Instructional Implementation Sequence:
Attaining the CCSS Mathematical Practices
Engagement Strategies

Strategy:	Description	Practice	Degree	Matrix Code*
Initiating think, pair–share	Pair-Share, or Think-Pair-Share, is a strategy easy to implement in any classroom at any grade level or subject. This strategy does not require any other change in pedagogy or materials. For pair – share, teachers merely ask a question or assign a problem and allow students to think and work with a partner for one to three minutes before requesting an answer to the question or problem. In think – pair – share students are given a brief period of time to think independently before working with a partner. While effective in results, this strategy is a significant first step in engaging all students in classroom instructional activities.	• Make sense of problems. • Critique the reasoning of others	• Explain their thought processes in solving a problem one way. • Understand and discuss other ideas and approaches.	1a Initial 3b Initial
Showing thinking in Classrooms	Teachers need to work toward higher degrees of student involvement in classroom activities. Once pair – share is incorporated into classroom routines, teachers need to incorporate additional strategies that promote "every pupil response" (EPR). EPR strategies include such responses as "thumbs up/thumbs down," or use of individual white boards for noting answers. Students are also pressed to be more aware of their thinking and express their thinking in more detail. Students are routinely asked to share their thinking in mathematics classrooms. However, what is routinely accepted as thinking is actually process description. Students merely provide the steps they used to solve the problem, not their reasoning and thinking about how they knew which processes to use. In order to reveal student thinking, more challenging, open-ended problems are needed.	• Construct viable arguments. • Attend to precision.	• Explain their thinking for the solution they found. • Communicate their reasoning and solution to others.	3a Initial 6 Initial

Strategy:	Description	Practice	Degree	Matrix Code*
Questioning and wait time	As thinking is increased in mathematics classroom, better questioning and wait time are required. Teachers need to provide thought provoking questions to students and then allow the students time to think and work toward an answer.	• Make sense of problems • Persevere in solving them • Construct viable arguments. • Critique the reasoning of others	• Explain their thought processes in solving a problem in several ways. • Stay with a challenging problem for more than one attempt. • Explain their thinking with accurate vocabulary both their own thinking and thinking of others. • Explain other student's solutions and identify strengths and weaknesses of the solution.	1a Initial 1b Initial 3a Intermediate 3b Initial
	Empowerment Strategies			
Grouping and engaging problems	The strategy of "grouping and engaging problems" is a significant shift in pedagogy and materials. Students are given challenging problems to work, and allowed to work on the problem in a group of two, three, or four. Challenging mathematics problems take time, effort, reasoning, and thinking to solve.	• Make sense of problems • Persevere in solving them • Reason abstractly and quantitatively. • Reason abstractly and quantitatively. • Construct viable arguments.	• Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways. • Try several approaches in finding a solution, and seek only hints if stuck. • Reason with models or pictorial representations to solve problems. • Translate situations into symbols for solving problems. • Justify and explain, with accurate language and vocabulary, why their solution is correct.	1a Advanced 1b Intermediate 2 Initial 2 Intermediate 3a Advanced

Grouping and engaging problems (continued)	The strategy of "grouping and engaging problems" is a significant shift in pedagogy and materials. Students are given challenging problems to work, and allowed to work on the problem in a group of two, three, or four. Challenging mathematics problems take time, effort, reasoning, and thinking to solve.	• Critique the reasoning of others. • Models with mathematics. • Uses appropriate tools strategically. • Uses appropriate tools strategically. • Look for and express regularity in repeated reasoning.	• Compare and contrast various solution strategies and explain the reasoning of others. • Use models to represent and solve a problem, and translate the solution to mathematical symbols. • Use the appropriate tool to find a solution. • Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. • Look for patterns, and use "if-then" reasoning strategies for obvious patterns.	3b Advanced 4 Initial 5 Initial 5 Intermediate 8 Initial
Using questions and prompts with groups	Once students are provided with opportunities to solve challenging problems in groups, teachers need to increase their ability to ask supporting questions that encourage students to continue working, provide hints or cues without giving students the answers, and ask probing questions to better assess student thinking and current understanding.	• Models with mathematics. • Look for and make use of structure.	• Use models and symbols to represent and solve a problem, and accurately explain the solution representation. • Look for structure within mathematics to help them solve problems efficiently.	5 Intermediate 7 Initial

