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Pre-algebra and Geometry: Teaching Pivotal Content for Elementary Grades

Session #412

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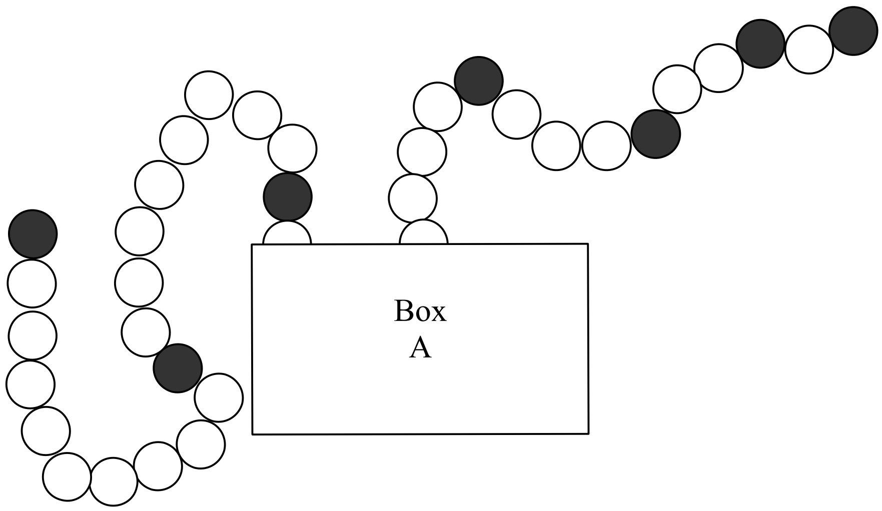
Beads in a Box

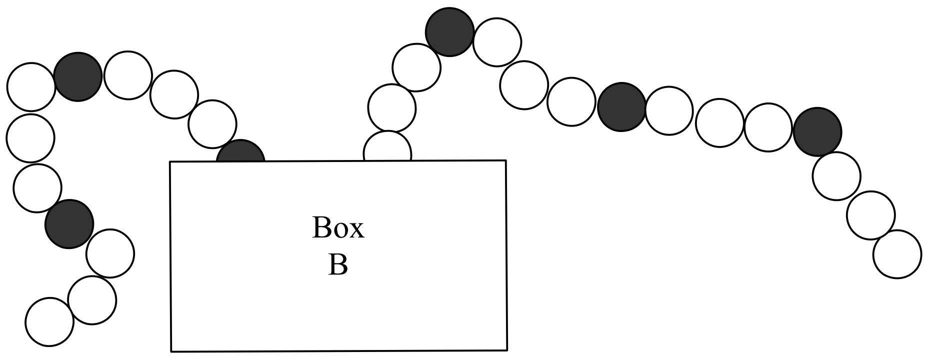
Use pictures, numbers and words to thoroughly explain your answers and work.

A. The beads form a chain and flow into and then out of box A. How many beads are there in the chain? How many are white? How many are Black



Beads in a Box

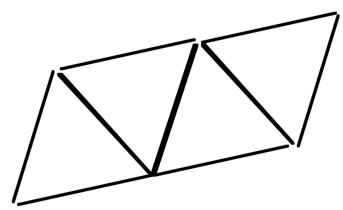
Use pictures, numbers and words to thoroughly explain your answers and work.
B. How many beads in box B? How may are white? How many are black?



C. Make a sketch or use the two-sided counters to design your own beads-in-a-box problem with a pattern of your own creation. After you have created your pattern, share your creation with everybody.

Pattern Block Trains

A. Suppose you were to connect 100 green triangular pattern blocks in a train. What will be the measure of the perimeter? Is there a relationship between the number of triangles and the perimeter of the design? Be prepared to share your strategy for solving this challenge.



B. What if you repeated the activity with each of the other five pattern blocks? Would the answers be the same as in part A?

C. What if you repeated the activity making a 100-block train repeating the following four blocks a square, a trapezoid, a triangle and a pentagon in the same order? How would your answer differ?

Similarity with Pattern Blocks

Materials
Pattern blocks, Paper for recording data, string, meter stick or tape measure.

The Plan

1. Begin with one square from the set of pattern blocks. Using only orange squares make another larger square using as few small squares as possible. Still using only orange squares make the next largest square using as few small squares as possible. There are now three squares (a single square, four squares, nine squares). For each of the three figures record the length of a side, perimeter, and area. What patterns can be found in these number sequences? What will be the next number in each sequence? Why?

2. Leave the squares intact on the table. Repeat this activity but instead of orange squares use just triangles, or just blue rhombi, or red trapezoids. So use trapezoid to make the next largest trapezoid. How are the number patterns the same or different as they were for the square?

