# Mathematical Humor with a Point 

## Communicating Effectively with Students



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## Other Books by David B. Spangler:

Math for Real Kids: Problems, Applications, and Activities for Grades 5-8, Good Year Books, © 1997, 2005

Strategies for Teaching Whole Number Computation: Using Error Analysis for Intervention and Assessment, Corwin Press, © 2010

Strategies for Teaching Fractions: Using Error Analysis for Intervention and Assessment, Corwin Press, © 2011

# Using Measurement to Put LOTTERY PROBABILITIES 

 INTO PERSPECTIVE TEACHER's NOTES- NCTM Standards: Measurement; Probability; Number and Operations
- Common Core

State Standards:
Measurement and Data; Statistics and Probability; Model with mathematics.

- Mathematical Topics:

Convert inches to miles; convert centimeters to kilometers; convert kilometers to miles; read mileage tables; understand basic probability concepts; use a linear representation to model a situation

- Grouping of Students: Work in pairs or individually


## BACKGROUND

A lottery is a tax on people who are bad at math.
-Dana Blankenhorn, business journalist (1955- )

The goal of this activity is to use concepts from measurement to provide a visualization for lottery probabilities. Students should conclude from their linear representations that "investing" money in a lottery (beyond, say, a token amount) is not a wise financial decision. This should come to light as they think about trying to pick the "lucky cup"-at $\$ 1$ a pick-from a line of cups stretched for thousands of miles.

The probabilities provided in this activity lesson were in use as of mid-2007. Probabilities are adjusted from time to time. For the latest information on Powerball, go to http://www.powerball. com/. For the latest information on Mega Millions, go to http://www.megamillions.com/.

Students should use a calculator when they convert from inches to feet to miles. Before students use the mileage tables to determine their "itineraries," you may want them to refer to a map or globe to estimate and show how far they think the cups will extend-beginning from, say, their home city.

When students work with the mileage tables, encourage them to use trial-and-error to find the "itinerary" that is as close to the computed number of miles (above or below) as possible. Remind students that the cities must be connecting cities (such as Chicago to New York, New York to London, etc.) You may want to present this as a contest where students compete to come closest to the computed number of miles. For an additional challenge, you may want to require student "itineraries" to include at least, say, four cities.

Distances provided in mileage tables will vary among sources. The source for the mileage tables included with this activity lesson is the publication infoplease (http://www.infoplease. com/ipa/A0004594.html and http://www. infoplease.com/ipa/A0759496.html) for U.S. cities and world cities, respectively. Additional mileage tables are available at those two sites.

Students are able to complete this activity lesson without knowledge on how the probabilities are computed. The following mathematical background is provided for teacher edification purposes-and for students who are capable of understanding the probability concepts involved.

Determining lottery probabilities involves finding the total number of combinations of groups of numbers (of specific sizes) that are possible. Since the order in which the numbers are drawn does not matter, lottery probabilities are based on combinations (unordered groupings) rather than, say, on permutations (where order does make a difference). The formula involves the use of factorials-such as $n!$, read " $n$ factorial." The factorial of a positive integer $n$ is the product of all integers from $n$ to 1 . So $5!=5 \times 4 \times 3 \times$ $2 \times 1=120$.

Formula for finding the number of combinations of $n$ things taken $r$ at a time:

$$
{ }_{n} \mathrm{C}_{r}=\frac{n!}{(n-r)!\cdot r!}
$$

Illinois Lotto: This lottery uses 52 lottery balls numbered from 1-52. With each $\$ 1$ ticket, players choose two combinations of six numbers each. At each drawing, a set of six winning numbers is drawn. To win the jackpot, a player must match all six numbers. The number of combinations of 52 things taken 6 at a time is calculated below:

$$
{ }_{52} C_{6}=\frac{52!}{(52-6)!\cdot 6!}=20,358,520
$$

There are 20,358,520 combinations in all, so with 2 chances of winning on a $\$ 1$ ticket, a player has 1 chance in $10,179,260$ to win the jackpot.

Powerball: For a $\$ 1$ ticket, a player selects numbers from two sets of numbers. The player selects five whole numbers from 1-59, and one whole number, the Powerball, from 1-39. To win the jackpot, a player must match all six numbers. The total number of possible combinations is found by multiplying the number of combinations of 59 things taken 5 at a time by the number of combinations of 39 things taken 1 at a time:

$$
\begin{gathered}
{ }_{59} \mathrm{C}_{5} \times{ }_{39} \mathrm{C}_{1}=\frac{59!}{(59-5)!\cdot 5!} \times \frac{39!}{(39-1)!\cdot 1!}= \\
5,006,386 \times 39=195,249,054
\end{gathered}
$$

So, on a $\$ 1$ ticket, the probability of winning the Powerball jackpot is 1 in 195,249,054.

