# Genres of high cognitive demand mathematical tasks for your students

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*** What is a genre?**

**Genre (from French, "kind" or "sort", from Latin: genus (stem gener-), is the term for any category of literature or other forms of art e.g. music, and in general, any type of discourse, whether written or spoken, audial or visual, based on some set of stylistic criteria.**

Genres are formed by conventions that change over time as new genres are invented and the use of old ones are discontinued.

*** What is the cognitive demand of a task?**

**"The kind and level of thinking required of students in order to successfully engage with and solve the task."**

**Stein, Smith, Henningsen, & Silver, (2000)**

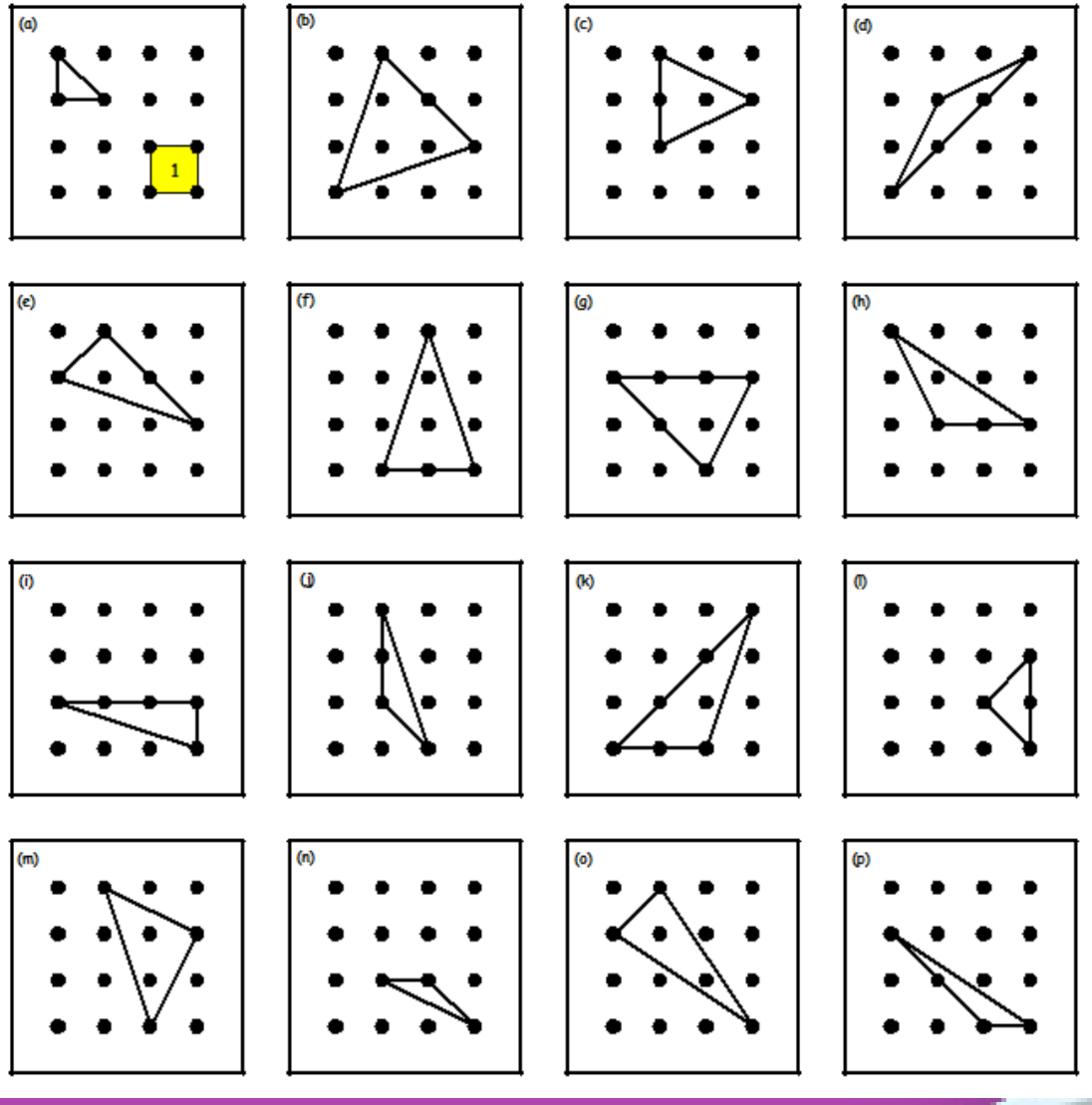
# Genres of tasks:

1) Sorting / classifying tasks 
2) Compare /contrast tasks 
3) Layering tasks 
4) Cognitive conflict tasks 
5) "Give examples" 
6) "Is it possible?" 
7) "What if not?" "What if…?" 
8) Look for the converse 
9) "Tell me about it" 

**Classification** is defined as
the ordering or arrangement of objects into sets (classes) on the basis of their relationships.

