

Open Your Eyes!

Seeing Linear Equations Differently

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NCTM National Conference
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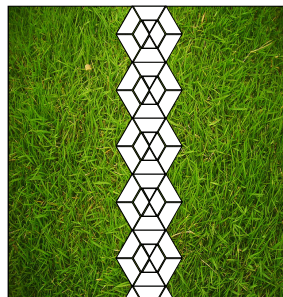


Tiling the Back Walk

Mrs. Ramirez is tiling her back walk and has designed her own hexagon mosaic tiles. They consist of: trapezoids, rhombi, and triangles.

The perimeter of the space she has outlined is 106 ft., and she only wants one tile across.

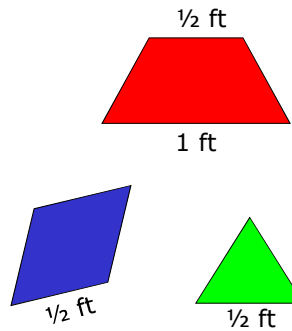
A sample piece of Mrs. Ramirez's outline



Tiling the Back Walk

Each mosaic tile contains:

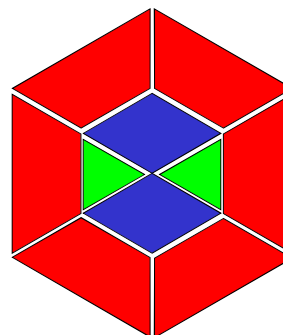
- 6 trapezoids
 - $\frac{1}{2}$ ft on the short sides
 - 1 ft on the long side
- 2 rhombi
 - $\frac{1}{2}$ ft on each side
- 2 triangles
 - $\frac{1}{2}$ ft on each side



Tiling the Back Walk

The cost for each individual tile is:

- Trapezoids - \$3/tile
- Rhombi - \$2/tile
- Triangles - \$1/tile

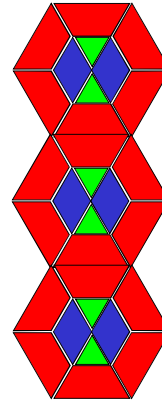


1 mosaic tile

Tiling the Back Walk

How much will it cost to tile Mrs. Ramirez's back walk?

\$624.00



Tiling the Back Walk

- What information is needed in order to solve this problem?
- Is there only one way to solve this problem?

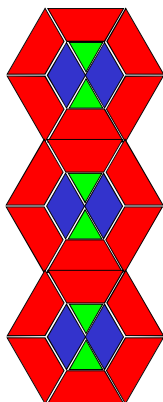
Functional Understanding

- 6th Grade: 6.EE.9
 - Use variable to represent two quantities in a real world problem that change in relationship to one another
- 7th Grade: 7.EE.4
 - Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about quantities
- 8th Grade: 8.F
 - Define, evaluate, and compare functions
 - Use functions to model relationships between quantities



Tiling the Back Walk

Finding the Perimeter...

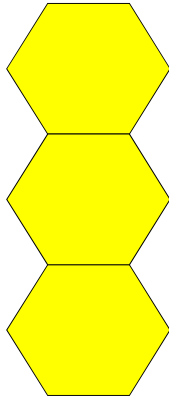


When thinking about the perimeter, let's use hexagon tiles.



Tiling the Back Walk

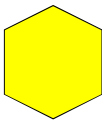
Finding the Perimeter...



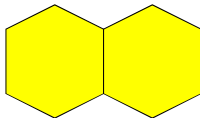
When thinking about the perimeter, let's use hexagon tiles.

Finding The Pattern

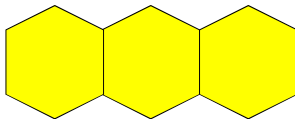
(1)



(2)



(3)



What "rule" describes the perimeter given any number of hexagons?



Students Interviewed

- 18 students in grades 6-8
- Variety of “readiness” levels as categorized by their teachers
- 6 different “rules” for the pattern



Student Solutions

- $y=4x+2$
- $y=6x-2(x-1)$
- $y=5+4(x-2)+5$
- $y=6+4(x-1)$
- $y=6x-2(x-2)-2$
- $y=5x-(x-2)$

Teaching Linear Equations

- What are some of the common student challenges with linear equations?

