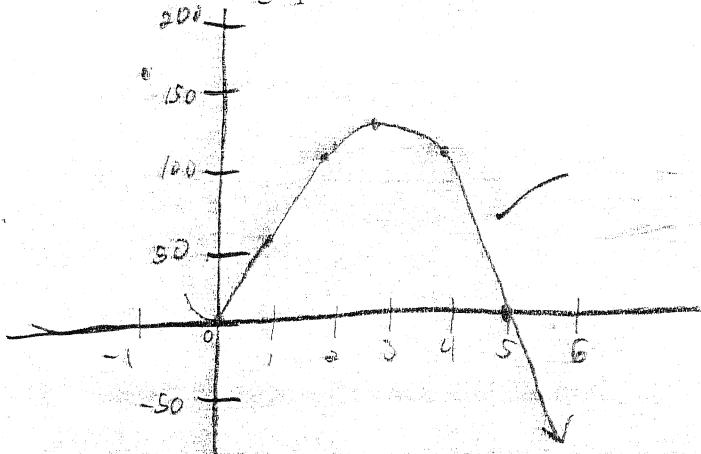


Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
$f(x)$	0	52	108	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = 108 \text{ miles}$$

At 2 pm (2 hours after noon) I will be 108 miles away from Morgantown.

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4.5) = -(4.5)^5 + 12(4.5)^4 - 54(4.5)^3 + 95(4.5)^2$$

$$f(4.5) = 78,468.75 \text{ miles}$$

At 4:30 pm (4.5 hours after noon) I will be 78,468.75 miles away from Morgantown.

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(1.5) = -(1.5)^5 + 12(1.5)^4 - 54(1.5)^3 + 95(1.5)^2$$

$$f(4) = 112$$

$$f(4) - f(1.5) = 112 - 84.65625$$

$$f(1.5) = 84.65625$$

$$\boxed{f(4) - f(1.5) = 27.34375 \text{ miles}}$$

Between 1:30 pm and 4:00 pm I traveled 27.3435 miles.

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(3) - f(2.5) = 126 - 121.09375$$

$$f(2.5) = 121.09375 \text{ miles}$$

$$f(3) = 126 \text{ miles}$$

between 2:30pm and 3:00pm I traveled 4,90625 miles.

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} x_1 &= 1 & f(1) &= 52 \\ x_2 &= 2.5 & f(2.5) &= 121.09375 \\ t_1 &= 52 & & \\ t_2 &= 121.09375 & \frac{121.09375 - 52}{2.5 - 1} &= \frac{69.09375}{1.5} = 46.0625 \text{ miles/hour} \end{aligned}$$

The average rate of change is 46,0625 miles per hour between 1:00pm and 2:30pm.

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} f(1.5) &= 84.65625 & \frac{126 - 84.65625}{3 - 1.5} &= \frac{41.34375}{1.5} = 27.5625 \text{ miles/hour} \\ f(3) &= 126 & & \end{aligned}$$

The average rate of change is 27,5625 miles per hour between 1:30pm and 3:00pm.

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\begin{aligned} f(2) &= 108 & \frac{121.09375 - 108}{2.5 - 2.0} &= \frac{13.09375}{.5} = 26.1875 \text{ miles/hour} \\ f(2.5) &= 121.09375 & & \end{aligned}$$

$f'(2) = 26.1875 \text{ miles/hour}$ The instantaneous rate of change at 2:00pm is 26,1875 miles per hour.

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\begin{aligned} f(2) &= 108 & \frac{108.03598 - 108}{2.001 - 2} &= \frac{0.3598}{.001} = 35.98 \text{ miles/hour} \\ f(2.001) &= 108.03598 & & \end{aligned}$$

The instantaneous rate of change at 2:00pm is 35.98 miles per hour.

11. Why is the estimate in #10 better than the estimate in #9?

The estimate in #10 is better because the instantaneous rate of change was found using a smaller margin of time than in #9. The closer the two variables of time are together the more precise the instantaneous rate of change will be. In #10 the time difference was (.001) and in #9 it was (.5).

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INTRODUCTION TO DERIVATIVES – PG. 3 OF 4

12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f(1.5) = 84.65625 \quad \frac{108 - 84.65625}{2 - 1.5} = \frac{23.34375}{.5} = 46.6875$$

$$f(2) = 108$$

$$f'(1.5) = 46.6875 \text{ miles/hour}$$

The instantaneous rate of change at 1:30pm is 46.6875 miles per hour.

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$f(1.5) = 84.65625 \quad \frac{84.661969 - 84.65625}{1.5001 - 1.5} = \frac{.005719}{.0001} = 57.19 \text{ miles/hour}$$

$$f(1.5001) = 84.661969$$

$$f'(1.5) = 57.19 \text{ miles/hour}$$

The instantaneous rate of change at 1:30pm is 57.19 miles per hour.

