Developing Mathematical Thinkers

Cathy Seeley (cseeley@utexas.edu)
Senior Fellow Emeritus, Charles A. Dana Center
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DISCUSSION

- Write 3-5 words or phrases that describe the kind of mathematical thinking you want students to be able to do.

From a presentation by Cathy Seeley, 2014

In this session...

- Thinking like a mathematician
- Nurturing talent
- Mathematical Habits of Instruction . . .

From a presentation by Cathy Seeley, 2014

Being a 'Doer' of Mathematics

"The only way to know mathematics
is to ***do*** mathematics."

Paul Halmos, mathematician

"It is pretty hard to ***understand*** mathematics
without ***doing*** some mathematics."

Jordan Ellenberg, mathematician and writer

From a presentation by Cathy Seeley, 2014

Thinking Like a Mathematician

- Doing real mathematics--figuring out hard problems
- Exploration that's sometimes messy;
  trying things that sometimes work, sometimes don't
- Reasoning, explaining and arguing
- Zooming in and zooming out
- Reflecting, considering, analyzing

From a presentation by Cathy Seeley, 2014

What math do all students need?

- The Big Three:
  - **Understand mathematics** (make sense of it)
  - **Do the arithmetic** (skills, facts, procedures)
  - **Use mathematics** (thinking, reasoning, applying, solving a range of problems)
- The New Basics: deep transferable skills for *versatilizing*:
  - Problem solving, reasoning, research, communication, creativity

From a presentation by Cathy Seeley, 2014

Mathematical Thinking

Overarching goal:

Students who can think mathematically . . .

From a presentation by Cathy Seeley, 2014

Premise:

What all students need for their future is as much about how they ***think*** as about what they ***know*** . . .

and helping every student succeed is as much about ***how*** we teach as about ***what*** we teach.

From a presentation by Cathy Seeley, 2014

Math Reasoning Inventory™

mathreasoninginventory.com

Marilyn Burns, PI

Funded by Gates Foundation

https://mathreasoninginventory.com/Home/AssessmentsOverview

DISCUSSION

- How did the teacher find out what Marisa was thinking?
- Had Marisa likely had experience developing mathematical habits of mind?

From a presentation by Cathy Seeley, 2014

The difference between...

- Learning clues, keys, and tricks vs. constructively struggling with good problems
- Learning how to do mathematical procedures vs. learning mathematical habits of mind

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Mind

- **Performing thought experiments**
- **Finding, articulating, and explaining patterns**
- Generalizing from examples; articulating generality in precise language
- Creating and using representations
- **Expecting mathematics to make sense**

From a presentation by Cathy Seeley, 2014

Al Cuoco, E. Paul Goldenberg, June Mark.
"Organizing a Curriculum around Mathematical Habits of Mind."
Mathematics Teacher May 2010

NCTM Process Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

***Principles and Standards for School Mathematics*, NCTM 2000**
(expanded from *Curriculum and Evaluation Standards for School Mathematics*, NCTM, 1989)

From a presentation by Cathy Seeley, 2014

Standards for Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. Reason abstractly **and quantitatively**.
3. **Construct viable arguments; critique others' reasoning.**
4. **Model with mathematics.**
5. Use appropriate tools **strategically**.
6. **Attend to precision.**
7. Look for and make use of **structure**.
8. Look for and make use of **regularity** in reasoning.

From a presentation by Cathy Seeley, 2014

TEKS Process Standards

(A) **apply mathematics** to problems arising in everyday life, society, and the workplace

(B) use a **problem-solving model** that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution

(C) select **tools**, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems

(D) **communicate** mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate

(E) create and use **representations** to organize, record, and communicate mathematical ideas

(F) analyze mathematical relationships to **connect and communicate mathematical ideas**

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

Problem Solving, Applications, Mathematical Modeling

- **Persevere in solving problems**
- Explain to selves; analyze givens, constraints; consider analogous problems; represent the situation in different ways; search for regularity / trends; plan solution path
- **Solve problems that apply the mathematics being learned**
- **Solve problems that may involve mathematics not yet learned**
- Use mathematical modeling to solve problems that may not be well defined; Interpret, reflect, fine-tune the model
- When solving problems, keep an eye on the big picture while also attending to the details (zooming in and out); shift perspective
- **Evaluate the reasonableness of results.**

From a presentation by Cathy Seeley, 2014

Reasoning, Explaining, Justifying

- **Includes quantitative reasoning**; make sense of quantities and relationships in problems
- Make conjectures and explore the truth of conjectures.
- Analyze situations; recognize and use counterexamples.
- **Justify conclusions and communicate/respond to arguments.**
- **Decide whether arguments of others make sense; ask useful questions to clarify or improve the arguments.**

From a presentation by Cathy Seeley, 2014

Representing, Connecting, Communicating Precisely

- Look for commonalities/relationships in similar problems or in mathematical ideas

- Coherently represent a problem; decontextualize and contextualize

- **Communicate precisely; make clear, effective arguments; use clear definitions**

- **Put precision in perspective (finding balance . . .)**

- **Does this make sense?**

From a presentation by Cathy Seeley, 2014

Tools

- Pencil/paper, manipulatives/concrete models, compass, protractor, calculator, spreadsheet, CAS, software, . . .
- Find and use external information (Internet, research, etc.)
- **Mental math**
- **Make decisions and understand limitations**

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Mind--Common Themes

- Solving problems
- Thinking, Reasoning, and Reflecting
- Discussing and Communicating
- Justifying and Explaining
- Generalizing from Patterns
- Connecting
- Making sense
- Patience, persistence

From a presentation by Cathy Seeley, 2014

DISCUSSION: Potential

- How smart do you think Marisa is?
- Why don't some students reach their mathematical potential?