Mega Millions: For a $\$ 1$ ticket, a player selects numbers from two sets of numbers. The player selects five whole numbers from 1-56, and one whole number, the Mega Ball, from 1-46. To win the jackpot, a player must match all six numbers. The total number of possible combinations is found by multiplying the number of combinations of 56 things taken 5 at a time by the number of combinations of 46 things taken 1 at a time.

$$
\begin{gathered}
{ }_{56} \mathrm{C}_{5} \times{ }_{46} \mathrm{C}_{1}=\frac{56!}{(56-5)!\cdot 5!} \times \frac{46!}{(46-1)!\cdot 1!}= \\
3,819,816 \times 46=175,711,536
\end{gathered}
$$

So, on a $\$ 1$ ticket, the probability of winning the Mega Millions jackpot is 1 in $175,711,536$. (What's "mega" about this game are the megaodds against winning!)


## MATHEMATICAL HUMOR

Probability of winning the Powerball jackpot with 1 ticket:

### 0.000000005

Probability of winning the Powerball jackpot with no tickets:

## $\mathbf{0 . 0 0 0 0 0 0 0 0 0}$

So, you are about as likely to win without a ticket as you are with a ticket.
To further illustrate the remote chance of winning the jackpot, have students read the number 0.000000005 ( 5 billionths; $1 \div 195,249,054 \approx 0.000000005$ ). So, with 1 ticket, a person has about 5 chances in 1 billion of winning. Thus, if a person were to purchase 10 tickets instead of 1 ticket, that person's chances of winning would improve tenfold to about 0.00000005 ( 5 chances in 100 million). If a person were to purchase 100 tickets instead of 1 ticket, that person would have about 7 chances in 10 million of winning. Despite these terrible probabilities, it is not uncommon to hear of people who "invest" \$100 or more in a given lottery. These people often say, "You have to play to win." But in reality, "You have to play to lose."

Question: How do you read "5!"?

## Answer: Five!

## SOLUTIONS

1a. Determining the number of inches of cups: 3 in. $\times 195,249,054=585,747,162 \mathrm{in}$.
Converting inches to feet:
$585,747,162 \mathrm{in} . \div 12 \approx 48,812,264 \mathrm{ft}$
Converting feet to miles:
$48,812,264 \mathrm{ft} \div 5,280 \approx 9,245 \mathrm{mi}$
b. "Itineraries" will vary. One possible solution:

Cleveland to Los Angeles ( $2,049 \mathrm{mi}$ );
Los Angeles to Mexico City ( $1,589 \mathrm{mi}$ );
Mexico City to Montreal ( $2,318 \mathrm{mi}$ );
Montreal to London (3,282 mi);
Total length of cups: $9,238 \mathrm{mi}$
2a. Determining the number of inches of cups: $3 \mathrm{in} . \times 175,711,536=527,134,608 \mathrm{in}$.
Converting inches to feet:
$527,134,608 \mathrm{in} . \div 12=43,927,884 \mathrm{ft}$
Converting feet to miles:
$43,927,884 \mathrm{ft} \div 5,280 \approx 8,320 \mathrm{mi}$
b. "Itineraries" will vary. One possible solution:

Indianapolis to Chicago ( 165 mi );
Chicago to Shanghai ( $7,061 \mathrm{mi}$ );
Shanghai to Manila ( $1,150 \mathrm{mi}$ ).
Total trip: $8,376 \mathrm{mi}$
c. Answers will vary. To determine the number of round trips, students may divide 8,320 by the distance between the two cities, and then divide by 2 . Another option is to multiply the distance between the two cities by 2 , and then divide 8,320 by that product.
3. Answers will vary. Most students should conclude that it is nearly impossible to win the jackpot in one of these lotteries; hence, it is not worth traveling any distance to buy tickets.

## EXTENSIONS

1. Have students use a globe, map, or other visual to present to the class their route for their cups for each lottery. The presentation should include a discussion of some of the cities, states, bodies of water, and countries that the trip goes through. (A student once commented, "It's going to be tough finding the 'winning cup'-since it is likely to be in the middle of the ocean!")
2. Have students repeat the activity lesson using cups with a metric dimension, say, 5 centimeters wide in diameter. Have students compute the length of the cups in centimeters, and then convert to kilometers (by dividing by 100,000 ). Have students convert the number of kilometers to miles by multiplying by 0.62 .
$\qquad$ Date $\qquad$

## Using Measurement to PUT LOTTERY PROBABILITIES into Perspective

Materials: globes or maps (optional)
When someone says, "You're more likely to be struck by lightning than you are to win a lottery," most people are not shocked or jolted. It's not striking news that lottery probabilities are very tiny.

