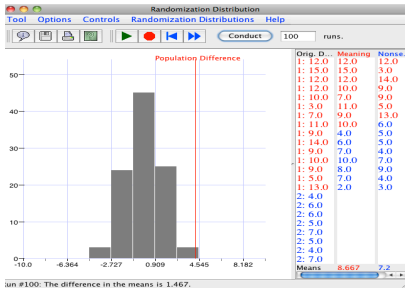
Making and Supporting Statistical Inference using Core Math Tools

2012 NCTM Annual Meeting & Exposition in Philadelphia, PA
Session 337
Patrick Hopfensperger
University of Wisconsin – Milwaukee
hopfensp@uwm.edu



Agenda for the Session

Overview of Core Math Tools

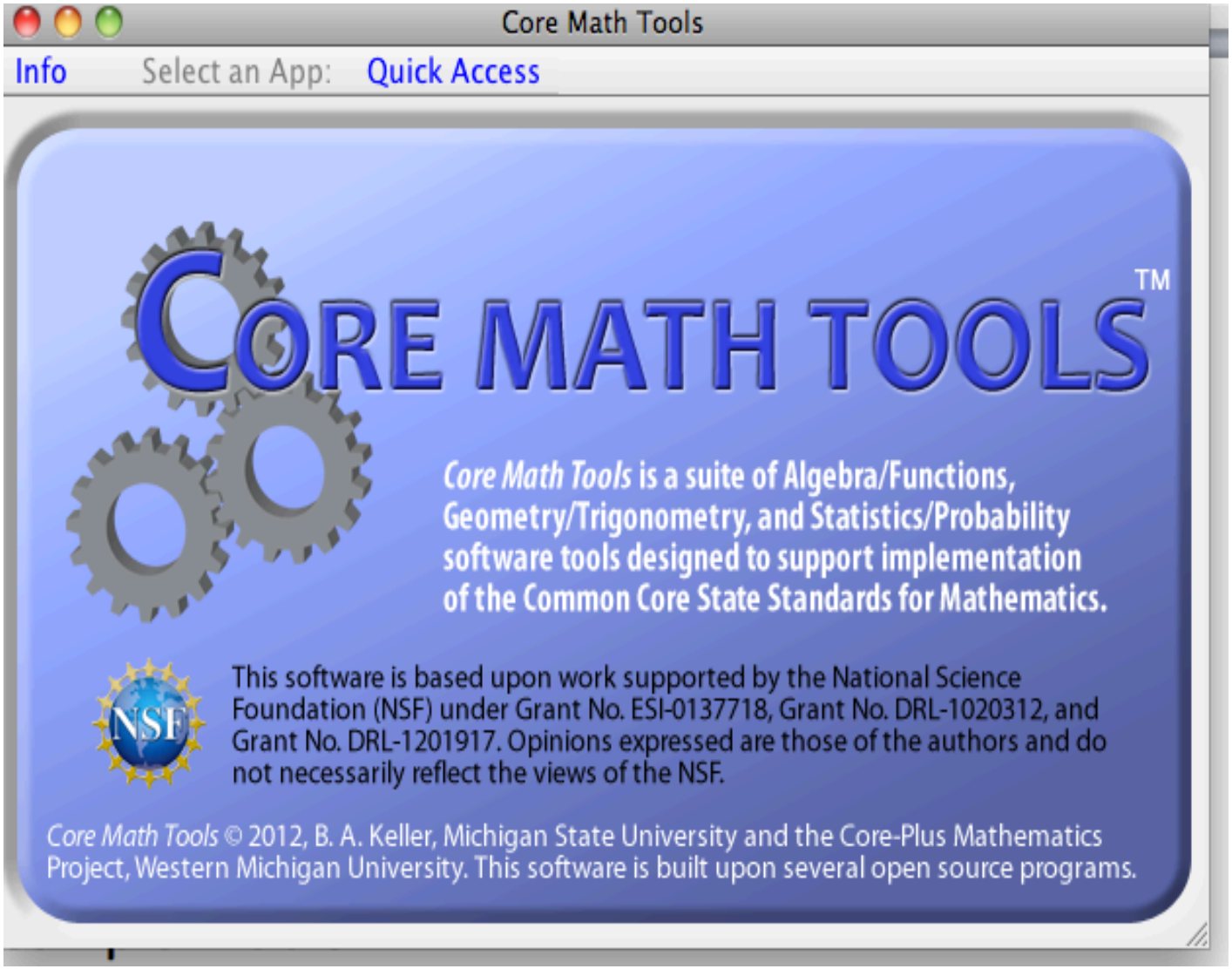
A brief look at the Simulation Application Tool

A brief look at the Statistics & Probability Tool

Mini-Lesson

Meaningful vs. Nonsense Words

Core Math Tools



Core Math Tools Use

Use by Teachers and Students:

Core Math Tools can be saved on computers and USB drives, making it possible to use them without internet access. Files can be saved and reloaded by students and teachers. Its portability allows easy access for students, teachers and parents outside the classroom. Core Math Tools will automatically check for updates when launched and Internet access is available.

www.nctm.org/coremathtools

Core Math Tools Home

Core Math Tools is a downloadable collection of interactive and dynamic software tools for algebra and functions, geometry, trigonometry, and statistics and probability. The tools are appropriate for use with any high school mathematics curriculum and compatible with the Common Core State Standards for Mathematics in terms of content and mathematical practices. Links in the chart below connect to particular components in the Core Math Tools Suite.

General Purpose Tools
CAS, Spreadsheet, Interactive Geometry, and Data Analysis

Custom Apps
Designed for focused explorations of specific topics

Advanced Apps
Designed for focused explorations of specific topics

Sample Lessons
Plans for lessons that use *Core Math Tools*

Data Sets
Downloadable data sets organized by data type

How To Pages
Hints and steps to do basic tasks with *Core Math Tools*

General Purpose Tools

Core Math Tools

Info Select an App: Quick Access

Algebra & Functions

CAS

Produce tables and graphs of functions, manipulate symbolic expressions, and solve equations and inequalities

Spreadsheet

Use familiar spreadsheet functions, insert class data or data from other sources, and employ data transformations.

Geometry & Trigonometry

Synthetic

Construct, measure, manipulate, transform, and animate geometric figures

Coordinate

Construct, measure, manipulate, transform, and animate geometric figures in a coordinate plane

Statistics & Probability

Data Analysis

Graphically display and analyze univariate and bivariate data

Simulation

Create and run simulations of probabilistic situations

General Purpose Tools

CAS, Spreadsheet, Interactive Geometry, Data Analysis and Simulation

Custom Apps

Focused exploration of specific topics such as triangle congruence conditions, sampling distributions, and linear programming

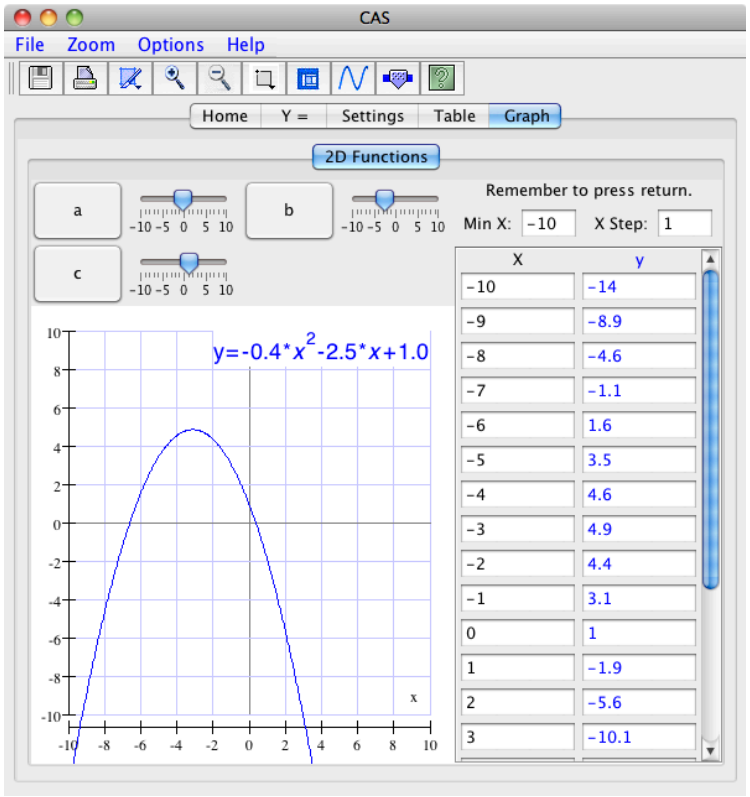
Advanced Apps

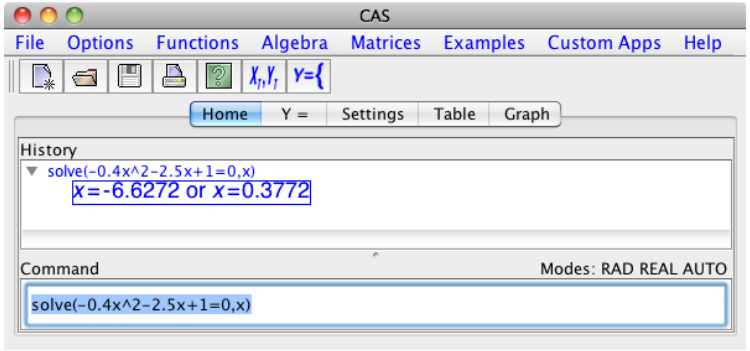
Tools for exploring post-CCSSM topics including vertex-edge graphs, contour diagrams, difference quotients, and cryptography

Three Families of Software

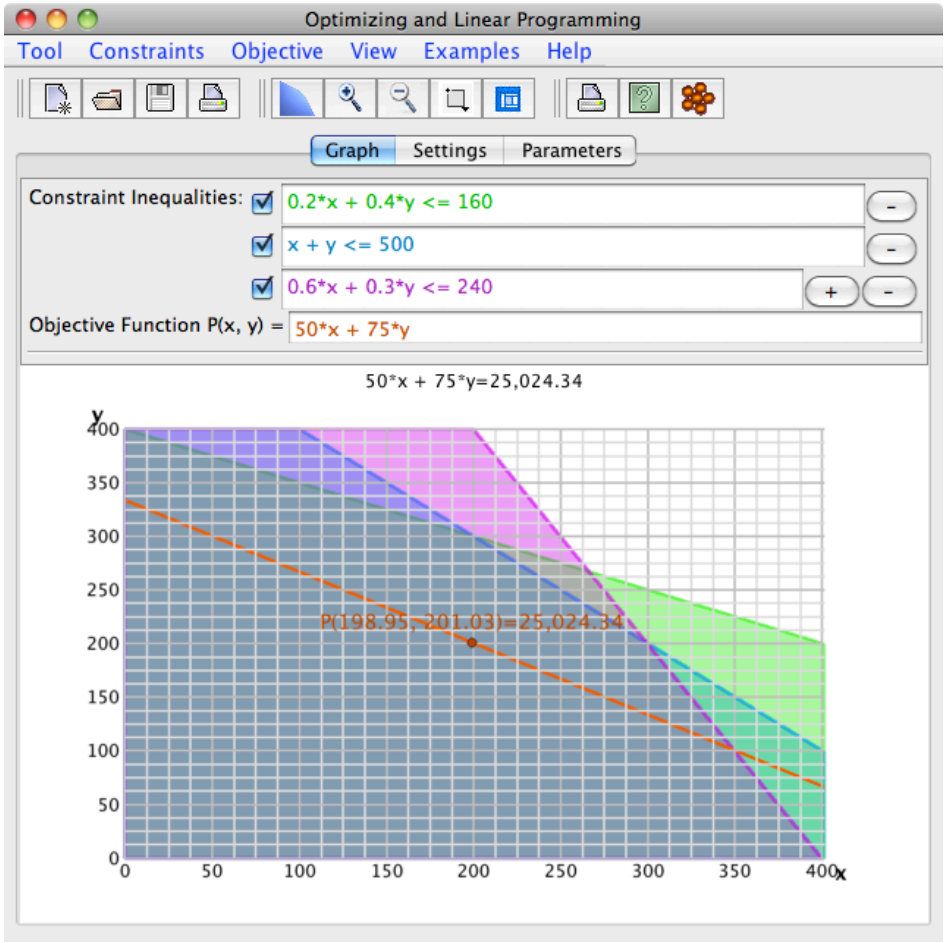
Algebra & Functions—The software for work on algebra problems includes an electronic spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities.

Algebra tools include an electronic spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities;



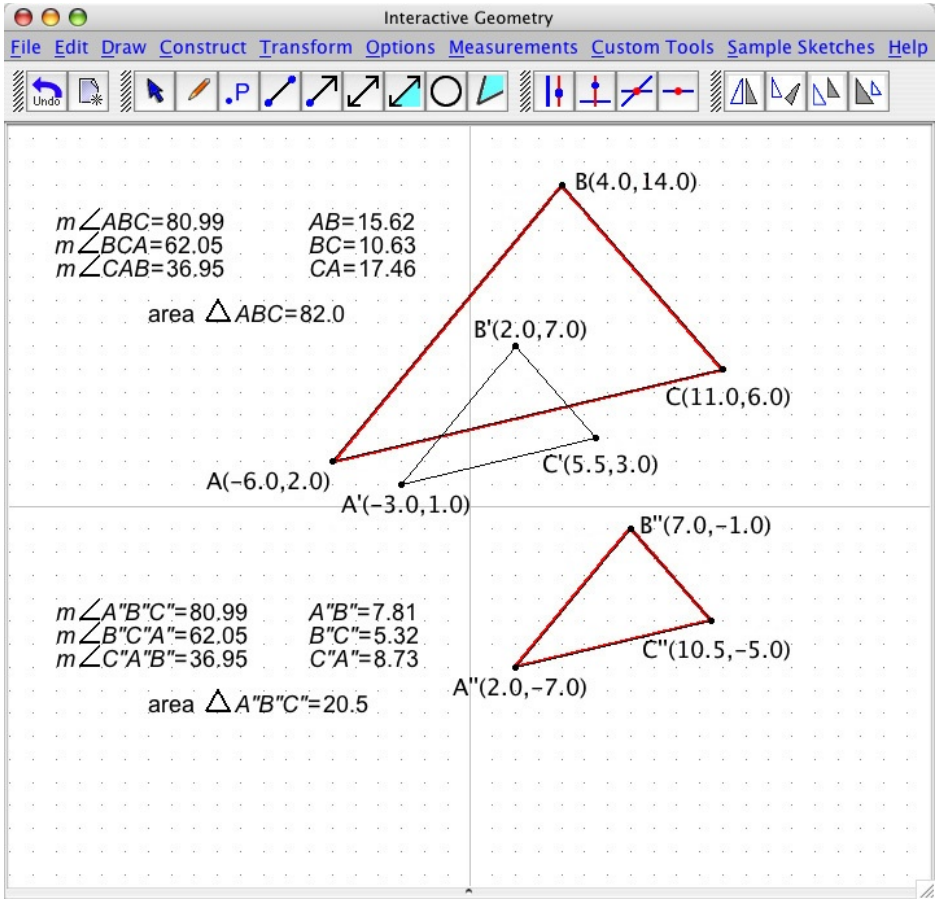


and custom apps supporting mathematical modeling.