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

The estimate in #13 is better because the difference in time used is much closer than in #12. The closer the estimate is depends on the difference of time used. The closer the two x values are to each other the closer the estimate will be. To produce an exact answer, the derivative equation of $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$ will need to be found and used.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f(4) = 112 \quad \frac{111.95996 - 112}{4.001 - 4} = \frac{-0.0404}{.001} = -40.04 \text{ miles/hour}$$

$$f(4.001) = 111.95996$$

$$f'(4) = -40.04 \text{ miles/hour}$$

The instantaneous rate of change at 4:00pm is -40.04 miles per hour.

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$f(4) = 112 \quad \frac{111.996 - 112}{4.0001 - 4} = \frac{-0.004}{.0001} = -40 \text{ miles/hour}$$

$$f(4.0001) = 111.996$$

$$f'(4) = -40 \text{ miles/hour}$$

The instantaneous rate of change at 4:00pm is -40 miles per hour.

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

✓ The answers in #15 and #16 are negative because the person traveling is returning home. These answers do not make sense because it is impossible to travel at a negative speed.

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18. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?

Based on the graph, why would the derivative be zero?

At $f(3.115)$ the graph's curve is at its maximum. The derivative is 0 because the person traveling is no longer driving away or toward home. At that exact time the person has turned around to head home.

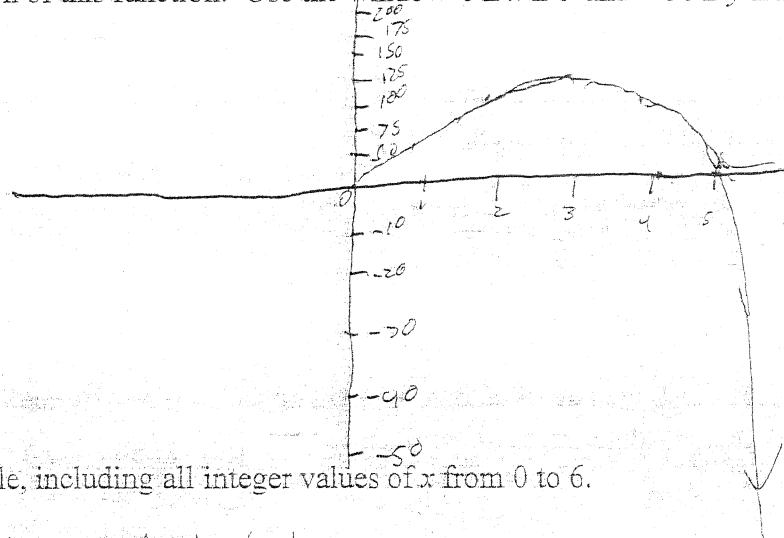
19. What does this point indicate about your trip? What happens before and after 3.115?

The point $f(3.115)$ indicates at this time the person traveling has stopped moving forward or back to home. Before 3.115 the person was traveling away from home. After 3.115 the person has began traveling back toward home.

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.

$\frac{f(2)}{2}$



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
y	0	52	108	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = -(2)^5 + 12(2)^4 - 54(2)^3 + 95(2)^2$$

$\frac{f(2)}{2}$

108 miles

this means 2 hours into my drive (2pm) I was 108 miles away from morgantown

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4.5) = -(4.5)^5 + 12(4.5)^4 - 54(4.5)^3 + 95(4.5)^2$$

$78,46875$

this means 4.5 hours into my trip I was 78,46875 miles away

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

This means how far I was at

4 hours into the trip (4pm) ~~I was 84,65625~~

Subtracted by how far I was

at 1.5 hours into trip I

was 27,34375 miles away

$$= 27,34375$$

$$112 - 84,65625 = 27,34375$$

miles

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$126 - 121.01375$$

$$\boxed{4.90625 \text{ miles}}$$

if you take the time at 3 and subtract it from 2.5 your 4.90625 miles away

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\frac{121.01375 - 82}{2.5 - 1}$$

$$\boxed{46.0625 \text{ miles}}$$

this means between 1 and 2.5 we were an average of 46.0625 miles away

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\frac{126 - 84.65625}{3 - 1.5}$$

$$\boxed{27.5625 \text{ miles}}$$

between 1.5 and 3 we were an average of 27.5625 miles away

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{111.39219 - 106}{2.1 - 2}$$

$$\boxed{33.9214 \text{ miles}}$$

this means we were miles every going about 33.9214 mph

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\frac{106.63579 - 108}{2.001 - 2}$$

$$\boxed{35.474002 \text{ miles}}$$

11. Why is the estimate in #10 better than the estimate in #9?

because the number that is used is closer to 2.0 which is going to give me a closer estimate to how far I was exactly 2 hours into my trip

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12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

this means at 1.30 ~~85.224554~~ $\frac{85.224554 - 84.65625}{1.51 - 1.5}$

I was going about 5,698.45549 mph

5.698455449

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

~~85.224554~~ $\frac{85.224554 - 84.65625}{1.501 - 1.5}$ 5.71677455

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

The answer is number 13 is closer to the correct answer because it is closer to exactly how fast at 1.5 hours away we can get close but its a limit so never will have the exact answer this way to produce the exact answer we would need the slope at 1.5

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

~~107.56719 - 112~~ $\frac{-44.3281}{4.1 - 4}$

This means between at 4 hours in we were going -44.3281 mph.

