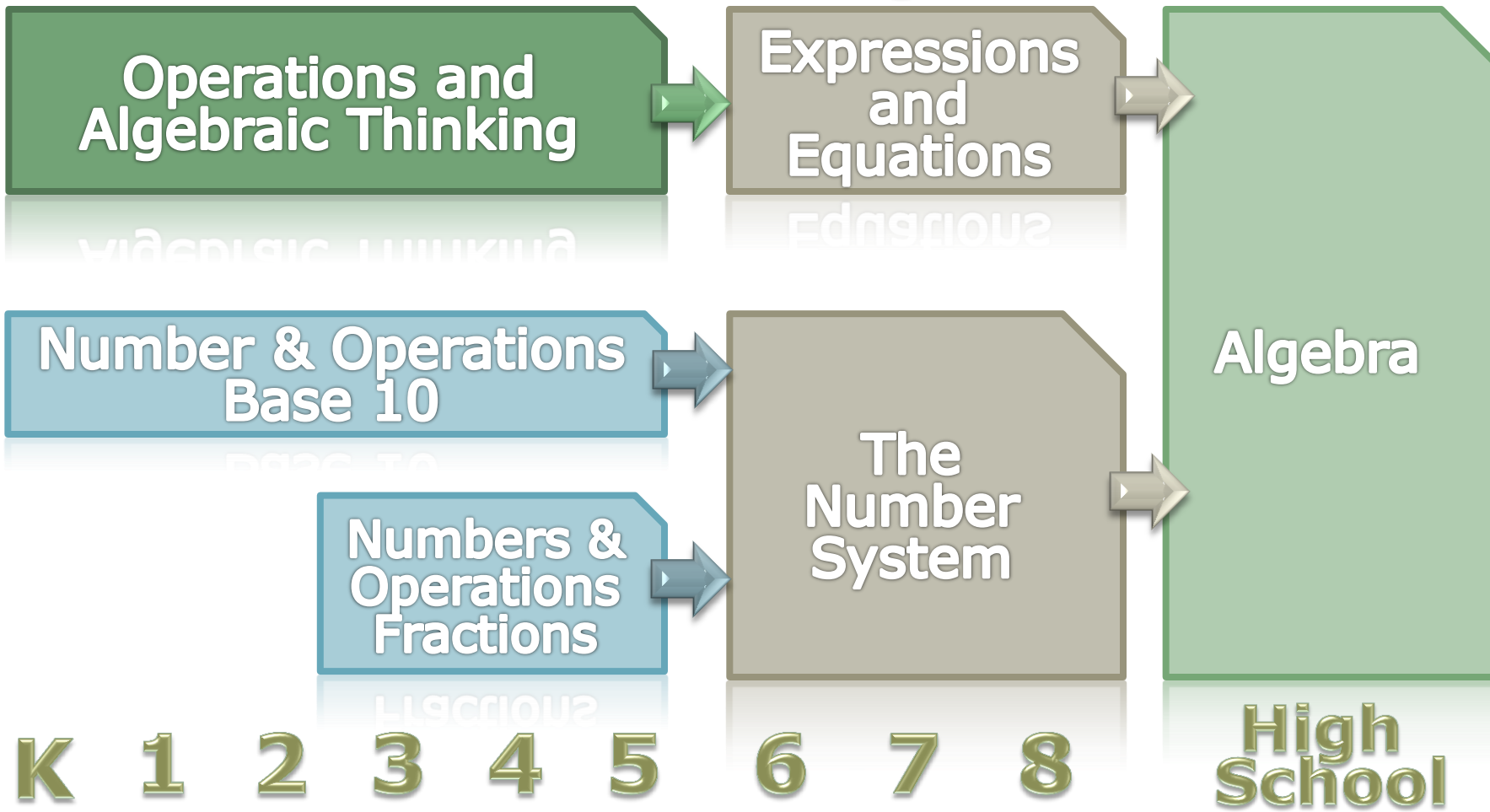
By 2018, we need 22 million new college degrees . . .





**COLLEGE
BEGINS
IN
KINDERGARTEN**

Focusing Attention within Number and Operations



Progressions to Algebra

	K	1	2	3	4	5	6	7	8
Know number names and the count sequence		Represent and solve problems involving addition and subtraction		Represent & solve problems involving multiplication and division		Understand the place value system	Apply and extend previous understandings of multiplication and division to divide fractions by fractions		
Count to tell the number of objects		Understand and apply properties of operations and the relationship between addition and subtraction		Understand properties of multiplication and the relationship between multiplication and division	Use the four operations with whole numbers to solve problems	Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of numbers to the system of rational numbers	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers	Work with radical and integer exponents
Compare numbers		Add and subtract within 20	Represent and solve problems involving addition and subtraction	Multiply & divide within 100	Generalize place value understanding for multi-digit whole numbers	Use equivalent fractions as a strategy to add and subtract fractions	Understand ratio concepts and use ratio reasoning to solve problems		Understand the connections between proportional relationships, lines, and linear equations**
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from		Work with addition and subtraction equations	Add and subtract within 20	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Use place value understanding and properties of operations to perform multi-digit arithmetic	Extend understanding of fraction equivalence and ordering	Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Analyze proportional relationship and use them to solve real-world and mathematical problems	Analyze and solve linear equations and pairs of simultaneous linear equations
Work with numbers 11-19 to gain foundations for place value		Understand place value	Use place value understanding and properties of operations to add and subtract	Develop understanding of fractions as numbers	Build fractions from unit fractions by applying and extending previous understandings of operations	Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Apply and extend previous understandings of arithmetic to algebraic expressions	Use properties of operations to generate equivalent expressions	Define, evaluate, and compare functions
		Use place value understanding and properties of operations to add and subtract	Relate addition and subtraction to length	Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Understand decimal notation for fractions, and compare decimal fractions	Graph points in the coordinate plane to solve real-world and mathematical problems*	Reason about and solve one-variable equations and inequalities	Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Use functions to model relationships between quantities
		Measure lengths indirectly and by iterating length units		Geometric measurement: understand concepts of area and relate area to multiplication and to addition			Represent and analyze quantitative relationships between dependent and independent variables		

FOCUSING AND CONNECTING TO WHAT MATTERS MOST

- Connecting Multiplication, Squares and Square Roots

7th Grade: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$. . . Evaluate square roots of small perfect squares and cube roots of small perfect cubes.