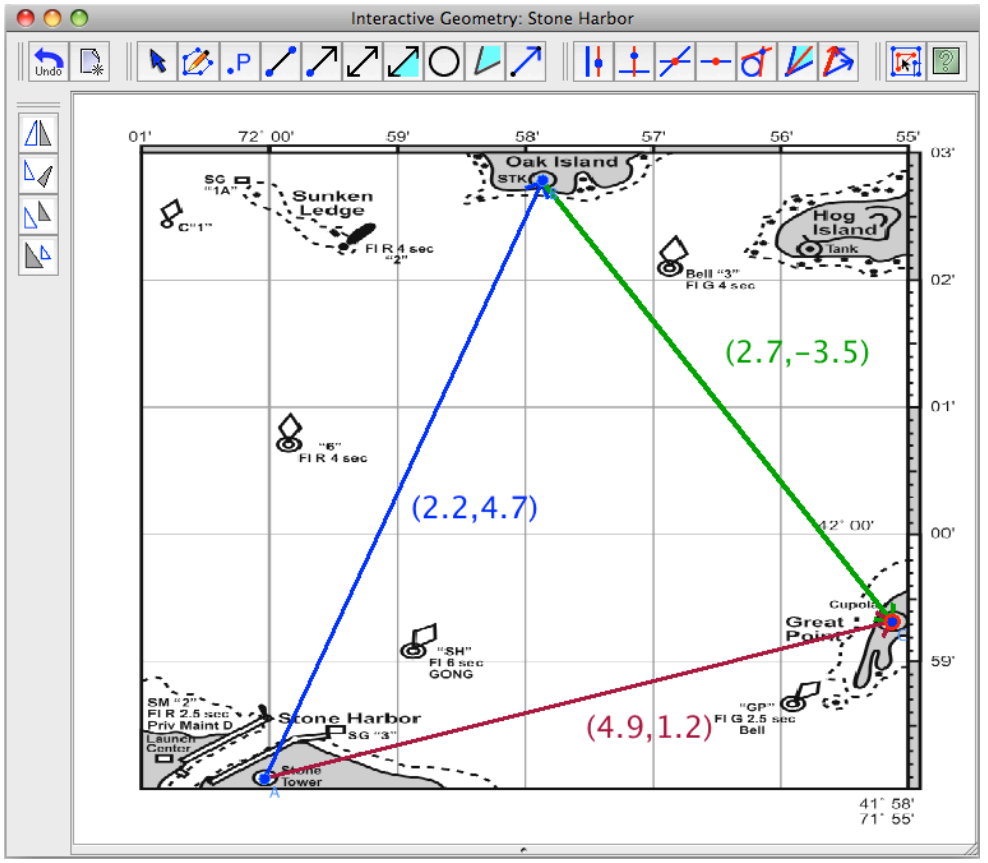


- ***Geometry & Trigonometry***—The software for work on geometry problems includes an interactive drawing program for constructing, measuring, and manipulating geometric figures and a set of custom apps for exploring properties of two- and three-dimensional figures.

Geometry tools include an interactive drawing tool for constructing, measuring, manipulating, and transforming geometric figures, a simple object-oriented programming language for creating animation effects,

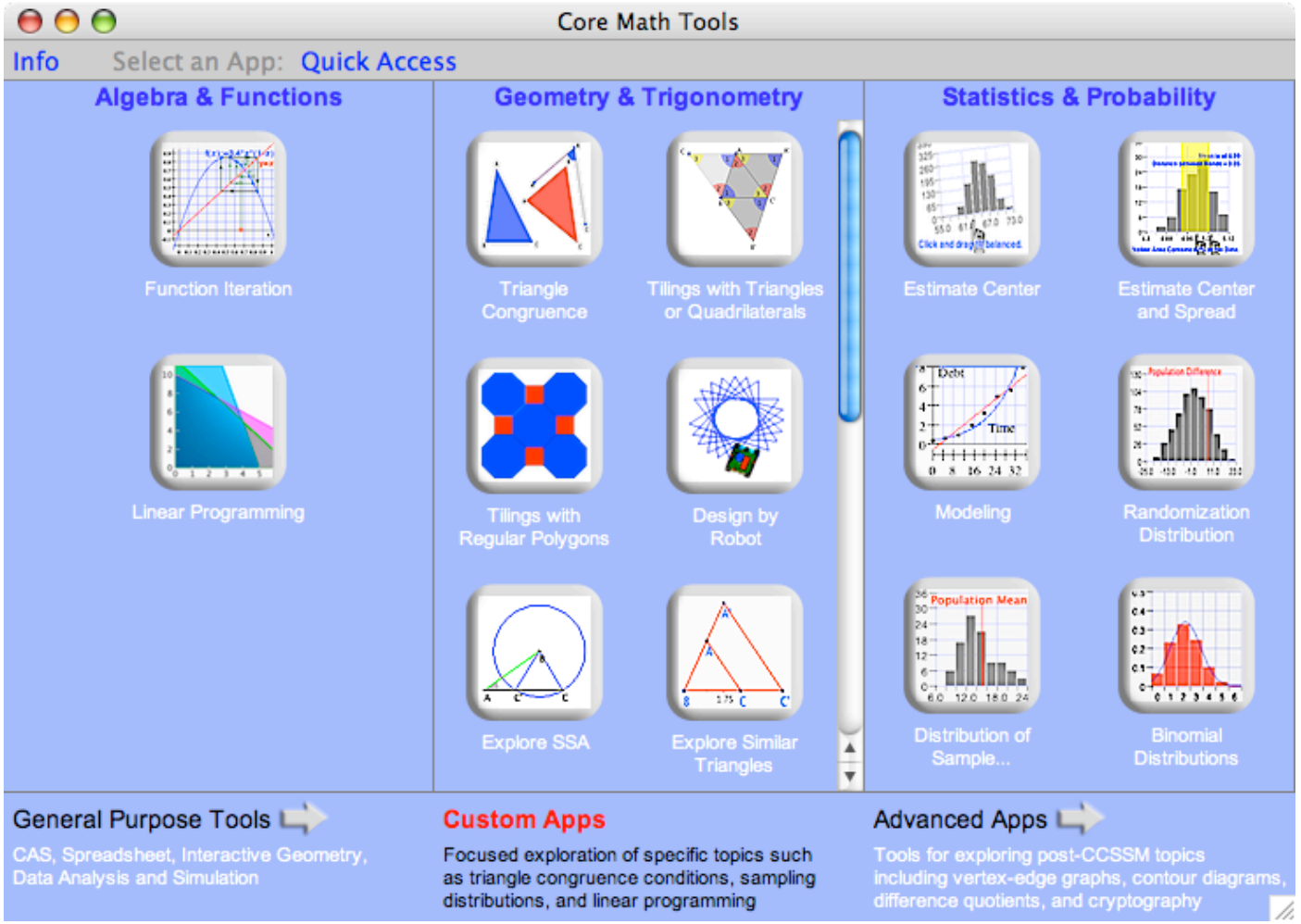


and custom apps for studying geometric models of contextual situations, physical mechanisms, tessellations, and special shapes.

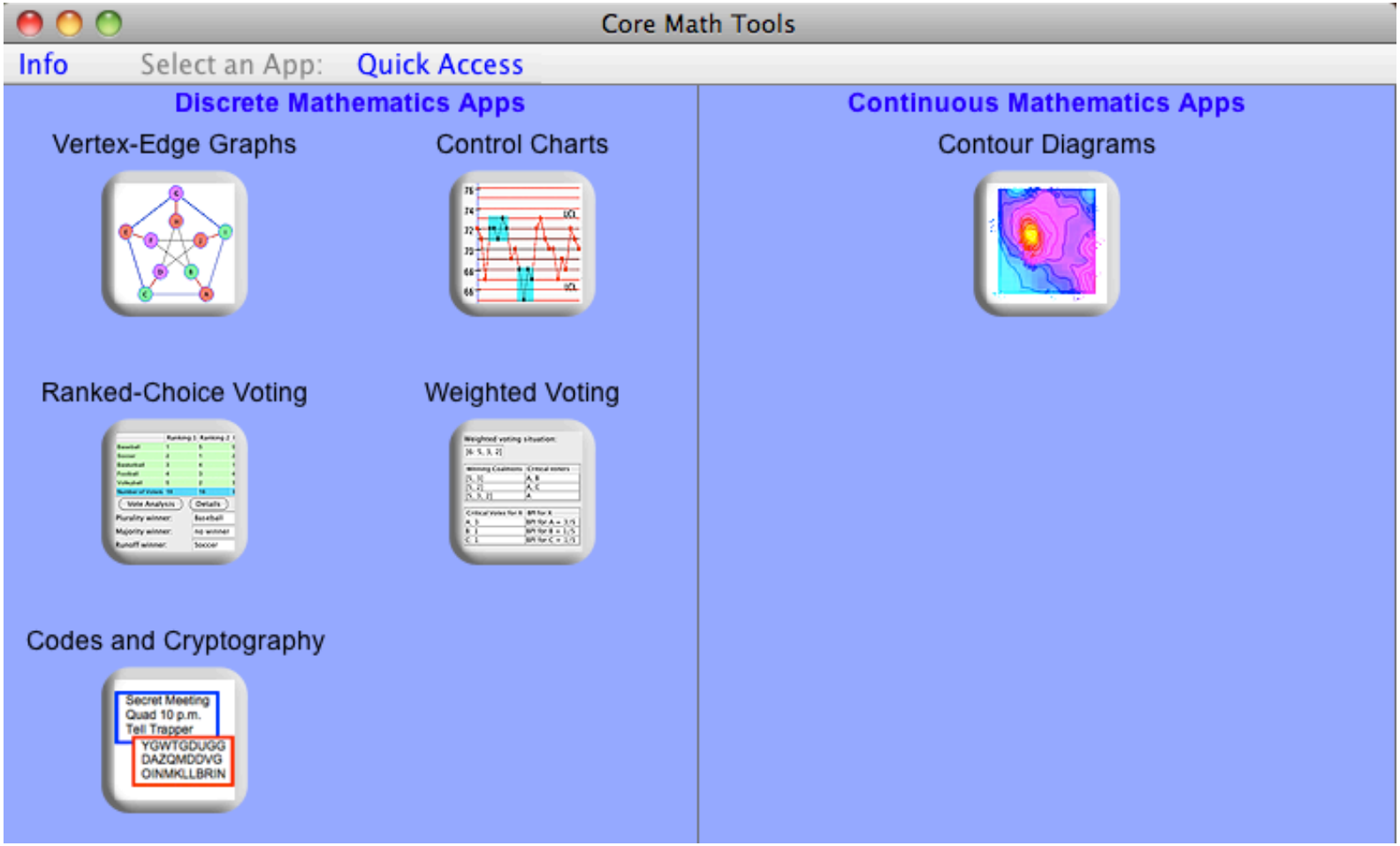


- ***Statistics & Probability***—The software for work on data analysis and probability problems provides tools for graphic display and analysis of data, simulation of probabilistic situations, and mathematical modeling of quantitative relationships.

Custom apps

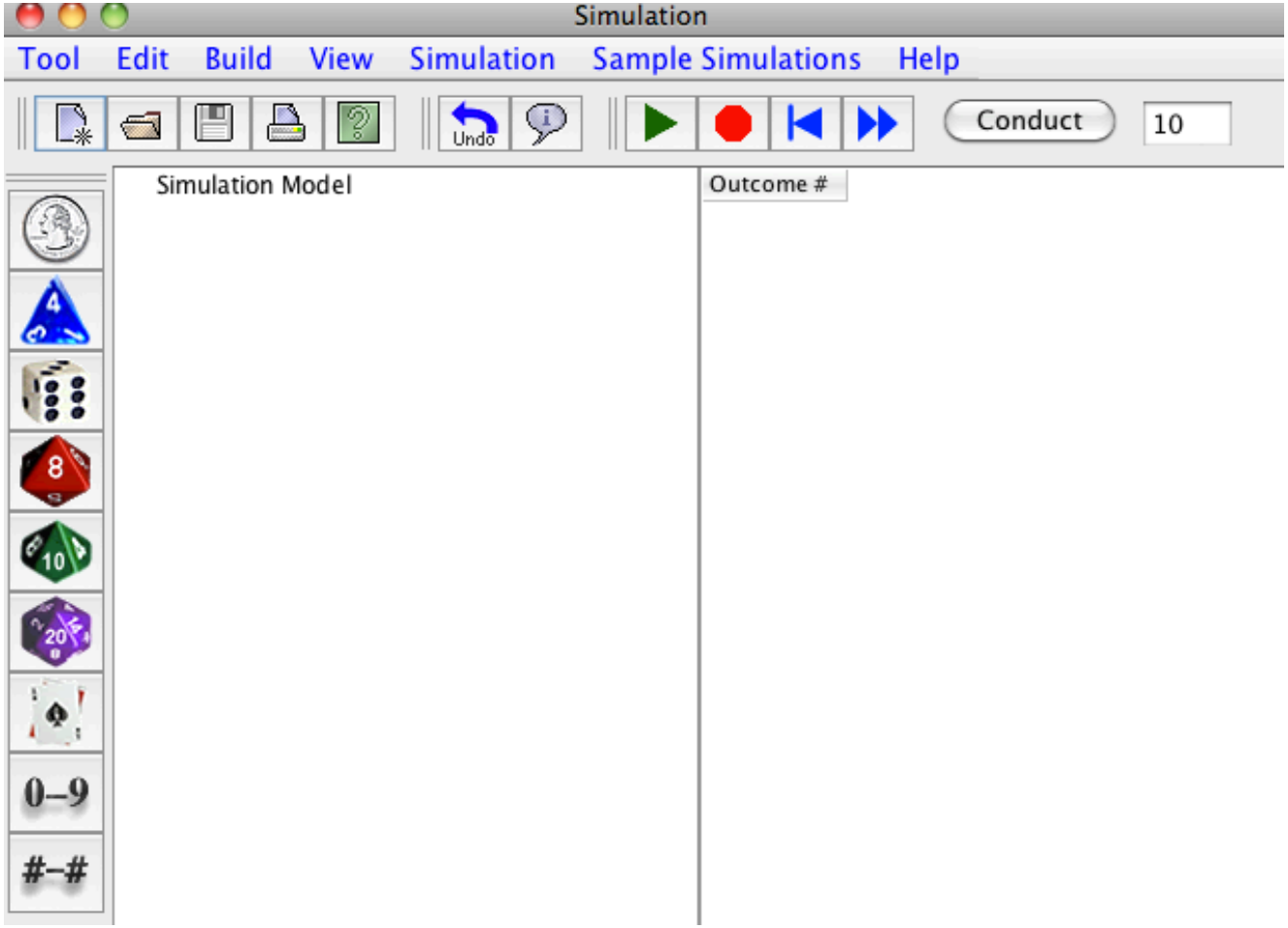

Core Math Tools
Info
Select an App: Quick Access
Algebra & Functions
Function Iteration
Linear Programming
Geometry & Trigonometry
Triangle Congruence
Tilings with Triangles or Quadrilaterals
Tilings with Regular Polygons
Design by Robot
Explore SSA
Explore Similar Triangles
Statistics & Probability
Estimate Center
Estimate Center and Spread
Modeling
Randomization Distribution
Distribution of Sample...
Binomial Distributions
General Purpose Tools
CAS, Spreadsheet, Interactive Geometry, Data Analysis and Simulation
Custom Apps
Focused exploration of specific topics such as triangle congruence conditions, sampling distributions, and linear programming
Advanced Apps
Tools for exploring post-CCSSM topics including vertex-edge graphs, contour diagrams, difference quotients, and cryptography