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$\frac{111.5654779 - 112}{4.01 - 4}$ -40.4122091

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

+2
Yes because the slope is going down therefore it's negative

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INTRODUCTION TO DERIVATIVES – PG. 4 OF 4

18. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?

Based on the graph, why would the derivative be zero?

x2 It stops going up and starts going down. The derivative is 0 because it's not going up or down.

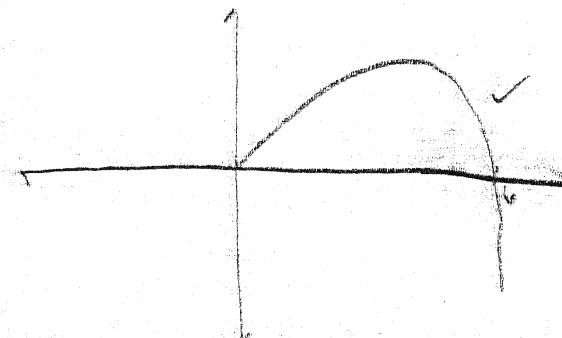
19. What does this point indicate about your trip? What happens before and after

3.115? This is the turning point ~~in~~ in the trip

x2 before this they were getting farther away and after they started coming closer to mountain.

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
$f(x)$	0	52	108	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = -(2)^5 + 12(2)^4 - 54(2)^3 + 95(2)^2$$

= 108 miles away from Morgantown

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4.5) = -(4.5)^5 + 12(4.5)^4 - 54(4.5)^3 + 95(4.5)^2$$

= 78.47 miles away from Morgantown. You are driving back to Morgantown.

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$= -(4)^5 + 12(4)^4 - 54(4)^3 + 95(4)^2 - -(1.5)^5 + 12(1.5)^4 - 54(1.5)^3 + 95(1.5)^2$$

$$112 = 84.66$$

27.34 miles

Between hours 1.5 and 4,
we traveled 27.34 miles.

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} &= -(3)^5 + 12(3)^4 - 54(3)^3 + 95(3)^2 - -(2.5)^5 + 12(2.5)^4 - 54(2.5)^3 + 95(2.5)^2 \\ &= 126 - 121.09 \\ &= 4.91 \text{ miles} \end{aligned}$$

Between hours 2.5 and 3,
we traveled 4.91 miles.

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$f'(x) = \frac{f(2.5) - f(1)}{2.5 - 1} = \frac{121.09 - 52}{2.5 - 1} = \frac{69.09}{1.5} = 46.06 \text{ mph}$$

Between hours 1 and 2.5,
our average speed was 46.06 mph.

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$f'(x) = \frac{f(3) - f(1.5)}{3 - 1.5} = \frac{126 - 84.66}{3 - 1.5} = \frac{41.34}{1.5} = 27.56 \text{ mph}$$

Between hours 1.5 and 3,
our average speed was 27.56 mph.

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f'(2) = -(2.01)^5 + 12(2.01)^4 - 54(2.01)^3 + 95(2.01)^2$$

$$= 108.36 = \star$$

miles away from
Morgantown

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\begin{aligned} &= -x^5 + 12x^4 - 54x^3 + 95x^2 \\ &= -5x^4 + 48x^3 - 162x^2 + 190x \\ &= -5(2)^4 + 48(2)^3 - 162(2)^2 + 190(2) \end{aligned}$$

$$= 36.01$$

11. Why is the estimate in #10 better than the estimate in #9?

#10 is better than #9 because 9 is just a guess,
using 2.01 which is close, but not exact and 10 is
an actual answer

12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$= -(1.5)^5 + 12(1.5)^4 - 54(1.5)^3 + 95(1.5)^2$$

$$= 84.66 = \star$$

miles away from
Morgantown 1.5 hours away

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$\begin{aligned} &= -x^5 + 12x^4 - 54x^3 + 95x^2 \\ &= -5x^4 + 48x^3 - 162x^2 + 190x \\ &= -5(1.5)^4 + 48(1.5)^3 - 162(1.5)^2 + 190(1.5) \end{aligned} = 57.19$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

Estimate #13 is better than #12 because we have an exact answer for #13. For #12, we can get very close using a number such as 1.49999. For an exact answer, we must find the derivative.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$= -(4)^5 + 12(4)^4 - 54(4)^3 + 95(4)^2$$

$$= 112 = \star$$

miles away from
Morgantown 4 hours away

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$\begin{aligned} &= -x^5 + 12x^4 - 54x^3 + 95x^2 \\ &= -5x^4 + 48x^3 - 162x^2 + 190x \\ &= -5(4)^4 + 48(4)^3 - 162(4)^2 + 190(4) \end{aligned} = -40$$

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

Slope \downarrow

They make sense for this problem because at hour 4, we are heading back to Morgantown, making our graph have a negative slope.