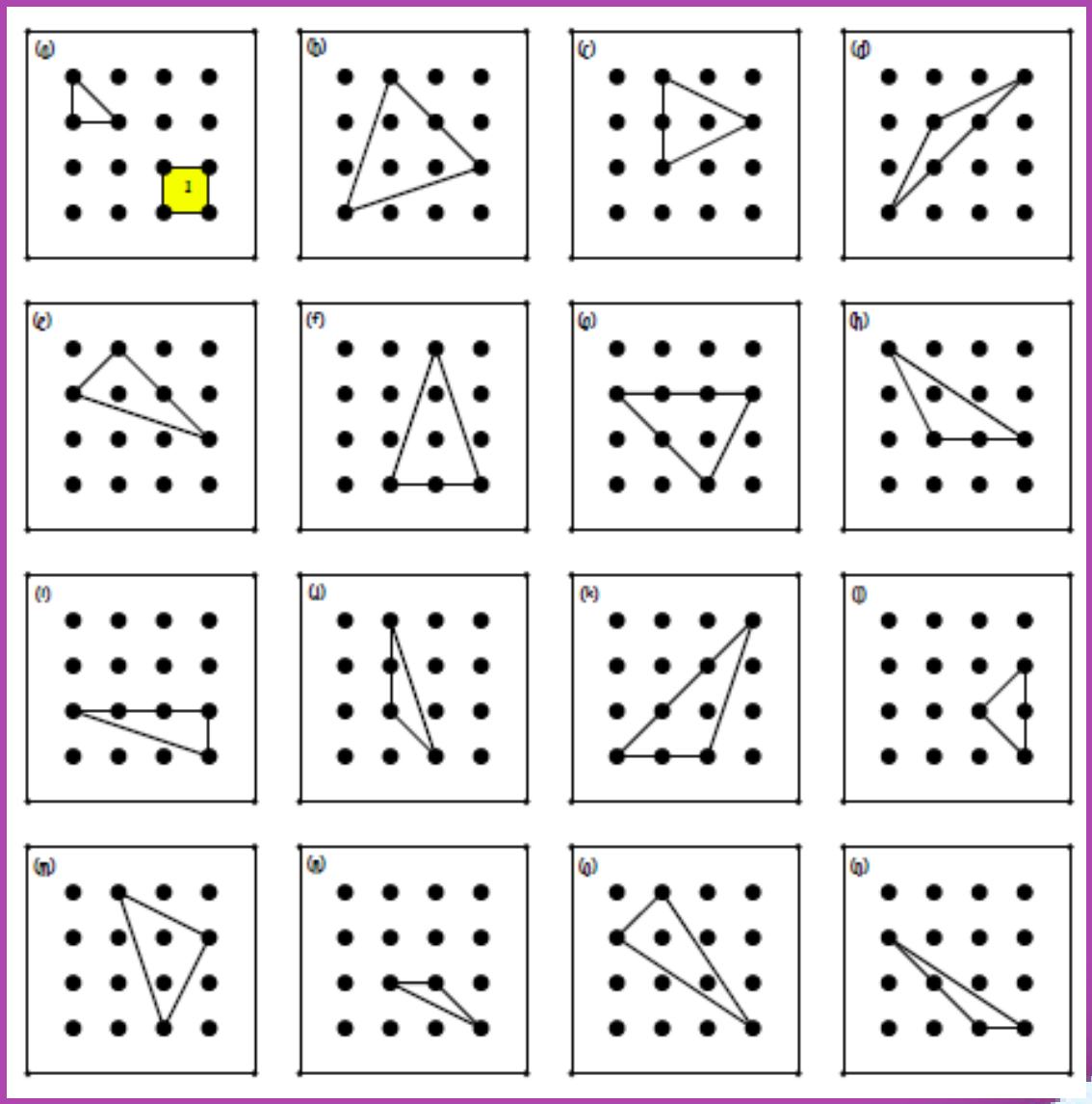
If the criteria for the ordering are given, it is a **sorting** task.

The term **classification** refers to the process and also denotes the end-product of that process.