Strategy:	Description	Practice	Degree	Matrix Code*
Allowing Students to Struggle	Students learn to persevere in solving challenging mathematics problems by being allowed to struggle with challenging problems. Students need to understand that mathematical problems do not usually have a quick, easy solution. Effective effort is a life-skill and should be learned interdependently and independently. Appropriate degree of difficulty is foremost on teachers' minds. If the problem is too easy, students do not need to struggle. If the problem is far too difficult, students are not capable of solving the problem. Teachers need to balance working in groups and working independently, and be able to quickly adjust grouping strategies as the need arises.	• Persevere in solving them.	• Struggle with various attempts over time, and learn from previous solution attempts.	1b Advanced
		• Model with mathematics.	• Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem.	4 Advanced
		• Use appropriate tools strategically.	• Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution.	5 Advanced
		• Attend to precision.	• Incorporate appropriate vocabulary and symbols in communicating their reasoning and solution to others.	6 Intermediate
		• Look for and make use of structure.	• Compose and decompose number situations and relationships through observed patterns in order to simplify solutions.	7 Intermediate
		• Look for and express regularity in repeated reasoning.	• Find and explain subtle patterns.	8 Intermediate

Strategy:	Description	Practice	Degree	Matrix Code*
Encouraging Reasoning	Students need to be encouraged to carefully think about mathematics, and to understand their level of knowledge. They also need to be able to accurately communicate their thinking. Reasoning, in this context, is used to convey having students stretch their understanding and knowledge to solve challenging problems. Reasoning requires students to pull together patterns, connections, and understandings about the rules of mathematics, and then apply their insight into finding a solution to a difficult, challenging problem.	• Reason abstractly and quantitatively. • Attend to precision. • Look for and make use of structure. • Look for and express regularity in repeated reasoning.	• Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. • Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. • See complex and complicated mathematical expressions as component parts. • Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function.	2 **Advanced** 6 **Advanced** 7 **Advanced** 8 **Advanced**

* See companion document

Standards of Student Practice in Mathematics Proficiency Matrix

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Standards of Student Practice in Mathematics Proficiency Matrix

	Students:	(I) = Initial	(IN) = Intermediate	(A) = Advanced
1a	**Make sense of problems**	Explain their thought processes in solving a problem one way. *(Pair – Share)* *	Explain their thought processes in solving a problem and representing it in several ways. *(Question/Wait time)* *	Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways. *(Grouping/Engaging)* *
1b	**Persevere in solving them**	Stay with a challenging problem for more than one attempt. *(Question/Wait time)* *	Try several approaches in finding a solution, and only seek hints if stuck. *(Grouping/Engaging)* *	Struggle with various attempts over time, and learn from previous solution attempts. *(Show Thinking)* *
2	**Reason abstractly and quantitatively**	Reason with models or pictorial representations to solve problems. *(Grouping/Engaging)* *	Are able to translate situations into symbols for solving problems. *(Grouping/Engaging)* *	Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. *(Encourage Reasoning)* *
3a	**Construct viable arguments**	Explain their thinking for the solution they found. *(Show Thinking)* *	Explain their own thinking and thinking of others with accurate vocabulary. *(Question/Wait time)* *	Justify and explain, with accurate language and vocabulary, why their solution is correct. *(Grouping/Engaging)* *
3b	**Critique the reasoning of others**	Understand and discuss other ideas and approaches. *(Pair – Share)* *	Explain other students' solutions and identify strengths and weaknesses of the solution. *(Question/W) ait time**	Compare and contrast various solution strategies and explain the reasoning of others. *(Grouping/Engaging)* *
4	**Model with Mathematics**	Use models to represent and solve a problem, and translate the solution to mathematical symbols. *(Grouping/Engaging)* *	Use models and symbols to represent and solve a problem, and accurately explain the solution representation. *(Question/Prompt)* *	Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem. *(Show Thinking)* *

