ESSENTIAL UNDERSTANDINGS IN GRADES 9–12 STATISTICS: PREPARING FOR THE CCSS

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Introduction

- Statistics recommended in HS curriculum for many years
  - *Curriculum and Evaluation Standards* (NCTM 1989)
  - *Principles and Standards for School Mathematics* (NCTM 2000)
- Widespread implementation of these standards incorporated inconsistently
- **Common Core State Standards in Mathematics** gives statistics a more prominent role

Introduction

- Statistics often presented as a loose collection of graphical and numerical methods
- Little or no underlying theory, applications, or connections between concepts
- Common Core State Standards
  - beyond mechanical and computational aspects
  - **conceptual understanding** necessary for sound statistical reasoning

Essential Understanding Series

- NCTM's *Essential Understanding* series addresses topics crucial to student development but are often difficult to teach
- 16 volumes planned
  - Statistics (Grades 9-12)
  - Statistics (Grades 6-8)
- *Big Ideas* and *Essential Understandings*
- Not textbooks – resources for teachers
- Focus on concepts, rather than mechanics

Big Idea 1

Data consist of structure plus variability.

Big Idea 1

- **Data = structure + variability**

- Mean, Median, Quartiles
- Function that describes the relationship between two variables

Mathematical Model

- Mean (μ)
- $y = f(x)$
- $y = a + bx$

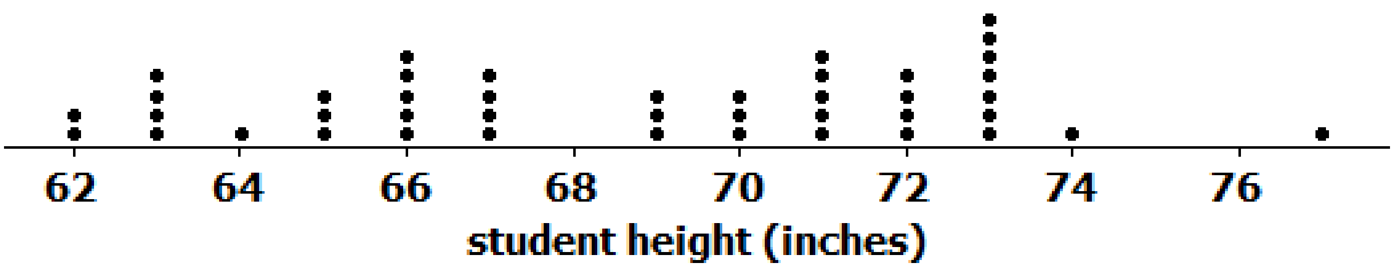
Big Idea 1

- **Data = structure + variability**

  - Extends a mathematical model
  - Allows us to get a better idea of the big picture
  - Use measures such as standard deviation to quantify variability