Because lottery probabilities are so tiny, they are outside of our everyday experience. Yet some people spend a lot of money on lotteries. This activity lesson uses concepts from measurement to help us visualize and "put into perspective" probabilities from three different lotteries.

## AN EXAMPLE: The Illinois LOtto

The Illinois Lotto uses 52 lottery balls numbered from $1-52$. With each $\$ 1$ ticket, players choose two combinations of six numbers each. At each drawing, a set of six winning numbers is drawn. To win the jackpot, a player needs to match all six numbers. (Players do not have to match numbers in the order drawn.)

With a $\$ 1$ ticket, the probability of matching all six numbers is 1 in $10,179,260$. To help get a picture of what 1 chance in $10,179,260$ might look like, let's imagine how far $10,179,260$ objects placed in a line might stretch. Since this model, or representation, involves using a "line," we call the model a linear representation.


## Linear Representation Of THE ILLINOIS LOTTO JACKPOT PROBABILITY:

Think of placing 10,179,260 paper cups, each 3 inches in diameter, in a straight line.
(a) How many miles long would that be?
(b) Between which two cities might this line of cups extend?
(c) How does this relate to your chances of winning the Illinois Lotto jackpot?

## ANsWERS TO EXAMPLE ON THE ILLINOIS LOTTO:

(a) $10,179,260$ cups, each 3 inches in diameter, would extend $3 \times 10,179,260$, or $30,537,780$ inches. Converting this to feet gives us $30,537,780 \div 12$, or $2,544,815$ feet. Dividing this result by 5,280 yields about 482 miles.
(b) This is the approximate distance between Chicago and Memphis!
(c) Suppose someone randomly places a pea under one of those cups-and you are given ONE chance to select the cup with the pea. Your chance of selecting that cup is the same as your chance of winning the Lotto jackpot on a $\$ 1$ play.

## QUESTIONS

## POWERBALL

Powerball is a lottery played in 42 states, Washington, D.C., and the U.S. Virgin Islands.
For a $\$ 1$ ticket, a player selects numbers from two sets of numbers. The player selects five whole numbers from 1-59, and one whole number, the Powerball, from 1-39. To win the jackpot, a player must match all six numbers. For a $\$ 1$ ticket, the probability of winning the Powerball jackpot is 1 in 195,249,054.
1a. How many miles long would a line of 195,249,054 paper cups, each 3 inches in diameter, extend?
b. Use the mileage tables included with this activity lesson, or another source, to map out a distance to represent the line of cups. Record your "itinerary" with mileage distancesgoing from one connecting city to the next-in the space below. Try to get as close as possible (above or below) to the number of miles in question 1a. If a globe or map is available, you may want to locate the cities in your "itinerary" to get a feel for the distances involved.

## MEGA MILLIONS

Mega Millions is a lottery that is played in California, Georgia, Illinois, Maryland, Massachusetts, Michigan, New Jersey, New York, Ohio, Texas, Virginia, and Washington.
For a $\$ 1$ ticket, a player selects numbers from two sets of numbers. The player selects five whole numbers from 1-56, and one whole number, the Mega Ball, from 1-46. A player needs to match all six numbers to win the jackpot. For a $\$ 1$ ticket, the probability of winning the Mega Millions jackpot is 1 in 175,711,536.
2a. How many miles long would a line of $175,711,536$ paper cups, each 3 inches in diameter, extend?
b. Use mileage tables to map out a distance to represent the line of cups. Record your "itinerary" with mileage distances-going from one connecting city to the next-in the space below. Try to get as close as possible (above or below) to the number of miles in question 2 a .
c. Select a city in the United States other than your own. How many round trips between your home city (or a city near you) and the selected city would you need to make in order to cover the number of miles determined in question 2a? Round to the nearest tenth.
3. From time-to-time, the jackpot for Powerball or Mega Millions grows to be in the hundreds of millions of dollars. In order to participate in an out-of-state lottery, some people drive from one state to another. (For example, a person living in Indiana may drive to Illinois to buy a ticket for Mega Millions.) In your opinion, is it worth "going to great lengths" to buy a lottery ticket? Explain.