Your task:

1) Sort these triangles by their area.
2) Classify these triangles using at least two different criteria.




| area ($u^2$) | triangle |
|---|---|
| 0.5 | (a)(n) |
| 1 | (j)(l)(p) |
| 1.5 | (d)(i) |
| 2 | (c)(e)(h) |
| 2.5 | (o)(m) |
| 3 | (g)(k)(f) |
| 3.5 | ? |
| 4 | (b) |

| | Acute | Right | Obtuse |
|---|---|---|---|
| Equilateral | - | | |
| Isosceles | (b) (c) (f) (o) | (a) (l) (m) | (d) |
| Scalene | (g) | (e) (i) | (h) (j) (k) (n) (p) |

## Why do we classify?

i) To organize the objects in order to achieve economy of memory;

ii) To achieve ease of manipulation of the objects in a class;

iii) To learn and describe the structure of and relationships among the objects in a class;

iv) To be able to compare and contrast the different classes;

v) To generate hypotheses about the objects in the class;

Compare contrast tasks consist of the analysis of two "objects" in order to identify common attributes and different characteristics

*When you compare "objects", you look for their similarities--the properties that make them alike, attributes that both of these objects have.

*When you contrast them, you look at their differences, properties that one of the "objects" has but the other has not.

Compare & Contrast

properties of the rational functions

Domain is a subset of R

May have an horizontal asymptote

Graph may not intersect the y-axis

Graph may intersect the x-axis many times

Not necessarily monotonic

continuous at each point of their domain

differentiable at each point of their domain

Domain is R

Have an horizontal asymptote

Graph intersect the y-axis

Graph intersect the x-axis at most once

monotonic

properties of the exponential functions $y = A \cdot B^{x+D} + C \quad (B>0)$

## Now it is your turn:

properties of a dilation

properties of a rotation

## Other concepts to compare / contrast:

- The factors and the multiples of a natural number
- The prime numbers and the composite numbers
- The rational numbers and the irrational numbers
- A rhombus and a rectangle
- A kite and a trapezoid
- A parabola and an hyperbola
- Arithmetic sequences and geometric sequences
- ?

**Layering Tasks** consist of a set a problems that have to be:

a) solved and

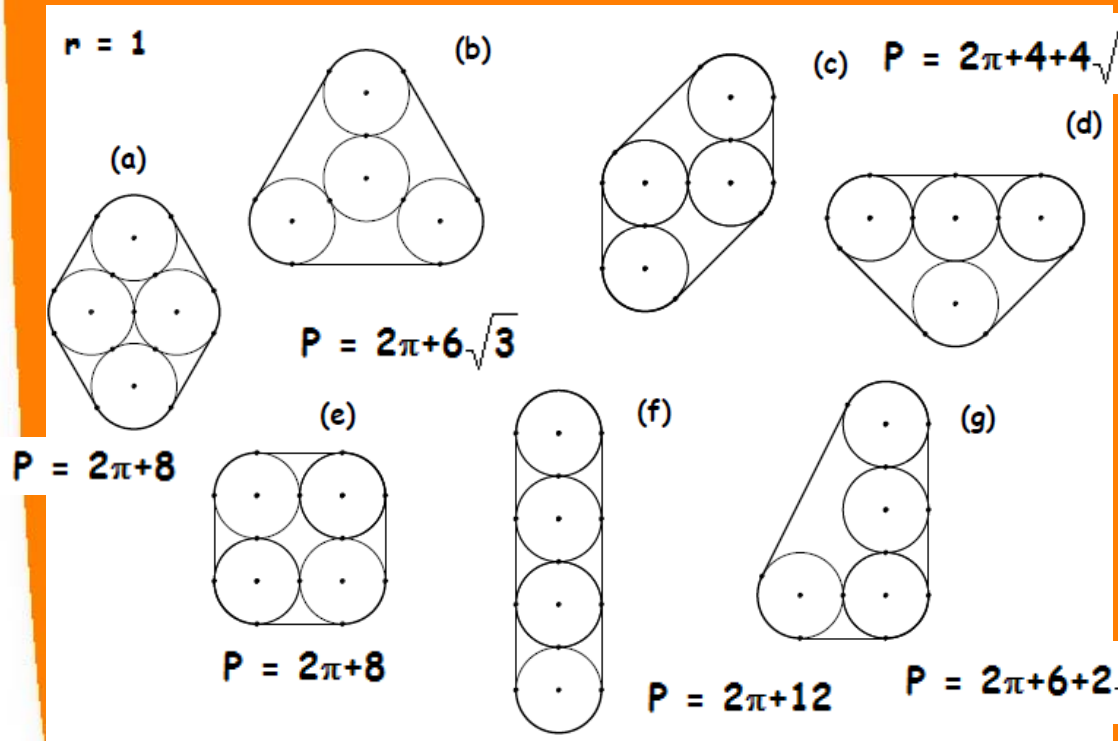
b) arranged in increasing order of difficulty.

