A Pedi for the Lady...

And Other Area/Volume Activities

Investigations that provide a foundation for Calculus

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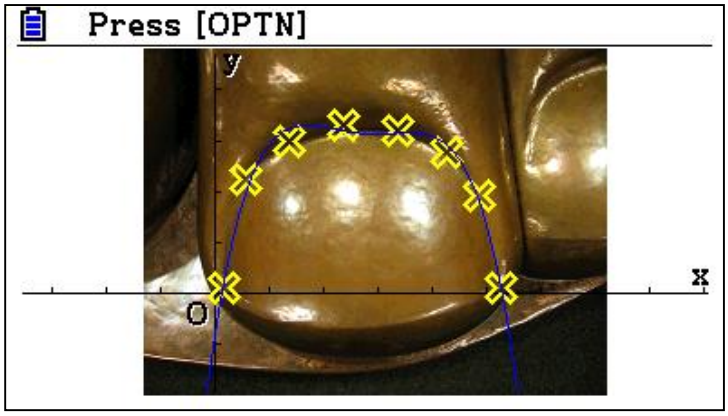
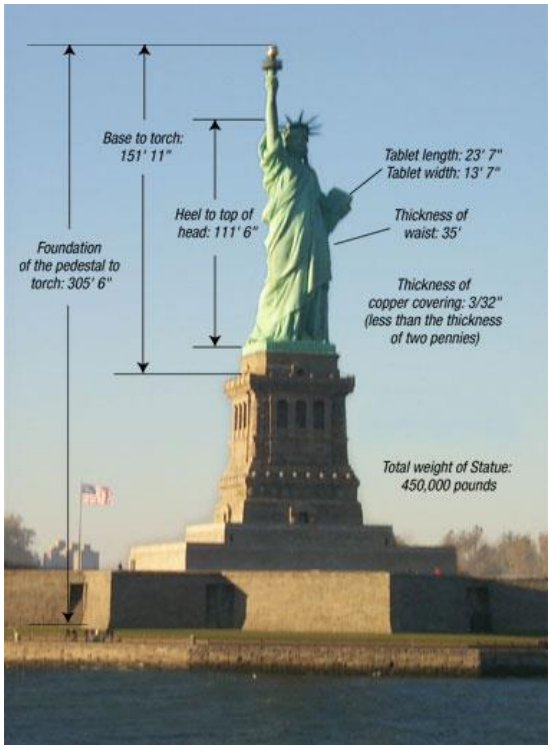
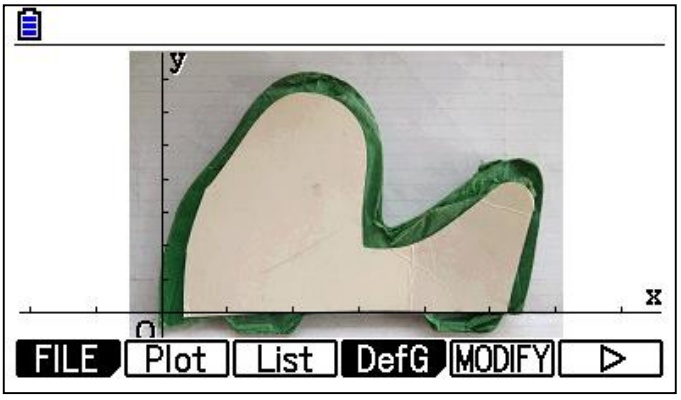
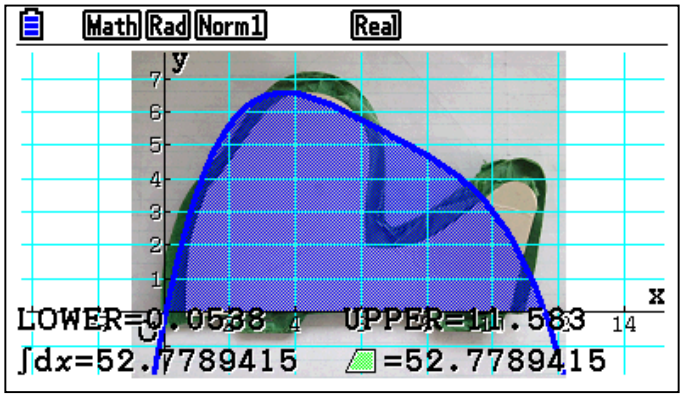
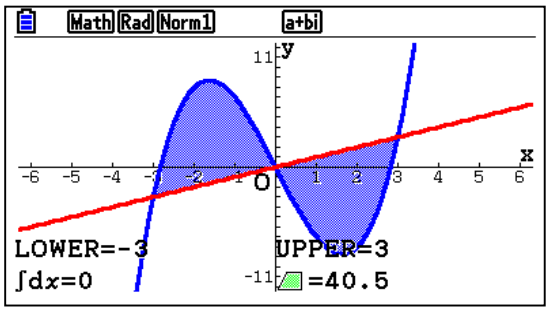