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INTRODUCTION TO DERIVATIVES - PG. 4 OF 4

- X 8. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?
Based on the graph, why would the derivative be zero?

-2

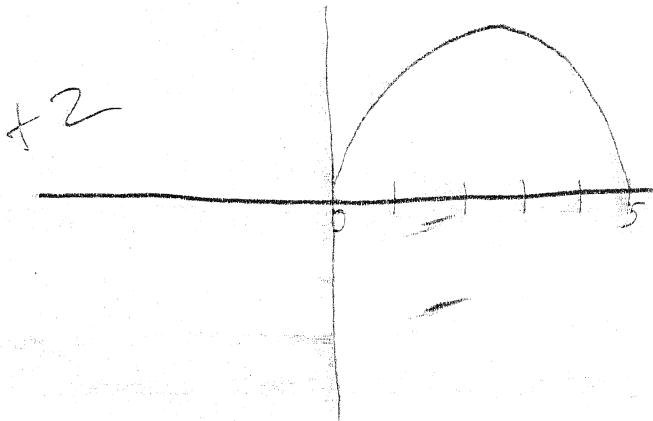
The graph at $f(x) = 3.115 = 0$ because that is the farthest point we are away from Morgantown and we are stopped.

19. What does this point indicate about your trip? What happens before and after 3.115?

This point indicates our farthest point away from Morgantown. Before $f(x) = 3.115$, we are traveling away and after $f(x) = 3.115$, we are traveling back.

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5
y	0	52	108	126	112	0

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = 108 \text{ miles}$$

In 2 hours (2:00) you are 108 miles from Morgantown.

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4.5) = 78,469 \text{ miles}$$

In 4.5 hours you are 78,469 miles from Morgantown.

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4) - f(1.5) = 112 - 84.656 = 27,344 \text{ miles}$$

from 1:30 to 4:00 you are 27,344 net miles from Morgantown.

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$126 - 121.094 = 4.906 \text{ net miles} \quad \begin{array}{l} \text{from } 2:30 - 3:00 \\ \text{you are } 4.906 \\ \text{net miles from} \\ \text{morgantown} \end{array}$$

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(2.5) - f(1)}{2.5 - 1} = \frac{121.094 - 52}{1.5} = 46.063 \text{ mph} \quad \begin{array}{l} \text{from } 1:00 - 2:30 \\ \text{you drove} \\ \text{an averaged} \\ 46.063 \text{ mph} \end{array}$$

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(3) - f(1.5)}{3 - 1.5} = \frac{126 - 84.656}{1.5} = 27.56 \text{ mph} \quad \begin{array}{l} \text{from } 3:00 - 1:30 \\ \text{you averaged} \\ 27.56 \text{ net miles per} \\ \text{hour away from} \\ \text{morgantown} \end{array}$$

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(2.01) - f(2)}{2.01 - 2} = \frac{108.36 - 108}{0.01} = f'(2) \approx 35.79 \text{ miles per hour}$$

At 2:00 you are traveling about 35.79 miles per hour

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\frac{f(2.001) - f(2)}{2.001 - 2} = \frac{108.03598 - 108}{0.001} = f'(2) \approx 35.979 \text{ mph}$$

11. Why is the estimate in #10 better than the estimate in #9?

It is more accurate due to being closer to 2. 2.001 is not as close to 2 as 2.001

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INTRODUCTION TO DERIVATIVES – PG. 3 OF 4

12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f'(1.5) = \frac{f(1.5) - f(1.49)}{1.5 - 1.49} = \frac{84.656 - 84.0824}{0.01} = 57.36 \text{ mi/hr}$$

At 1:30 you are travelling about 57.36 miles per hour

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$\frac{f(1.5) - f(1.499)}{1.5 - 1.499} = \frac{84.656 - 84.59904}{0.001} = 56.96 \text{ mi/hr}$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

#13 is closer to 1.5 than #12. You can get

Innately closer to 1.5 using this technique. In order to get an exact answer you need to find the derivative.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(4.01) - f(4)}{4.01 - 4} = \frac{111.5999 - 112}{0.01} = -40.4122 \text{ mi/hr} \quad f'(4) = -40.41$$

- At 4:00 you are traveling at about -40.4122 miles per hour back to Morgantown

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$\frac{111.9599 - 112}{0.001} \approx 40.041 \quad f'(4) \approx 40.041$$

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

Yes when talking about velocity from morgantown,

& 2 Velocity reaches a maximum point between 2 - 3 hours then goes negative because you are driving back to Morgantown.