## Air Distances between U.S. Cities in Statute Miles

| Cities | Birmingham | Boston | Buffalo | Chicago | Cleveland | Dallas | Denver |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Birmingham, Ala. | - | 1,052 | 776 | 578 | 618 | 581 | 1,095 |
| Boston, Mass. | 1,052 | - | 400 | 851 | 551 | 1,551 | 1,769 |
| Buffalo, N.Y. | 776 | 400 | - | 454 | 173 | 1,198 | 1,370 |
| Chicago, III. | 578 | 851 | 454 | - | 308 | 803 | 920 |
| Cleveland, Ohio | 618 | 551 | 173 | 308 | - | 1,025 | 1,227 |
| Dallas, Tex. | 581 | 1,551 | 1,198 | 803 | 1,025 | - | 663 |
| Denver, Colo. | 1,095 | 1,769 | 1,370 | 920 | 1,227 | 663 | - |
| Detroit, Mich. | 641 | 613 | 216 | 238 | 90 | 999 | 1,156 |
| El Paso, Tex. | 1,152 | 2,072 | 1,692 | 1,252 | 1,525 | 572 | 557 |
| Houston, Tex. | 567 | 1,605 | 1,286 | 940 | 1,114 | 225 | 879 |
| Indianapolis, Ind. | 433 | 807 | 435 | 165 | 263 | 763 | 1,000 |
| Kansas City, Mo. | 579 | 1,251 | 861 | 414 | 700 | 451 | 558 |
| Los Angeles, Calif. | 1,802 | 2,596 | 2,198 | 1,745 | 2,049 | 1,240 | 831 |
| Louisville, Ky. | 331 | 826 | 483 | 269 | 311 | 726 | 1,038 |
| Memphis, Tenn. | 217 | 1,137 | 803 | 482 | 630 | 420 | 879 |
| Miami, Fla. | 665 | 1,255 | 1,181 | 1,188 | 1,087 | 1,111 | 1,726 |
| Minneapolis, Minn. | 862 | 1,123 | 731 | 355 | 630 | 862 | 700 |
| New Orleans, La. | 312 | 1,359 | 1,086 | 833 | 924 | 443 | 1,082 |
| New York, N.Y. | 864 | 188 | 292 | 713 | 405 | 1,374 | 1,631 |
| Omaha, Nebr. | 732 | 1,282 | 883 | 432 | 739 | 586 | 488 |
| Philadelphia, Pa. | 783 | 271 | 279 | 666 | 360 | 1,299 | 1,579 |
| Phoenix, Ariz. | 1,456 | 2,300 | 1,906 | 1,453 | 1,749 | 887 | 586 |
| Pittsburgh, Pa. | 608 | 483 | 178 | 410 | 115 | 1,070 | 1,320 |
| St. Louis, Mo. | 400 | 1,038 | 662 | 262 | 492 | 547 | 796 |
| Salt Lake City, Utah | 1,466 | 2,099 | 1,699 | 1,260 | 1,568 | 999 | 371 |
| San Francisco, Calif. | 2,013 | 2,699 | 2,300 | 1,858 | 2,166 | 1,483 | 949 |
| Seattle, Wash. | 2,082 | 2,493 | 2,117 | 1,737 | 2,026 | 1,681 | 1,021 |
| Washington, D.C. | 661 | 393 | 292 | 597 | 306 | 1,185 | 1,494 |

Source: From infoplease. Available at http://www.infoplease.com/ipa/A0004594.html. Used with permission. Additional mileage tables for distances between U.S. cities are available at the above Web site.