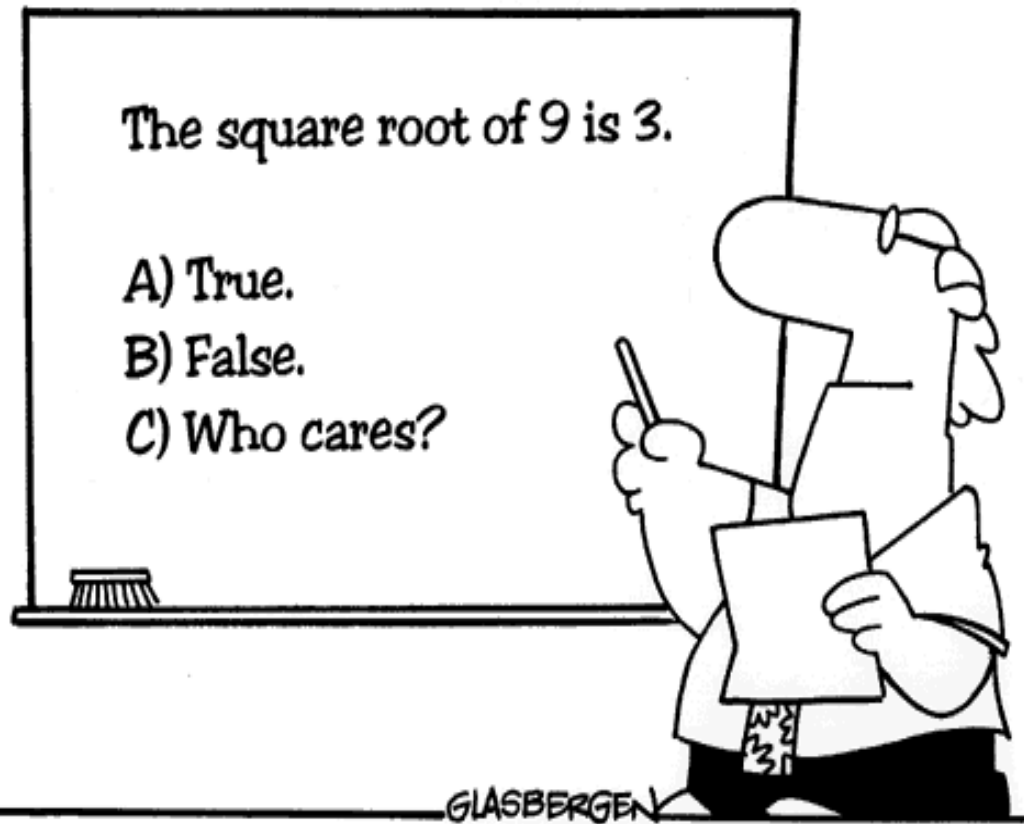
8th Grade: Know that there are numbers that are not rational, and approximate them by rational numbers.

Use rational approximations of irrational numbers to compare the size of irrational numbers

. . .

Roots

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Many students actually look forward to Mr. Atwadder's math tests.

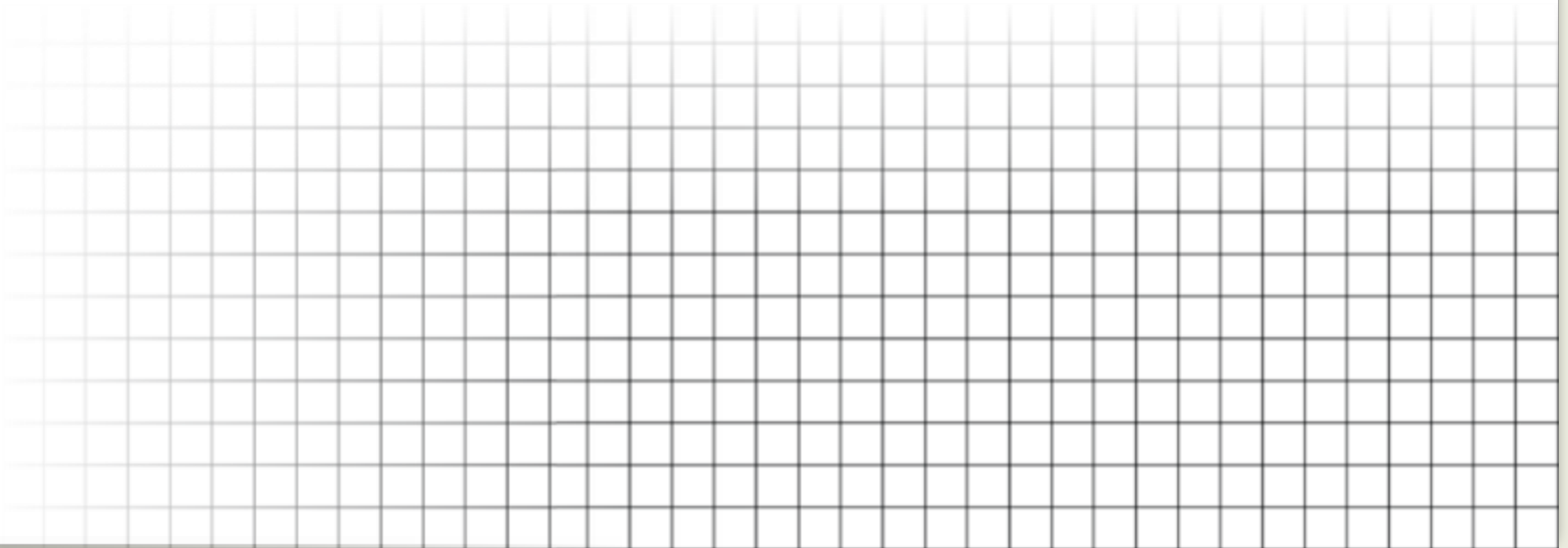
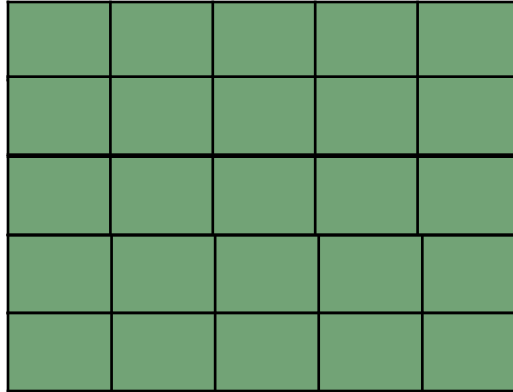
Roots