Advanced Apps

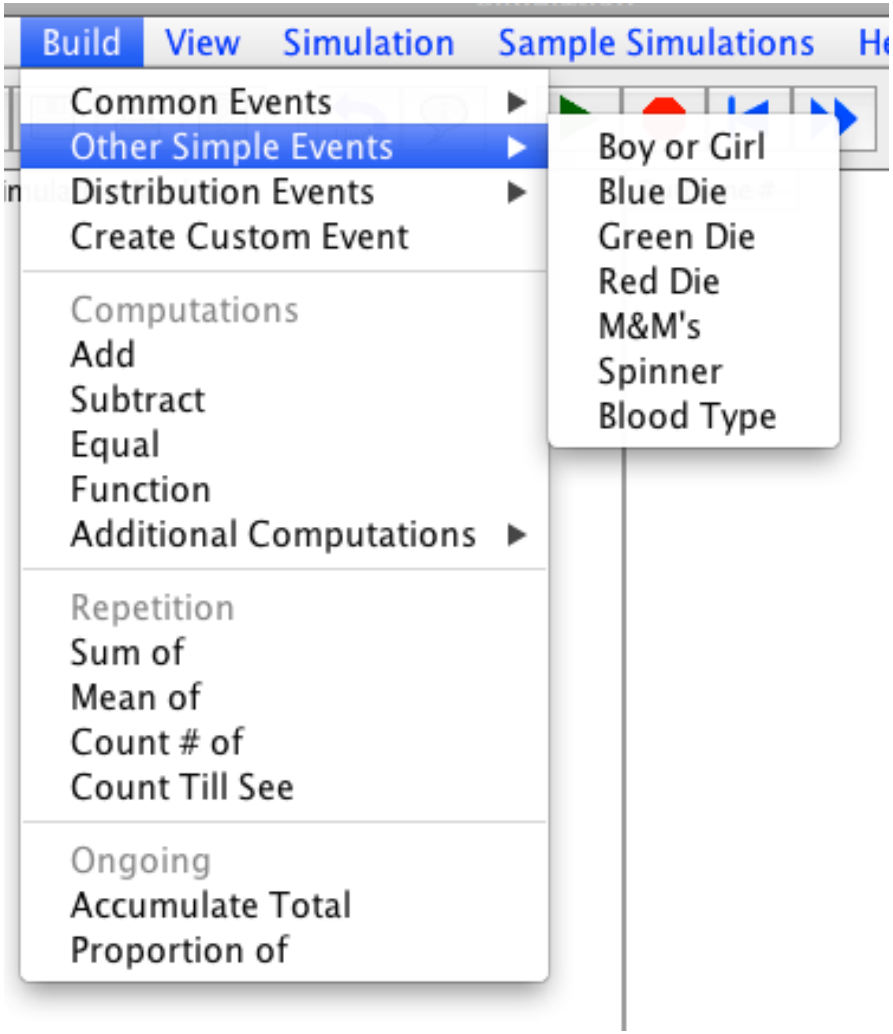


Simulation Tool

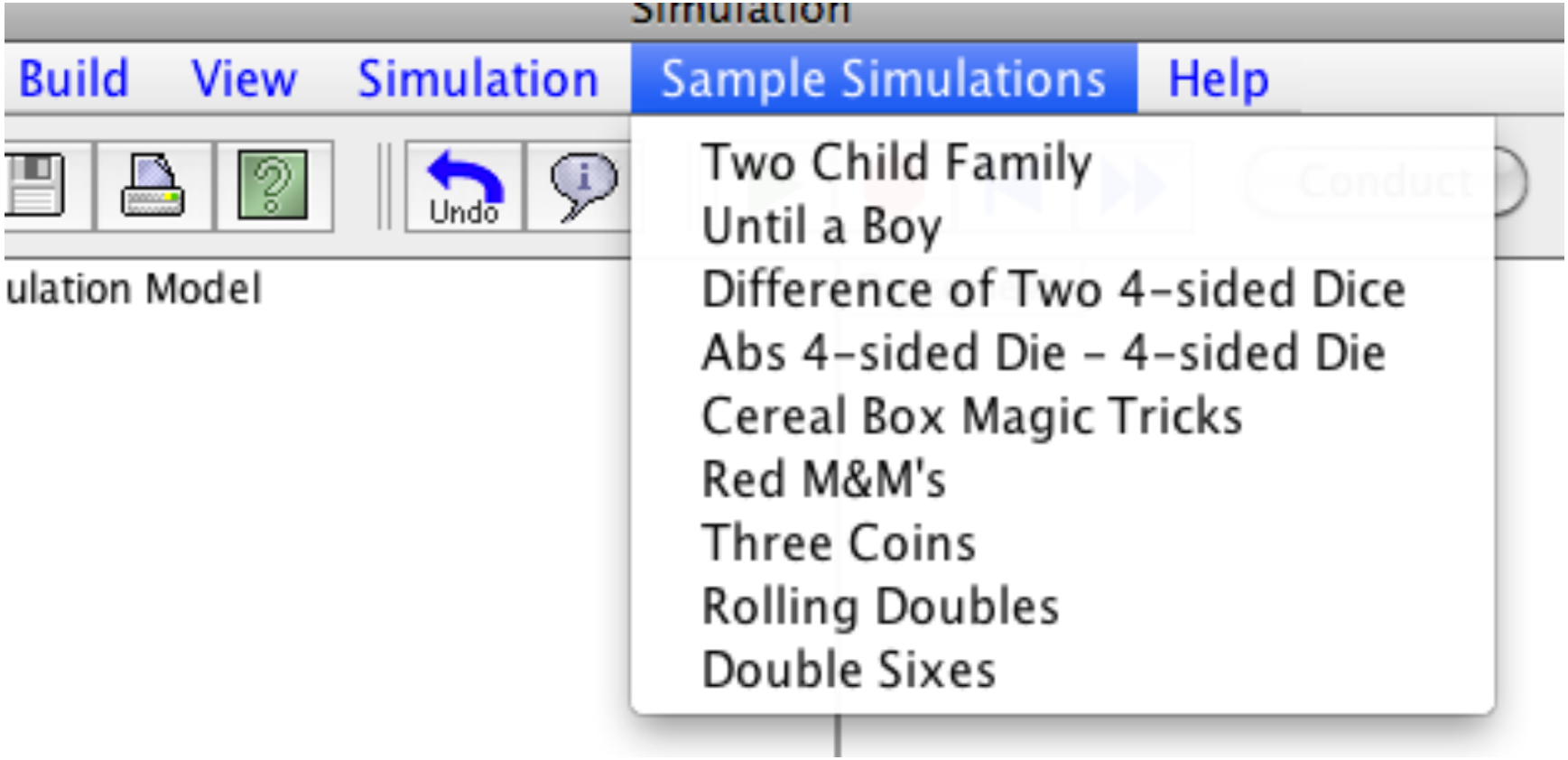
Simulation


Simulation
Tool
Edit
Build
View
Simulation
Sample Simulations
Help
Undo
Conduct
10
Simulation Model
Outcome #
0–9
#–#

Simulation menus

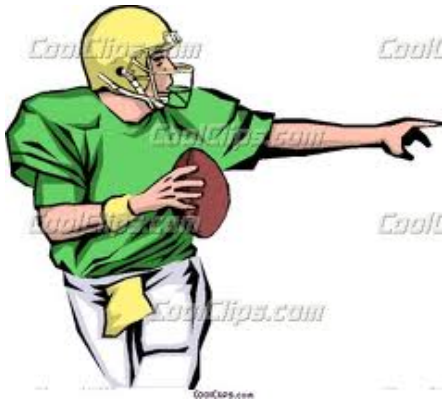


Simulation Menus

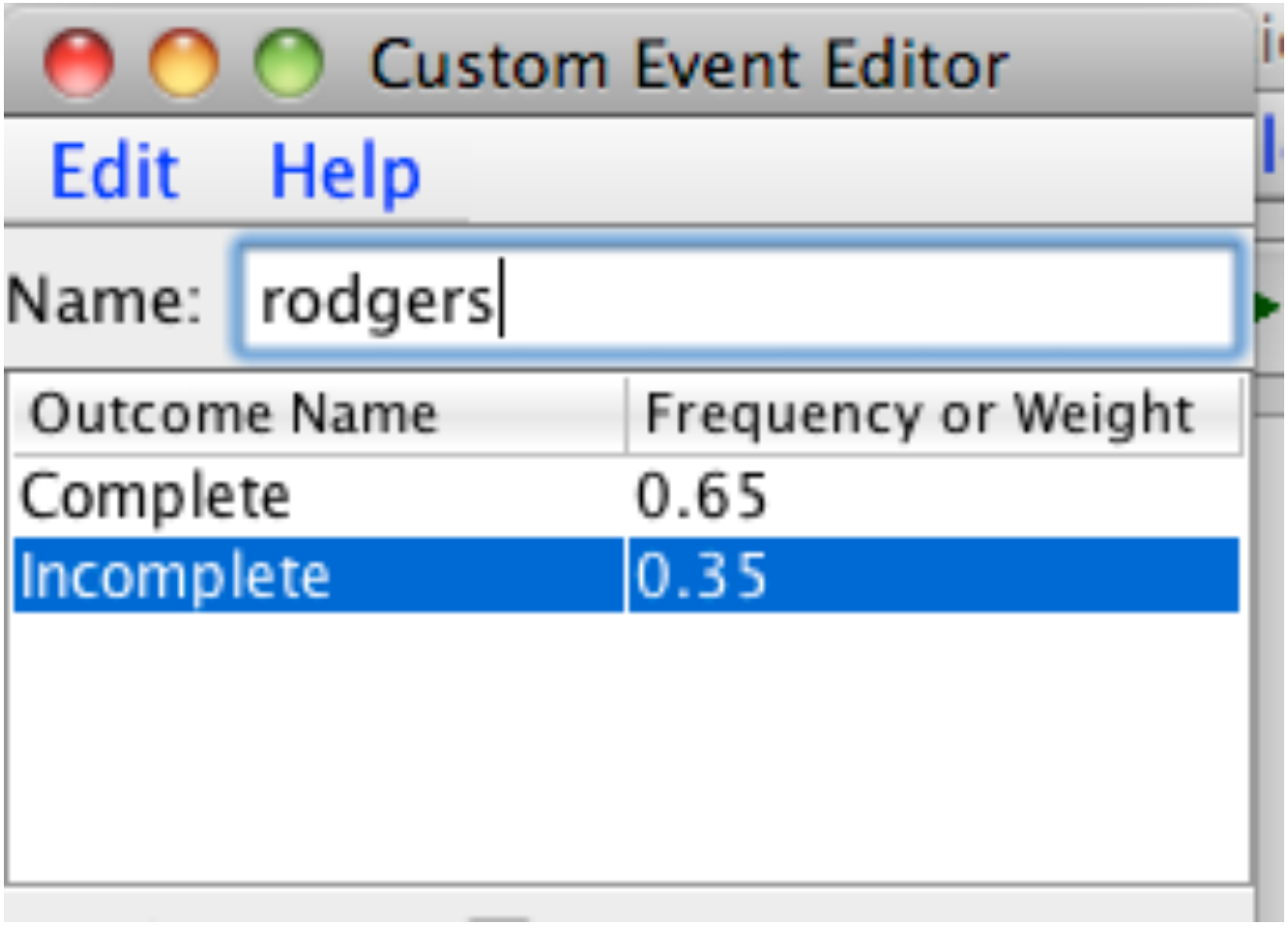


Example

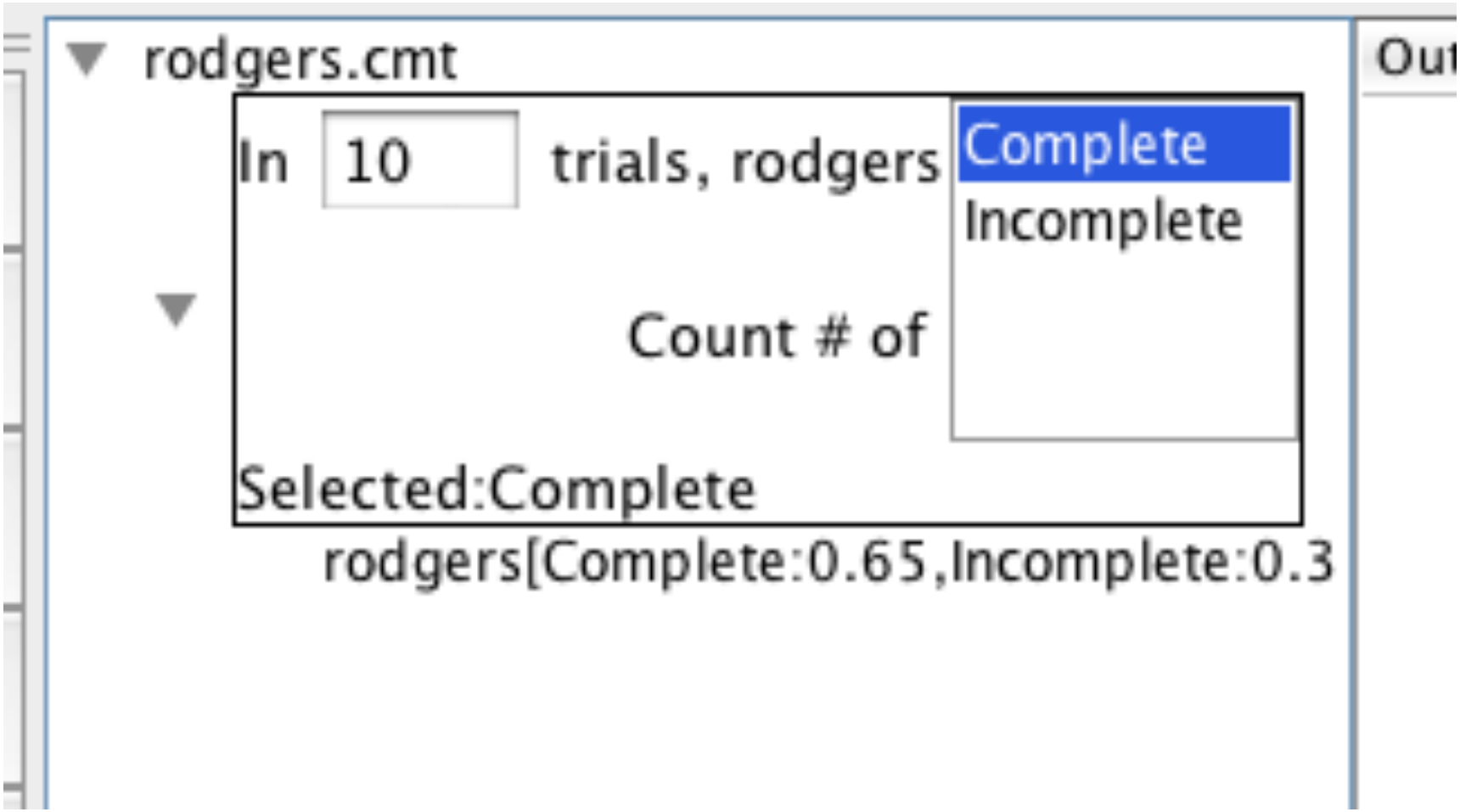
Aaron Rodgers – QB for Green Bay Packers completes about 65% of passes that he throws. Suppose he makes 10 passes in a game. Estimate the probability that he completes at least 7 of the 10 passes.



