# The Median-Median Line: Connecting Data, Geometry, and Algebra 

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## The need for a task

MM2D2. Students will determine an algebraic model to quantify the association between two quantitative variables.

Gather and plot data that can be modeled with linear and quadratic functions.

Examine the issues of curve fitting by finding good linear fits to data using simple methods such as the median-median line and "eyeballing."

## The Need for a Task

Establish three median points from the data: $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right),\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$

Write the equation of the medianmedian line in the form $y=a x+b$ where

- $a=\frac{y_{3}-y_{1}}{x_{3}-x_{1}}$ and
- $b=\frac{y_{1}+y_{2}+y_{3}-a\left(x_{1}+x_{2}+x_{3}\right)}{3}$


## The Common Core State Standards

G-CO: 10) Prove theorems about triangles. Theorems include: the medians of a triangle meet at a point.

S-ID: 6) Fit a linear function for a scatter plot that suggests a linear association.

S-ID: 7) Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Look for and express regularity in repeated reasoning

## Warm Up

- Write the equation of a line passing through $(-3,8)$ and $(5,9)$.
- Find the length and midpoint of the segment with endpoints $(-3,8)$ and $(5,9)$.

Draw a scalene triangle. How do you locate the centroid of the triangle? What are the properties of the centroid of a triangle?

## Warm Up Solutions

## Properties of the Centroid:



- It is the weighted center of the triangle.
- The centroid divides the median into segments in a 1:2 ratio, with the longer section adjacent to the vertex and the shorter section adjacent to the midpoint.
- Connecting the centroid to each vertex divides the triangle into three triangles of equal area.


## The Situation

- The NBA is trying to provide relevant information to potential team owners.
- Reaching purchase agreements requires being able to predict the value of a team.
- Data is provided showing the revenue produced by each franchise and the team's overall value. The NBA wants to be able to use this information to predict the value of any team based on its revenue.
- You are going to develop this model using statistical methods.


## Investigation

- Does this data appear to have a linear association? Describe the scatter plot.

Use the provided noodles to approximate a line that best represents the data in your scatter plot. Write the equation of your line.

## The Median-Median Line

$\%$ The median-median line is a specific line that can be used to represent linearly associated data.
: In order to find the median-median line, you must divide the data into three groups and then find points that represent the medians (both vertically and horizontally) of these three sections of data.
: Once the three median points are found, they form a triangle.
:The median-median line is parallel to one side of this triangle and passes through the centroid of the triangle.

## The Median-Median Line

$\%$ Why is the use of three median points important to finding a line to represent the linear relationship?

## The Median-Median Line

- Divide your data into three groups with as close to the same number in each group as possible.
- If you cannot divide it evenly, make the leftmost and the rightmost groups have the same number of data points.
- Draw vertical lines on your scatter plot marking the divisions between the three groups.
- Note: Two identical x-values must be in the same group.
- Find the point that is the median of the $x$-values and the median of the $y$-values for each group. Describe how you would do this both using the graph and using the list of data.
- Label the three median points M1, M2 and M3, with M1 as the leftmost point and M3 as the rightmost point.


## My Solution



## My Solution



## My Solution

| Left |  | Middle |  | Right |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Value <br> (y) | Revenue (x) | Value <br> (y) | Revenue $(x)$ | Value <br> (y) | Revenue (x) |
| 174 | 63 | 230 | 82 | 274 | 98 |
| 188 | 70 | 236 | 85 | 275 | 102 |
| 196 | 70 | 239 | 85 | 282 | 102 |
| 199 | 70 | 244 | 91 | 283 | 105 |
| 202 | 72 | 249 | 94 | 284 | 109 |
| 208 | 72 | 272 | 94 | 328 | 109 |
| 216 | 75 | 278 | 96 | 338 | 117 |
| 218 | 78 | 280 | 97 | 356 | 119 |
| 227 | 80 | 290 | 97 | 401 | 149 |
| 258 | 80 |  |  | 447 | 160 |
| $(72,205)$ |  | $(94,249)$ |  | $(109,306)$ |  |

## The Median-Median Line

- These three points form a triangle. The centroid of this triangle is the weighted center of the data.
- The median-median line will be parallel to the line containing M1 and M3 and will pass through the centroid of the triangle.
- Use your knowledge of algebra and geometry to write the equation for the median-median line for this data. Show your calculations along with a graph in the coordinate plane showing the triangle and the median-median line.


## Follow Up Questions

- How does the median-median line compare to the line that you drew just by guessing?
- What algebraic tools did you use in your process for writing the equation of the median-median line?

What geometric tools did you use in your process for writing the equation of the median-median line?

- How would you calculate the equation of the medianmedian line if the three points (M1, M2, and M3) happen to be collinear?
- How can the NBA use the median-median equation you found to provide information to potential owners?


## Revisiting the Formula

Establish three median points from the data: $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right),\left(\mathrm{x}_{2}, \mathrm{y}_{2}\right),\left(\mathrm{x}_{3}, \mathrm{y}_{3}\right)$

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Look for and express regularity in repeated reasoning

## Questions?

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## A Possible Method

- Find the slope of the line containing M1 and M3.
(2.73)

Find the midpoint of the segment with endpoints M1 and M3. $(90.5,255.5)$

- Find the distance from the midpoint to the opposite vertex. (7.38)
- Find the centroid. We know that the centroid is $1 / 3$ of the distance just calculated away from the midpoint along the median. We can find this by solving a system of equations or with other methods.
(91.67, 253.34)
- My Equation of the Median-Median Line:

$$
y=2.73 x+3.06
$$