- Comes from the Latin word *RADIX* meaning *root*
- 1557 in *The Whetstone of Witte* by Robert Recorde
 - "Thei onely haue rootes, whiche bee made by many multiplications of some one number by itself"
- Symbol traceable to Christoff Rudolff *Die Coss* in 1525
 - Used because it resembled a small *r*
 - The vinculum was added later



Focus on what matter most

1x5
2x5
3x5
4x5
5x5



Finding a square root

Connecting multiplication to higher math concepts . . .

$$\sqrt{58}$$

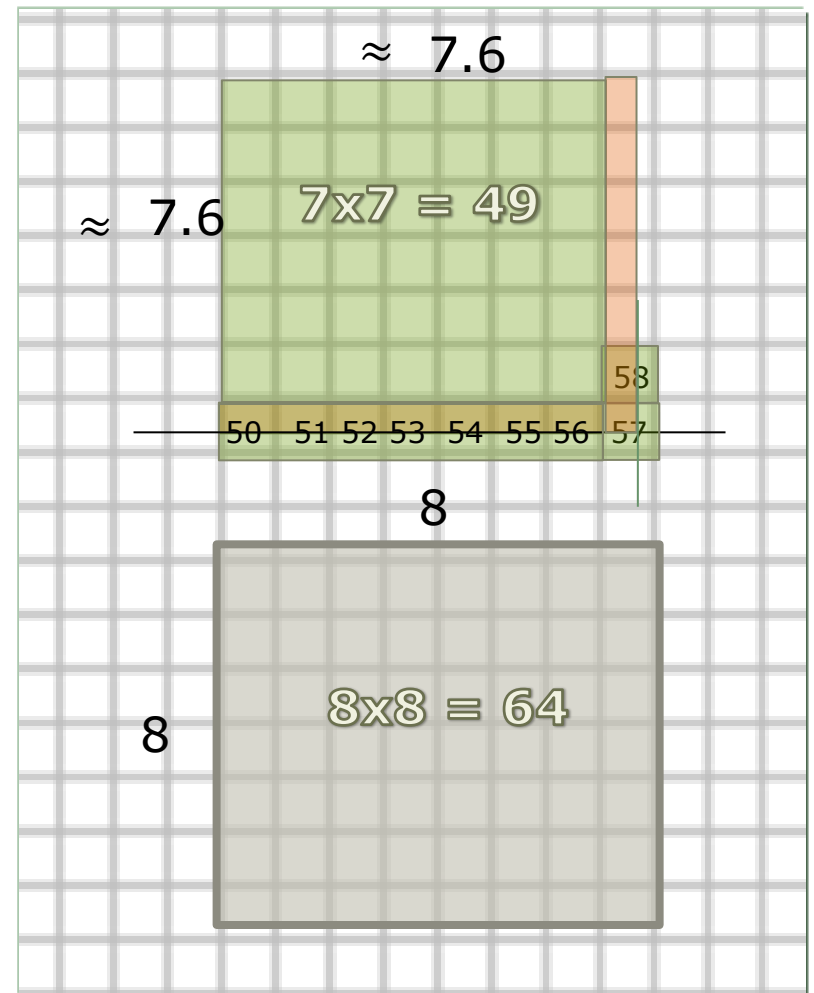
Smallest square:

Largest square:

$$49 < 58 < 64$$

$\sqrt{58}$ is between 7 and 8.

$$\sqrt{58} \approx 7.6$$



FOCUSING AND CONNECTING TO WHAT MATTERS MOST

3rd Grade: Compare two fractions with the same numerator or the same denominator by reasoning about their size.

4th Grade: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $\frac{1}{2}$.

Are your students struggling with fractions?

I can't estimate the sum of $1\frac{1}{12}$ plus $17\frac{1}{18}$

$\frac{1}{5}$ is greater than $\frac{1}{2}$

$\frac{1}{7}$ is the same as 0.7

There are no numbers between $\frac{1}{3}$ and $\frac{2}{3}$ on the number line



Fraction Riddle

I'm a fraction with a value less than $\frac{1}{2}$.

Both my numerator and denominator are one-digit primes.

What three fractions could I be?

Fractions

- Latin *frangere* (to break), often called “broken numbers
- Fibonacci (1202) *Liber abaci* generally used *fractio* and first used the fraction bar
- Robert Recorde in *Ground of Artes* (1575) "A Fraction in deede is a broken number"
- Chaucer (~1400) - The earliest recorded sense of the word is "an aliquot part of a unit, a fraction or subdivision"