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INTRODUCTION TO DERIVATIVES – PG. 4 OF 4

18. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?

Based on the graph, why would the derivative be zero?

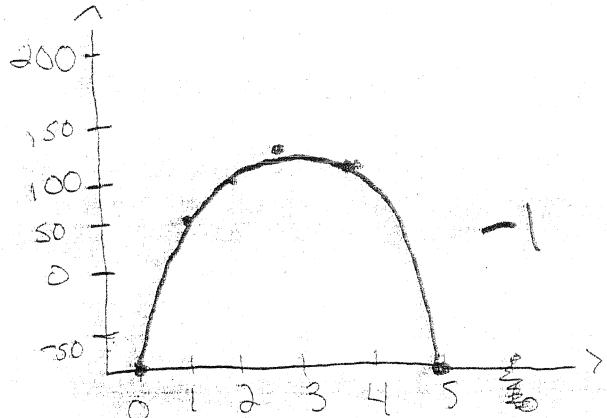
✓ The graph reaches its highest point. The derivative is zero because its velocity reaches its maximum speed away from Morgantown.

19. What does this point indicate about your trip? What happens before and after 3.115?

✓ You are as far away from Morgantown on this trip as you can be. Before 3.115 you are reaching the furthest point and after 3.115 you are on your way back from the furthest point.

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
$f(x)$	0	52	108	126	112	0	8

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$f(2) = 108$ mi. You are 108 miles away from Morgantown after driving for 2 hours.

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$f(4.5) \approx 78.5$ mi. You are 78.5 miles away from Morgantown after driving for 4.5 hours.

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4) = 112$$

$$f(1.5) \approx 85$$

$$f(4) - f(1.5) = 27 \text{ mi.}$$

You traveled 27 miles between 1:30 and 4:00.

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(3) = 126 \quad \text{You traveled 5 miles between}$$

$$f(2.5) \approx 121 \text{ mi. } 2:30 \text{ and } 3.$$

$$f(3) - f(2.5)$$

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$f(1) = 52 \quad 121 - 52$$

$$f(2.5) = 121 \quad \frac{121 - 52}{2.5 - 1} = 46 \text{ mph}$$

You were traveling about 46 mph between 1 and 2:30.

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$f(1.5) = 85 \quad 126 - 85$$

$$f(3) = 126 \quad \frac{126 - 85}{3 - 1.5} \approx 27 \text{ mph}$$

You were traveling about 27 mph between 1:30 and 3:00.

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\cancel{\text{approx}} \quad f(2) = 108 \quad \frac{111 - 108}{1} = 30 \text{ mph}$$

$$\cancel{\text{approx}} \quad f(2.1) = 111$$

$f'(2) \approx 30 \text{ mph}$ You are

traveling

aprox. 30 mph at
2:00

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\cancel{\text{approx}} \quad -x^5 + 12x^4 - 54x^3 + 95x^2$$

$$f'(2) \approx 36 \quad (-5x^4 + 12(4x^3) - 54(3x^2) + 95(2x)) \approx 36 \text{ mph}$$

11. Why is the estimate in #10 better than the estimate in #9?

#10 is taking the actual derivative as opposed to #9 taking an approximate one

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12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f(1.5) \approx 85 \quad f'(1.5) \approx 50 \text{ mph}$$

$$f(1.6) \approx 90 \quad \frac{90-85}{1.6-1.5} \approx 50$$

You are traveling approx. 50 mph at 1:30.

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$\text{Follows } -x^5 + 12x^4 - 54x^3 + 95x^2$$

$$f'(1.5) \approx 57 \text{ mph} - (5x^4) + 12(4x^3) - 54(3x^2) + 95(2x) \approx 57 \text{ mph}$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

#13 takes the actual derivative. #12 you can get as close as possible using numbers closer and closer together, but it will never produce an exact answer. To produce an exact answer, you would need to use the actual derivative.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f(4) = 112 \quad \frac{106-112}{4-1} \approx 40 \quad f'(4) \approx 40 \text{ mph}$$

You are traveling approx. 40 mph at 4:00.

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$\text{Follows } f'(4) \approx 40 \text{ mph} - 5x^4 + 12(4x^3) - 54(3x^2) + 95(2x)$$

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

✓ Because you began driving back to Morgantown at this time because according to the graph in #1, your distance away from Morgantown is decreasing.

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8. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?

Based on the graph, why would the derivative be zero?