Custom Event Editor



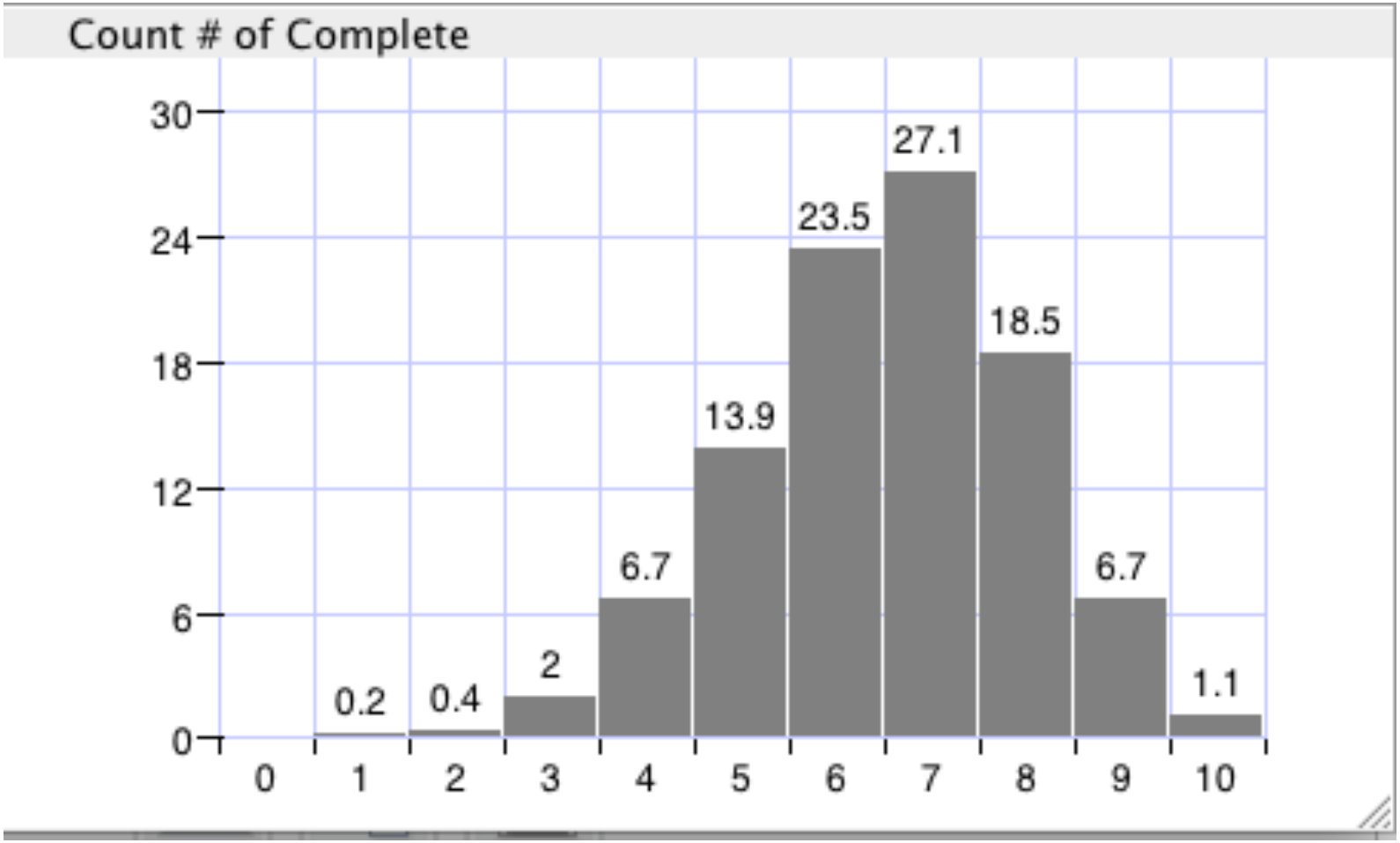
Count number of successes



Conduct the Simulation

Outcome #	rodgers	Count # of Complete
1	Complete	6
2	Incomplete	
3	Complete	
4	Complete	
5	Incomplete	
6	Complete	
7	Incomplete	
8	Complete	
9	Incomplete	
10	Complete	

Repeat a large number of times

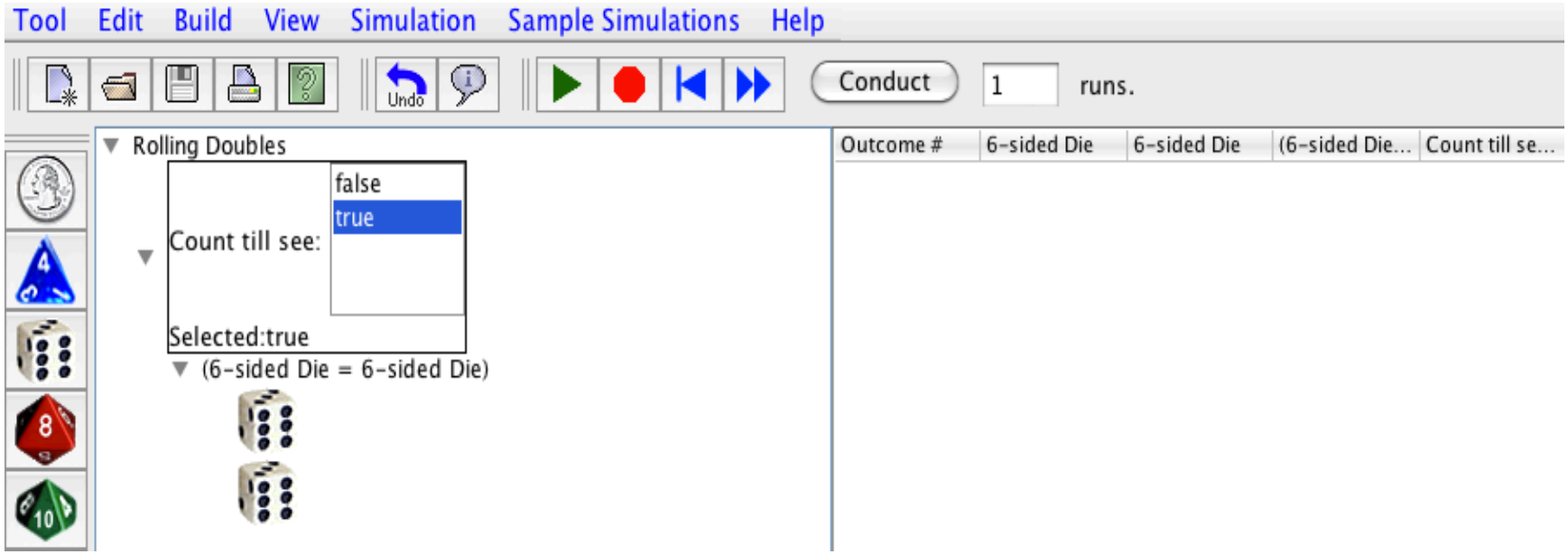


Example

On average, how many times must you roll two fair 6-sided die before you see doubles?

Core Math Tools

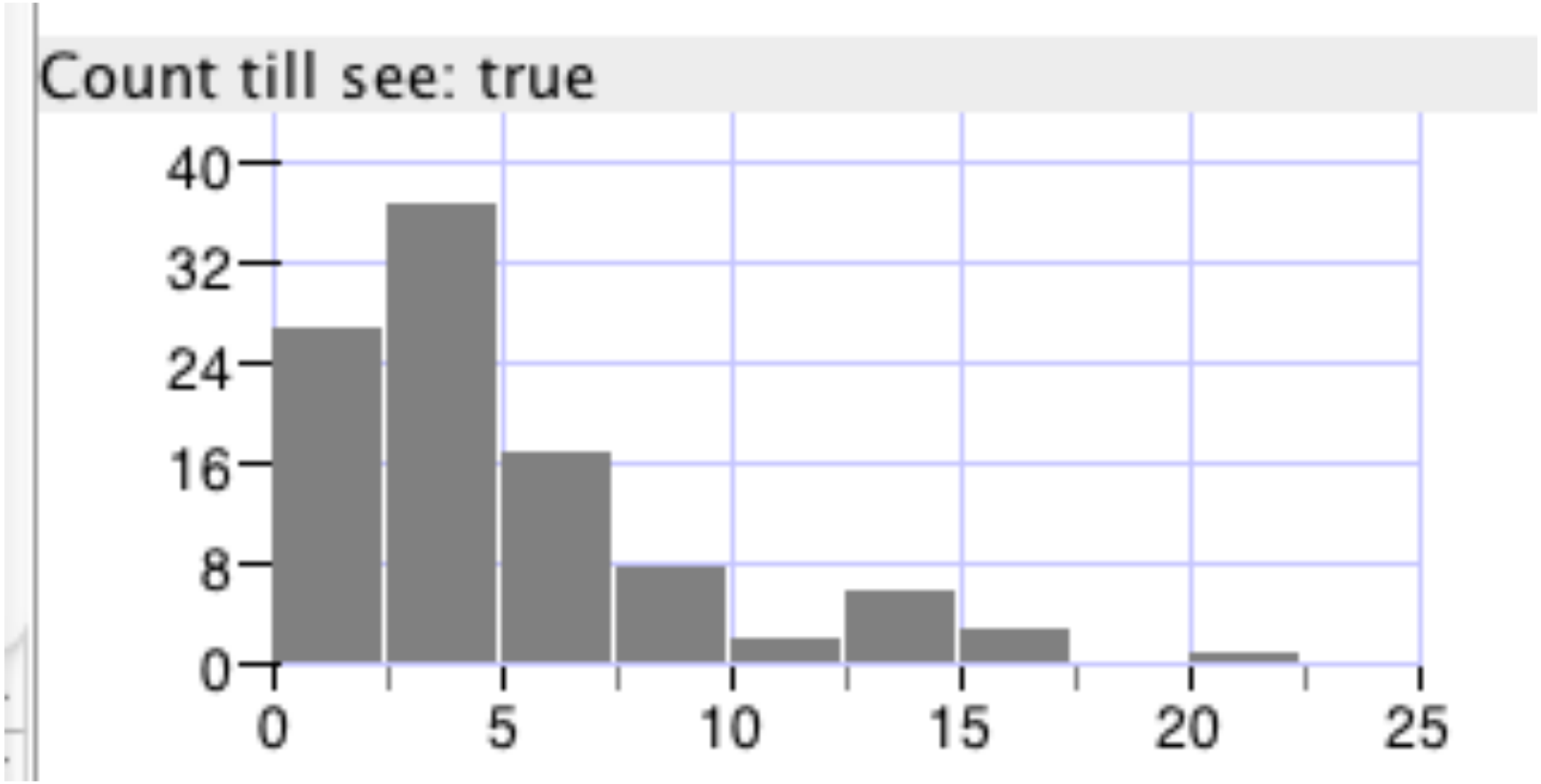
Simulation Setup



Results

Outcome #	6-sided Die	6-sided Die	(6-sided Die...	Count till se...
1	6	2	false	13
2	4	3	false	
3	3	5	false	
4	3	2	false	
5	5	6	false	
6	4	3	false	
7	6	3	false	
8	3	2	false	
9	3	1	false	
10	2	6	false	
11	3	4	false	
12	2	1	false	
13	1	1	true	

Histogram



Summary Statistics

Count till see: true

n = 101
mean = 5.39
minimum = 1
q1 = 2
median = 4
q3 = 7
maximum =21
sample standard deviation = 4.22
sample variance =17.84