5	**Use appropriate tools strategically**	Use the appropriate tool to find a solution *(Grouping/Engaging)* *	Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. *(Grouping/Engaging)* *	Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution. *(Show Thinking)* *
6	**Attend to precision**	Communicate their reasoning and solutions to others *(Show Thinking)* *	Incorporate appropriate vocabulary and symbols when communicating with others. *(Allowing Struggle)* *	Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. *(Encourage Reasoning)* *
7	**Look for and make use of structure**	Look for structure within mathematics to help them solve problems efficiently (such as 2 x 7 x 5`` which has the same value as 2 x 5 x 7, so instead of multiplying 14 x 5, which is (2 x 7) x 5, the student can flexibly mentally and calculate 10 x 7. *(Question/Prompt)* *	Compose and decompose number situations and relationships through observed patterns in order to simplify solutions. *(Allowing Struggle)* *	See complex and complicated mathematical expressions as component parts. *(Encourage Reasoning)* *
8	**Look for and express regularity in repeated reasoning**	Look for obvious patterns, and use the if/then reasoning strategies for obvious patterns. *(grouping/engaging)* *	Find and explain subtle patterns. *(Allowing Struggle)* *	Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function. *(Encourage Reasoning)* *

* See companion document: *Instructional Implementation Sequence to Engage in Practices*

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Questions to Help Elementary Students become Proficient with the
8 Standards for Mathematical Practice

#1. Make sense of problems and persevere in solving them.

1. What is the question you are trying to answer?
2. What information is important? (consider marking the text/close reading)
3. How is the problem like another problem you have solved?
4. How did you start? Who started a different way? (intentional interruption)
5. What did you do when you were stuck? What else can you try?
6. How did you get that answer? Does it make sense with the problem?
7. Why is that true?

#2. Reason abstractly and quantitatively.

1. Can you summarize this context in a math sentence?
2. Can you write an equation to show this situation?
3. What do the numbers you have here stand for (refer to)?
4. What if you started with ______ instead of ______?
5. Does what you have written down or said make sense of this?
6. What do the symbols that you used mean? (+, –, x, ÷ , =)?

#3. Construct viable arguments and critique the reasoning of others.

1. How can you convince me that your strategy and answer make sense?
2. Do you think your partner's solution makes sense? Why or why not?
3. How can you disagree or ask questions in a friendly, nice, and helpful way?
4. Can you explain to the class why your way (method) works?
5. How would you restate what you just said so more students understand?
6. Can you explain your partner's thinking and answer?
7. Can you restate what your classmate just said? What question do you have for him or her about that answer, strategy, or thinking?

#4. Model with Mathematics

1. How can you represent the situation using a picture, table, manipulatives, counters, expression or equation? (start with word sentences then number sentences)
2. What word and number sentences represent your drawing?
3. What other number sentences that would correctly represent your drawing?
4. Does your expression or equation match the situation? How does it do that?
5. Can you explain how each number and symbol relates to the context?
6. How can you use math (number sentence) to model the problem or context? (e.g., two cookies on each of 5 plates: "5 x 2 = 10")

#5. Use appropriate tools strategically

1. How did you use that tool to help you solve the problem? (picture, sketch, manipulative, string, counter, straight edge, table, graph, ruler, folded paper, color, angle ruler, etc.)
2. Why did you choose that tool?
3. What did the tool help you discover?
4. What tools have you considered using?
5. Are there limits to this tool?
6. Why is this ______ an appropriate tool for this situation?
7. What technology or other tool might be a better for this problem?

#6. Attend to precision.

1. What does this statement (or symbol, term, vocabulary) mean?
2. What new math words did you hear today? How could you use them?
3. Is the method you used an efficient one? How so?
4. Is there a more efficient way?

   (*If a student does not understand why it works, then it is <u>not</u> efficient for him or her*)
5. How could you state your conclusion more precisely?
6. How are these statements (or symbols or words) similar and different?
7. Are the calculations accurate and labeled? Is it accurate enough for this context?