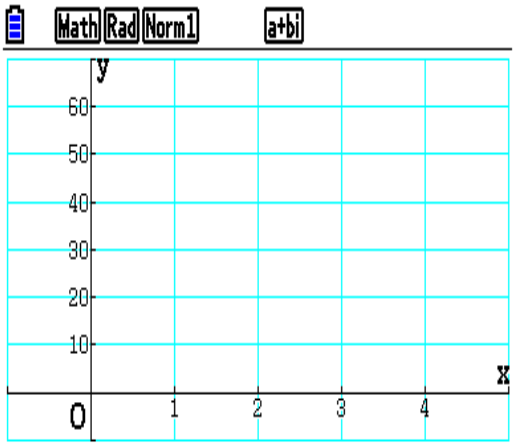






Did you know that D=*RxT* is connected to velocity, integration, and the area under a curve? Let's see how this simple concept is considered to be *CALCULUS!*

1. Texas Pete is on I-20 moving west at the rate of 60 mph. At 1:00, he presses the cruise control on the car and then lets the car travel at this *steady* rate for 3 hours.



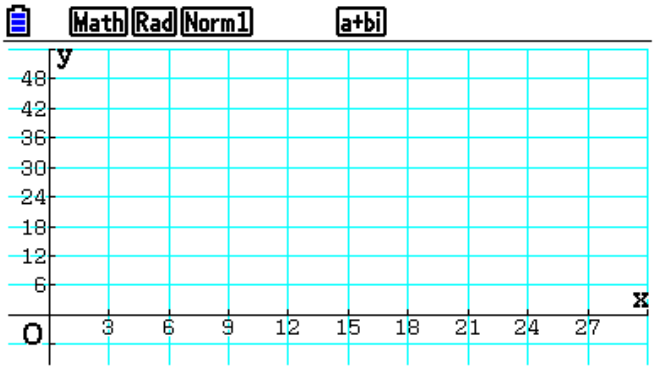
a) Draw a graph for this problem representing the 3 hours from 1:00.

b) Find the area trapped between the graph and the x-axis.

c) Describe the meaning of the area in context to this problem.

d) Find the slope of the graph and describe its meaning with respect to the problem.

2. Professor Ben Dover is going from home to work. It is a straight path from his home to work with no stops along the way. It takes him 6 minutes to get to 48 mph. He then travels at 48 mph for 18 minutes and then it takes him another 3 minutes to come to a stop.



a) Draw the graph of this problem.

b) Find the area under the curve.

c) Determine the meaning of the area in context to the problem.

d) Find the slope of the first six minutes and the last three minutes. What do these numbers represent?