Students may be asked also to add

problems of their own and place them in the sequence created.

Four congruent circular cylinders were "tied" with a string following these designs.

In each case, the length of the string is calculated.




Your task:

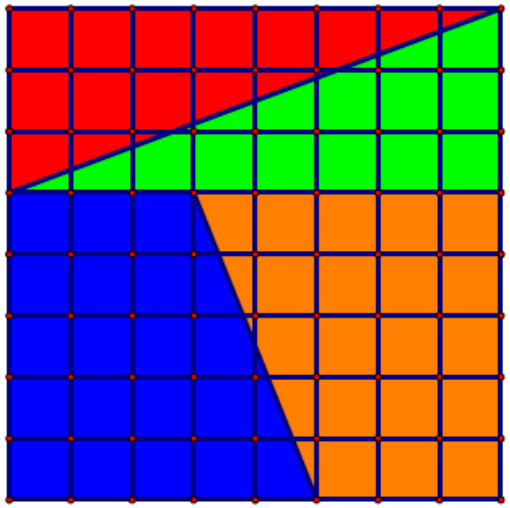
1) Arrange the designs in increasing order of difficulty for the calculation of its perimeter.
2) Justify your ordering.
3) Add a design of your own.

## Cognitive conflict

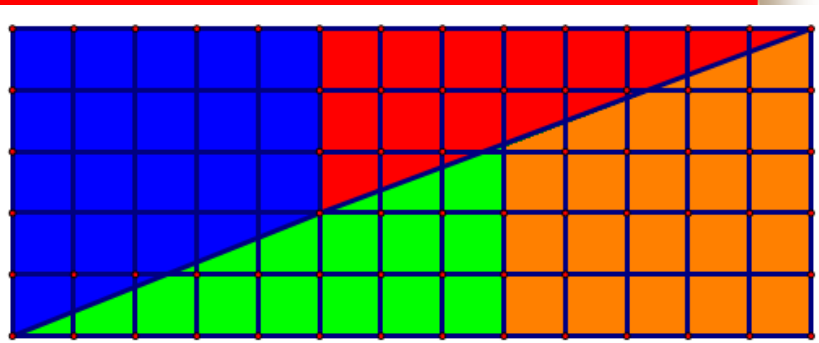
"Smooth seas do not make skillful sailors."

## Task: Resolve the conflict

$$6 \underset{(1)}{=} \sqrt{36} \underset{(2)}{=} \sqrt{(-9)(-4)} \underset{(3)}{=} \sqrt{-9} \cdot \sqrt{-4} \underset{(4)}{=} 3i \cdot 2i \underset{(5)}{=} -6$$