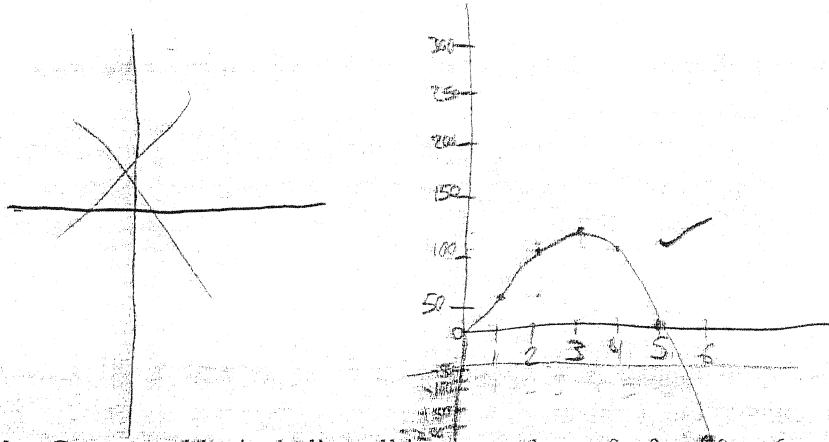
-2 The graph does not move. The derivative is 0 because you are not traveling at this point in time.

9. What does this point indicate about your trip? What happens before and after 3.115?

-2 This point indicates that you are not travelling at this point in time. Before this point, you are simply approaching this point. After this point, you are not traveling

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
$f(x)$	0	52	122	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = 108 \text{ miles at } 2:00 \text{ pm after 2 hours of driving.}$$

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4.5) = 78.5 \text{ miles after 4.5 hours of driving.}$$

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(4) - f(1.5) = 112 - 89.65625 = 27.34 \text{ miles after 2.5 hours of driving.}$$

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(3) - f(2.5) = 126 - 121.18 = 4.81 \text{ miles after } .5 \text{ hours of driving}$$

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(b) - f(a)}{b - a} = \frac{121.1 - 52}{2.5 - 1} = \frac{69.1}{1.5} = 46.0666667 \frac{\text{mi}}{\text{hr}}$$

is the AROC

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(b) - f(a)}{b - a} = \frac{126 - 84.65625}{3 - 1.5} = \frac{41.34375}{1.5} = 27.5625 \frac{\text{mi}}{\text{hr}}$$

is the AROC.

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{2.01 - 2}{2.01 - 2} = \frac{108.357902 - 108}{.01} = 35.7902 \frac{\text{mi}}{\text{hr}}$$

$f'(2) = 35.7902 \frac{\text{mi}}{\text{hr}}$ you were driving about that many $\frac{\text{mi}}{\text{hr}}$ after 2 hours.

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$f(2) = -5(2)^4 + 48(2)^3 - 162(2)^2 + 190(2)$$

$$= 36$$

11. Why is the estimate in #10 better than the estimate in #9?

If is a better estimate because it is a whole number rather than a decimal also meaning number 10 is the exact number and #9 is an estimate.

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12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(1.5) - f(1.05)}{1.5 - 1.05} = \frac{84.65625 - 55.555555555}{.45} = 29.18070656 \frac{\text{mi}}{\text{hr}}$$

You were driving about that many $\frac{\text{mi}}{\text{hr}}$ after 1.5 hours

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$f(1.5) = -5(1.5)^4 + 48(1.5)^3 - 162(1.5)^2 + 140(1.5)$$

$$= 56.1875$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

#13 is better than the estimate in #12 because it is an exact estimate. Other than being very close to the estimate. During these techniques you may obtain an extremely close estimate giving you an appropriate answer.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(4.01) - f(4)}{4.01 - 4} = \frac{1115958779 - 112}{.01} = f'(4) = -90.41221 \frac{\text{mi}}{\text{hr}}$$

You were going about that many $\frac{\text{mi}}{\text{hr}}$ after 4 hrs so it looks like the driver must have stopped.

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$f(4) = -5(4)^4 + 48(4)^3 - 162(4)^2 + 140(4)$$

$$= -40$$

- Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

The answers are negative because the driver must have taken a break during this time. It makes sense on the problem because after 4 on the graph, S is 0 making 4 have to be negative.

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For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?

Based on the graph, why would the derivative be zero?

- 2 The graph is negative at this point. The derivative is zero because when you plug 3.115 in the equation that is what it rounds to.

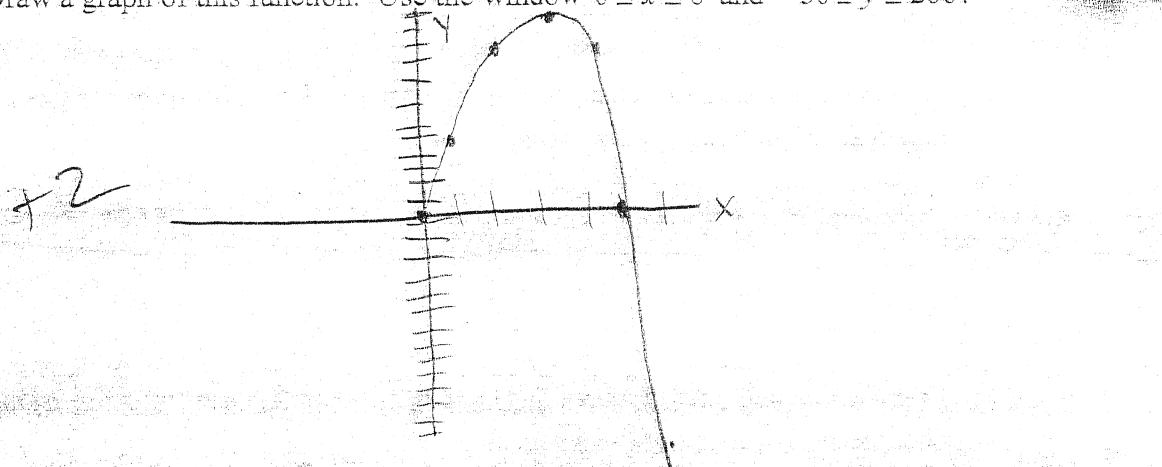
What does this point indicate about your trip? What happens before and after 3.115?