Big Idea 1

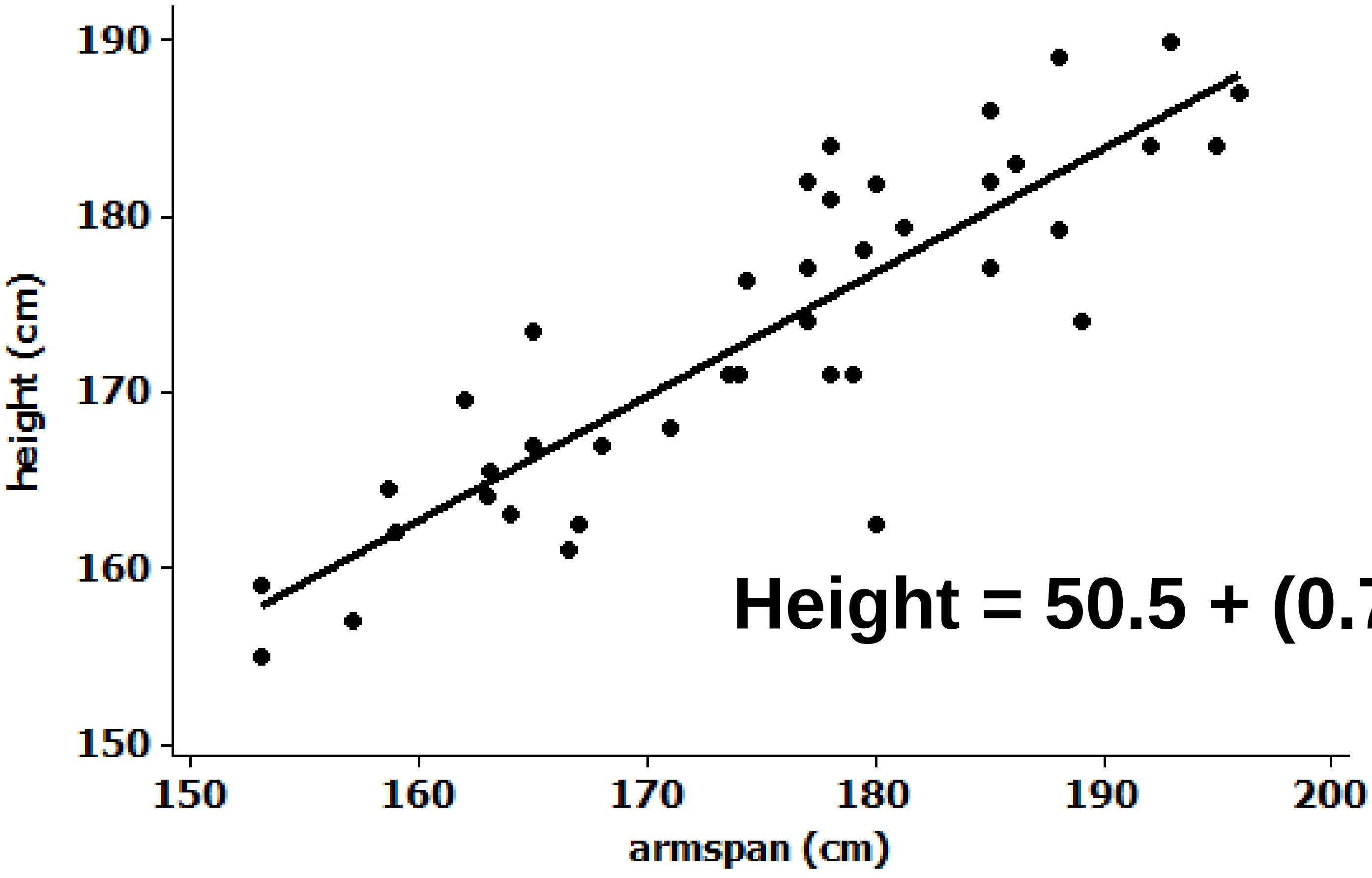
- Data = structure + variability



μ = 68.7 inches

Big Idea 1

- Data = structure + variability



Height = 50.5 + (0.7)(armspan)

Big Idea 3

Hypothesis tests answer the question, “Do I think this could have happened by chance?”

Big Idea 3

- **Hypothesis testing**
  - ***Logic*** is straightforward
  - ***Formal process*** dominates at the expense of conceptual understanding
- Use **data** to compare two competing claims, called hypotheses
- "Which hypothesis is more plausible given the data"

Big Idea 3

- Company manufactures bags of chocolates
- Company **claims** that each bag contains 50% dark chocolates, and 50% milk chocolates
- You prefer dark chocolate, and you really want a dark chocolate right ***now***

Big Idea 3

- Reach into the bag, pull out a chocolate, and it is a milk chocolate

milk

milk

milk

milk

Big Idea 3

- Are you just **unlucky**, or are you beginning to doubt the manufacturer’s claim?
- How many milk chocolates would you have to select in a row before you would ***reject*** the manufacturer’s claim?

Big Idea 3


Suppose claim is true
Examine data "evidence"
Convincing evidence against claim
Conclusion: Reject claim
Not convincing evidence against claim
Conclusion: Fail to reject claim

Big Idea 5

To evaluate an estimator, you have to consider bias, precision and the sampling method.

Big Idea 5

- Big Idea 5 combines the concepts introduced in Big Ideas 1 through 4
- Creates a coherent framework relating
  - how data are collected
  - various estimators
  - sample size

Big Idea 2

Distributions describe variability.