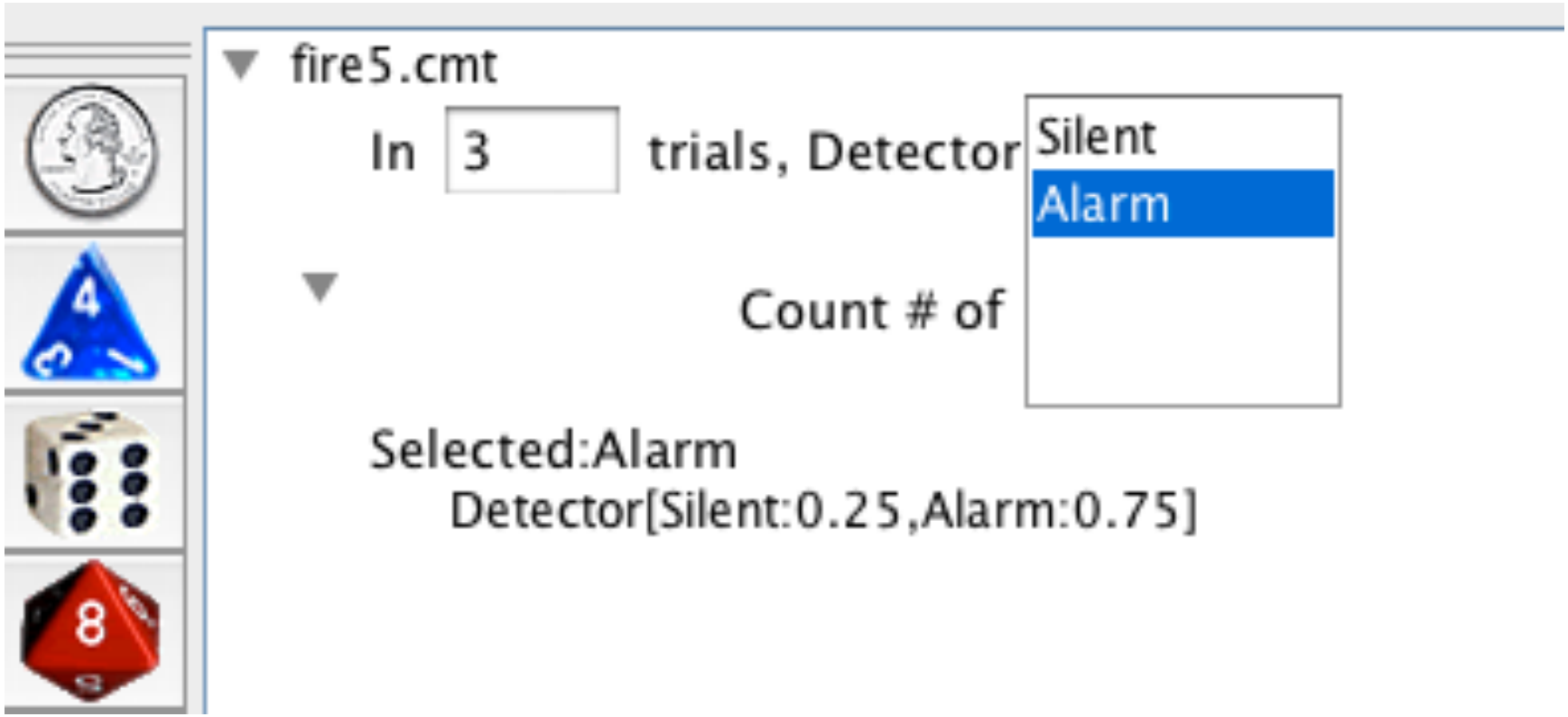
Fire Alarms Simulation

From Navigating Through Probability 9-12 (NCTM)

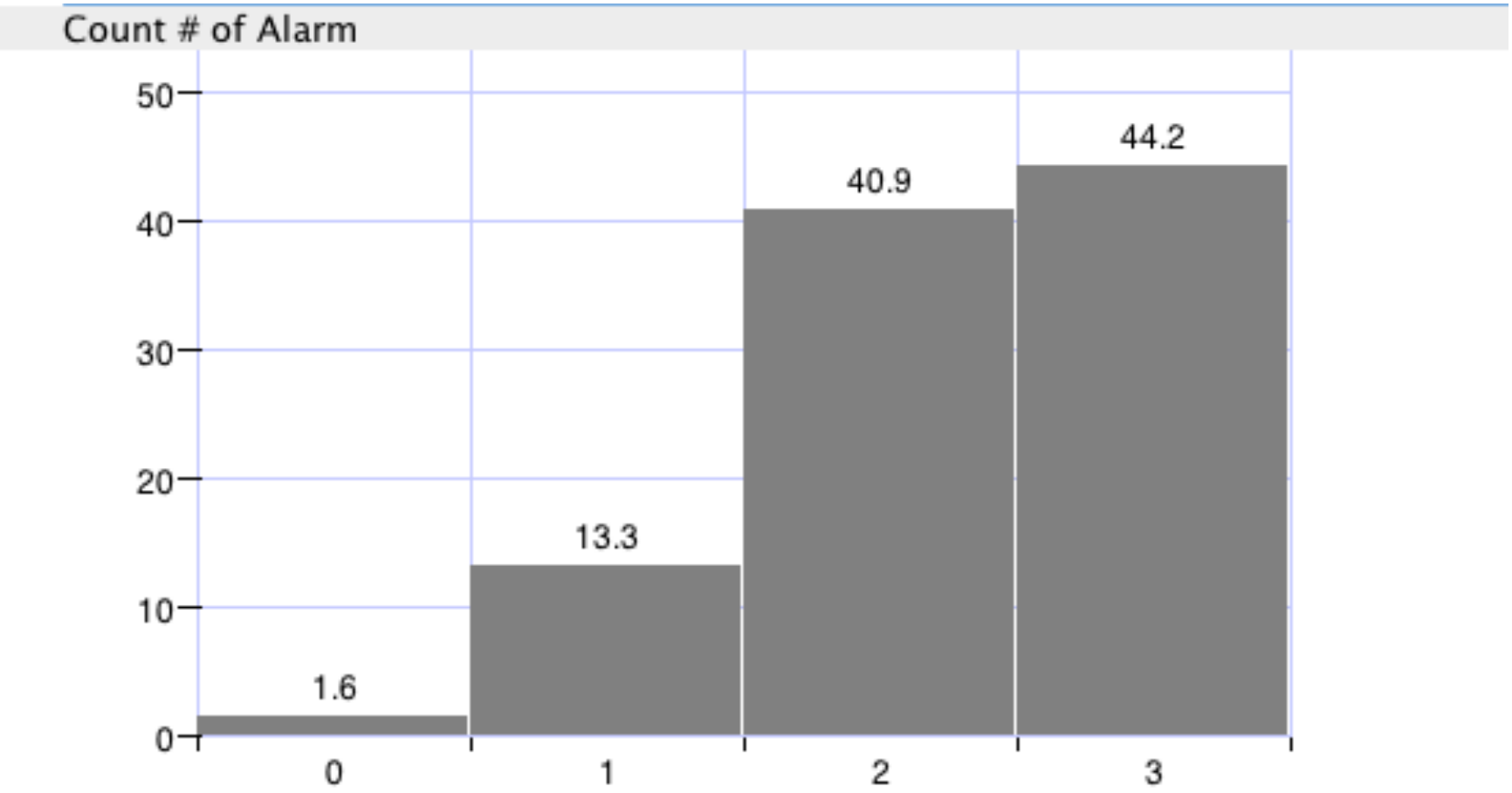
A local high school installed 3 fire alarms in the cafeteria. Each alarm is estimated to sound the alarm for a fire 75% of the time.

What is the probability that at least one of the alarms will go off if a fire starts in the cafeteria?

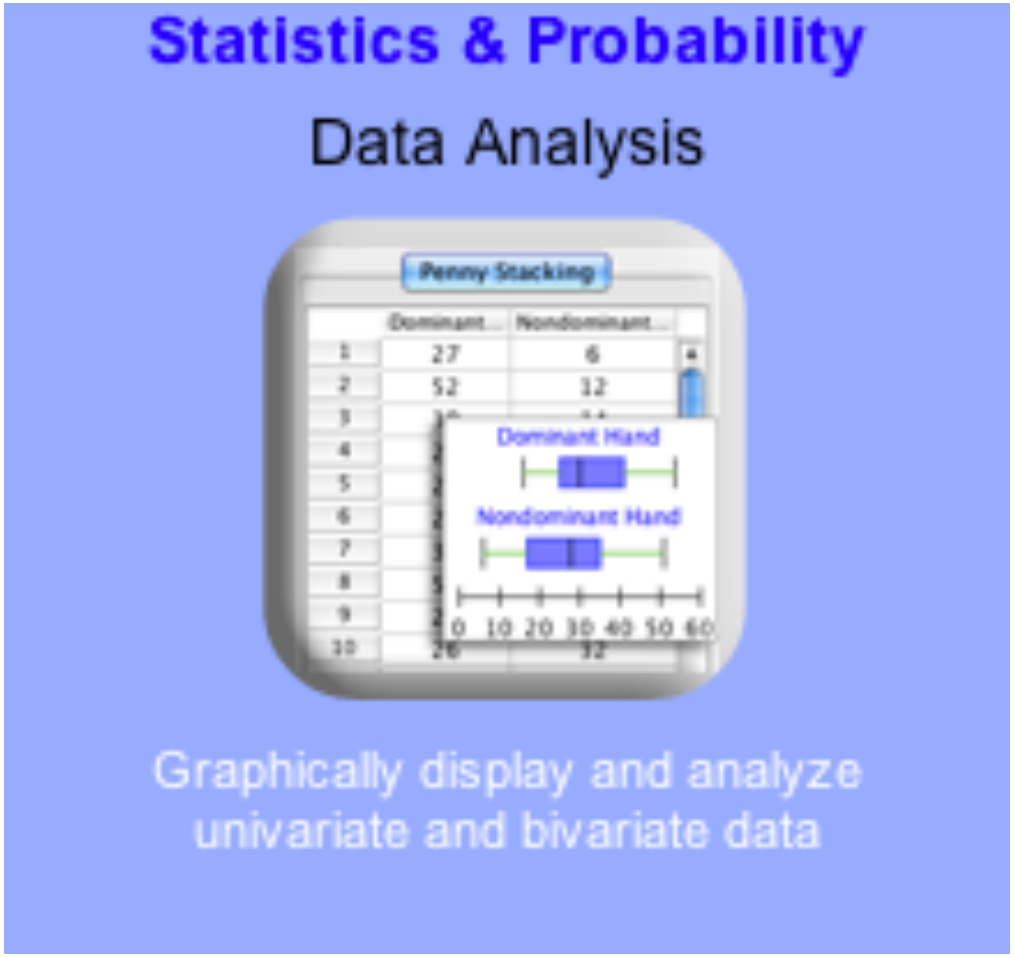
Build Custom Event



Conduct trials



Data Analysis tool



Analyzing Univariate Data

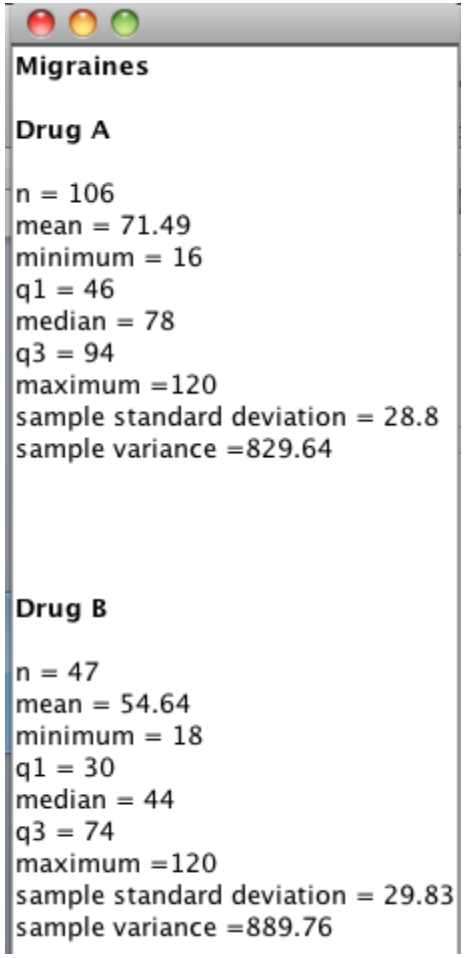
Migraines

Time (mins.) passed to get relief from a migraine headache for two different medications.

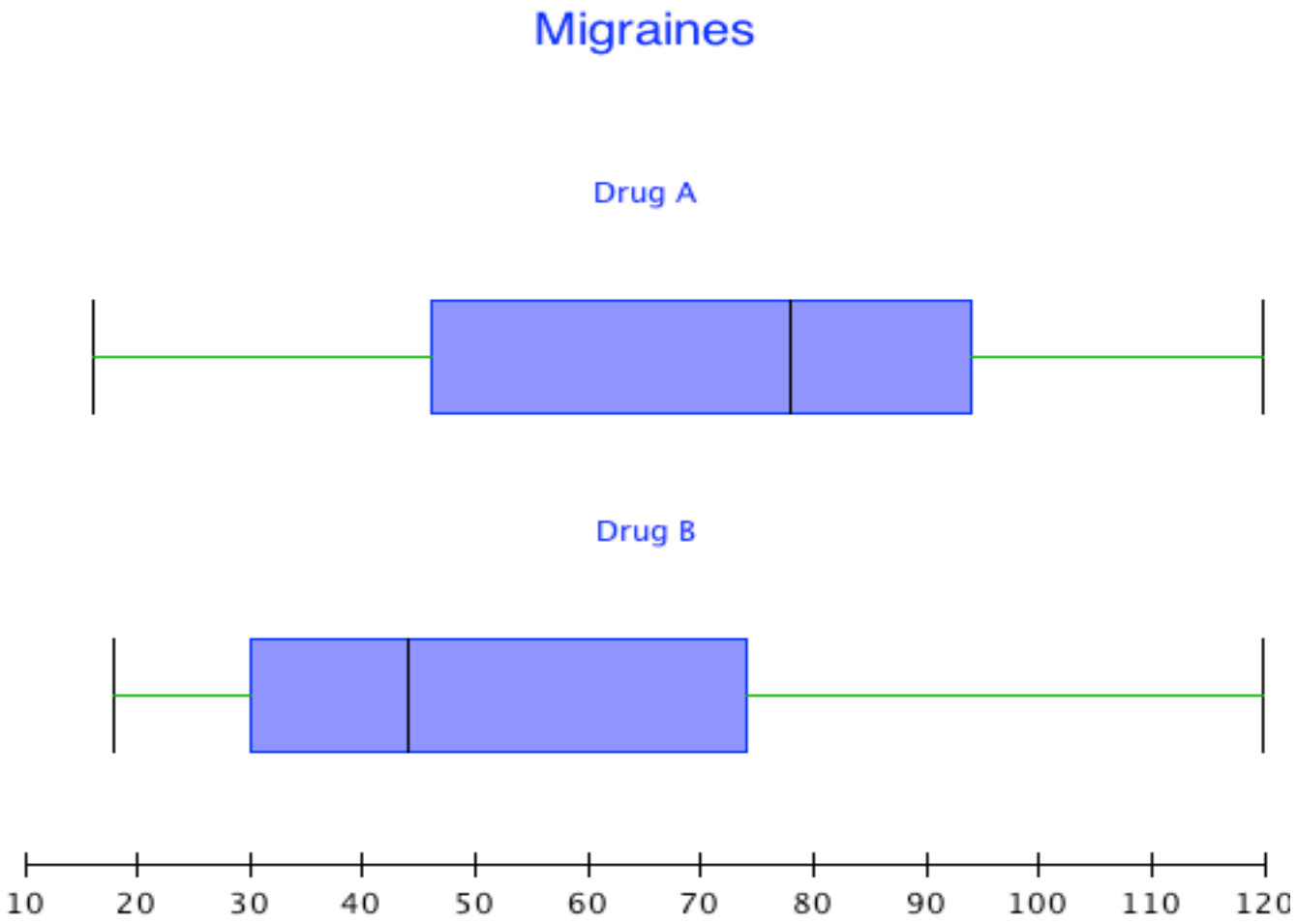
	Drug A	Drug B
1	16	18
2	18	20
3	18	20
4	20	22
5	22	24
6	22	24
7	24	24
8	24	26
9	26	26
10	26	30
11	28	30

Source: Navigating through data analysis in Grades 6–8 (CD ROM). Bright, G. (2003). Reston, VA: NCTM.

