The Intersection of Math, Music and Technology

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Technology + Math Education = A Long History

- Chalk, slates
- Textbooks
- Pencils (with erasers!)
- Slide Rules
- Calculators
- Computers
- The Internet
- Smartboards

What has this modern technology been used for?

- Mainly to help in *computations* (e.g., Wolfram|Alpha)
- Secondarily, as a *resource*
- More recently: as an individual *tutor* (e.g., Khan Academy, Brightstorm, TeacherTube)
- The power of computers to visualize AND to synthesize sounds has been underutilized – where might this fit into math?

A ~~common~~ sad story

- So I'm teaching sinusoidal functions to my algebra 2 students for the Nth year in a row….
- And from the moment I begin, we're all mutually bored.
- There MUST be a better way, right? After all, it IS the 21st century…

What do we usually talk about when we talk about sinusoids?

- Period, amplitude, phase, sinusoidal axis
  - GRAPHING! Lots, and lots, and lots of graphing! (and radians!!!)
- We assume prior knowledge of:
  - Right triangle trigonometry
  - "Common Trig Values" (sin(45°), etc.)
  - Quadrant rules and function definitions

  (sin θ = y/r, etc.)

Why sinusoids? In the style of Milne…

- Because they're Important!

  (they must be, because we devote several chapters to them!)
- Because learning to Verify their Identities is Good Practice!

  (practice for what, I wonder?)
- Because they're Needed for Calculus!

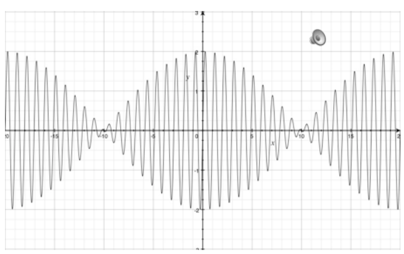
  (actually, in calculus, they're usually simply treated as another example of a function that can be integrated, differentiated, or power-series'd)
- Because they're Interesting!

  (this is actually true….!)

What else do we spend our time doing with sinusoids?

- Formulas!
  - Sums & differences
  - Identities: reciprocal, Pythagorean, quotient, cofunction, parity
  - Double- and half-angle formulas
  - Sum-to-product and product-to-sum
- Modeling (maybe…)
- Could we use modern technology to illustrate these ideas somehow?

For example, how about teaching the sum-to-product formulas this way:



Connections: Sinusoids & Music

- Music is the single most important example in our students' lives of a periodic, sinusoidally-based function
- It's a topic that for almost all students immediately grabs their interest (even if they're not "musical" themselves)
- It actually is something we teachers can use to do DEMOS (demos?) with.

Why should we connect them?

- Think back to your own high school experiences:
  - If you studied a musical instrument, you learned how to read music, how to play your instrument, etc.
  - A good question: why did you choose your particular instrument?
    - "I liked its sound" – you were thinking about its *timbre*
    - Timbre connects nicely to sinusoids and Fourier series!
- If you didn't play an instrument, what attracted you to music? Rhythm? The melody? All of these have mathematical connections!

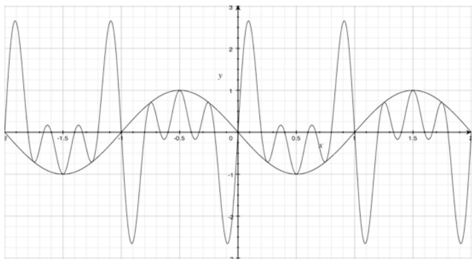
The perception of sound (after Loy, "Musimathics")

- Sounds can be thought of in 6 "dimensions":
  - Frequency (perceived as **pitch**)
  - The point at which the sound begins (**onset**)
  - Amplitude or intensity (perceived as **loudness**)
  - The length of time that the sound lasts (the **duration**)
  - The change in the sound's intensity over time (the **envelope** of the sound)
  - The quality of the sound – that which distinguishes a trumpet from an oboe, for example (the **wave shape)**
  - These are the most important descriptors of sound

Sound Perception

- The ear – can be thought of as a receiver – translates information about incoming sounds into the six "dimensions" we discussed
- Objective measures of sound perception and music are difficult, and a major research topic.
  - Examples:
    - pitch and loudness (as perceived) are not linear functions of frequency and amplitude (and they actually influence each other!)
    - One will perceive sounds that "aren't there": experiment of Seebeck on missing fundamental (Audacity Demo)

The Missing Fundamental



The Generation of Sound

- Understanding of vibrating systems is critical for both generation and detection of sound
- Connections between math and physics: Every object that has any elastic properties vibrates at a particular fundamental frequency – and this frequency is dependent on a property of the material:
  - Springs: $f \propto \sqrt{1/m}$
  - Strings: $f \propto 1/L$
  - Helmholtz resonator: $f \propto \sqrt{A/LV}$
- The frequency of vibration can be related to sinusoidal functions in the usual way ($x(t) = \sin(2\pi ft)$, where x is displacement)

Musical Vibrating Systems

- Stringed instruments
  - Categorized in several ways:
    - How they are played (bowed, picked, struck)
    - How they choose pitch (unstopped, stopped fretted, stopped unfretted)
    - If sound can be continuously produced (e.g., plucked vs. bowed)
- Percussion instruments
  - 1-dimensional (bars)
  - 2-dimensional (membranes and plates)
- Wind instruments (brass, woodwinds, flutes)