Air Distances between U.S. Cities in Statute Miles

| Cities | Detroit | El Paso | Houston | Indianapolis | Kansas City | Los Angeles | Louisville |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Birmingham, Ala. | 641 | 1,152 | 567 | 433 | 579 | 1,802 | 331 |
| Boston, Mass. | 613 | 2,072 | 1,605 | 807 | 1,251 | 2,596 | 826 |
| Buffalo, N.Y. | 216 | 1,692 | 1,286 | 435 | 861 | 2,198 | 483 |
| Chicago, III. | 238 | 1,252 | 940 | 165 | 414 | 1,745 | 269 |
| Cleveland, Ohio | 90 | 1,525 | 1,114 | 263 | 700 | 2,049 | 311 |
| Dallas, Tex. | 999 | 572 | 225 | 763 | 451 | 1,240 | 726 |
| Denver, Colo. | 1,156 | 557 | 879 | 1,000 | 558 | 831 | 1,038 |
| Detroit, Mich. | - | 1,479 | 1,105 | 240 | 645 | 1,983 | 316 |
| El Paso, Tex. | 1,479 | - | 676 | 1,264 | 839 | 701 | 1,254 |
| Houston, Tex. | 1,105 | 676 | - | 865 | 644 | 1,374 | 803 |
| Indianapolis, Ind. | 240 | 1,264 | 865 | - | 453 | 1,809 | 107 |
| Kansas City, Mo. | 645 | 839 | 644 | 453 | - | 1,356 | 480 |
| Los Angeles, Calif. | 1,983 | 701 | 1,374 | 1,809 | 1,356 | - | 1,829 |
| Louisville, Ky. | 316 | 1,254 | 803 | 107 | 480 | 1,829 | - |
| Memphis, Tenn. | 623 | 976 | 484 | 384 | 369 | 1,603 | 320 |
| Miami, Fla. | 1,152 | 1,643 | 968 | 1,024 | 1,241 | 2,339 | 919 |
| Minneapolis, Minn. | 543 | 1,157 | 1,056 | 511 | 413 | 1,524 | 605 |
| New Orleans, La. | 939 | 983 | 318 | 712 | 680 | 1,673 | 623 |
| New York, N.Y. | 482 | 1,905 | 1,420 | 646 | 1,097 | 2,451 | 652 |
| Omaha, Nebr. | 669 | 878 | 794 | 525 | 166 | 1,315 | 580 |
| Philadelphia, Pa. | 443 | 1,836 | 1,341 | 585 | 1,038 | 2,394 | 582 |
| Phoenix, Ariz. | 1,690 | 346 | 1,017 | 1,499 | 1,049 | 357 | 1,508 |
| Pittsburgh, Pa. | 205 | 1,590 | 1,137 | 330 | 781 | 2,136 | 344 |
| St. Louis, Mo. | 455 | 1,034 | 679 | 231 | 238 | 1,589 | 242 |
| Salt Lake City, Utah | 1,492 | 689 | 1,200 | 1,356 | 925 | 579 | 1,402 |
| San Francisco, Calif. | 2,091 | 995 | 1,645 | 1,949 | 1,506 | 347 | 1,986 |
| Seattle, Wash. | 1,938 | 1,376 | 1,891 | 1,872 | 1,506 | 959 | 1,943 |
| Washington, D.C. | 396 | 1,728 | 1,220 | 494 | 945 | 2,300 | 476 |

Source: From infoplease. Available at http://www.infoplease.com/ipa/A0004594.html. Used with permission. Additional mileage tables for distances between U.S. cities are available at the above Web site.

## Air Distances between World Cities in Statute Miles

| Cities | Berlin | Buenos Aires | Cairo | Calcutta | Cape Town | Caracas | Chicago |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Berlin | - | 7,402 | 1,795 | 4,368 | 5,981 | 5,247 | 4,405 |
| Buenos Aires | 7,402 | - | 7,345 | 10,265 | 4,269 | 3,168 | 5,598 |
| Cairo | 1,795 | 7,345 | - | 3,539 | 4,500 | 6,338 | 6,129 |
| Calcutta | 4,368 | 10,265 | 3,539 | - | 6,024 | 9,605 | 7,980 |
| Cape Town | 5,981 | 4,269 | 4,500 | 6,024 | - | 6,365 | 8,494 |
| Caracas | 5,247 | 3,168 | 6,338 | 9,605 | 6,365 | - | 2,501 |
| Chicago | 4,405 | 5,598 | 6,129 | 7,980 | 8,494 | 2,501 | - |
| Hong Kong | 5,440 | 11,472 | 5,061 | 1,648 | 7,375 | 10,167 | 7,793 |
| Honolulu | 7,309 | 7,561 | 8,838 | 7,047 | 11,534 | 6,013 | 4,250 |
| Istanbul | 1,078 | 7,611 | 768 | 3,638 | 5,154 | 6,048 | 5,477 |
| Lisbon | 1,436 | 5,956 | 2,363 | 5,638 | 5,325 | 4,041 | 3,990 |
| London | 579 | 6,916 | 2,181 | 4,947 | 6,012 | 4,660 | 3,950 |
| Los Angeles | 5,724 | 6,170 | 7,520 | 8,090 | 9,992 | 3,632 | 1,745 |
| Manila | 6,132 | 11,051 | 5,704 | 2,203 | 7,486 | 10,620 | 8,143 |
| Mexico City | 6,047 | 4,592 | 7,688 | 9,492 | 8,517 | 2,232 | 1,691 |
| Montreal | 3,729 | 5,615 | 5,414 | 7,607 | 7,931 | 2,449 | 744 |
| Moscow | 1,004 | 8,376 | 1,803 | 3,321 | 6,300 | 6,173 | 4,974 |
| New York | 3,965 | 5,297 | 5,602 | 7,918 | 7,764 | 2,132 | 713 |
| Paris | 545 | 6,870 | 1,995 | 4,883 | 5,807 | 4,736 | 4,134 |
| Rio de Janeiro | 6,220 | 1,200 | 6,146 | 9,377 | 3,773 | 2,810 | 5,296 |
| Rome | 734 | 6,929 | 1,320 | 4,482 | 5,249 | 5,196 | 4,808 |
| San Francisco | 5,661 | 6,467 | 7,364 | 7,814 | 10,247 | 3,904 | 1,858 |
| Shanghai | 5,218 | 12,201 | 5,183 | 2,117 | 8,061 | 9,501 | 7,061 |
| Stockholm | 504 | 7,808 | 2,111 | 4,195 | 6,444 | 5,420 | 4,278 |
| Sydney | 10,006 | 7,330 | 8,952 | 5,685 | 6,843 | 9,513 | 9,272 |
| Tokyo | 5,540 | 11,408 | 5,935 | 3,194 | 9,156 | 8,799 | 6,299 |
| Warsaw | 320 | 7,662 | 1,630 | 4,048 | 5,958 | 5,517 | 4,667 |
| Washington, D.C. | 4,169 | 5,218 | 5,800 | 8,084 | 7,901 | 2,059 | 597 |