This point indicates that I am slowly starting to slow down.

- 2 Before 3.115 I am going fast, making progress. After I am slowing down

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.



2. Create a table, including all integer values of x from 0 to 6.

x	0	1	2	3	4	5	6
$f(x)$	0	52	108	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$f(2) = -(2)^5 + 12(2)^4 - 54(2)^3 + 95(2)^2$$

$$x^2 = 108 \quad \text{In 2 hours you drive 108 miles}$$

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$(4.5) = -(4.5)^5 + 12(4.5)^4 - 54(4.5)^3 + 95(4.5)^2$$

$$= 78.46875 \quad \text{In 4.5 hours you drive 78.5 miles}$$

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} f(4) - f(1.5) &= 112 - 84.65625 \\ &= 27.34375 \\ &= 27.34 \end{aligned}$$

Between 1.5 and 4 hours

You drive 27.34 miles

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(3) - f(2.5) = 126 - 121.09375$$
$$= 4.90625$$

Between 2.5 and 3 hours you drive 4.91 miles

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(1) - f(2.5)}{1 - 2.5} = \frac{52 - 121.09375}{-1.5} = \frac{-69.09375}{-1.5} = 46.0625$$

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(1.5) - f(3)}{1.5 - 3} = \frac{84.65625 - 126}{-1.5} = \frac{-41.34375}{-1.5} = 27.5625$$

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(2.02) - f(2)}{2.02 - 2} = \frac{108.44 - 108}{0.02} = 35.58$$

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\frac{f(2.01) - f(2)}{2.01 - 2} = \frac{108.36 - 108}{0.01} = 35.79$$

11. Why is the estimate in #10 better than the estimate in #9?

Using $f(2.01)$ rather than $f(2.02)$ is better because it is closer to the whole number 2.

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INTRODUCTION TO DERIVATIVES – PG. 3 OF 4

12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(1.52) - f(1.5)}{1.52 - 1.5} = \frac{85.79206472 - 84.65625}{0.02} = 56.79$$

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$\frac{f(1.51) - f(1.5)}{1.51 - 1.5} = \frac{85.22614554 - 84.65625}{0.01} = 28.49$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

Using $f(1.51)$ is closer to $f(1.5)$ than $f(1.52)$ so you get a more accurate answer. You would need to find the smallest possible number to 1.5 to get an exact answer.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(4.02) - f(4)}{4.02 - 4} = \frac{111.1934307 - 112}{0.02} = -40.83$$

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$\frac{f(4.01) - f(4)}{4.01 - 4} = \frac{111.5958779 - 112}{0.01} = -40.41$$

17. Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

On the graph, $f(4)$ is a negative slope

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INTRODUCTION TO DERIVATIVES – PG. 4 OF 4

18. For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?
Based on the graph, why would the derivative be zero?

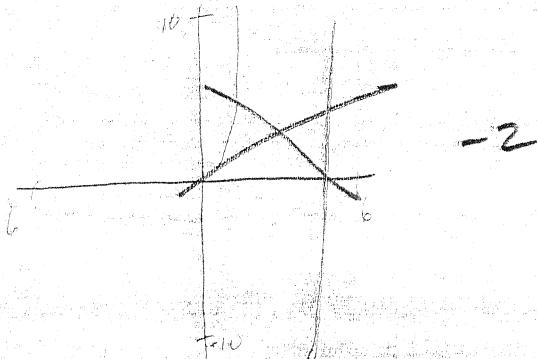
The graph of $f(3.115)$ is negative, it is the highest point on the graph

19. What does this point indicate about your trip? What happens before and after 3.115? It's the fastest point on your trip.

Suppose you start driving away from Morgantown in a car at 12:00 noon. At any time x hours after your trip starts, your distance away from Morgantown (in miles) is given by the function $f(x) = -x^5 + 12x^4 - 54x^3 + 95x^2$. You drive for 5 hours.