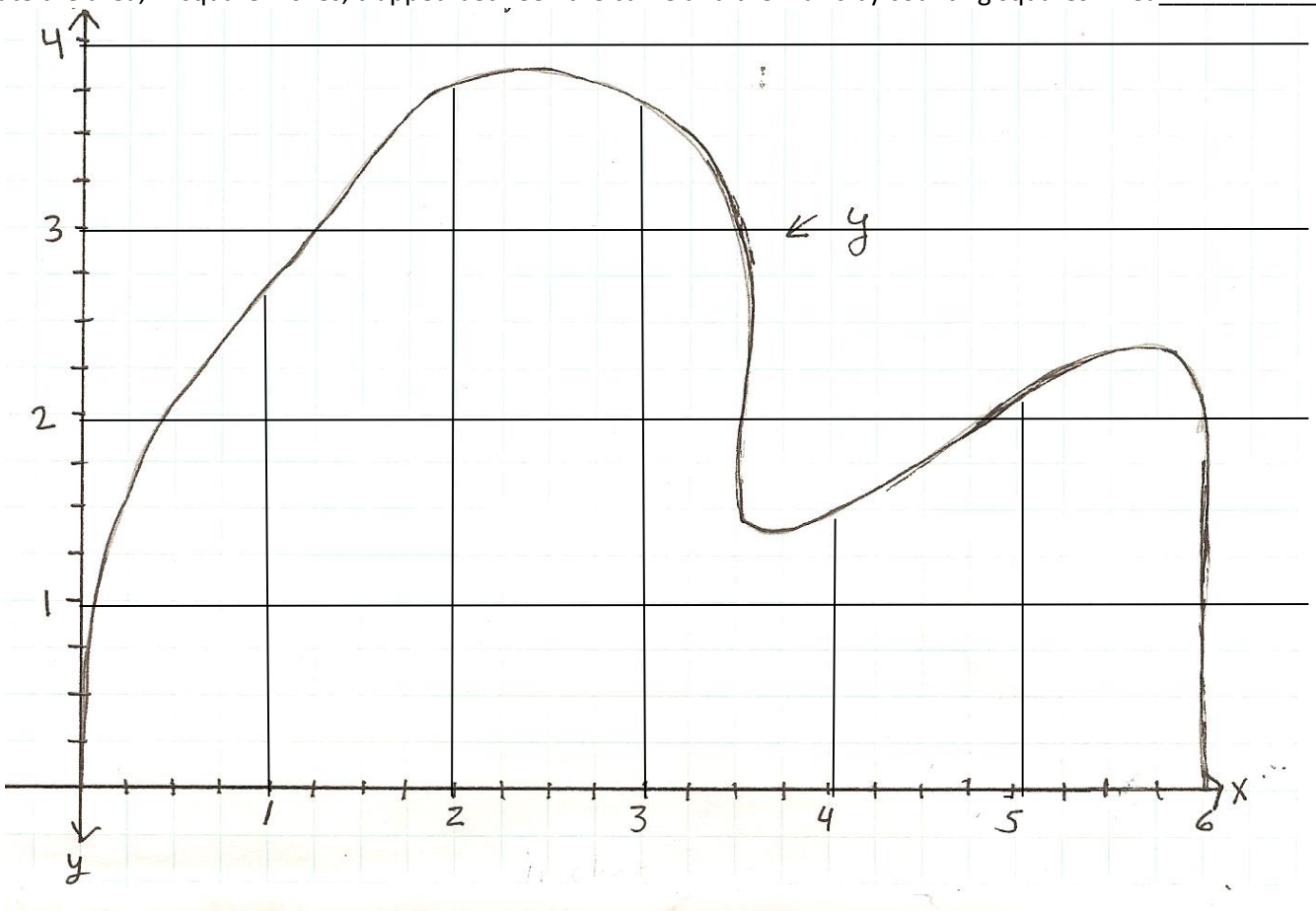
3. Sasquatch is filling his hot tub with hot water using his on-demand water heater. As time passes, the hot water is dispersed more slowly. The table below states the rates at which the heater fills the

hot tub at 20 minute intervals. If the hot tub initially had 10 gallons in it, how many gallons are in it at the end of 120 minutes?

Time min	0	20	40	60	80	100	120
Rate gal/min	50	46	46	40	35	30	20

Slicing a Mushroom: Finding Area and Volume Activity

1. Estimate the area, in square inches, trapped between the curve and the x-axis by counting squares. Area_____



2. Lightly color the **area** covered by using the **Midpoint** Riemann Sum Method with 6 sub-intervals, each 1-inch long. Use a different color for each rectangle. Estimate and record the area covered by this method. _____

3. Find the *polynomial* equation that best fits the mushroom's shape by using the regression feature of your calculator. Regression is located in the **Statistics** mode Use the points from the integer and midpoint values on the graph.

State the value and meaning of r^2 : _____. *Y1*=_____

4. Graph the equation from step 3 using the **Graph** mode.

Find the area by "integrating" between the roots of Y1: **Area** = $\int_a^b Y1\,dx =$ _____

What do the roots represent in context to this graph? _____

Find the area by using this integral: **Area**: $\int_0^6 Y1\,dx =$ _____

What is the significance of 0 and 6 on this integral? _____

5. Compare the calculations for the area. What could be done to make these estimates better?

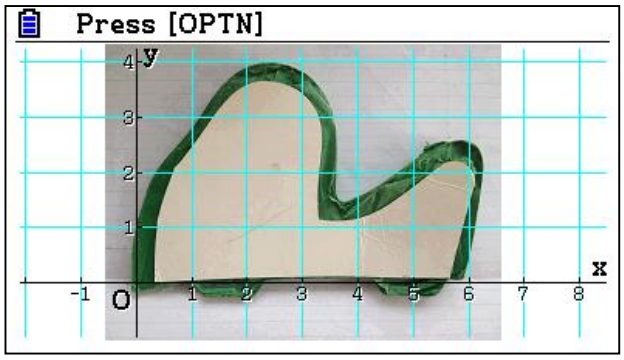
6. Now…. Imagine the shape being spun about the *x-axis* in such a way that it forms a ***solid***. Find the **volume** of this solid using the same 6 slices as in step 2 where the height of the graph at each midpoint represents the length of the radius of each cylinder.