Descriptive Statistics



Graphs



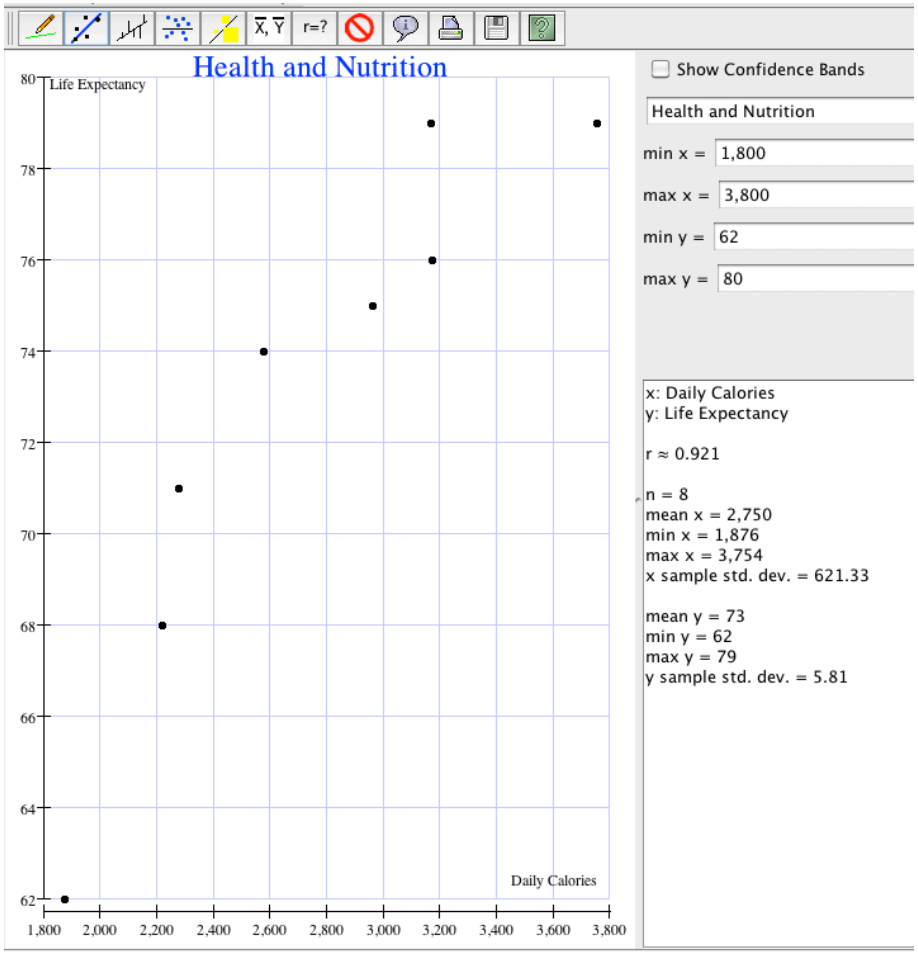
Analyzing Bivariate Data

- Health and Nutrition
- The data in the table show how average daily food supply (in calories) is related to life expectancy (in years) and infant mortality rates (in deaths per 1,000 births) in a sample of countries in the western hemisphere. (Source: World Health Organization Global Health Observatory Data Repository; www.populstat.info/Americas)

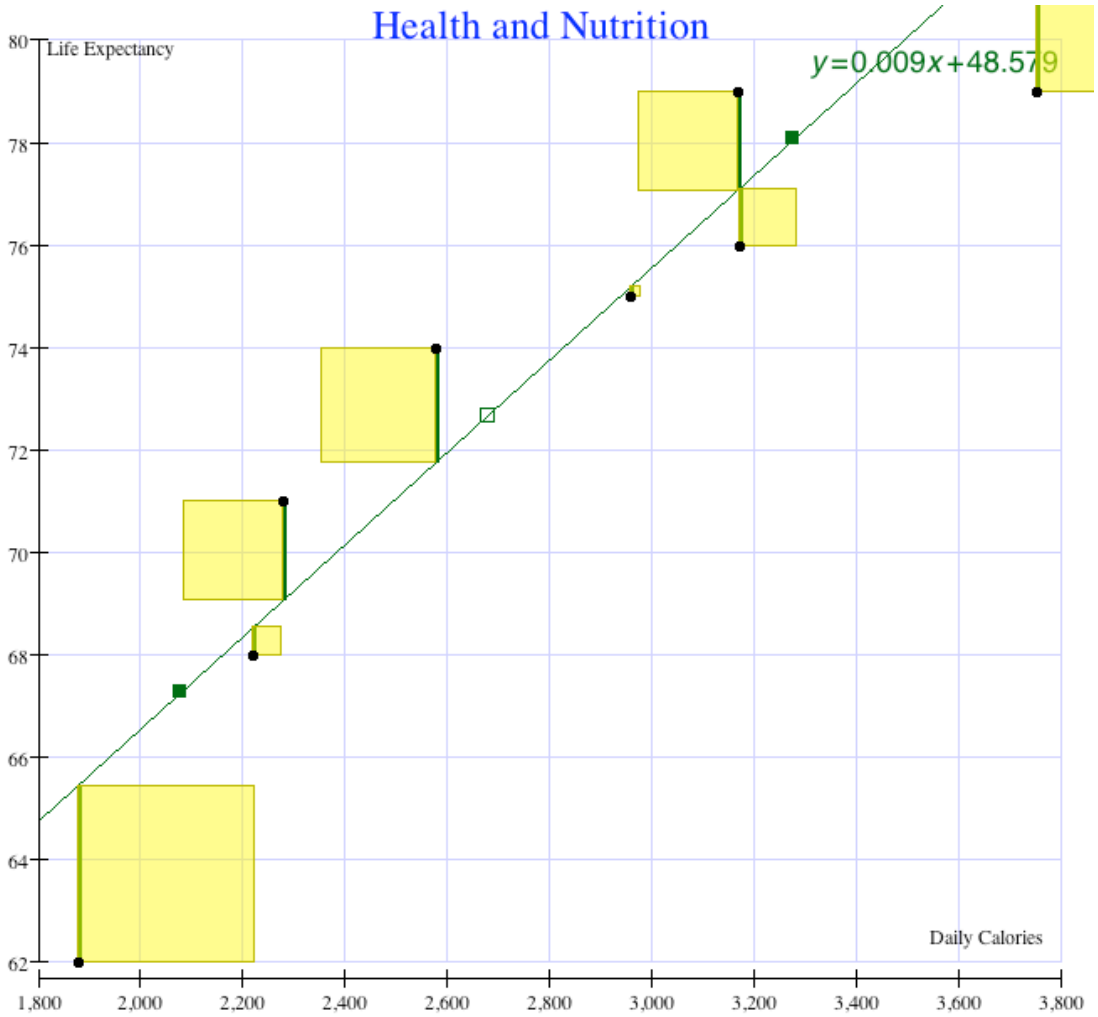
Relationship between daily calories and Life expectancy

	Country	Daily Calories	Life Expectancy	Infant Mortality
1	Argentina	2,959	75	13
2	Bolivia	2,219	68	40
3	Canada	3,167	79	7
4	Dominica...	2,277	71	27
5	Haiti	1,876	62	64
6	Mexico	3,171	76	15
7	Paraguay	2,577	74	19
8	United St...	3,754	79	7

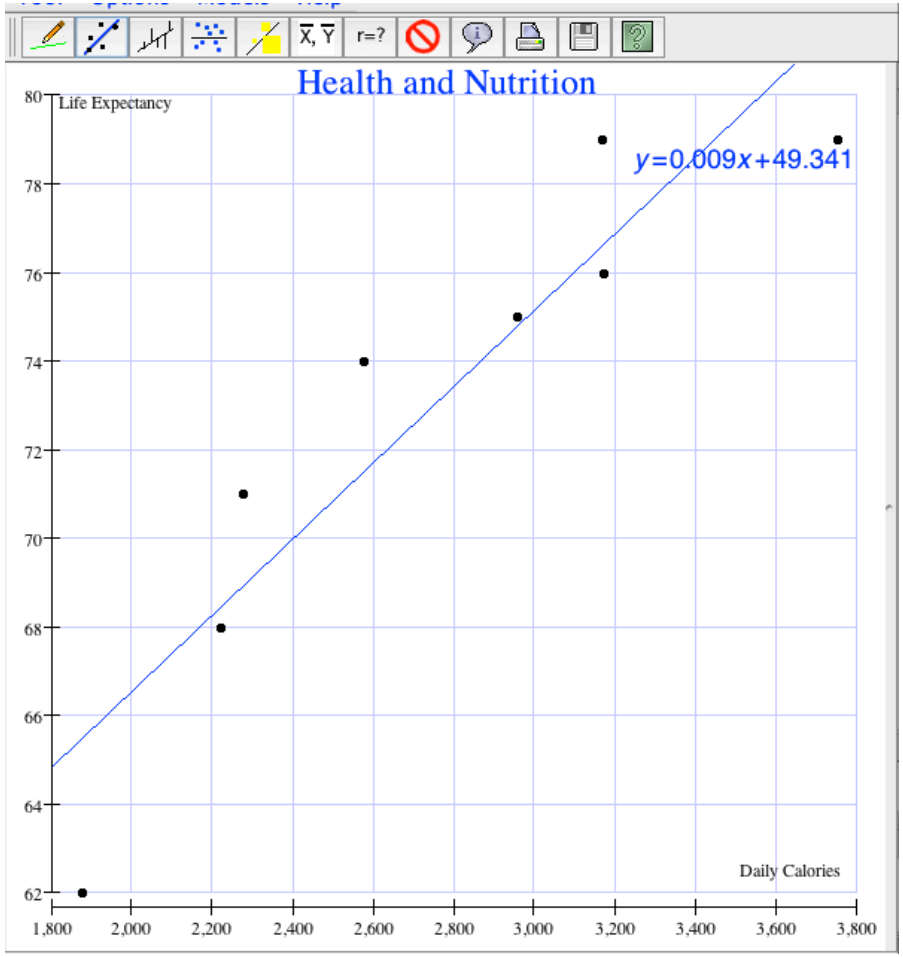
Scatterplot



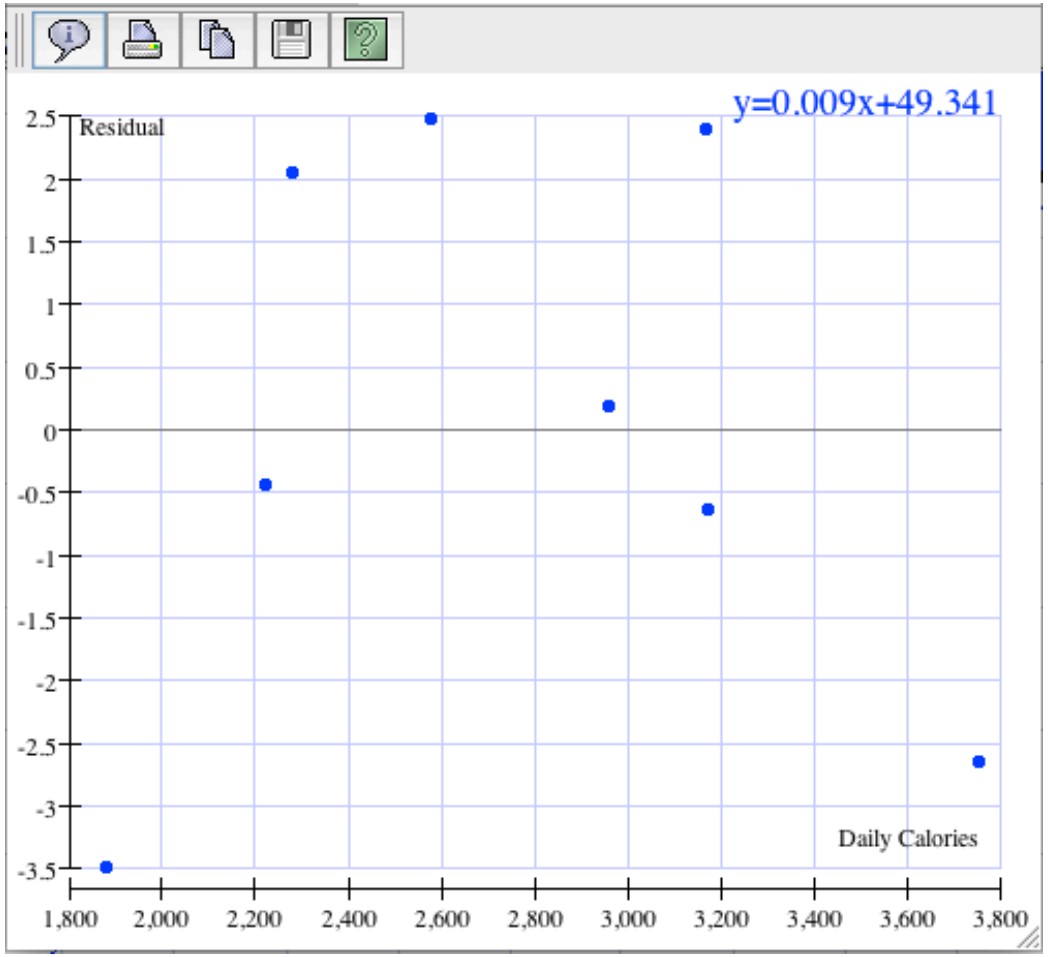
Moveable Line
show residuals and squares