Recursive

x	y
1	3
2	5
3	7

Closed

x	y
1	3
2	5
3	7



Recursive to Slope-Intercept

$$y = mx + 4$$

- What does Joi know?
- What misconceptions does Joi have?






Slope-Intercept Form

$$y = 4x + 2$$

- What does Joi understand about the construction of her “rule”?
- What misunderstandings does Joi still have?



Conceptual and Procedural Knowledge

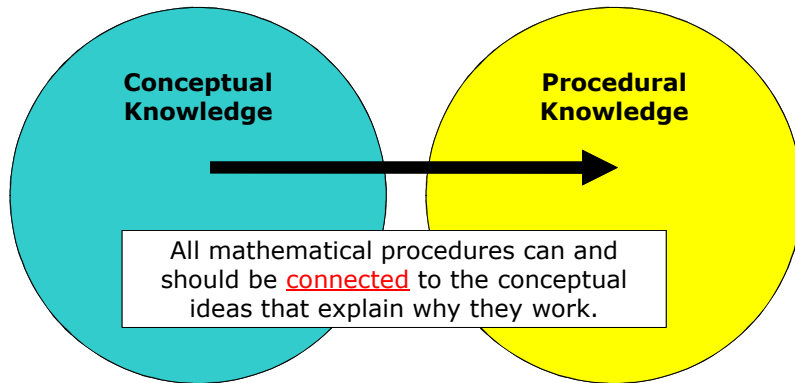
Conceptual Knowledge

Logical relationships constructed internally and existing in the mind as a part of a network of ideas.

Procedural Knowledge

Knowledge of rules, procedures and symbolism used in carrying out routine mathematical tasks.

Conceptual and Procedural Knowledge

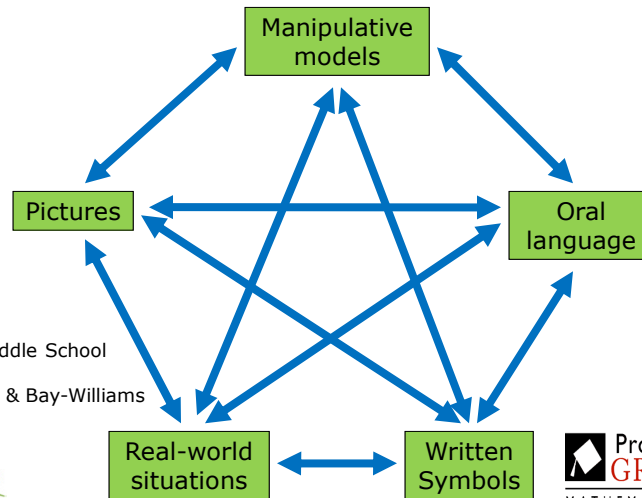


Developing Conceptual Understanding

“Communication can support students’ learning of new mathematical concepts as they act out a situation, draw, use objects, give verbal accounts and explanations, use diagrams, write and use mathematical symbols.”

National Council of Teachers of Mathematics
“Principles and Standards for School Mathematics”, p. 61

Multiple Representations



Taken from:
"Elementary and Middle School
Mathematics"
Van de Walle, Karp, & Bay-Williams



Question

How can the utilization of geometric patterns help students build conceptual understanding of linear functions?