#7. Look for and make use of structure.

1. How can you use what you already know about numbers to help you here?
2. Why is this expression equivalent to this other expression? (3 + 1 = 2 + 2)
3. What pattern exists in your work?
4. Does this problem remind you of another one? How are they alike?
5. What are possible answers for this problem? (none, one, multiple, infinite answers) What information or rules tell you this?
6. Why do you think this works?

#8. Look for and express regularity in repeated reasoning.

1. Can you look at this and get some new ideas?
2. What repetitions do you notice? What does that tell you?
3. How can you generalize?
4. Is there a shortcut you could use? Under what conditions does it work?
5. What conclusion/conjectures would you propose based on these examples?
6. Is this always true? (Here? Never? Sometimes? Always?)How do you know?
7. Are there other problems we could solve using this same strategy?

Questions That Help Grade 6-8 Students become Proficient in the 8 Standards of Mathematical Practice

#1. Make sense of problems and persevere in solving them.

1. What is the question you are trying to answer?
2. How is this problem like another problem you have solved?
3. How did you start? Who started a different way? (intentional interruption)
4. What did you do when you were confused or stuck? What else can you try?
5. How did you reach that conclusion? Why is that true? Does that answer the original question? How does it relate to the original context or question?

#2. Reason abstractly and quantitatively.

1. Can you write words, an expression or equation to show this situation?
2. Does what you have recorded make sense of this situation?
3. What do the numbers, letters, and signs, including equal sign, represent?
4. What if you started with ______ rather than with ______?

#3. Construct viable arguments and critique the reasoning of others.

1. How would you convince me that your strategy and answer makes sense?
2. Do you think your partner's solution makes sense? Why or why not?
3. How can you disagree in a constructive, nice, and helpful way?
4. Can you explain to the class your partner's solution and why it works?
5. How would you restate what your classmate just said?
6. Earlier, a student did this. Right or wrong? What was the thinking?
7. What properties or rules makes this work or not work?
8. What questions do you have for [the student who just explained]

#4. Model with Mathematics

1. Can you use a picture, table or graph to represent the context/problem?
2. How can you use a math sentence to model (represent) this situation? (move from word sentences to a mix of words, numbers, & symbols, then to number sentences)
3. Does your expression or equation (each of the numbers, letters, symbols and quantities in parentheses) match the context?
4. Can a different expression or equation also represent the context?
5. Can you explain to your partner how each piece of this expression relates to the context? Does your partner see it the same way?
6. Can this expression or equation be made into an equivalent one?
   Can it be simplified? Is it possible to simplify it further?

#5. Use appropriate tools strategically

1. How did you use that tool (picture, sketch, manipulative, straight edge, table, graph, ruler, folded paper, color, angle ruler, etc.) to help you solve the problem? Who used a different tool?
2. What technology did you use? Who used different technology?
3. What did the tool help you discover? What are its limits?
4. What other tools or technology could have been considered?
5. Why were these tools appropriate, or not, for this situation or context?
6. Are there better tools or technology for this particular problem?

#6. Attend to precision.

1. What does this statement (the words, symbols, and/or terms) mean?
2. What new math vocab words did you hear of use today?
   What did they describe? How could you use them?
3. Is the method you used an efficient one? How so?
   (If a student does <u>not</u> understand why it works, then it is <u>not</u> efficient for him or her.)
4. Is there a more efficient way to say or do that?
5. How could you state your conclusion more precisely?
6. How are these statements (or symbols or words) similar and different?
7. Is your answer correct & labeled? Is it accurate enough for this context?

#7. Look for and make use of structure.

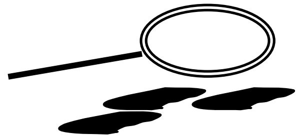
1. What do you already know that will help you solve this?
2. Does this problem remind you of another one? What do you notice or wonder? (This requires deeper thinking than "what is same and/or different")
3. What patterns do you see in this situation or in your work?
4. Why is this expression equivalent to this other expression?
5. What are possible answers for this problem? (zero, one, multiple, infinite many)
   What information tells you this?
6. Where can you use number properties? (Commutative, associative, identity, distributive…...)
7. Why and where do you think this works? Here, never, sometimes, always?