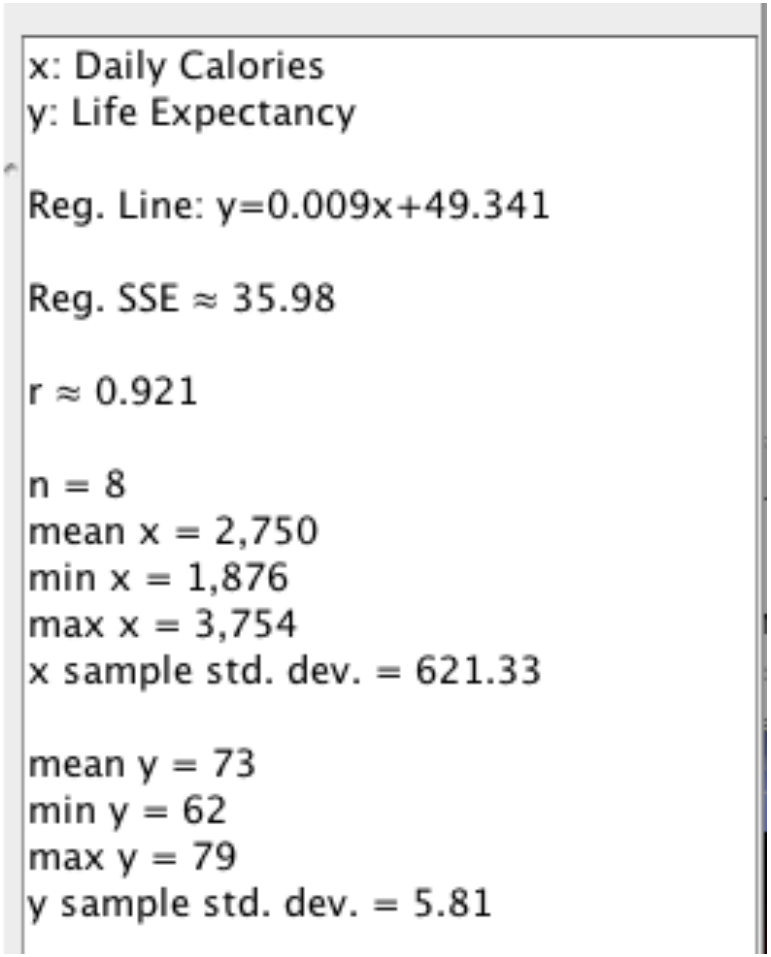
Least Squares Regression Line



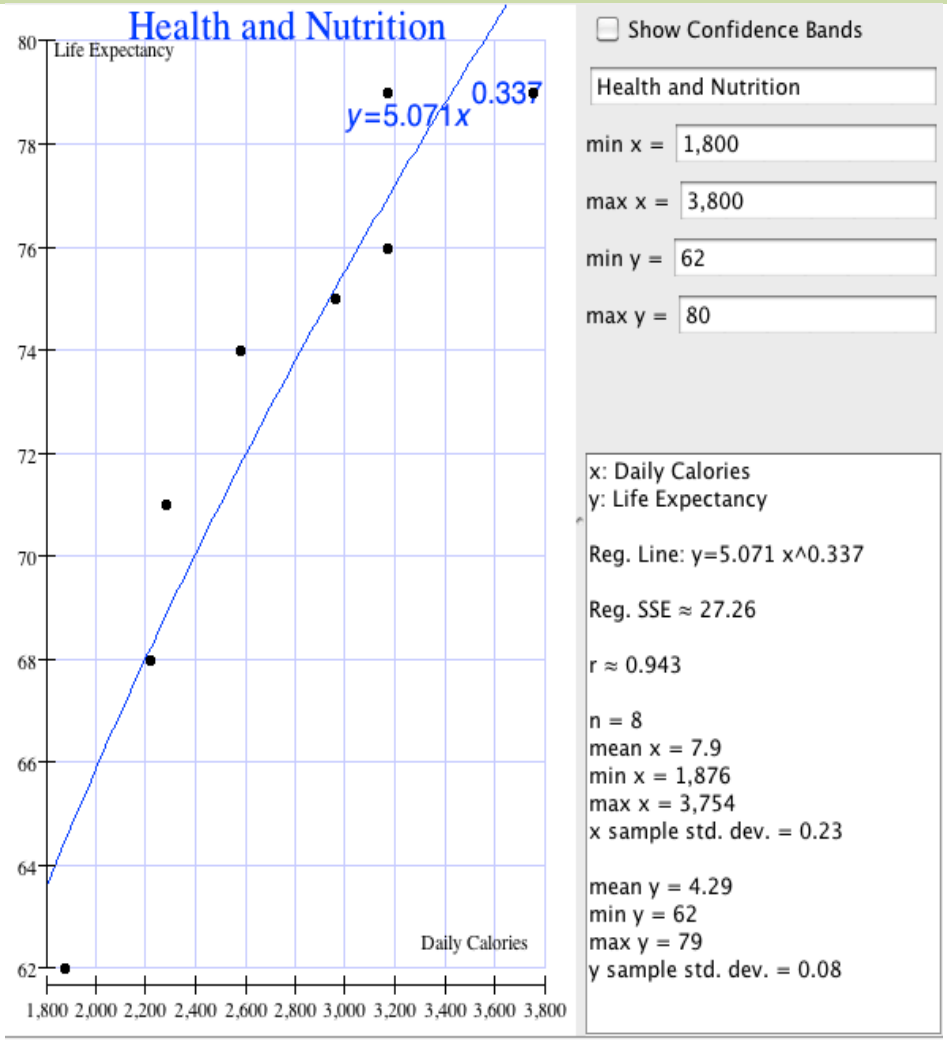
Residual Plot



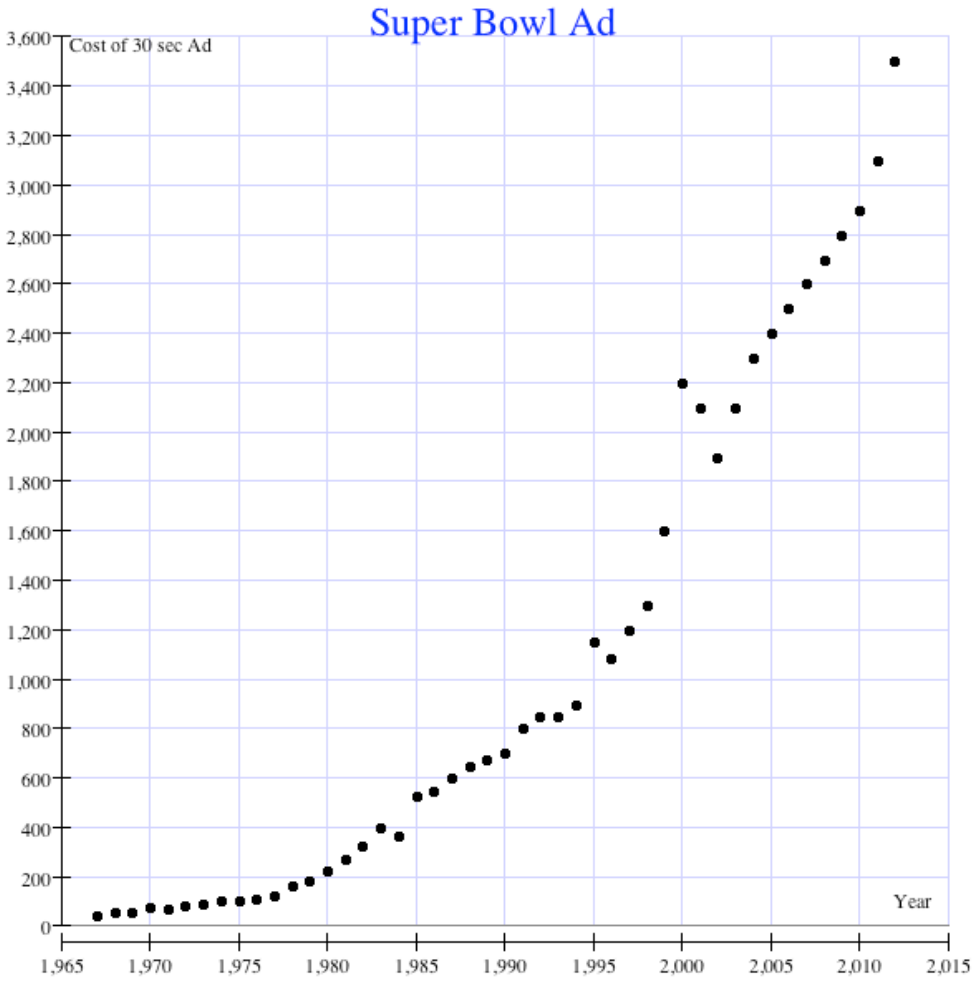
Plot Summary



Another Model



Exponential Growth?



Investigation:

Memorizing Words

From Focus in High School Mathematics Reasoning and Sense Making (NCTM)

7th grade CCSSM

7.SP.3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

CCSSM High School

S-IC-5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Student Experiment

A ninth-grade class of thirty students was randomly divided into two groups of fifteen students. One group was asked to memorize the list of meaningful words; the other group was asked to memorize the list of nonsense words. The number of words correctly recalled by each student was tabulated, and the resulting data are as follows:

Results from a Grade 9 Class

Number of meaningful words recalled:

12, 15, 12, 12, 10, 3, 7, 11, 9, 14, 9, 10, 9, 5, 13

Number of nonsense words recalled:

4, 6, 6, 5, 7, 5, 4, 7, 9, 10, 4, 8, 7, 3, 2

Analyze the Data

- Enter data into Column A and B. Find the five number summary, mean, and standard deviation for both lists of data.

- Construct parallel box plots

- On the basis of the summary statistics and the display, what observations can be made regarding how the students assigned the meaningful words performed compared with how the students assigned the nonsense words performed?

Descriptive Statistics

Meaning

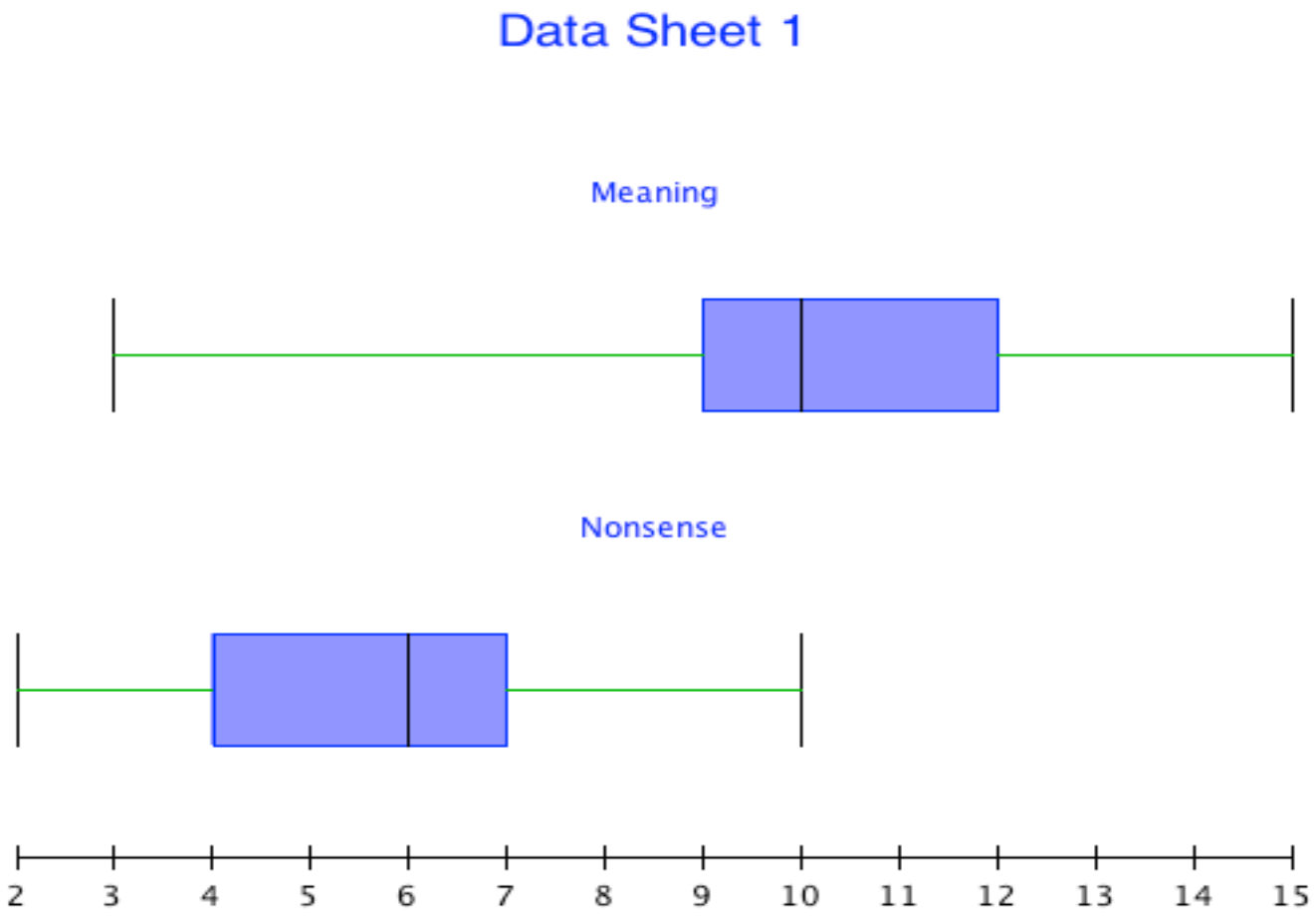
n = 15
mean = 10.07
minimum = 3
q1 = 9
median = 10
q3 = 12
maximum =15
sample standard deviation = 3.26
sample variance =10.64

Summary Statistics

Nonsense

n = 15
mean = 5.8
minimum = 2
q1 = 4
median = 6
q3 = 7
maximum =10
sample standard deviation = 2.24
sample variance =5.03

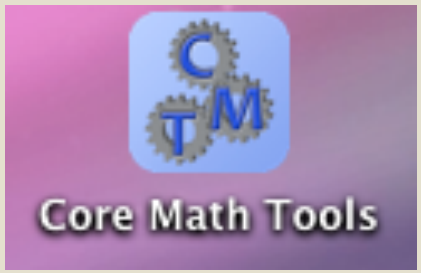
Parallel Box Plots



- Calculate the Interquartile range (IQR) for both sets of data.

- How many IQR's are the medians separate by?

- Do you think this is a significant spread?