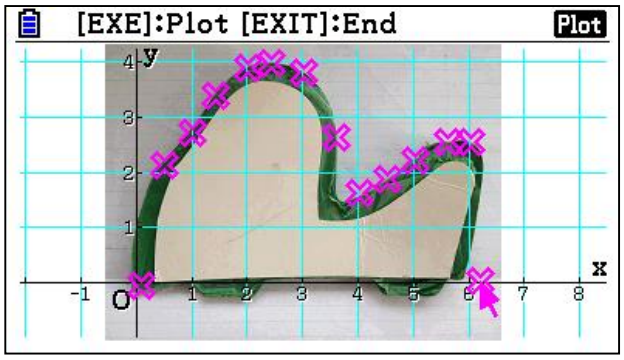
Volume of a cylinder: V=_____ The **Volume** of this figure is _____ by this method.

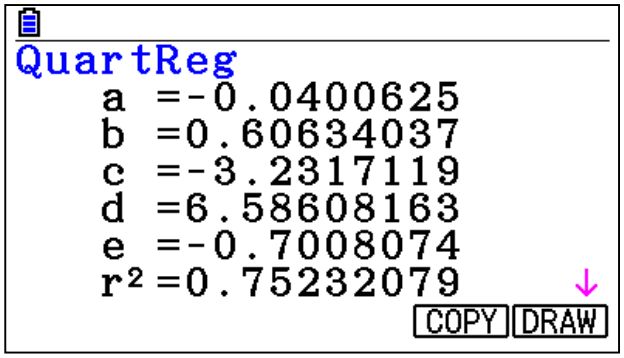
7. Using the equation from step 3, find the volume of this solid by "integrating" between the x-intercepts.

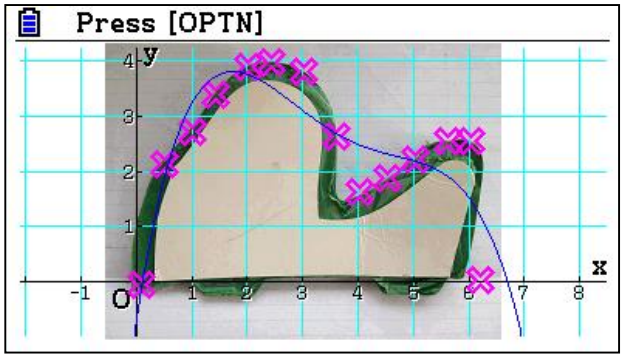
Volume= $\pi\int_0^6 (Y1)^2\,dx =$ _____ What does $Y1$ represent in this expression? _____

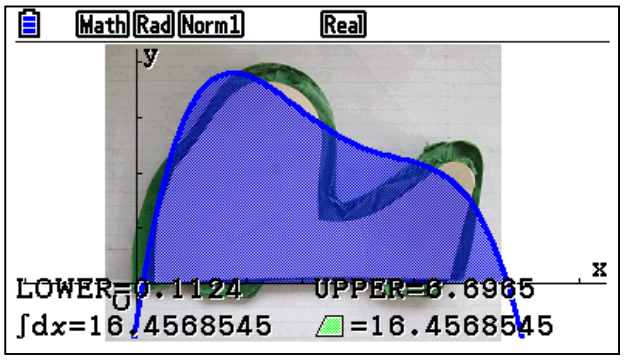
Using the Casio Picture Converter, any picture can be imported, and then many explorations can be done. Which of the following regression equations would be the best estimate of the area? Explain!

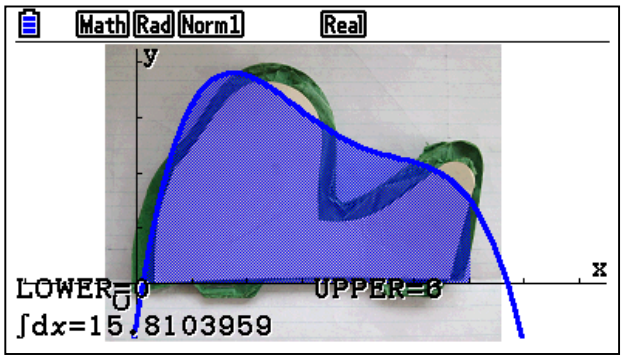












The volume could then be estimated by substituting the quartic equation into $V = \pi \int_0^6 (Y1)^2 \, dx$ and calculating the integral using the **Run** mode. Based on the derived equation for Y1, the volume is approximately 145.651 cubic inches.