Village Mathematics
Teach the village to educate the child

Project GRAD
MATHEMATICS

SEEING the Pattern

(2)

$6+6-2$

(3)

$6+6+6-4$

(4)

$6+6+6+6-6$


Village Mathematics
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How much to subtract

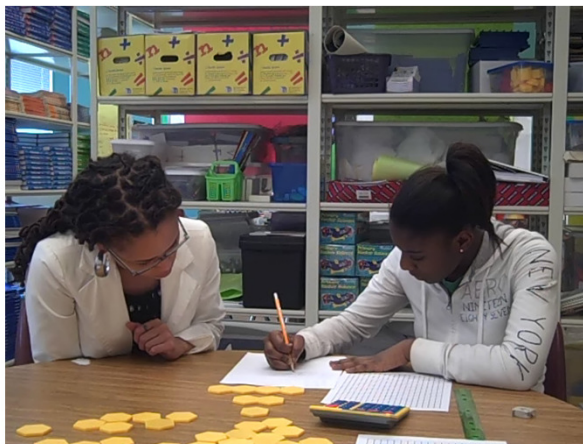
$$\begin{aligned} 2 \text{ tiles} &= 6 + 6 - 2 \\ 3 \text{ tiles} &= 6 + 6 + 6 - 4 \\ 4 \text{ tiles} &= 6 + 6 + 6 + 6 - 6 \end{aligned}$$

Village Mathematics
Teach the village to educate the child

Project GRAD
MATHEMATICS

Patterns in the Groups of 2

$$\begin{aligned}2 \text{ tiles} &= 6 + 6 - 2^{19} \\3 \text{ tiles} &= 6 + 6 + 6 - 4^{29} \\4 \text{ tiles} &= 6 + 6 + 6 + 6 - 6^{39} \\9 \text{ tiles} &= 6 \times 9 - 16^{89} \\20 \text{ tiles} &= 6 \times 20 - 38 = 82^{199} \\100 \text{ tiles} &= 6 \times 100 - 99 \times 2 = \\&= 6 \times 100 - (100 - 1) \times 2\end{aligned}$$



Completed Equation

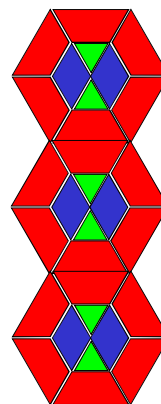
$$x = 6x - (x-1) \times 2$$

- What is her understanding about the construction of this equation?
- How is this understanding different than her understanding of her first equation?

Tiling the Back Walk

Be sure to include the following 3 representations:

- Pictorial representation of how you see the pattern growing
- Function to represent how the perimeter pattern grows
- Verbal description of your solution



Party with Peeps

I am having a party, and I want to give 2 Peeps to each of my guests as part of a party favor bag. Because the Peep rush has ended, Haribo is having a special promotion. For each package of Peeps that you order, you get 2 Peeps as a gift.



Party with Peeps

- How many packages of Peeps should I order for my 84 guests?
- How many Peeps will I be paying for and how many will be free?





Keys to Success for Students

- Making and using predictions
- Documenting mathematically
- Discussing and explaining thinking
- Moving from the concrete to the abstract



Keys to Success for Teachers

- Listen then respond
- Think like your students
- Explain as a way of building on their knowledge
- Be patient!



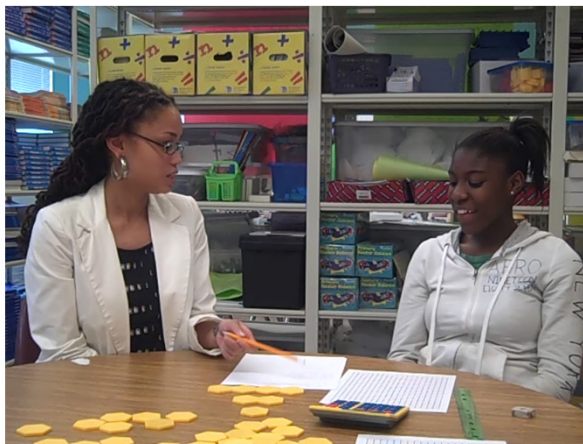
What are the benefits and challenges of teaching this way?

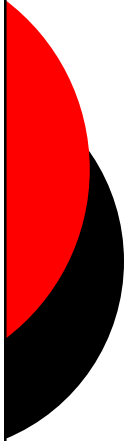
Benefits

- Teaching for understanding
- Problem solving
- Connection to prior concepts
- Teaches to multiple learning styles

Challenges

- Instructional Time
- Classroom Management
- Planning





Thank You!

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