Building *Common Core Mathematical Practices* through Early Algebra

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- How are *Common Core* Mathematical Practices being developed in your school/district?
- How are *Common Core* Standards emphasized in relation to the Mathematical Practices?

COMMON CORE
MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Early Algebra

- Generalizing mathematical relationships
- Representing generalizations
- Justifying generalizations
- Reasoning with generalizations

Of the mathematical content in grades K-5, early algebra is most able to comprehensively address the Mathematical Practices of the *Common Core*.

Early Algebra can occur in a variety of mathematical domains, including:

1. functional thinking

2. equivalence, expressions, equations, inequalities

3. (generalized) arithmetic

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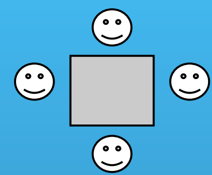
2. equivalence, expressions, equations, inequalities

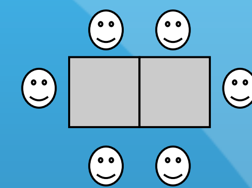
3. (generalized) arithmetic (if time)

Brady Problem

Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables. He can seat 4 people at one square table in the following way:



If he joins another square table to the first one, he can seat 6 people:



a) If Brady keeps joining square tables in this way, how many people can sit at:

3 tables? 4 tables? 5 tables? Record your responses in the table below and fill in any missing information:

b) **Do you see any patterns in the table? Describe them.**

c) Find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables. Describe your rule in words.

d) Describe your relationship using variables. What do your variables represent?

e) If Brady has 10 tables, how many people can he seat? Show how you got your answer.

How does this task help students develop *Common Core* Mathematical Practices?

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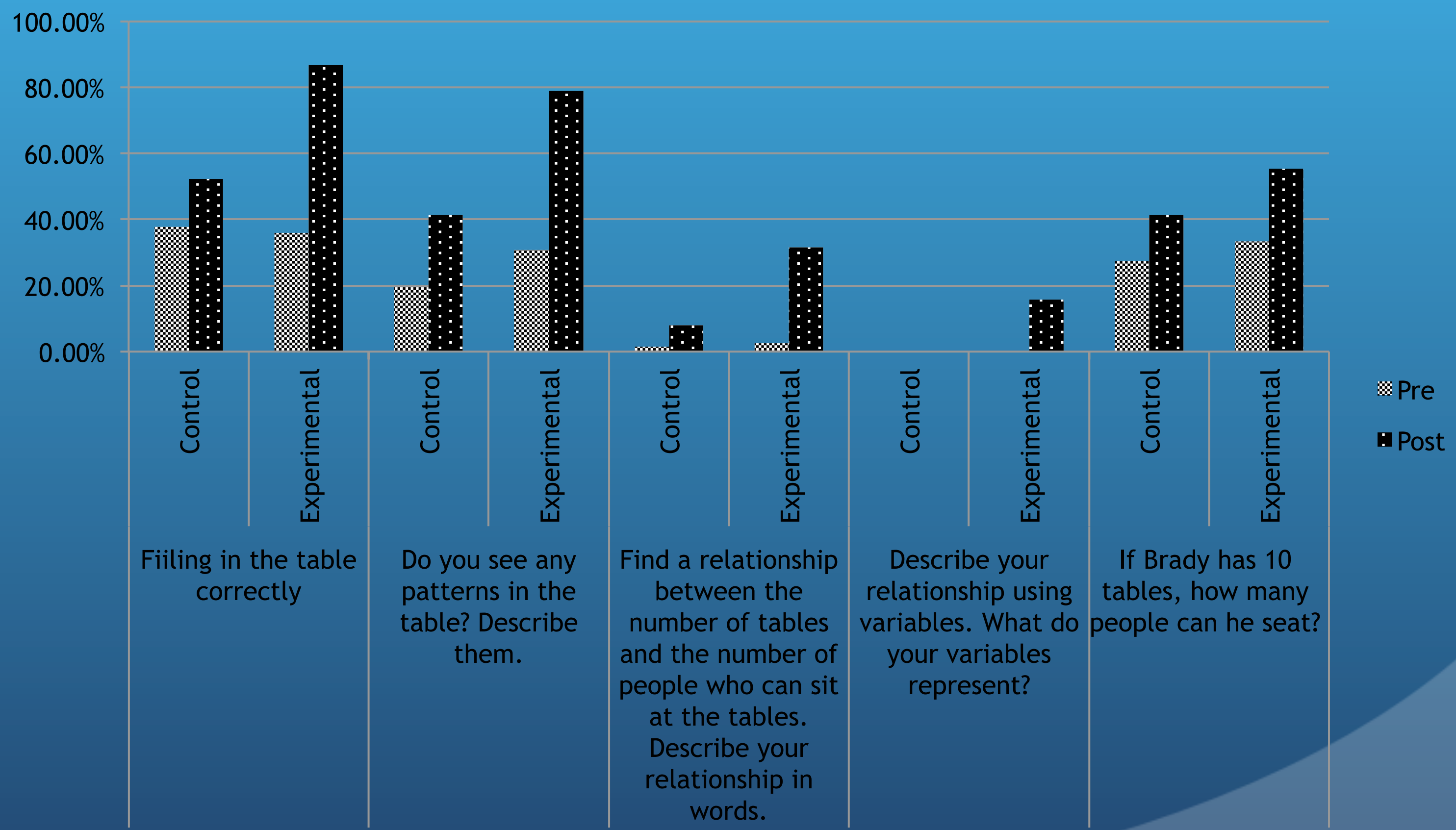
Can elementary students do tasks like this?