Proper and Improper Fractions and Mixed Number

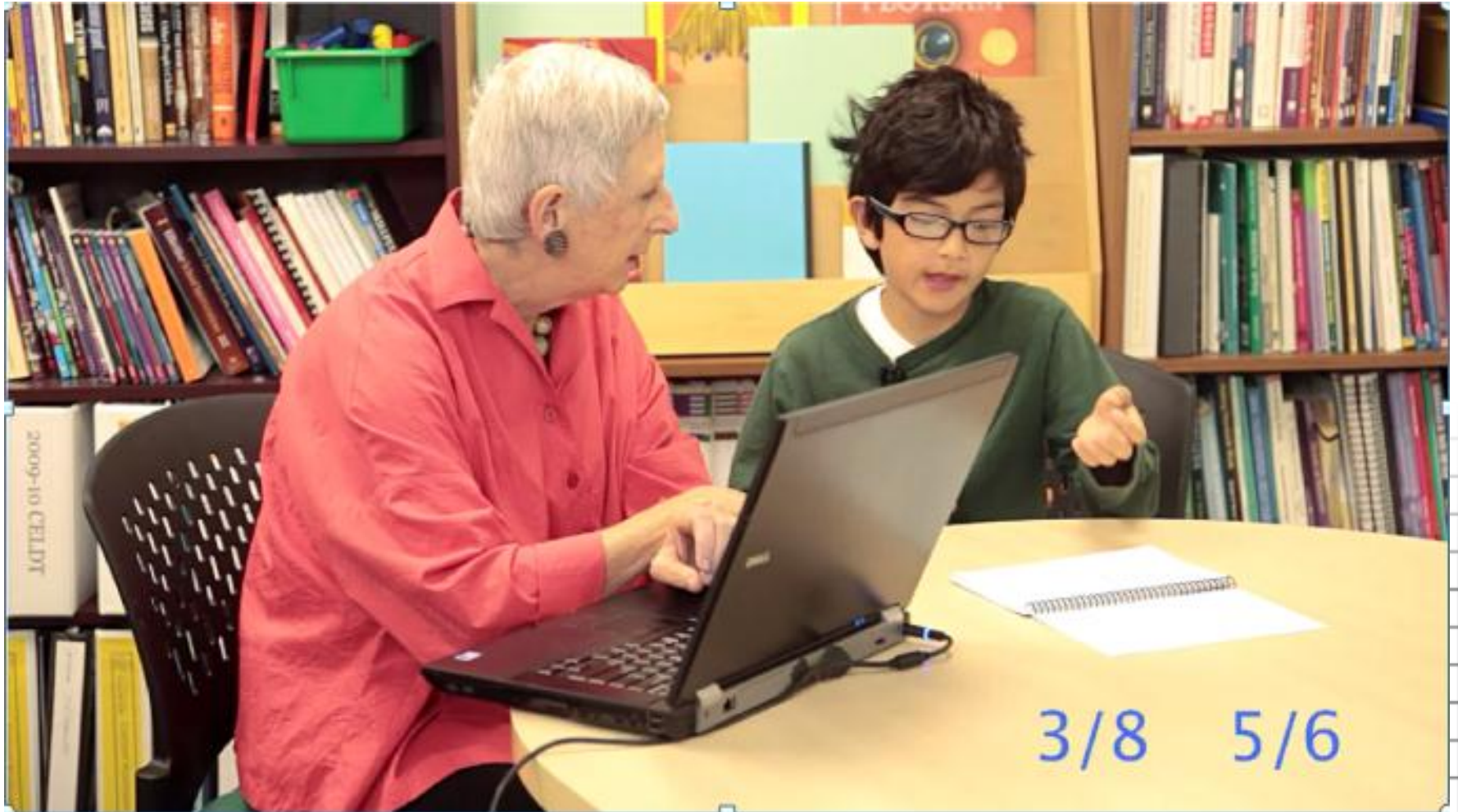
- **IMPROPER:** Robert Recorde (1542) "An Improper Fraction...that is to saye, a fraction in forme, which in dede is greater than a Unit."
- **PROPER:** Samuel Jeake *Arithmetic* (1701) "Proper Fractions always have the Numerator less than the Denominator, for then the parts signified are less than a Unit . . ."
- **MIXED NUMBER** by Robert Recorde (1542) "mixt numbers (that is whole numbers with fractions)"

Let's do some more math . . .

Which is greater

$$\frac{3}{8} \text{ or } \frac{5}{6}$$

Let's see how Zakari did it . . .



Foundations for Computing: Comparing Fractions

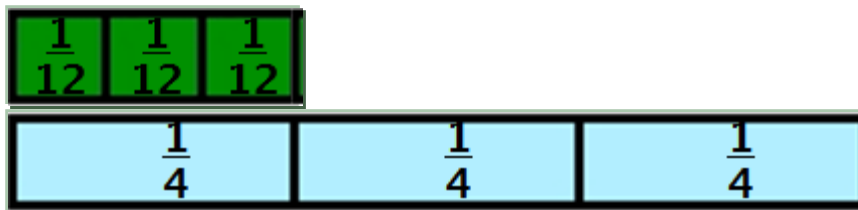
Strategy 1: Compare unit fractions

$$\frac{1}{8} < \frac{1}{2}$$



Strategy 2: Compare fractions with common numerators

$$\frac{3}{12} < \frac{3}{4}$$



Strategy 3: Compare fractions with common denominators

$$\frac{1}{4} < \frac{2}{4}$$

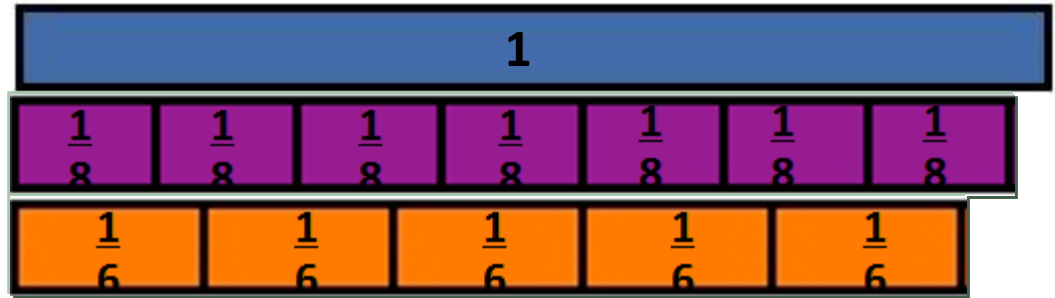


Foundations for Computing: Comparing Fractions

Strategy 4:

Compare fractions one unit fraction from 1

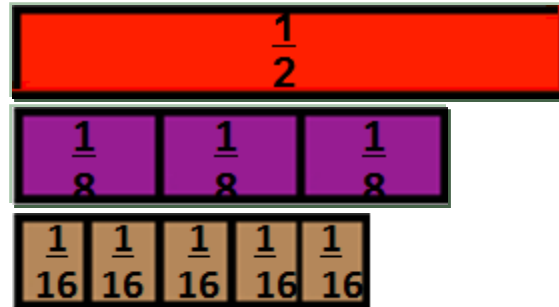
$$\frac{7}{8} > \frac{5}{6}$$



Strategy 5:

Compare fractions to $\frac{1}{2}$

$$\frac{3}{8} > \frac{5}{16}$$

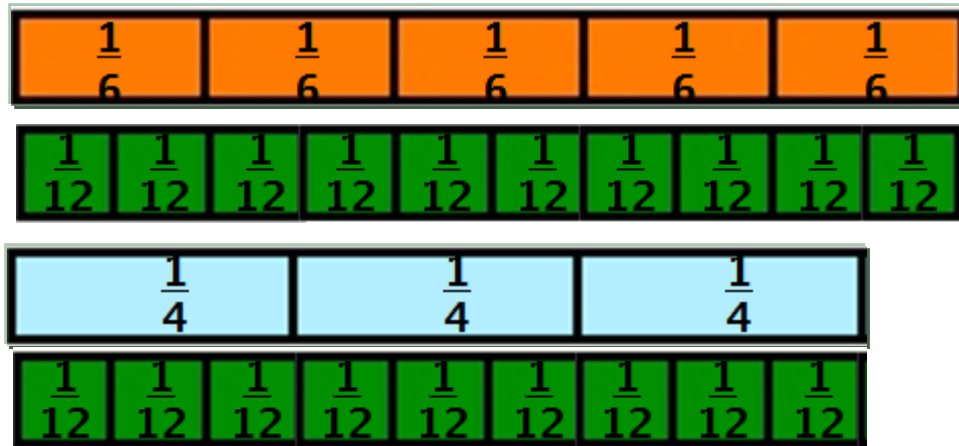


Foundations for Computing: Comparing Fractions

Strategy 6:

Change fractions to equivalent fractions

$$\frac{3}{4} > \frac{5}{6}$$



Foundations for Computing: Fraction Families on a Multiplication Chart

12 by 12 Multiplication Table

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48



1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
5	10	15	20	25	30	35	40	45	50	55	60

9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

FOCUSING AND CONNECTING TO WHAT MATTERS MOST

- Connecting operations of whole numbers to fractions

5th Grade: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

6th & 7th Grade: Apply and extend previous understandings . . .

Foundations for Computing: Applying the Big Ideas

- 10 is the organizer of our number system
- Numbers can be composed and decomposed

$$\begin{array}{r} \textcircled{8} + 6 = \\ 8 + 2 + 4 \\ \text{---} \\ 10 + 2 = 14 \end{array}$$

Foundations for Computing: Applying the Big Ideas to Fractions

- 1 is the organizer of fractions
- Fractions can be composed and decomposed

$$\begin{array}{r} 8 + 4 = \\ \quad \quad \quad \wedge \\ 8 + 2 + 2 \\ \quad \quad \quad \vee \\ 10 + 2 = 12 \end{array}$$

$$\begin{array}{r} \textcircled{\frac{4}{5}} + \frac{3}{5} = \\ \quad \quad \quad \wedge \\ \frac{4}{5} + \frac{1}{5} + \frac{2}{5} = \\ \quad \quad \quad \vee \\ \frac{5}{5} + \frac{2}{5} = 1 \frac{2}{5} \end{array}$$

FOCUSING AND CONNECTING TO WHAT MATTERS MOST

- Connecting operations of whole numbers to fractions

4th Grade: Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. . . . Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.