$S = 8^2 = 64$ units

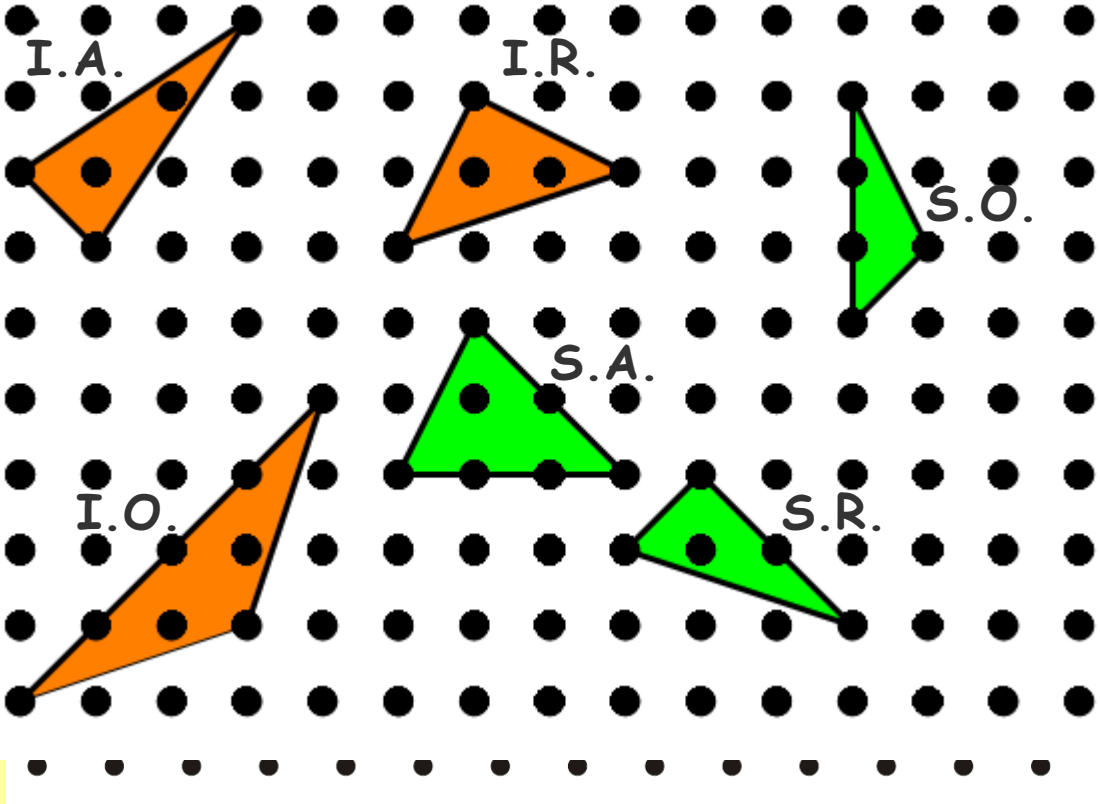


$S = 5x13 = 65$ units

64 = 65?

## Give examples:

In a traditional geoboard,
construct the 7 different types of triangles

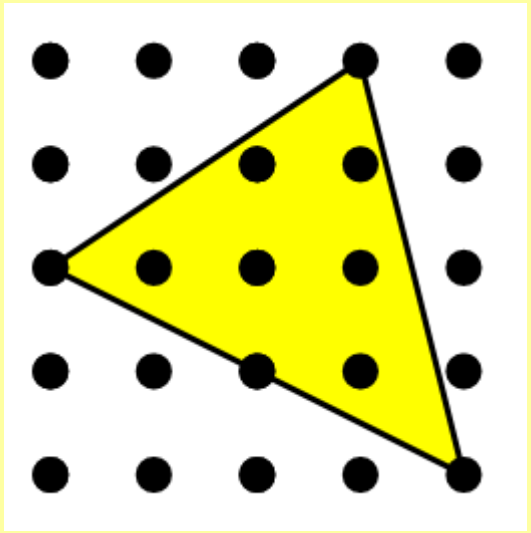


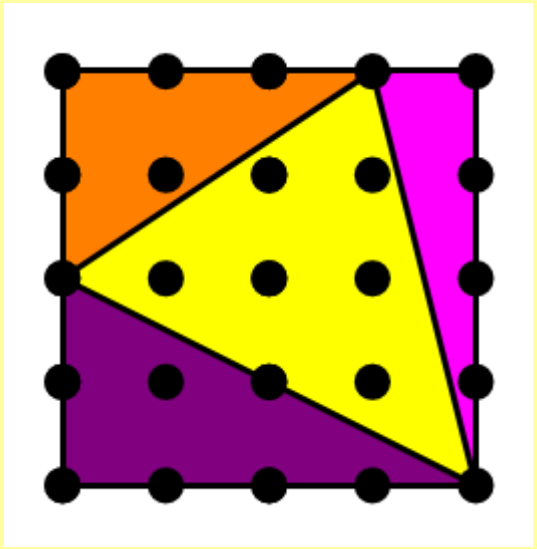

**Is it possible?**

# Equilateral triangles in a traditional geoboard: NO!!

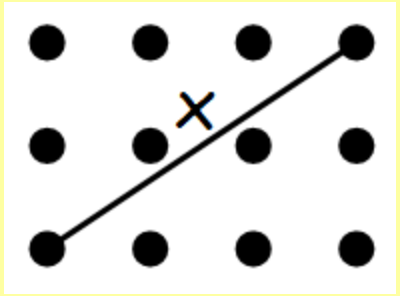
Are you sure?