Big Idea 2: Distributions Describe Variability

Being able to think about data in terms of distributions and to distinguish between the different ways distributions are used (to describe the values in a population, the values in a sample or the values of a statistic for different possible samples) are key to understanding statistical inference.

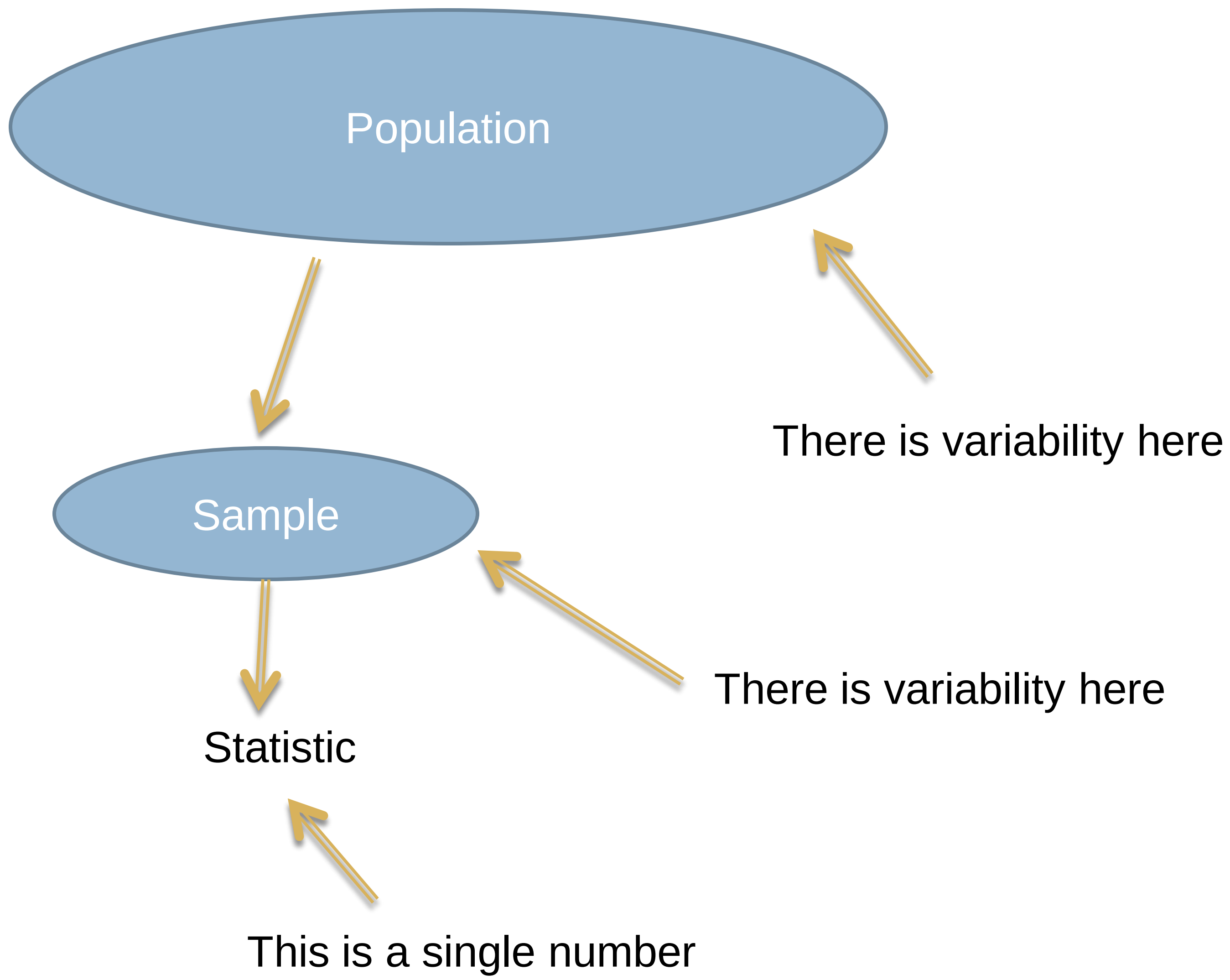
Life would be simple if…

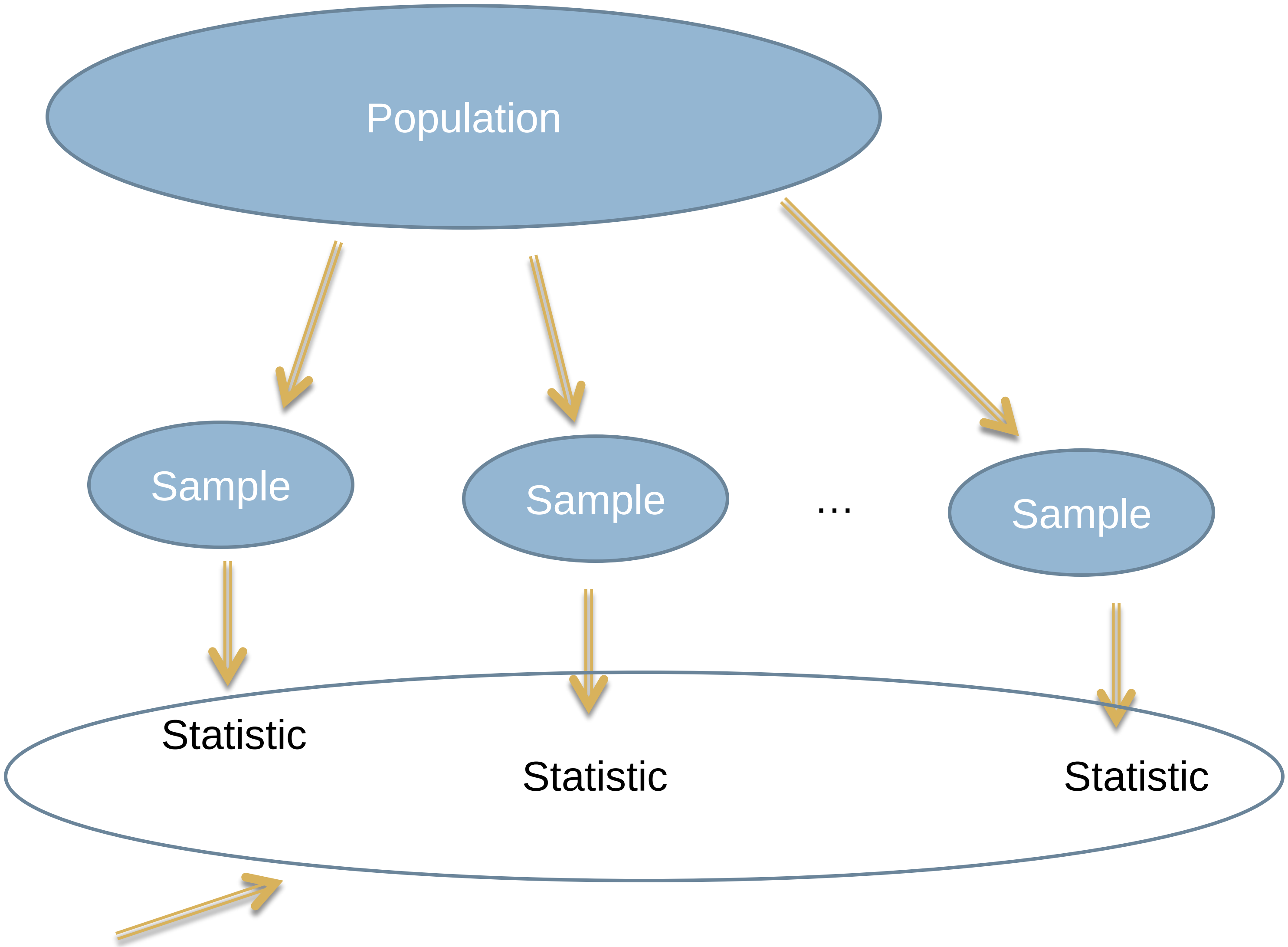
- there was no variability!
  - Simple because it would be easy to draw conclusions based on data.
  - Simple, but BORING!
- As Statistics people we
  - LOVE variability
  - Are OK with being wrong 5% of the time!