Source: From infoplease. Available at http://www.infoplease.com/ipa/A0759496.html. Used with permission. Additional mileage tables for distances between world cities are available at the above Web site.

Air Distances between World Cities in Statute Miles

| Cities | Hong Kong | Honolulu | Istanbul | Lisbon | London | Los Angeles | Manila |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Berlin | 5,440 | 7,309 | 1,078 | 1,436 | 579 | 5,724 | 6,132 |
| Buenos Aires | 11,472 | 7,561 | 7,611 | 5,956 | 6,916 | 6,170 | 11,051 |
| Cairo | 5,061 | 8,838 | 768 | 2,363 | 2,181 | 7,520 | 5,704 |
| Calcutta | 1,648 | 7,047 | 3,638 | 5,638 | 4,947 | 8,090 | 2,203 |
| Cape Town | 7,375 | 11,534 | 5,154 | 5,325 | 6,012 | 9,992 | 7,486 |
| Caracas | 10,167 | 6,013 | 6,048 | 4,041 | 4,660 | 3,632 | 10,620 |
| Chicago | 7,793 | 4,250 | 5,477 | 3,990 | 3,950 | 1,745 | 8,143 |
| Hong Kong | - | 5,549 | 4,984 | 6,853 | 5,982 | 7,195 | 693 |
| Honolulu | 5,549 | - | 8,109 | 7,820 | 7,228 | 2,574 | 5,299 |
| Istanbul | 4,984 | 8,109 | - | 2,012 | 1,552 | 6,783 | 5,664 |
| Lisbon | 6,853 | 7,820 | 2,012 | - | 985 | 5,621 | 7,546 |
| London | 5,982 | 7,228 | 1,552 | 985 | - | 5,382 | 6,672 |
| Los Angeles | 7,195 | 2,574 | 6,783 | 5,621 | 5,382 | - | 7,261 |
| Manila | 693 | 5,299 | 5,664 | 7,546 | 6,672 | 7,261 | - |
| Mexico City | 8,782 | 3,779 | 7,110 | 5,390 | 5,550 | 1,589 | 8,835 |
| Montreal | 7,729 | 4,910 | 4,789 | 3,246 | 3,282 | 2,427 | 8,186 |
| Moscow | 4,439 | 7,037 | 1,091 | 2,427 | 1,555 | 6,003 | 5,131 |
| New York | 8,054 | 4,964 | 4,975 | 3,364 | 3,458 | 2,451 | 8,498 |
| Paris | 5,985 | 7,438 | 1,400 | 904 | 213 | 5,588 | 6,677 |
| Rio de Janeiro | 11,021 | 8,285 | 6,389 | 4,796 | 5,766 | 6,331 | 11,259 |
| Rome | 5,768 | 8,022 | 843 | 1,161 | 887 | 6,732 | 6,457 |
| San Francisco | 6,897 | 2,393 | 6,703 | 5,666 | 5,357 | 347 | 6,967 |
| Shanghai | 764 | 4,941 | 4,962 | 6,654 | 5,715 | 6,438 | 1,150 |
| Stockholm | 5,113 | 6,862 | 1,348 | 1,856 | 890 | 5,454 | 5,797 |
| Sydney | 4,584 | 4,943 | 9,294 | 11,302 | 10,564 | 7,530 | 3,944 |
| Tokyo | 1,794 | 3,853 | 5,560 | 6,915 | 5,940 | 5,433 | 1,866 |
| Warsaw | 5,144 | 7,355 | 863 | 1,715 | 899 | 5,922 | 5,837 |
| Washington, D.C. | 8,147 | 4,519 | 5,215 | 3,562 | 3,663 | 2,300 | 8,562 |

Source: From infoplease. Available at http://www.infoplease.com/ipa/A0759496.html. Used with permission. Additional mileage tables for distances between world cities are available at the above Web site.