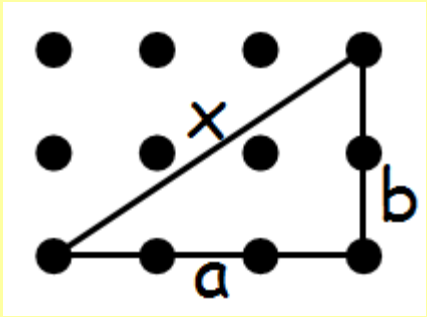
Why there are no equilateral triangles in that geoboard?





The area of any polygon on the geoboard is …

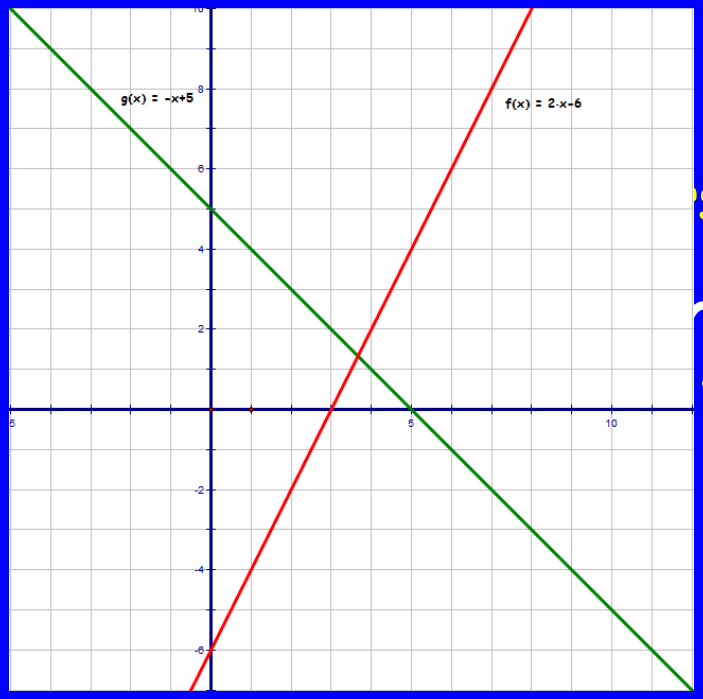


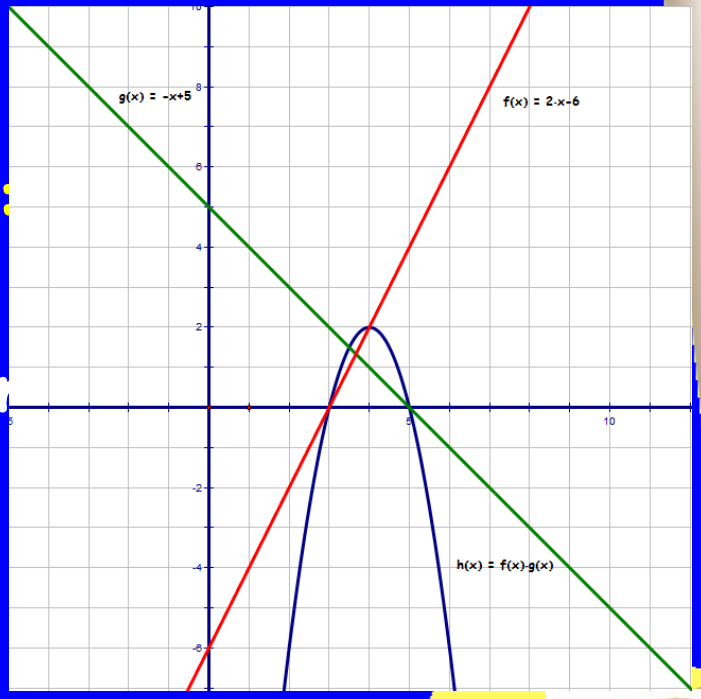




The square of the length of any segment on the geoboard is…

Look for the converse:

The product of two non-constant linear functions is a …….




# "Tell me about it"

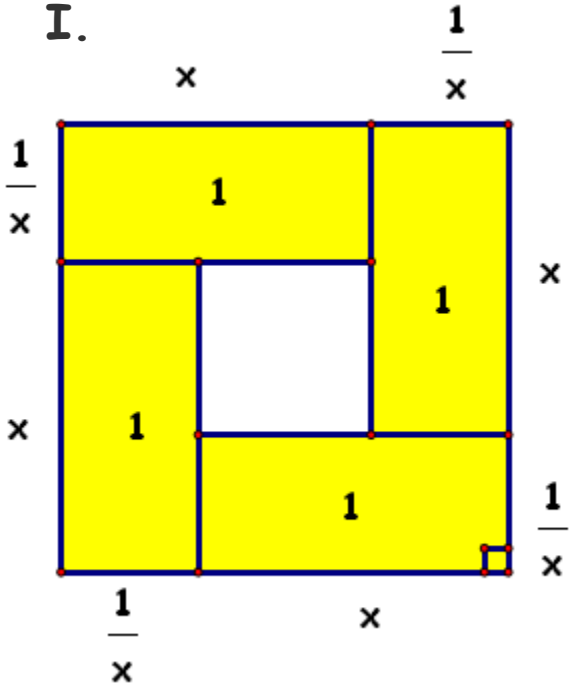
**The sum of a positive number and its reciprocal is at least two.**