Most math folks don't get this!

Distributions Describe Variability

- Variability in a population
- Variability in a sample
- Sample-to-sample variability in the values of a statistic


Population
Sample
Statistic
There is variability here
There is variability here
This is a single number



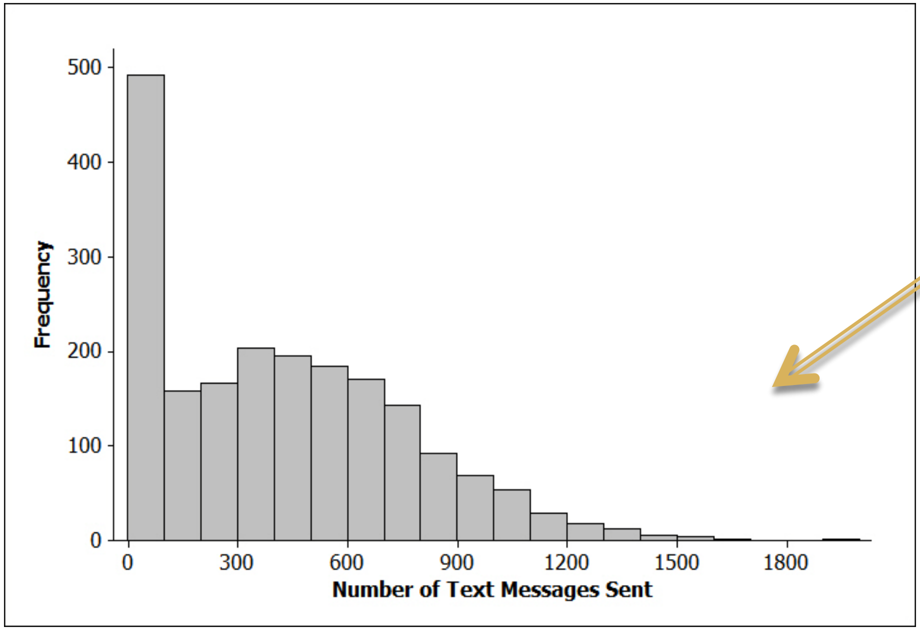
There is variability in these statistic values

Distributions Describe Variability

- Variability in a population
  - Population Distribution
- Variability in a sample
  - Sample Distribution
- Sample-to-sample variability in the values of a statistic
  - Sampling Distribution

Example

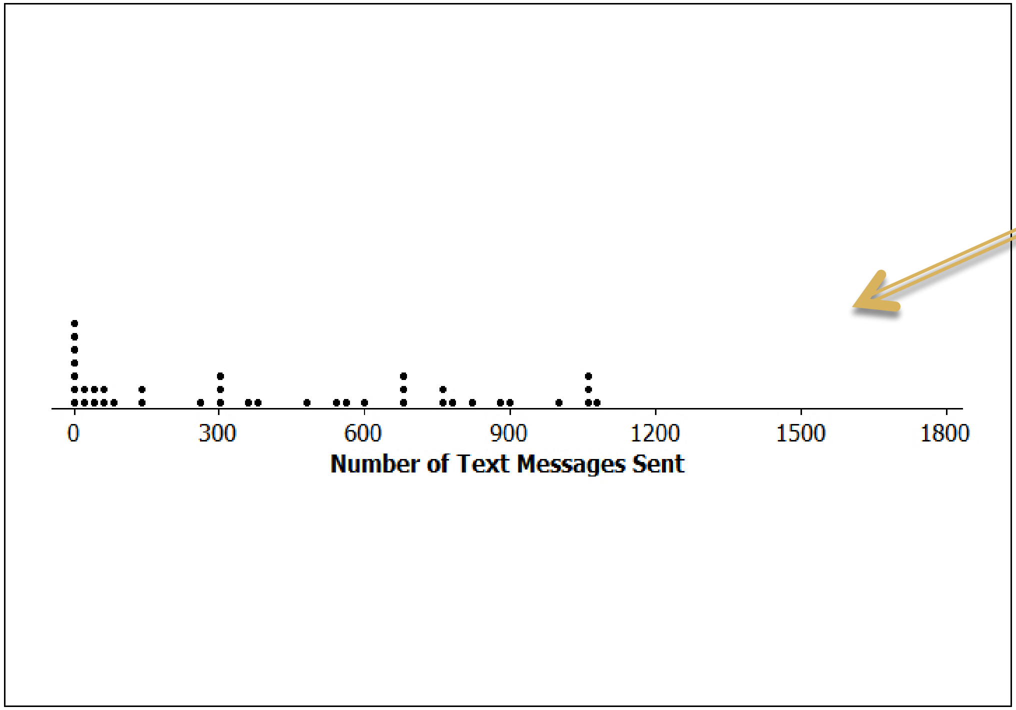
- Population of 2000 students
- Distribution of number of text messages sent in one month