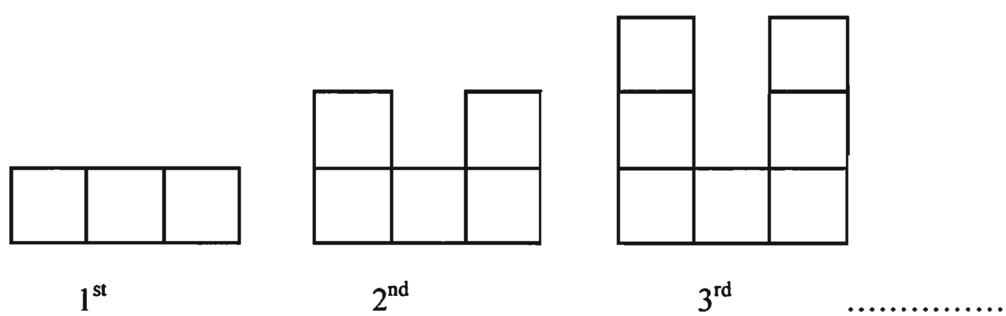
3. Now take a single square in your hand. Stand over the larger square made with four small squares. Place the single square in your hand between one of your eyes and the four squares on the table as you look down on them. At some point in your line of vision between your eye and the four squares, the single square should appear to cover the four squares exactly. Hold that position. Measure the distance between the eye and the small square and then measure the distance between the eye and the larger square on the table. How do the measures compare?

Three Student Examples

Three students, Sissy, Caloma, and Mellie each answered the questions below. You are not trying to grade them. You need to read all three answers and see if what they sat and do make sense to you. Be ready to ask about the parts of their work that does not make sense or that you think is not correct. The students have drawn lines from what they write to the picture so you can see the connections they are trying to make.

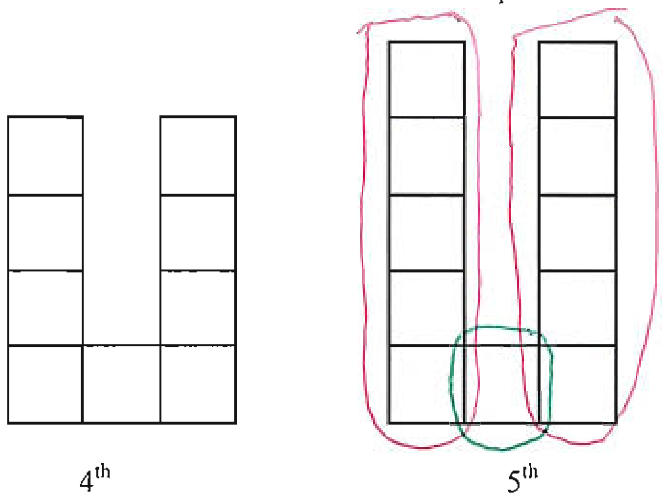
Here is the problem the students were answering:
The first three terms of a growing pattern are given.

a. Sketch the next two terms in this pattern
b. Describe this pattern. Connect your description to the term number and to the sketch.
c. Make a table that shows the total number of blocks in each term, the term number, and the change as it grows.
d. What will the 25th term look like and how many blocks would I need to make it?
e. Write both an explicit and iterative formula for finding the number of blocks when the term is any number



Sissy's Solution:

1. Sketch the next two terms in this pattern.



2. Describe this pattern and connect your description to the term number.
   Sissy's said:
   When I look at this pattern I see two equal vertical stacks of squares with one other square in the middle at the bottom. With each new term one more square is added to the vertical stacks, but the middle block does not grow. It is always just one square. I also see that the number of blocks in the vertical stacks is the same as the term number. So in the fifth term, there are 5 blocks in each vertical stack.

3. Make a table that shows the total number of blocks in each term, the term number, and the change as it grows.

   Sissy's Table

Term	1st	2nd	3rd	4th	5th	6th	…..	n
Total Squares	3	5	7	9	11	13		
Change		2	2	2	2	2	2	