Nelsen, R. (1993)

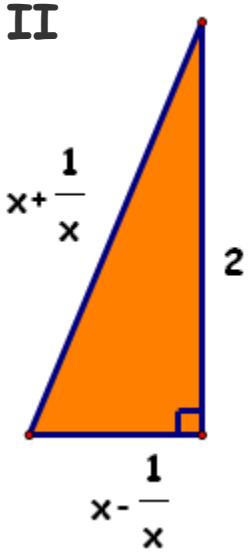
Proofs without words p.62

$$x>0 \Rightarrow x + \frac{1}{x} \geq 2$$

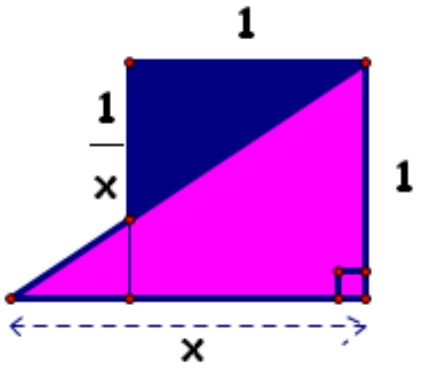
I.



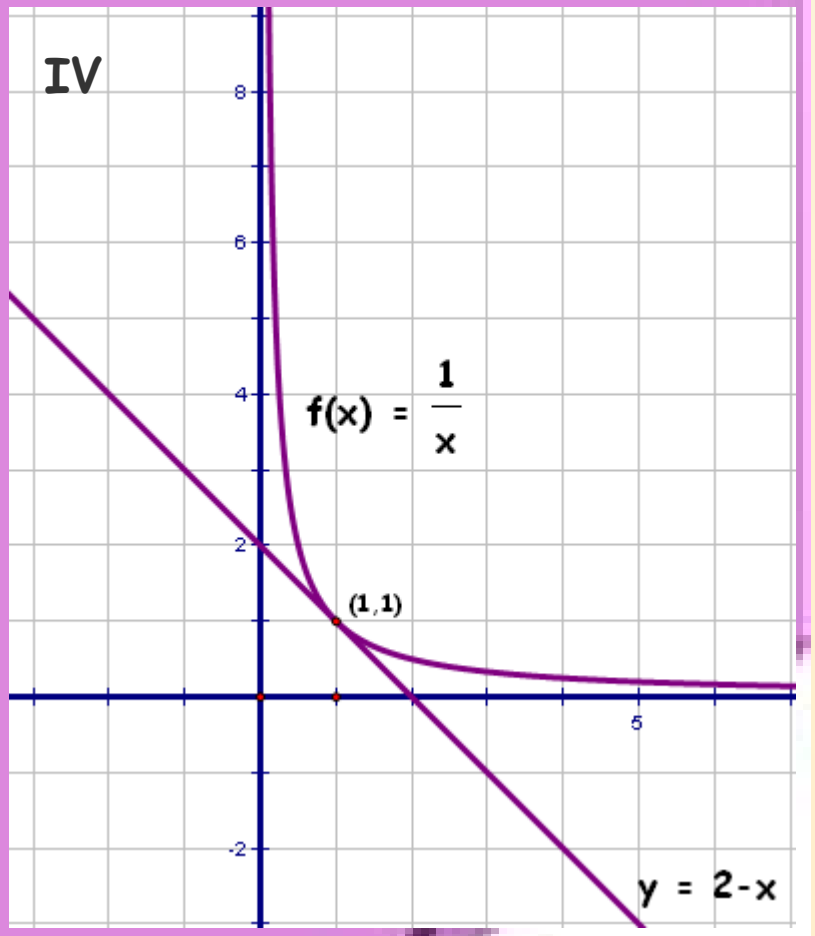

II




III




IV

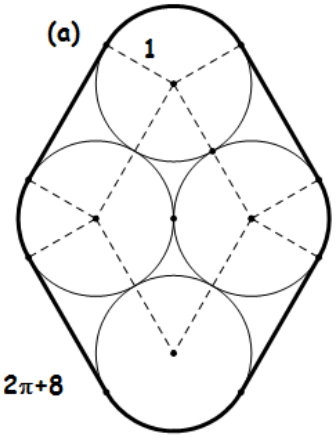


## Tasks of Higher Level of cognitive demand:

* Require complex and non-algorithmic thinking;
* Require students to explore and understand the nature of mathematical concepts, processes or relationships;
* Demand self-monitoring of own cognitive processes;
* Require students to analyze the task and actively examine tasks constrains that may limit possible solution strategies and solutions;