From a presentation by Cathy Seeley, 2014

Factors to consider

- Student factors: Motivation, intelligence, beliefs
- Teacher factors: Knowledge, expectations, beliefs
- Instructional factors
  - Nature of the task
  - Opportunities to struggle, think, figure things out, discuss
  - A classroom environment of trust, collaboration, respect, and (eventual) success, where perseverance and constructive feedback of each other are valued

From a presentation by Cathy Seeley, 2014

Intelligence

- Fixed vs. growth mindset . . .
  (Carol Dweck, *Mindset*, 1999)

- Your mindset influences confidence, perseverance, and your willingness to take risks

- From brain research:
  The activities a person engages in can change their intelligence.

- Who determines the activities a student engages in?

From a presentation by Cathy Seeley, 2014

Targeting beliefs with action

- Students' beliefs matter.
- Teachers' beliefs and actions matter.
- Modest interventions make a difference.

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High Expectations means...

- Challenging our habits and beliefs
- Setting challenging standards for all students
- Doing whatever it takes for students to achieve the standards
- Never thinking in advance that you know where they're headed or what they need
- Making sure they all get to struggle and succeed

From a presentation by Cathy Seeley, 2014

Answer-getting vs. learning mathematics

- USA:
  How can I teach my kids to get the answer to this problem?

- Japanese:
  How can I use this problem to teach the mathematics of this unit?

  – Devised methods for slowing down, postponing answer-getting

Phil Daro, 2012

From a presentation by Cathy Seeley, 2014

The difference between Japan and the US

- "You quit teaching too soon and go on to the next thing."
- "We finish."
- Finishing happens when students have learned.
- And learning is incomplete if students aren't developing mathematical thinking.

From a presentation by Cathy Seeley, 2014, Thanks to Phil Daro

Marisa didn't get to finish...

From a presentation by Cathy Seeley, 2014

Upside-down teaching

- From: *"I - We - You"*
- To: *"You - We - I"*
- *Or: "You - Y'all - We - I"*

Thanks to Phil Daro, Deborah Ball, Magdalene Lampert, and Cathy Seeley

From a presentation by Cathy Seeley, 2014

Upside-down teaching

- Start with a rich problem
- Engage students in dealing with the problem, constructively struggling with the problem and the mathematics
- Students discuss, compare, interact, question
- Teacher helps students connect and notice what they've learned

From a presentation by Cathy Seeley, 2014

Mathematical Habits of Instruction

- Use a problem-centered, upside-down teaching model
- Use appropriate technology appropriately
- Learn (and help students learn) to zoom out, zoom in, and go back and forth
- Help students learn to notice and use patterns, connections, and properties within and across mathematical topics and problems (mathematical structure)
- Use formative assessment to pay attention to learning

From a presentation by Cathy Seeley, 2014

Achievement Gap

From a presentation by Cathy Seeley, 2014

Untapped Potential

From a presentation by Cathy Seeley, 2014

What if we raise the floor AND the ceiling?

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Two Sides of Untapped Potential

- Bringing up all students to achieve their highest levels of mathematics and science--raising the floor
- Identifying the stars
- Raising the ceiling and letting them soar
- Untapped potential within each student, within groups of students, and at the school, district, state and national level--potential we haven't reached . . . YET.

From a presentation by Cathy Seeley, 2014

Untapped Potential

From a presentation by Cathy Seeley, 2014

Unlimited Potential

From a presentation by Cathy Seeley, 2014

Even our *best* students...

...will benefit from a strong, diverse, engaging, relevant classroom.

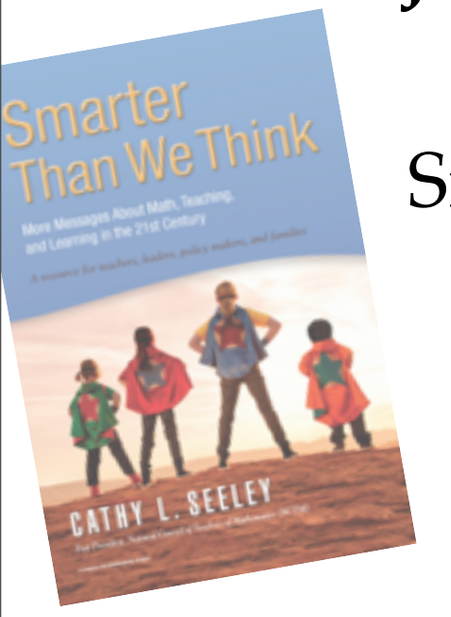
From a presentation by Cathy Seeley, 2014

Their future is in our hands

...and ours is in theirs

From a presentation by Cathy Seeley, 2014

E-mail for a copy of the slides: cseeley@utexas.edu



Just published April 2014: *Smarter Than We Think*

Messages from today...

Smarter Than We Think*, Upside-Down Teaching*,
Clueless, Mathematical Habits of Mind*,
Mathematical Habits of Instruction

Book Signing at Educators Outlet 12:15 today

Faster Isn't Smarter--

Messages About Math, Teaching, and Learning in the 21st Century

Seeley 2009

http://mathsolutions.com/fasterisntsmarter *(Download 5 messages)*

Cathy's websites:

http://cathyseeley.com http://csinburkinafaso.com