Musical Vibrating Systems

- These systems each have natural frequencies at which they **resonate**
- Musical systems produce even or odd multiples of a particular frequency as well:
  - A clarinet playing a 440 Hz note will also tend to generate frequencies of 1320 Hz, 2200 Hz ("odd harmonics"), etc.
  - The same is true for a flute – except it will also generate EVEN harmonics (88o Hz, 1760 Hz, etc.)
- A note played by a particular instrument is therefore a LINEAR COMBINATION of frequencies, each with different amplitudes (using a model of y =sin(2πft)): For example:

  Clarinet A-440: a*sin(880πt) + b*sin(2640πt) + c*sin(4400πt)…

  Flute A-440: d*sin(880πt) + e*sin(1760πt) +f*sin(2640πt)+…
- The presence of even harmonics gives a flute a different characteristic sound from a clarinet – a different **TIMBRE!**
- This is a rich area for mathematics students to explore –what sorts of sounds are generated when you add together different harmonics with different amplitudes? (This is essentially Fourier analysis!)

Fourier Analysis? Really?

- Any periodic function that is assumed to repeat indefinitely (with period [-π,π]) without alteration can be modeled as a Fourier series:

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos(nx)+\sum_{n=1}^{\infty}b_n\sin(nx)$$

- From the perspective of a precalculus student, this simply means that a periodic function can be built from sums of sinusoids!
- But visualization and auditory examples are critical….as well as opportunities for experimentation!

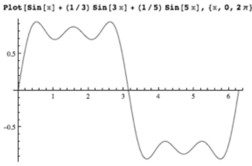
Technology, Music and Sinusoids

- The goal: to use technology that is readily available to demonstrate interesting connections between sinusoids and sound.
- In particular, we want to be able to:
  - Demonstrate how sounds are created from basic "pure sines" (Audacity, Mathematica)
  - Investigate what actual sounds "look like" and what they're built from (Audacity, GarageBand)

What if we want to build sounds from formulas?

Mathematica does a very nice job with this:

```
Plot[Sin[x] + (1/3) Sin[3 x] + (1/5) Sin[5 x], {x, 0, 2 π}]
```




```
Play[Sin[2 π * 220 t] + (1/3) Sin[2 π * 660 t] + (1/5) Sin[2 π * 1100 t], {t, 0, 2}]
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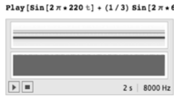


Mathematica (www.wolfram.com)

- A computational software program that is immensely powerful.
- Lots of possible ways for mathematics teachers to use it, including lots of demos at the site http://demonstrations.wolfram.com – they do not require having a license for Mathematica to use
- Let's "see" a couple of examples involving sinusoids and sound!

Audacity (http://audacity.sourceforge.net)

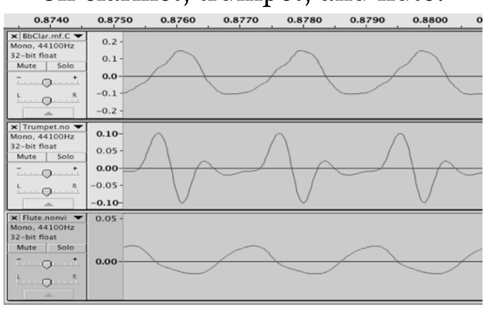
- From the website:

"Audacity is a free, easy-to-use and multilingual audio editor and recorder for Windows, Mac OS X, GNU/ Linux and other operating systems. You can use Audacity to:
  - Record live audio.
  - Convert tapes and records into digital recordings or CDs
  - Edit Ogg Vorbis, MP3, WAV or AIFF sound files.
  - Cut, copy, splice, or mix sounds together.
  - Change the speed or pitch of a recording. And more!"
- Note that many schools may already have this program installed – commonly used in foreign language.

How do we use Audacity to create and visualize sounds?

- Pure sine tones
- Mixtures of sine tones
- “Synthetic” tones (square waves, sawtooth waves)
- Analyses of “actual instrument” sounds, both visual and spectral

An example: The same note (C5) on clarinet, trumpet, and flute:



Summary of Audacity

- Easy to create simple waveforms
- Can import samples of musical instruments to hear AND see the differences in the waveforms

(http://theremin.music.uiowa.edu/MIS.html)

- Can do simple spectral analysis
- Can apply pitch changes, effects to existing sounds, can “see” what they do to the waves

GarageBand (Mac OS X only)

- A software program by Apple that allows users to create music or podcasts (Wikipedia entry)
- Ships with all new Macs
- Fairly easy to use – can use software instruments as well as real instruments
- Part of the iLife suite for Macintosh
- Possible PC equivalent: Mixcraft 5 (see www.garagebandforwindows.com for more info)

How can GarageBand and Audacity play together?

- We can use GarageBand to generate a variety of "software instrument" sounds, or real sounds such as an electric guitar
- We can then import those sounds into Audacity for analysis and comparison
- Students get a very clear visual idea that timbre is something physical – something connected to sums of sinusoids!

Conclusion

- There are many ways that we can bring technology into the classroom – but bringing music and technology in together can give our students a deeper understanding of sinusoids, their utility, and what all of these "different properties" actually mean.
- Thank you very much for your time!