Going Beyond an Informal Approach

Going beyond an informal approach

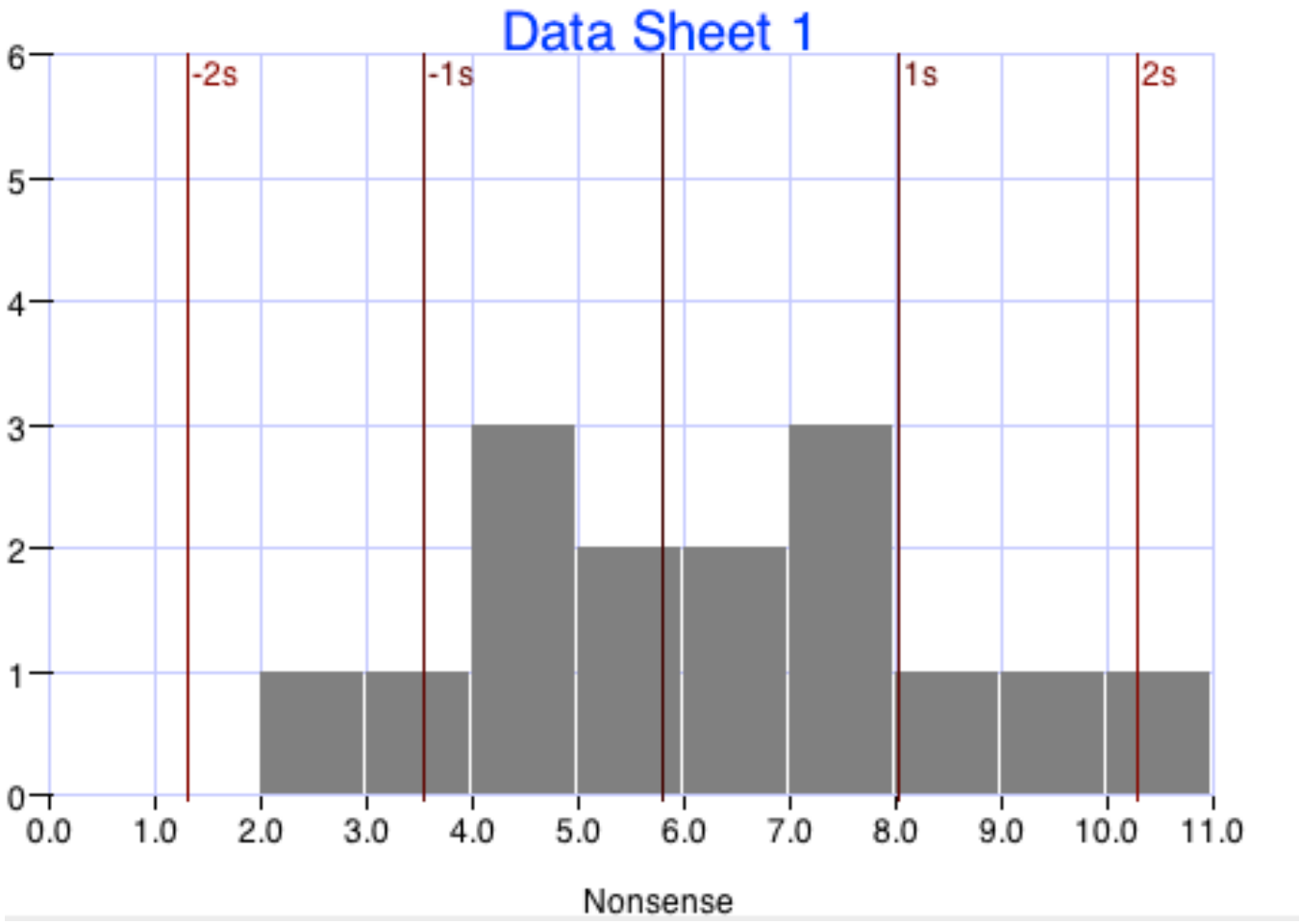
Question:

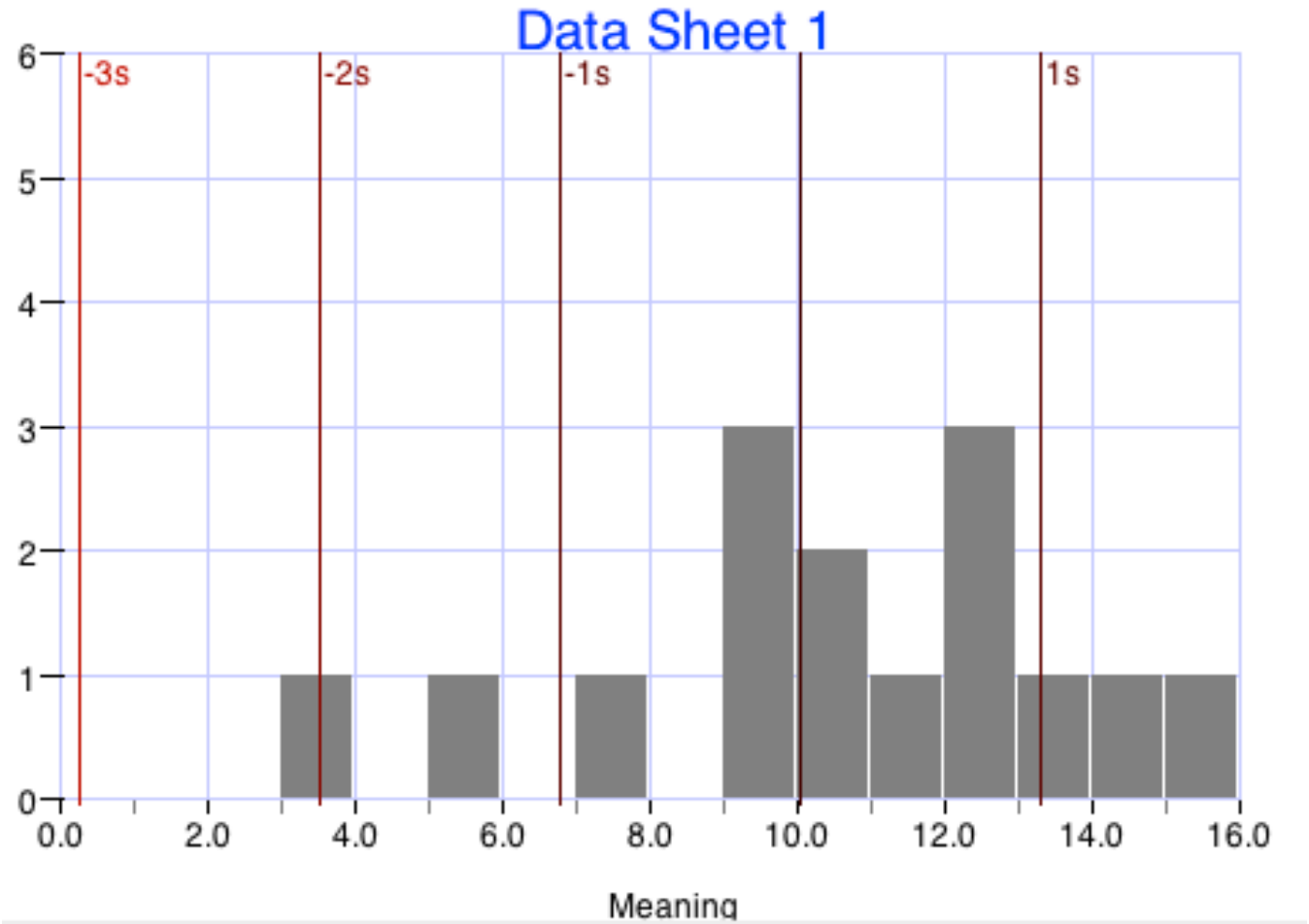
Is the difference between the means a significant difference?

Visually See the Difference

- Construct a histogram of each list of data.

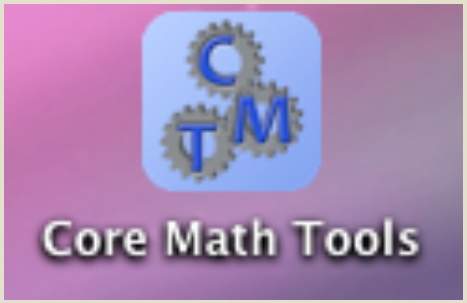
- Make a sketch of your histogram and mark the mean and one standard deviation above and below the mean on the histogram.


Data Sheet 1
-2s
-1s
1s
2s
6
5
4
3
2
1
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
11.0
Nonsense


Data Sheet 1
-3s
-2s
-1s
1s
6
5
4
3
2
1
0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
Meaning

The two means differ by 4.27 words.

Question: Is this a significant difference?



Randomization Test

Set-up for the experiment

Assumption:

Assume there is no difference between the mean number of meaningful words and the mean number of nonsense words.

This would mean that the spelling list a person received had nothing to do with how many words they were able to memorize. If a person memorized 5 words from the meaningful list that person would have memorized around 5 words from the nonsense list.

Question is:

- How often would we see a difference in the means as large or larger than 4.27 words assuming the list the students received didn't matter?
- How likely is it to see a difference of 4.27 or more purely by chance?
- If the probability of seeing a difference of 4.27 purely by chance is very low then this would mean a significant difference.

- Shuffle the 30 cards together and then “deal” the cards into two piles of 15 cards.
- Designate one pile A (meaningful) and the other B (nonsense)

- Enter the scores written on the cards from pile A into Column C.
- Enter the scores written on the cards from pile B into Column D

- Find the mean of each list and then find the difference in the means (mean of A – mean of B)
- Report the difference between the means.
- Repeat this procedure many more times.

Randomization Test

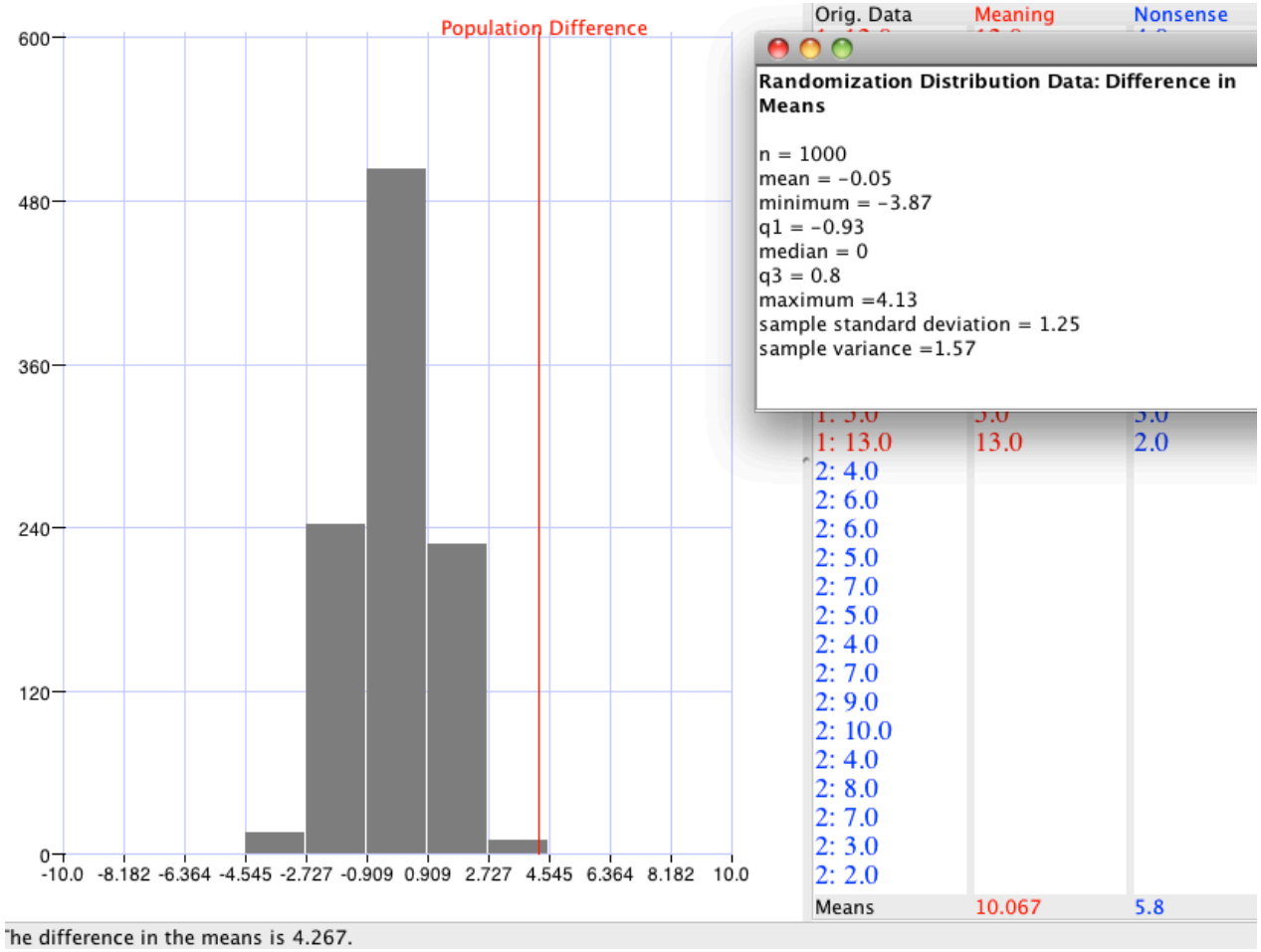
Core Math Tools

Using Core Math Tools to run this simulation that we did with the cards a large number of times.

Setup of Randomization Test

Orig. Data	Meaning	Nonsense
1: 12.0	12.0	4.0
1: 15.0	15.0	6.0
1: 12.0	12.0	6.0
1: 12.0	12.0	5.0
1: 10.0	10.0	7.0
1: 3.0	3.0	5.0
1: 7.0	7.0	4.0
1: 11.0	11.0	7.0
1: 9.0	9.0	9.0
1: 14.0	14.0	10.0
1: 9.0	9.0	4.0
1: 10.0	10.0	8.0
1: 9.0	9.0	7.0
1: 5.0	5.0	3.0
1: 13.0	13.0	2.0
2: 4.0		
2: 6.0		
2: 6.0		
2: 5.0		
2: 7.0		
2: 5.0		
2: 4.0		
2: 7.0		
2: 9.0		
2: 10.0		
2: 4.0		
2: 8.0		
2: 7.0		
2: 3.0		
2: 2.0		

Results


Population Difference
600
480
360
240
120
0
-10.0 -8.182 -6.364 -4.545 -2.727 -0.909 0.909 2.727 4.545 6.364 8.182 10.0
Randomization Distribution Data: Difference in Means
n = 1000
mean = -0.05
minimum = -3.87
q1 = -0.93
median = 0
q3 = 0.8
maximum =4.13
sample standard deviation = 1.25
sample variance =1.57
Orig. Data
Meaning
Nonsense
1: 13.0
13.0
2.0
2: 4.0
2: 6.0
2: 6.0
2: 5.0
2: 7.0
2: 5.0
2: 4.0
2: 7.0
2: 9.0
2: 10.0
2: 4.0
2: 8.0
2: 7.0
2: 3.0
2: 2.0
Means
10.067
5.8
he difference in the means is 4.267.

Analyzing the Differences

- Describe the histogram
  - What do the values represent?
  - Where do the values center?
  - Does this value make sense?
- Where does the value of 4.27 words (actual difference) fall in this distribution?
- Is this difference likely to have happened by chance?
- What conclusions can we draw?

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Questions?

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