#8. Look for and express regularity in repeated reasoning.

1. What repetitions/iterations/recursive patterns do you notice?
   How do they help you explain the context? How do they help you?
2. How can you generalize that? Can it be applied to other numbers?
3. What conjectures or conclusions would you propose based on this work?
4. Is there a shortcut you could use? Under what conditions does it work?
5. What conjecture/conclusions could be based on what you see?
6. Is this always true? Sometimes true? Never? How do you know?
7. Are there other problems we could solve using this same strategy?

Math Constructive Conversation Skills Poster

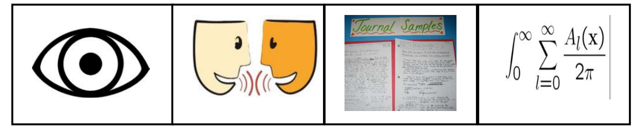
Think Together to Analyze a Problem & Prepare to Solve it



Prompt starters:
What are we trying to do?
What is the problem asking?
How does the problem begin? What happens?
What do we need to know?
How can we break this down?
What type of problem is this?
What patterns do we notice?
What's our plan for solving it?
Can you say what you just did?
What is your estimate for the answer?

Response starters:
In order to ___, we need to …
In other words,
More specifically, it is … because…
Let's see, it is similar to the problem that the teacher did because…
It is important to ____ because
Let's stay focused on ….
Let's get back to the idea of…
In future problems we will remember to…

Use Multiple Approaches & Representations

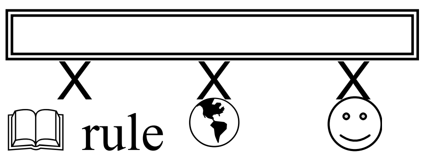


Prompt starters:
How else can we show this?
How can we draw this?
What symbols can we use?
How can we explain this to others?
How can we write what we are thinking/doing?
How can we translate this into math?
Let's back up and try a different way.
Which method is most useful? Why?

Response starters:
Maybe we can use…
Another way to show this is…
In math symbols we could write…
We can draw it like this because it says…
Let's try to… and see what happens.

Problem/Task

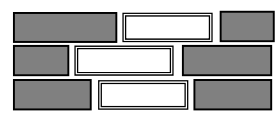
Explain & Support Reasoning

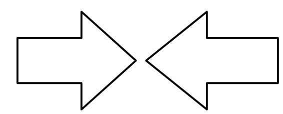


Prompt starters:
Can you explain why you…?
What does that mean?
What are you doing?
What math rule are you using?
Can you give an example?
How does the sample problem help us?
What are examples of this problem from real life?
Can you clarify where you…?
How did you get this answer?

Response starters:
If we ____, then we need to ___ because…
A key mathematical principle is making sure that you…
In real life this is similar to when you want to…
An example from my life is
One case that illustrates this is…
In math, we always need to…
Let me show you what I mean.

Connect to, Build on, & Challenge Partner Ideas





Prompt starters:
How can we add to the idea of…
What do you think about this strategy for solving it?
What else could support this idea?
Do you agree?
What contradicts this? What are other points of view?
What did we learn from doing this problem? How will it help in life?
Let's create a similar problem.

Response starters:
That reminds me of…
I want to add on to your idea of…
That idea connects to…
I see it a different way,
On the other hand, …
That makes me think of…
We can agree that…

Routine Question Sequence

Who got an answer?

Who got a different answer?

Who got a different answer?

Who would like to explain how someone might get one of the answers?

Who got it another way?

How might someone get this one?

Who saw another way?

To tape on an Index Card:

Routine Question Sequence

Who got an answer?

Who got a different answer?

Who got a different answer?

Who would like to explain how someone might get one of the answers?

Who got it another way?

How might someone get this one?

Who saw another way?