1. Draw a graph of this function. Use the window $0 \leq x \leq 6$ and $-50 \leq y \leq 200$.

$$f(5) = -5^5 + 12(5)^4 - 54(5)^3 + 95(5)^2 = 0$$



2. Create a table, including all integer values of x from 0 to 6.

0	1	2	3	4	5	6
0	52	108	126	112	0	-468

3. Find $f(2)$. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} f(2) &= -2^5 + 12(2)^4 - 54(2)^3 + 95(2)^2 \\ &= -32 + 192 - 432 + 380 = 108 \text{ miles after 2 hrs.} \end{aligned}$$

4. Find $f(4.5)$. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} f(4.5) &= -4.5^5 + 12(4.5)^4 - 54(4.5)^3 + 95(4.5)^2 \\ &= -1845.3 + 4920.75 - 4920.75 + 1923.75 = 78.45 \text{ miles} \\ &\quad \text{You Are 78.45 miles} \end{aligned}$$

5. Find $f(4) - f(1.5)$. Give units for the answer, and interpret in terms of the problem.

$$\begin{aligned} f(4) &= -4^5 + 12(4)^4 - 54(4)^3 + 95(4)^2 \\ &= 1024 + 3072 - 3456 + 1520 = 2160 \end{aligned}$$

$$\begin{aligned} f(1.5) &= -1.5^5 + 12(1.5)^4 - 54(1.5)^3 + 95(1.5)^2 \\ &= -7.59 + 60.75 - 182.25 + 213.75 = 84.66 \\ f(4) &\quad f(1.5) \\ 2160 - 84.66 &= 2075.34 \text{ miles} \end{aligned}$$

6. Find $f(3) - f(2.5)$. Give units for the answer, and interpret in terms of the problem.

$$f(3) = -3^5 + 12(3)^4 - 54(3)^3 + 95(3)^2 =$$

$$=$$

$$f(2.5) = -2.5^5 + 12(2.5)^4 - 54(2.5)^3 + 95(2.5)^2$$

$$= 121.094 \text{ miles}$$

7. Find the average rate of change of $f(x)$ from 1 to 2.5. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(2.5) - f(1)}{2.5 - 1}$$

$$= \frac{121.09 - 52}{1.5} = 46.06 \text{ miles}$$

8. Find the average rate of change of $f(x)$ from 1.5 to 3. Give units for the answer, and interpret in terms of the problem.

$$\frac{f(3) - f(1.5)}{3 - 1.5} = \frac{126 - 84.65625}{1.5}$$

$$= 27.5625 \text{ miles}$$

9. Find an estimate the instantaneous rate of change of $f(x)$ at 2. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f'(2) = \frac{f(2.01) - f(2)}{2.01 - 2} = \frac{108.3579022 - 108}{0.01} = 35.7902 \text{ miles}$$

10. Find a better estimate for the instantaneous rate of change of $f(x)$ at 2.

$$\frac{f(2.0001) - f(2)}{2.0001 - 2} = \frac{108.00259979 - 108}{0.0001} = 35.9979 \text{ miles}$$

11. Why is the estimate in #10 better than the estimate in #9?

Because the numbers being used in the equation for $f(b)$ are more exact.

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12. Find an estimate the instantaneous rate of change of $f(x)$ at 1.5. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$\frac{f(1.501) - f(1.5)}{1.501 - 1.5} = \frac{84.7134 - 84.656}{.001} = 57.4 \text{ miles}$$

13. Find a better estimate for the instantaneous rate of change of $f(x)$ at 1.5.

$$\frac{f(1.50001) - f(1.5)}{1.50001 - 1.5} = \frac{84.6562181 - 84.656}{.00001} = 8.2187 \text{ miles}$$

14. Why is the estimate in #13 better than the estimate in #12? How close of an estimate may be obtained using these techniques? What would you need to do produce an exact answer?

$f(b)$ in #13 is more exact. The more you extend/make more exact $f(b)$, the closer the estimate will be, however, it's impossible to find an exact answer because you can always keep making the number smaller.

15. Find an estimate the instantaneous rate of change of $f(x)$ at 4. Give a symbol for this answer. Also, give units and interpret in terms of the problem.

$$f'(4) = \frac{f(4.001) - f(4)}{4.001 - 4} = \frac{111.9599 - 112}{.001} = -40.1 \text{ miles}$$

16. Find a better estimate for the instantaneous rate of change of $f(x)$ at 4.

$$= \frac{f(4.00001) - f(4)}{4.00001 - 4} = \frac{111.9996 - 112}{.00001} = -40 \text{ miles}$$

- Why are your answers for #15 and #16 negative? Does this make sense for the problem you are working on?

-2

Because at 4 they may have stopped indicating a break in the graph.

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- X For this function, $f'(3.115) = 0$. What happens to the graph of $f(x)$ at 3.115?
Based on the graph, why would the derivative be zero?

-2

Based on the graph there is

a break in the trip, it stops.

- X What does this point indicate about your trip? What happens before and after 3.115?

-2

It shows a stop in the trip, there is rapid change on both sides of 3.115.