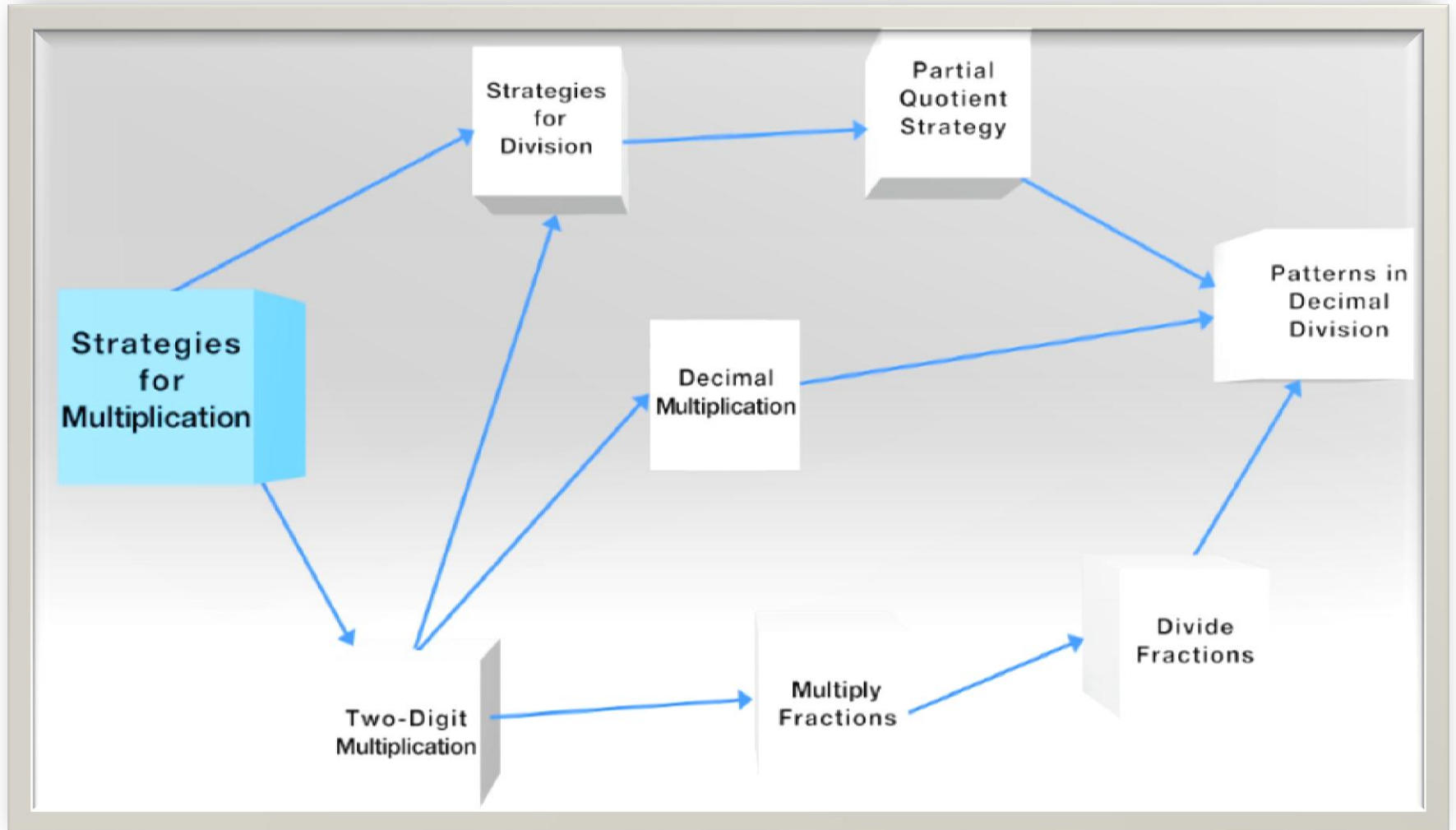
FOCUSING AND CONNECTING TO WHAT MATTERS MOST

- Connecting operations of whole numbers to fractions

5th Grade: Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.

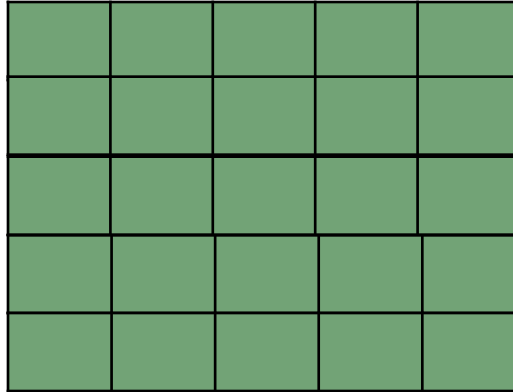
6th & 7th Grade: Apply and extend previous understandings . . .

Foundational Progressions



Focus on what matter most

1x5
2x5
3x5
4x5
5x5



Connecting multiplication to higher math concepts . . .

$$\frac{1}{2} \times 5$$



$$2 \frac{1}{2}$$

$$24 \times \frac{3}{4}$$

$$= 24 \times 3 \times \frac{1}{4}$$

$$= 72 \times \frac{1}{4}$$

$$= \frac{72}{4} = 18$$

FOCUSING AND CONNECTING TO WHAT MATTERS MOST

- Connecting operations of whole numbers to fractions

5th Grade: Interpret division of a unit fraction by a non-zero whole number, and compute such quotients . . . Interpret division of a whole number by a unit fraction, and compute such quotients.

6th & 7th Grade: Apply and extend previous understandings . . .

Dividing Fractions

Imagine you are beginning to teach students division with fractions.

What would you do to introduce this concept to students?

How would you present the following problem?

$$1\frac{3}{4} \div \frac{1}{2}$$

Introducing Division of Fractions

What is the common phrase we hear teachers say when teaching students to divide fractions?

“Ours is not to reason why,
simply invert and multiply!”

What we should be saying is “dividing by a number is equivalent to multiplying by its reciprocal.”

Introducing Division of Fractions

Dividing by 2 is the same as multiplying by $\frac{1}{2}$, therefore dividing by $\frac{1}{2}$ is the same as multiplying by 2.

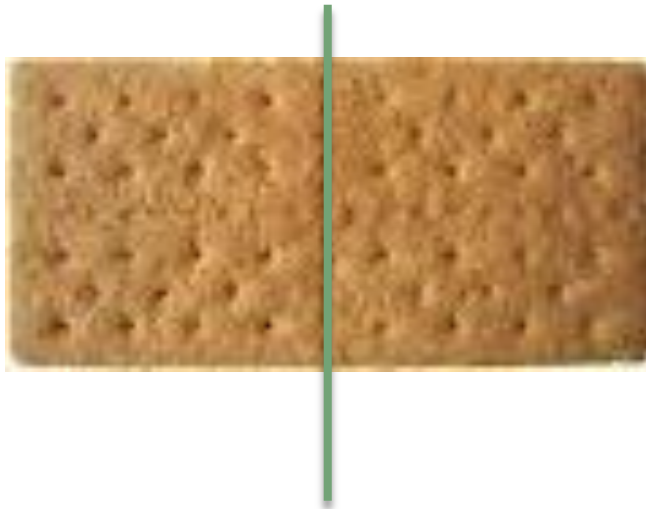
INPUT	EQUATIONS	OUTPUT
1		$\frac{1}{2}$
$\frac{1}{2}$		$\frac{1}{4}$
$\frac{1}{4}$		$\frac{1}{8}$
$\frac{1}{5}$		$\frac{1}{10}$
16		8

Introducing Division of Fractions

How many different ways can we solve the problem $1\frac{3}{4} \div \frac{1}{2}$?