3^{rd}-grade students' performance on Brady Problem

- No significant differences at pre-test between experimental and control.
- Experimental students significantly outperformed controls at post-test on items 10a-d (resp., $p<0.001$, $p<0.001$, $p<0.01$, and $p<0.05$); outperformed controls on 10e, but not significantly.



GROWING TRAIN PROBLEM

There was a train that ran the same route everyday. As it went along, it picked up two train cars at each stop.

1. How many train cars did it have at stop 1? How many train cars did it have at stop 2? How many train cars did it have at stop 3?

2. Organize your information in a function table (t-chart). Write an equation that shows the relationship between the values in your table for each set of values.

3. Find a relationship between the number of stops and the total number of cars on the train. Represent your rule in words and variables.

4. If we count the engine, how would this affect your function table? How would this affect your rule?

In your experience, what grade level of student can solve this task?

What would their thinking look like?

How does this task help students develop *Common Core* Mathematical Practices?

COMMON CORE
MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.

- Explain the meaning of parts of an equation (function rule), including variables, in terms of the problem context
- Identify regularity in data by noticing structure in number sentences and representing this structure in a generalized form as a function rule, using symbolic notation
- Make sense of data in function tables (t-charts), including interpreting these data to find problem solutions

2. Reason abstractly and quantitatively.

- Make sense of co-varying quantities and the correspondence relationship between them
- <u>Decontextualizing</u> – abstract the problem situation about a train that grows in a specific, numerical way (at each stop, the train adds 2 more cars) and represent the functional relationship in a generalized symbolic form
- <u>Contextualizing</u> – explain at any point the meaning behind the symbols used to represent the relationship (‘v’ represents the number of stops the train makes)

COMMON CORE
MATHEMATICAL PRACTICES

3. Construct viable arguments and critique the reasoning of others.

- Reason inductively about a set of number sentences in order to find a generalized relationship (function rule)
- Build arguments that explain the correctness of a generalized rule based on the problem context from which data arose (e.g., in the rule v + v + 1 = r, '+1' represents counting the train engine)

4. Model with mathematics.

- Write number sentences ("addition equations") to describe relationships between specific numbers in co-varying data.
- Identify important data in a situation and map their relationship using tables and formulas.
- Analyze the relationship in data about co-varying quantities and draw conclusions about a generalized relationship.
- Interpret their results (function rule) within the problem context as a way to make sense of their model (e.g., does the model accurately reflect the situation?)

COMMON CORE
MATHEMATICAL PRACTICES

5. Use appropriate tools strategically.

- Use different representations such as natural language, algebraic notation, and tables to reason about co-varying relationship and navigate between representations. Use of tables as a strategic choice to organize co-varying data.

6. Attend to precision.

- Understand that a variable represents a quantity, not an object and accurately state the meaning of symbols they choose (‘r’ represents the number of stops, not the actual stops).
- Represent function rule with precision (completeness and consistency of representation)--as an an equation (‘R + R = V’), not an expression (‘R + R’) or in syncopated language (“the number of cars is R + R)

7. Look for and make use of structure.

- Able to discern a pattern or structure in co-varying quantities

8. Look for and express regularity in repeated reasoning.

- Notice that calculations are repeated in number sentences expressing a relationship between two co-varying values and, from this, can abstract the function rule and express this regularity in symbolic notation (“R + R = V” or “R + R + 1 = V”)

- Your Questions?
- Your Comments?

Additive Inverse

A. Find the missing numbers:

0 = 35 – ___

___ – 247 = 0

___ = 78 – 78

B. What do you notice? What can you say about what happens when you subtract a number from itself? Describe your conjecture in words.

C. Represent your conjecture using a variable. Why did you use the same variable? What does it mean to use the same variable in an equation?

D. Can you express your conjecture a different way using the same variable and number?

E. For what numbers is your conjecture true? Is it true for all numbers? Use numbers, pictures, or words to explain your thinking.

G. Callie's mother has some juice boxes in her pantry. Callie's friends come over to play and her mother gives everyone a juice box. She doesn't have any left. Write an equation that represents this situation.