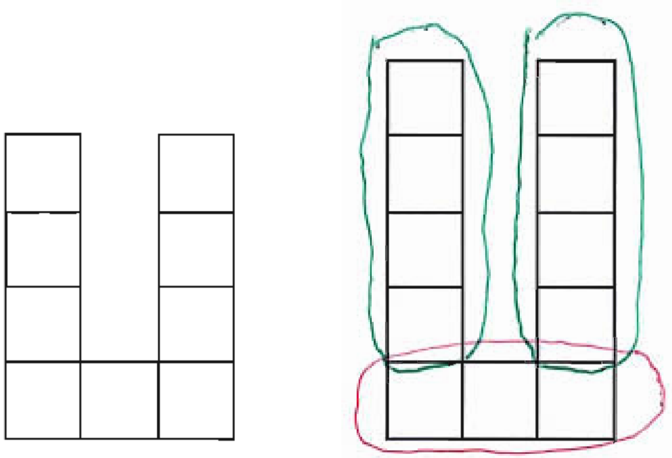
4. What will the 25^{th} term look like and how many blocks are needed to make it?
   Sissy's said:
   I see that the number of blocks in the vertical stacks is the same as the term number. So the 25^{th} term would have 25 blocks in each vertical stack. That is 50 blocks. And then the one in the middle makes 51 blocks all together.

5. Write a formula for finding the number of blocks when the term is any number.

   This is Sissy's explicit formula:
   I see that the number of blocks in the vertical stacks is the same as the term number. So the 25^{th} term would have 25 blocks in each vertical stack. That is 50 blocks. And then the one in the middle makes 51 blocks.
   Explicit Formula $2T + 1$

Caloma's Solution:

1. Sketch the next two terms in this pattern.



4th 5th

2. Describe this pattern and connect your description to the term number.

   Caloma said:
   I looked at this a different way than Sissy. When I look at it I see a horizontal group of three blocks along the bottom. Those three stay the same throughout the pattern. On top of the ends of the bottom row are two stacks the same size. I see the pattern grows because each of the stacks gets one new block on top with each new term. The number of blocks stacked on top of one end is one less than the term number.

3. Make a table that shows the total number of blocks in each term, the term number, and the change as it grows.

   Caloma's Table

Term	1st	2nd	3rd	4th	5th	6th	…..	n
Total Squares	3	5	7	9	11	13		
Change		2	2	2	2	2		

4. What will the 25th term look like and how many blocks would I need to make it?

   Caloma said:
   The bottom row always has three blocks. The number of blocks stacked on the ends of these three is one less than the term number.
   For the 25th term, the number of blocks on the ends is 25 - 1 =24. So all together there are 3 + 24 + 24= 51

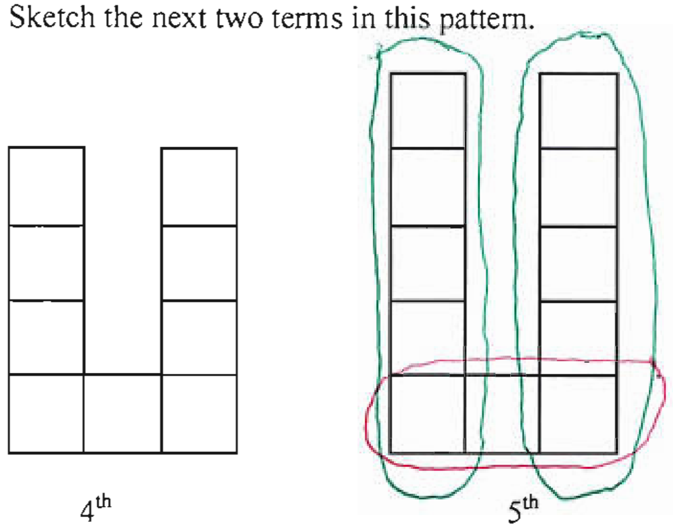
5. Write a formula for finding the number of blocks when the term is any number.

   Caloma's Iterative formula: Add two more blocks each time.

   Caloma's Explicit formula
   If T is the term number and there are two stacks we get T – 1 times 2. Then we need to add on the three at the bottom. So the explicit formula is: 2(T-1) + 3

Mellie's Solution:

1. Sketch the next two terms in this pattern.



2. Describe this pattern and connect your description to the term number.

   Mellie said:
   When I look at the pattern I see three parts - the right stack, the bottom row and the left stack. The bottom row does not grow, but the right and left stacks grow one each time. The bottom overlaps with the two stacks.
   The number of blocks in the right and left stacks is the same as the term number.