How does this compare to your answer in question 6?

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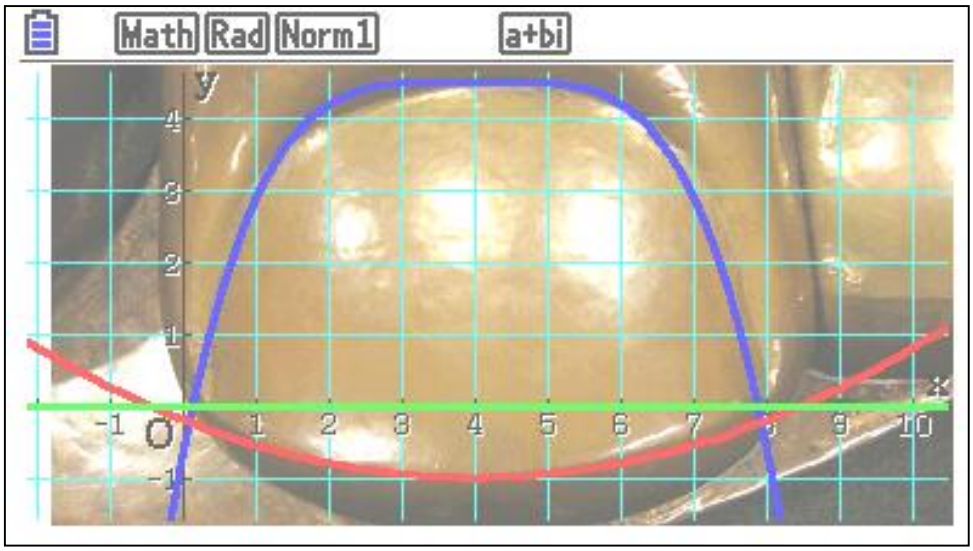

1. Estimate the area in square inches trapped <u>above</u> and <u>below</u> the **<u>QUARTIC</u>** curves outlining the shape of Lady Liberty's big toe nail by counting squares! Each unit in the picture is equivalent to about .45 feet. Round final answers to tenths place.

Area ≈ _____ square <u>units</u> Convert to square feet: Area ≈ _____ square <u>feet</u>

2. Color the area covered by your assigned Riemann Sum Methods using **<u>4</u>** equal sub-intervals.

Right-Hand= $\int_0^8 Y1\,dx \approx$ _____sq. **units**, Left-Hand= $\int_0^8 Y1\,dx \approx$ _____sq**. units**, and Trapezoid= $\int_0^8 Y1\,dx \approx$ _____sq. **units**

Right-Hand= $\int_0^8 Y1\,dx \approx$ _____sq. **feet**, Left-Hand= $\int_0^8 Y1\,dx \approx$ _____sq. **feet**, and Trapezoid= $\int_0^8 Y1\,dx \approx$ _____sq. **feet**

What is the significance of the 0 and 8 on the integral to this problem?_____

What could be done to make this method of estimating more accurate? _____

What *width* would each rectangle approach if an infinite number of rectangles were "squeezed" in between 0 and 8? _____

3. Find **Quartic** equations modeling the **toenail's** shape from above and below of the Statue of Liberty's big toenail. Enter the coordinates where x-values are integers into the regression feature of your calculator. Round values to the thousandth's place.

Upper Curve: Y1= _____ Lower Curve: Y2= _____

(Usually I do not give the values. I have students estimate values from the graph in order to have slightly different answers.)

x	y
0	-.2
1	2.8
2	4.2
3	4.4
4	4.5
5	4.4
6	4.2
7	3
8	-.2

x	y
0	-.1
1	-.6
2	-.8
3	-.9
4	-1
5	-.9
6	-.8
7	-.6
8	-.1

4. Enter an equation that represents the trapped area using the equations from step 3. Find this area by ***integrating*** between the x-coordinates of the intersection points. The x-coordinate of the intersection points provide the lower, a, and upper, b, values for the integral below.

a=_____ & b=_____ Area: $\int_a^b \left[(Y1)-(Y2)\right]dx \approx$ _____

5. An ounce of polish cost about **$25** per ounce. It will cover approximately **<u>20</u>** square feet. Use this information to come up with a reasonable estimate for painting the toenail on the largest toe. Then give an estimate for painting all ten toe nails. Provide justification and work for your estimate.

6. Research and the state 5 interesting facts about the Statue of Liberty that you did not already know.