Measurement Model

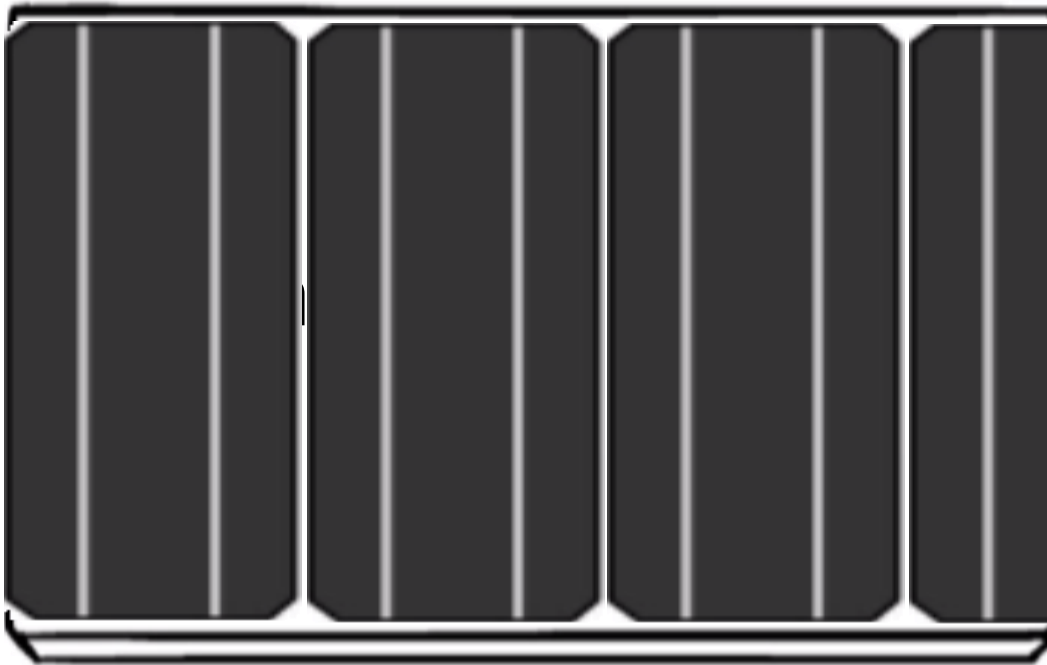
“How many $\frac{1}{2}$ s in $1\frac{3}{4}$?” (e.g., cups, graham crackers, piece of wood)



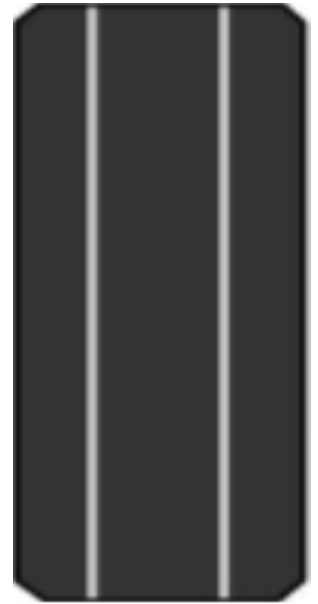
There are $3\frac{1}{2}$ pieces.

Measurement Model

$$1\frac{3}{4}m$$



$$\frac{1}{2}m$$



FOCUSING AND CONNECTING TO WHAT MATTERS MOST

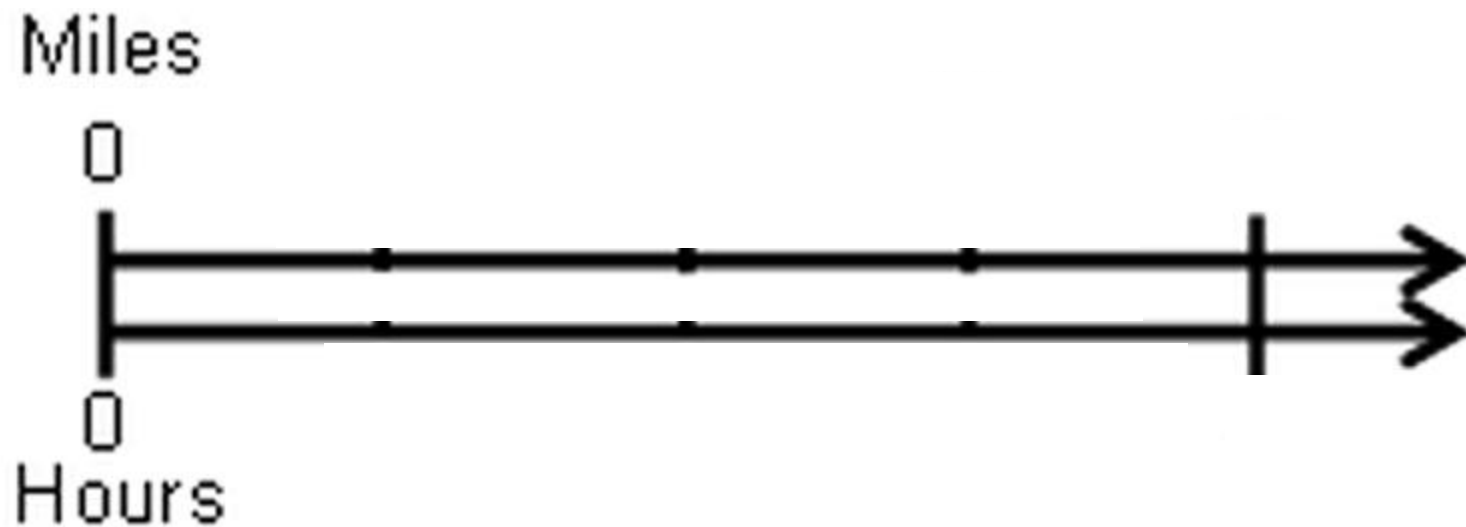
- Connecting number lines to ratios

6th Grade: Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.