3. Make a table that shows the total number of blocks in each term, the term number, and the change as it grows.

Term	Total # of Squares	Change
1st	3	> 2
2nd	5	> 2
3rd	7	> 2
4th	9	> 2
5th	11	> 2
6th	13	
. . .		
Nth		

4. What will the 25th term look like and how many blocks would I need to make it?

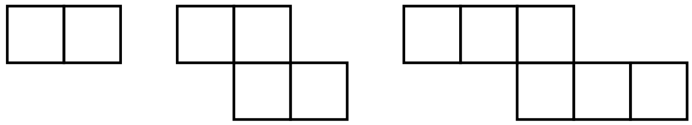
Mellie said:
I will describe my three parts. The number of blocks in the right and left stacks is the same as the term number. The right stack is 25 block, the left stack is 25 blocks. And the bottom is always 3. So there are 25 + 25 + 3 = 53 blocks to build the 25th term. But the two stacks and the bottom overlap by one block on the left and one on the right. That means I counted these twice, so I need to subtract 2 so I do not count them twice. So 53 -2 = 51.

5. Write a formula for finding the number of blocks when the term is any number.

   Mellie's Iterative formula: Add two more blocks each time.

   Mellie's Explicit formula:
   I see the pattern in three parts - the right stack, the bottom row and the left stack. The bottom row does not grow and is always 3. The number of blocks in the right and left stacks is the same as the term number so I will have T + T = 2T. So all together I have 3 + 2T. But the stacks and the bottom overlap by one block on the left and one on the right. That means I counted these twice, I need to subtract 2 so I do not count them twice.
   Explicit Formula: 3 + 2T - 2

A



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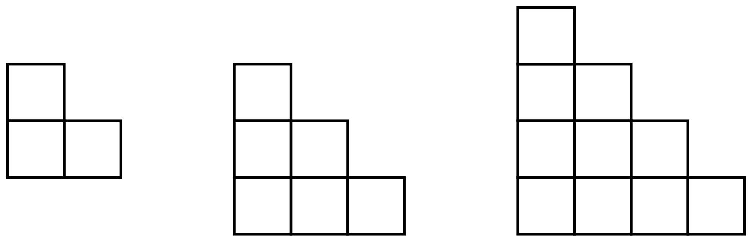

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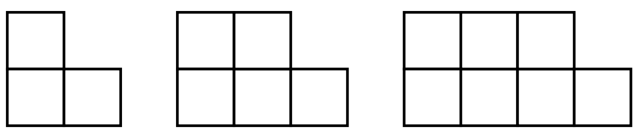

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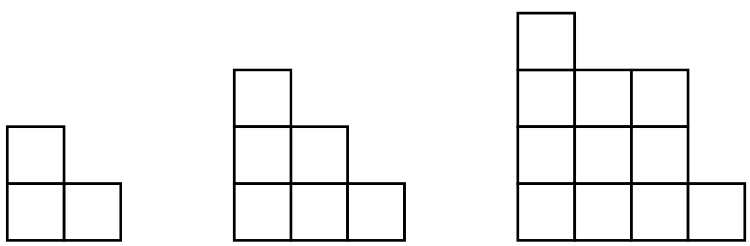

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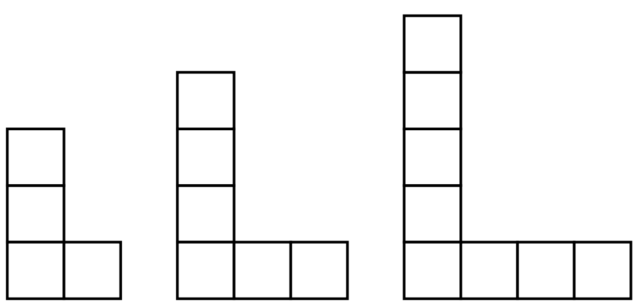


They all start the same way

The first two terms of a number pattern are

1, 2, ….

a) Write at least four *different* number patterns that begin with these two numbers (more if possible) by listing the next four terms in each pattern.
b) Write a general statement for how each term builds from the previous term.
c) Formulate a generalization to predict the 20th number in each of the patterns, the 100th number, the nth?
d) Represent each pattern with materials?