This is a population distribution

Example continued

- Random sample of 40 students

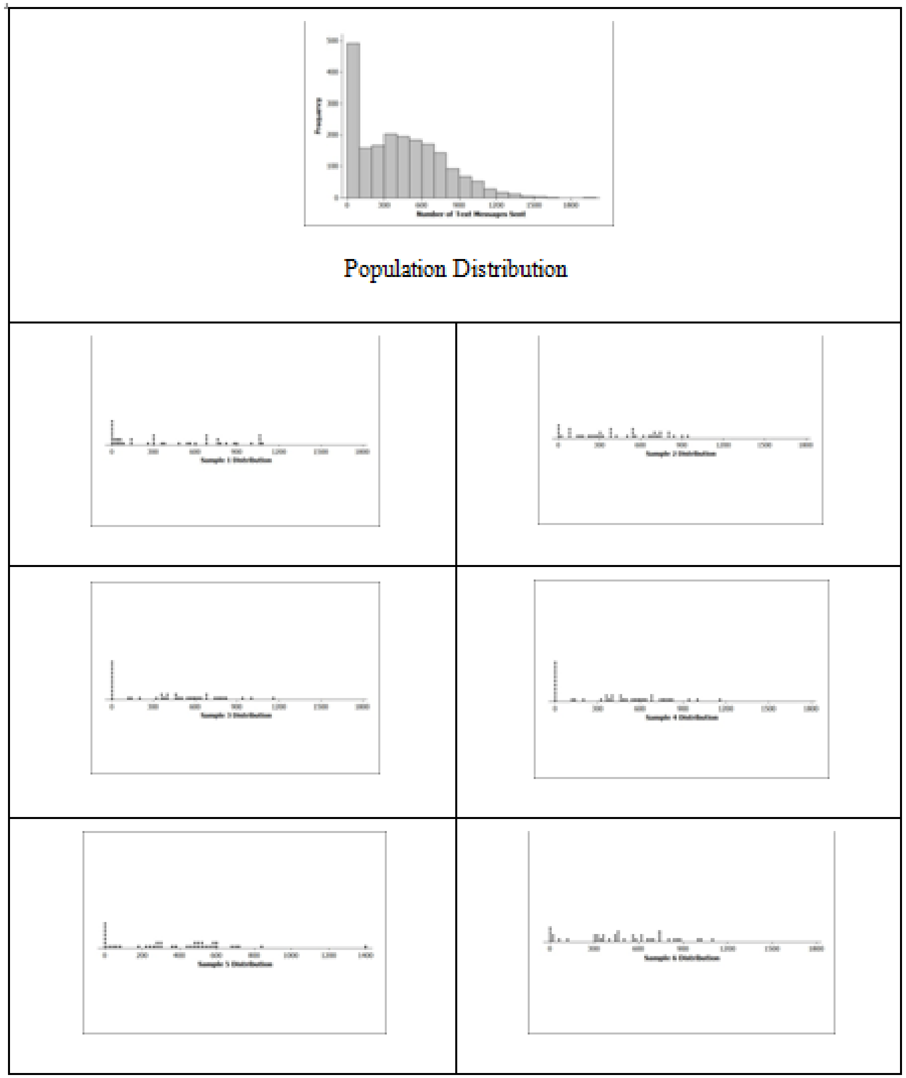


This is a sample distribution

Example continued

IF sample is selected at random from the population, we expect the sample distribution to resemble the population distribution