How does this task help students develop *Common Core* Mathematical Practices?

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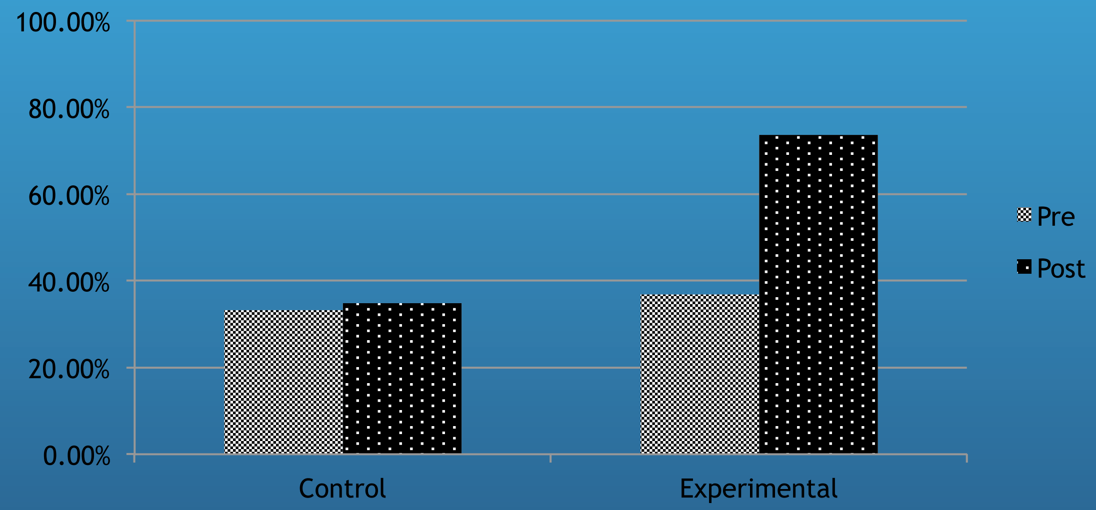
Can elementary students do tasks like this?

Marcy 's teacher asks her to figure out "23 + 15." She adds the two numbers and gets 38. The teacher then asks her to figure out "15 + 23." Marcy already knows the answer.

a) How does she know?

b) Do you think this will work for all numbers? If so, how do you know?

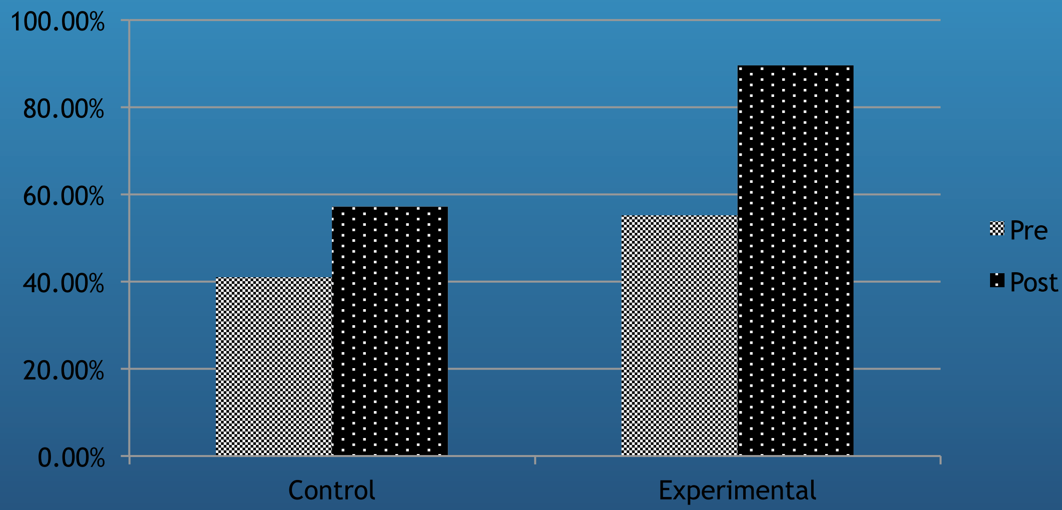


- No significant differences at pre-test between experimental and control.
- Experimental students <u>significantly</u> outperformed controls at post-test

Evelyn computes the following:
8 – 8 = ___
12 – 12 =___

She gets an answer of 0 each time. She starts to thinks that anytime you subtract a number from itself, the answer is 0. Which of the following best describes her thinking? Circle your answer.

a) $a + 0 = 0$
b) $a = b + a + b$
c) $a - a = 0$
d) $a \times 0 = 0$



- No significant differences at pre-test between experimental and control.
- Experimental students <u>significantly</u> outperformed controls at post-test ($p<0.001$)

Early algebra not only….

- develops children's algebra-readiness for middle grades,

it also

- provides significant opportunities for students to engage in and develop the Mathematical Practices advocated by the *Common Core*.