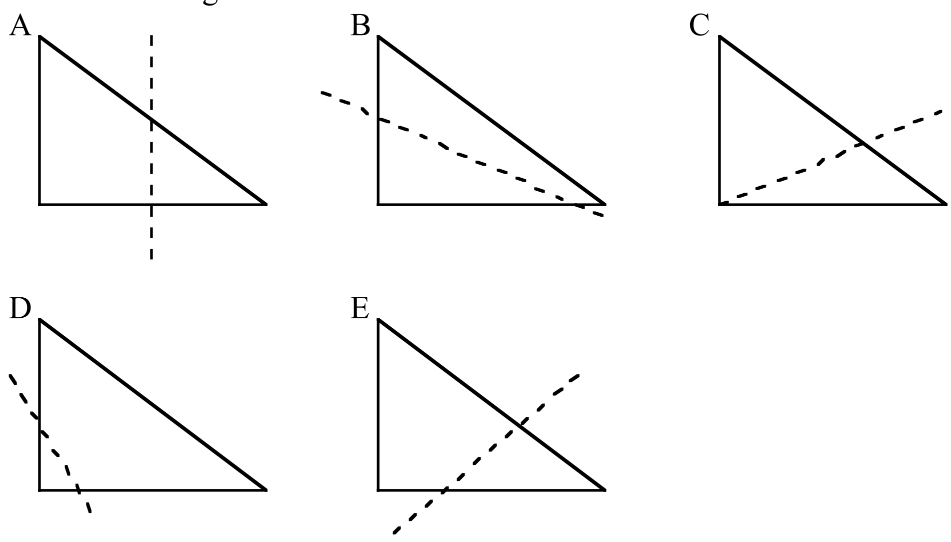
Decomposing Shapes

Investigate the number and the kinds of shapes that result when a given polygon is cut with a single, straight line. The challenge is to find all the different outcomes and to find a systematic way to proceed so that you know you have found them all.

Example:
There are five different outcomes.

1. When I slice triangle A I get a trapezoid and a right triangle.
2. When I slice triangle B I get a right triangles and a quadrilateral.
3. When I slice triangle C I get an acute triangles and an obtuse triangle.
4. When I slice triangle D I get a right triangle and a quadrilateral.
5. When I slice triangle E I get an isosceles triangle and a quadrilateral.

For the purpose of this investigation we will say that we really only have four outcomes since the decomposition of D and B give the same outcomes.



Your Challenge
You and your group have been assigned one kind of polygon to decompose. You should have many copies of this shape and scissors to cut.

1. Your task, like the example above, is to make a single straight cut through the polygon separating it into two other polygons. What are the names of these two polygons?
2. How many different pairs of polygons can you get by making cuts in different places?
3. How do you know if you have found all the possibilities?
4. Can you make a cut so the two resulting shapes are congruent?

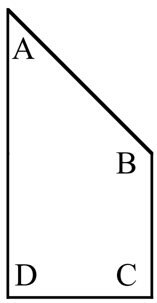
Use the following wordlist to label your decomposed shapes

acute triangle	scalene triangle,	isosceles triangle	right triangle
obtuse triangle	equilateral triangle	quadrilateral	square
parallelogram	rhombus	trapezoid	rectangle
pentagon	hexagon	isosceles trapezoid	

Translations – Reflections – Rotations
(slides – flips – turns)

The Plan

1. Have the students place the shape below cut from card stock or an index card in the center of the paper and trace around it. Labeling it with the letters will help when giving directions. This is center position is considered *home*.



2. Move the shape from the home position to a new position on the sheet of paper by following a set of directions. Trace around the shape at this new location and label the new location *A*. For example you may give directions like: *Slide the shape to the right about one inch. Rotate it a quarter turn counter-clockwise around point A. Flip the shape up about line BC.* See comment at bottom for more ideas.

3. Does the order in which you made the transformations matter? Use the same directions but complete them in a different order to see if you land at the same location.

4. Return to the home position and follow directions for three new positions to be labeled *B*, *C*, and *D*.

5. Have students write their your own descriptions for how to move from
   a. A to B.
   b. B to A
   c. C to B
   d. D to A.

Teacher Notes

Sample directions

Begin at home. Slide up about 2 inches. Flip to the left. Slide right to about the end of the paper. Trace and label this position A

Return to the home position. Flip across DC. Slide 6 cm to the east. Rotate a half turn about point A. Trace and label this position B.

Return home. Rotate 90 degrees counter-clockwise about point C. Flip down across the horizontal edge. Slide southwest to the corner. Trace and label this position C.
Return home. Slide left to the edge of the page. Rotate a quarter turn right about point C. Flip up across the top edge of the shape. Trace and label this point D.

For translations or slides you need to give details on which direction and how far. Directions are usually horizontal or vertical and are indicated by words like right, left, up, down, north, south, etc. (if you use words like north or south you will need to put a marker on the paper that indicates which direction is north.)

For reflections or flips you need to indicate the line about which the figure is to be reflected. Sometime you may need to also give the direction like right, left, up or down.

For rotations or turns you need to give instructions about the point about which to rotate the figure; the direction like clockwise or counter-clockwise; and the amount of turn like a quarter-turn or 45º.

For this activity a non-symmetrical shape is useful. If a shape like a square is used it is difficult to tell if it had been flipped or turned because of its symmetry.

Using transformations to see that two figures are congruent.

Place two copies of the quadrilateral in different locations and describe the set of flips, turns, sand slides needed to show the figures are the same shape and same size.