# Uncovering Humorous MATHEMATICAL BLUNDERS TEACHER's NOTES 

- NCTM Standards:

Number and Operations; Geometry; Algebra; Data Analysis and Probability

- Common Core

State Standards: Operations and Algebraic Thinking; Measurement and Data; Attend to precision.

- Mathematical Topics: Estimation; finding the percent increase; fraction/percent conversions; addition of fractions; ordering numbers; sum of the degree measures of the central angles of a circle; combining like terms; median; probability of compound events
- Grouping of Students: Work in pairs or small groups or individually


## BACKGROUND

Unfortunately, despite years of study and life experience in an environment immersed in quantitative data, many educated adults remain functionally innumerate.
-From The Case for Quantitative Literacy by Lynn Arthur Steen (2000)

For many years the author has been collecting mathematical blunders such as those in this activity lesson. The author has used such material in his classes, mathematics presentations at conferences, and after-dinner appearances to
draw attention to mathematics illiteracy in the real world-and to the ridiculous conclusions that result. Essentially, the use of this material illustrates the fact that the kinds of errors students make in the classroom are often repeated by the same people-as adults-in the real world of work. If students observe how foolish some people appear in the real world due to their mathematical illiteracy, perhaps this will provide some incentive for them to improve their performance as students in the classroom.

## MATHEMATICAL HUMOR

Sign at a grocery store: "Stock up and Save. Limit 1."
Sign at grocery store: "Open 24 hours on Labor Day: 9:00 A.m.-9:00 P.м."
Sign on road leading to tunnel:
"When this sign is under water, the tunnel is impassable."
Baseball player: "We just beat the Braves four games out of three."
Youngster at restaurant: "Do you give free refills?"
Waitress: "Yes, but you have to pay for them."
Sign in a subway car: "Illiterate? Write today for free help."
TV football announcer discussing cleat sizes during a nationally televised game: "Which is more, one-half inch or five-eighths inch?"

## SOLUTIONS

1a. The age of the dinosaur bones should be given as an estimate, not as an exact amount.
b. The dinosaur bones are about 70 million years old.
2 a. The new 2 -liter size contains $100 \%$ more than the 1 -liter size.
b. When some people see a 1 and a 2, they think " $\frac{1}{2}$," or $50 \%$. Others think twice the size means $50 \%$.

3a. Turning something around $360^{\circ}$ gets you back to where you started.
b. The player meant to say $180^{\circ}$. This would result in the opposite effect of the current situation.
4. Since $\frac{3}{4}+\frac{1}{3}=\frac{13}{12}$, a value greater than 1 , this is an impossible situation. The whole cannot be greater than $100 \%$.
5a. The sign attempts to find a sum of unrelated data involving different units of measure.
b. Students may solve the problem in any of these ways:
$49 \mathrm{~mm}+8 \mathrm{~cm}+20 \mathrm{~m}=0.049 \mathrm{~m}+$ $0.08 \mathrm{~m}+20 \mathrm{~m}$, or 20.129 m $49 \mathrm{~mm}+8 \mathrm{~cm}+20 \mathrm{~m}=4.9 \mathrm{~cm}+8 \mathrm{~cm}+$ $2,000 \mathrm{~cm}$, or $2,012.9 \mathrm{~cm}$ $49 \mathrm{~mm}+8 \mathrm{~cm}+20 \mathrm{~m}=49 \mathrm{~mm}+80 \mathrm{~mm}+$ $20,000 \mathrm{~mm}$, or $20,129 \mathrm{~mm}$
c. Convert all measurements so that they are expressed in the same unit.

6a. The "interval" between three-quarters and $75 \%$ is 0 . So the baseball player was redundant in saying that he was about threequarters to $75 \%$ ready to play. (Comedian David Letterman once said: "There is a new survey-apparently three out of every four people make up $75 \%$ of the population.")
b. $\frac{1}{2}$ and $50 \%$ name the same amount. Also, $\frac{1}{3}$ and $33 \frac{1}{3} \%$ name the same amount.

7a. By definition, $50 \%$ of students are above the median and $50 \%$ are below the median. So it is impossible for every child in the district to ever be above the district median.
b. Yes. Since, by definition, $50 \%$ of the students are above the district median test score, the district is halfway toward the $100 \%$ goal of School Board Member B. Unfortunately, the district will remain being halfway toward that goal.
8a. The answer choice "Not here" could potentially be any number-including those that are larger than the largest number in the list, 6.17. The question, "Which is the largest number?" considered together with the answer choice "Not here" might suggest that the question is getting at the fact that there is no largest number. The "correct" answer given by the answer key for this most confusing test item was 6.17.
b. Answers will vary. Answer choice E could be changed to a specific value. Another option is to change the question to "Which is the largest number listed below?"