7th Grade: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Solving Problems with a Double Number Line: Rates

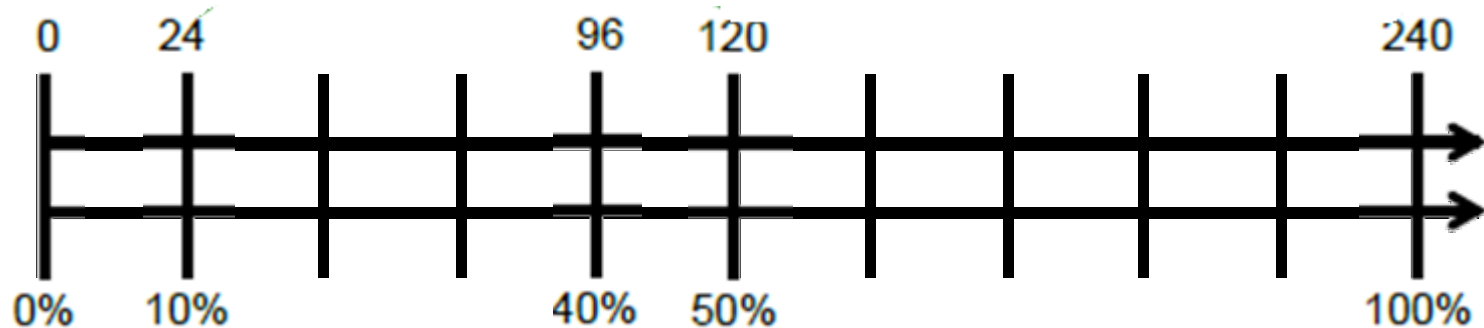
On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation?



Solving Problems with a Double Number Line: Percents

A part is a percent of a whole. If the whole of 240 is 100%, then the part shown for 24 students is 10%, for 120 students is 50%. What percent is represented by 96 students

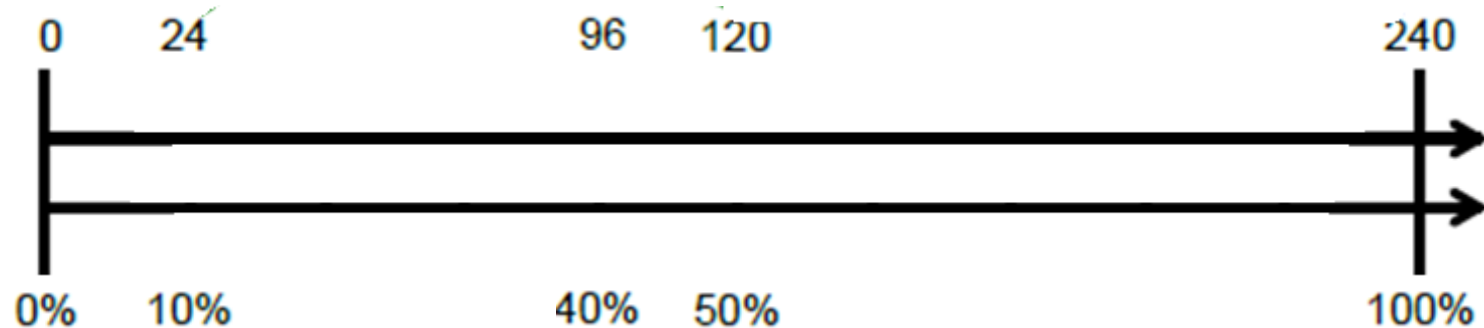
Number of Students



Solving Problems with a Double Number Line: Percents

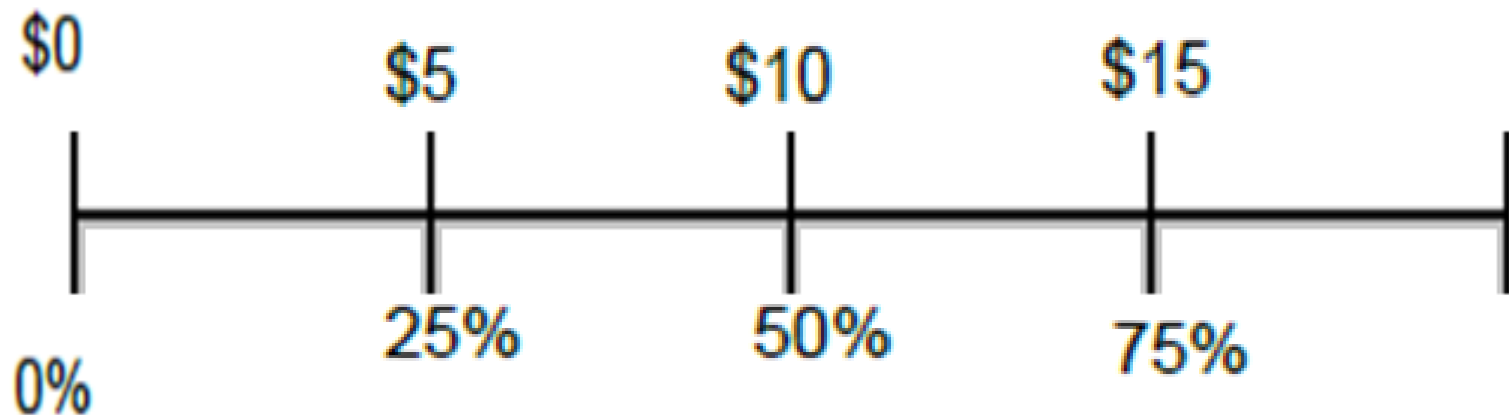
When the entire 6th grade of 240 students was polled, results showed that 96 students were dissatisfied with the music at a dance. What percentage of the 6th grade does this represent?

Number of Students



Solving Problems with a Double Number Line: Percents

Andrew was given an allowance of \$20. He used 75% of his allowance to go to the movies. How much money was spent at the movies?



When not knowing math can cost you \$15,000 . . .



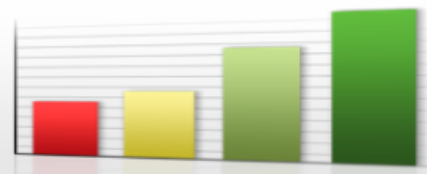
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