* Require considerable cognitive effort and may involve some level of anxiety because of the unpredictable nature of the solution process required

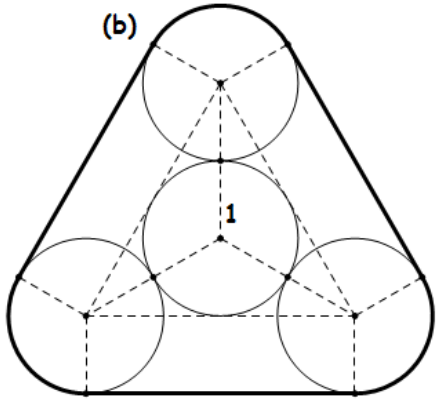
(Smith and Stein, 1998)

(a)



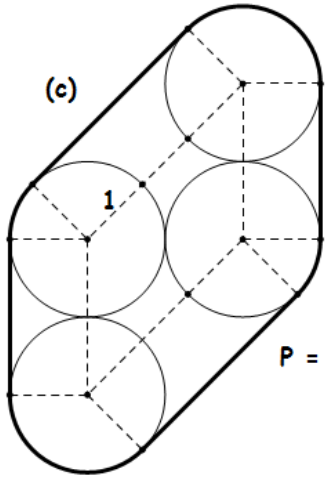

$P = 2\pi+8$

(b)



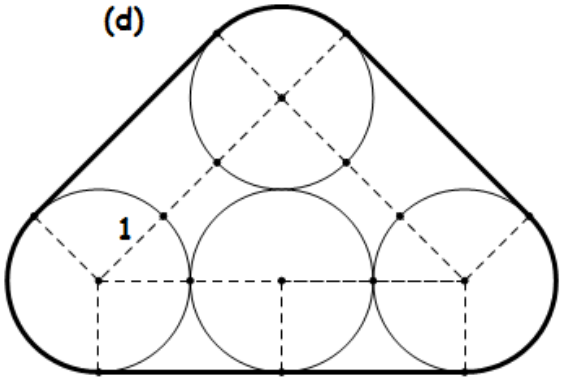

$P = 2\pi+6\sqrt{3}$

(c)



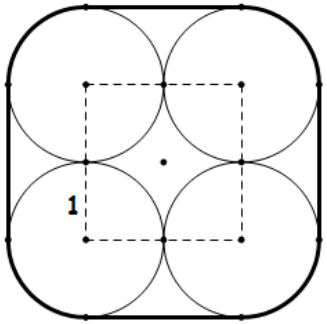

$P = 2\pi+4+4\sqrt{2}$

(d)



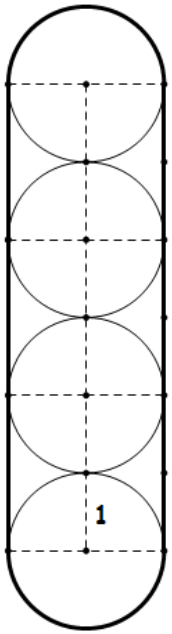

$P = 2\pi+4+4\sqrt{2}$

(e)



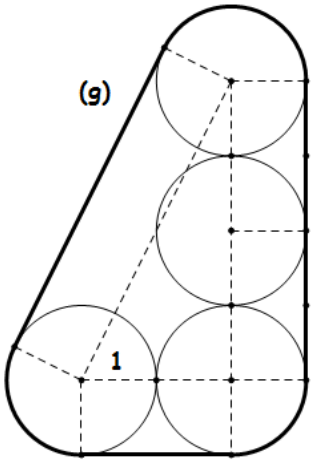

$P = 2\pi+8$

(f)




$P = 2\pi+12$

(g)




$P = 2\pi+6+2\sqrt{5}$