9a. 0.50
b. 0.50
c. $P($ no rain on both days $)=\frac{1}{2} \times \frac{1}{2}=\frac{1}{4}$ (or 25\%)
d. $P($ rain on at least on day $)=1-P($ no rain on both days) $=1-\frac{1}{4}=\frac{3}{4}$ (or $75 \%$ )
e. The situation involves compound probabilities that need to be multiplied—along with consideration given to the various possible cases (rain on one day but not on the other, and rain on both days). The weather forecaster added the probabilities to conclude that rain was certain over the weekend. This, of course, is incorrect because it was certainly possible for no rain to occur on either day.

## EXTENSION

Ask students to search for mathematical blunders that may appear in newspapers, magazines, store advertisements, newscasts, and more. Have them present the blunders to the class. Have students suggest ways to fix the blunders.
$\qquad$ Date $\qquad$

## Uncovering Humorous MATHEMATICAL BLUNDERS

The following mathematical blunders actually took place in the real world. Some are rather humorous in nature. The names of the people and companies guilty of the blunders have been omitted to "protect their identities."


1. Museum Docent: "These dinosaur bones are exactly 70,000,006 years old."

Museum Visitor: "How do you know that they are exactly that old?"
Museum Docent: "Well, 6 years ago when I got this job, they told me they were $70,000,000$ years old. So, $6+70,000,000=70,000,006$."
a. Uncover the docent's mathematical blunder.
b. What should the museum docent have said?
2.

## Try our new 2-liter size. It contains 50\% more than the 1 -liter size.

a. Uncover the mathematical blunder in this ad.
b. Why do you suppose this is such a common error?
3. Basketball Player: "Now that I'm joining the Mavericks, we're going to turn around the program 360 degrees."
a. Uncover the basketball player's mathematical blunder.
b. What do you think the basketball player meant to say?
$\qquad$
4.

## Job Opportunities Mathematics Research Assistant

This is a $\frac{3}{4}$ research and $\frac{1}{3}$ teaching position.

Uncover the mathematical blunder in this ad.
$\qquad$
5.

| SNOWMASS VILLAGE |  |
| :--- | ---: |
| POPULATION | 1,018 |
| ELEVATION | 8,388 |
| ESTABLISHED | 1967 |
| TOTAL | 11,373 |

a. This sign was probably created in jest. What mathematical "blunder" is being illustrated here?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
b. Find the sum of these measurements: $49 \mathrm{~mm}+8 \mathrm{~cm}+20 \mathrm{~m}$.
c. What did you do in order to find the sum?
$\qquad$
6. Baseball Player: "With my leg almost healed, I am about three-quarters to 75 percent ready to play."
a. Uncover the baseball player's mathematical blunder.
b. Circle all pairs of numbers below that name the same amount.
$\frac{1}{2}$ and $50 \% \quad \frac{1}{3}$ and $33 \frac{1}{3} \% \quad \frac{1}{8}$ and $8 \% \quad \frac{2}{5}$ and $25 \%$
7. School Board Member A (commenting on local test results): "Within three years, I expect every child in the district to be above the district median test score."
School Board Member B: "Well, I guess we're already halfway toward your goal."
a. Uncover the mathematical blunder committed by School Board Member A.
b. Was the comment made by School Board Member B accurate? Explain.
8. Item on a multiple-choice test:

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Which is the largest number?
A. 6.097
B. 6.0971
C. 6.096711
D. 6.17
E. Not here
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a. The answer given in the answer key for this item was $D, 6.17$. Uncover the mathematical blunder committed by the writer of this test item.
$\qquad$
$\qquad$
$\qquad$
b. How would you rewrite this test item? Include the answer for your test item.
9. Weather Forecaster: "There's a $50 \%$ chance for rain on Saturday, and a $50 \%$ chance for rain on Sunday. Well, I guess we're certain to have rain this weekend."
a. What is the probability for no rain on Saturday?
b. What is the probability for no rain on Sunday? $\qquad$
c. What is the probability for no rain on both days?

Hint: $P($ no rain on both days $)=P($ no rain on Sat. $) \times P($ no rain on Sun. $)$
d. What is the probability that there will be rain on at least one of the two days?
Hint: $P$ (rain on at least one day) $=1-P$ (no rain on both days)
e. Describe the weather forecaster's mathematical blunder.
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