Sample-to-Sample Variability

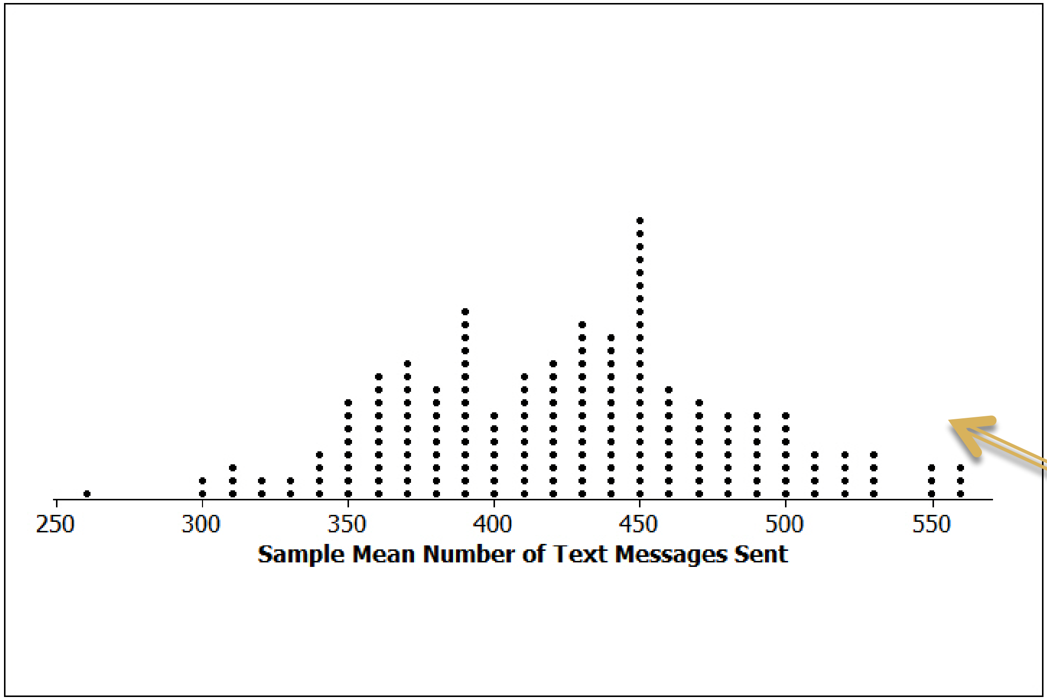


Example Continued

For this population and for these six samples:

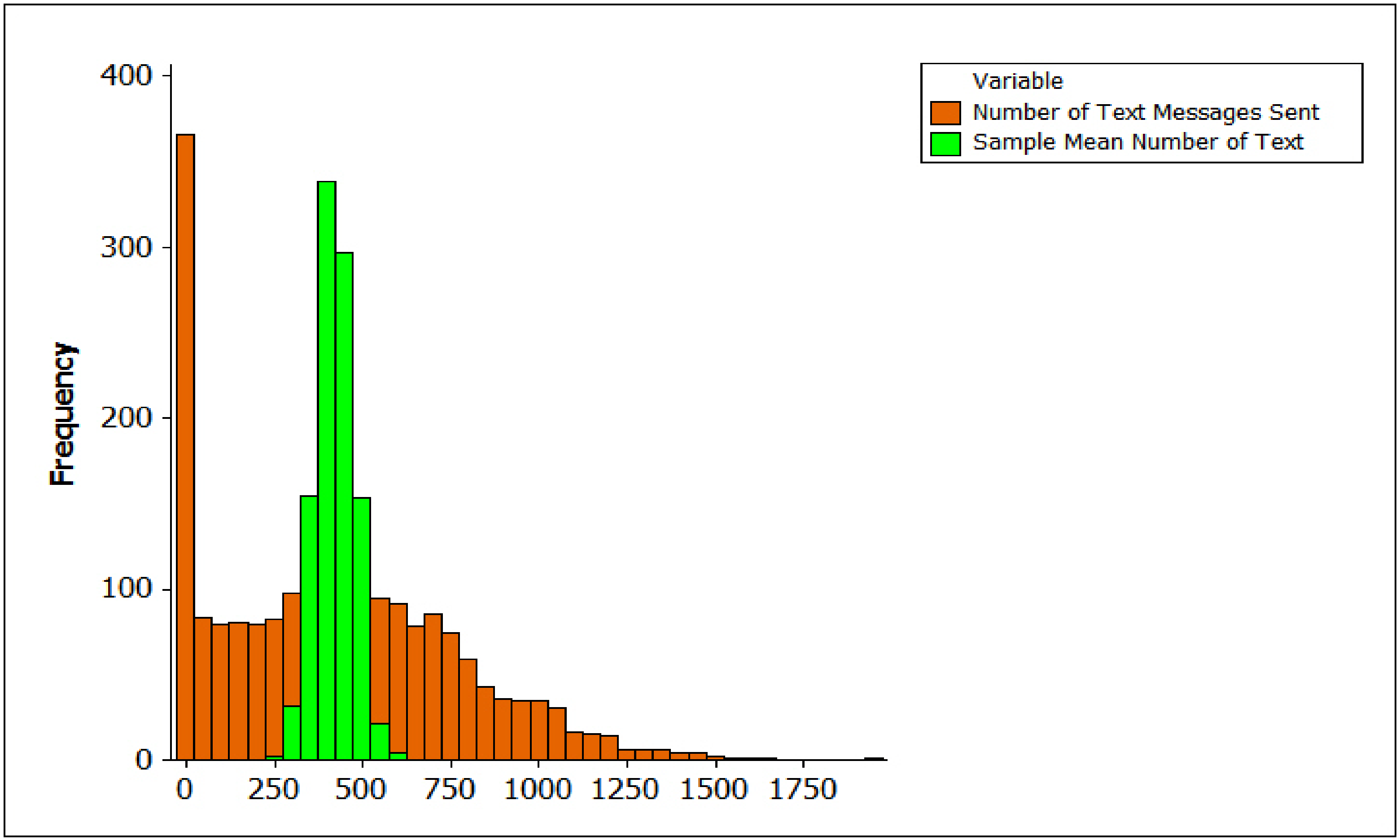
Population mean	**420.6**
Sample 1 mean	422.2
Sample 2 mean	418.2
Sample 3 mean	411.7
Sample 4 mean	564.2
Sample 5 mean	367.2
Sample 6 mean	484.7

For 200 Random Samples



Start of a Sampling Distribution

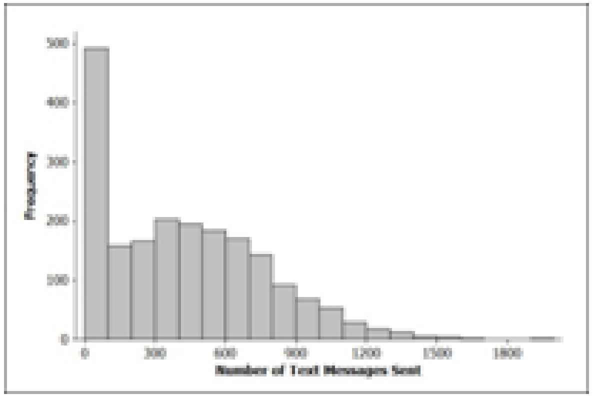
For 1000 Random Samples



Population Distribution

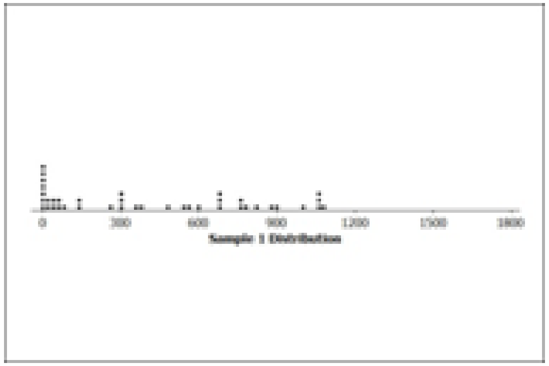
Population Distribution

The distribution of the values of some variable for the entire population. It describes individual-to-individual variability in the population. In most real-life contexts, the population distribution is unknown.

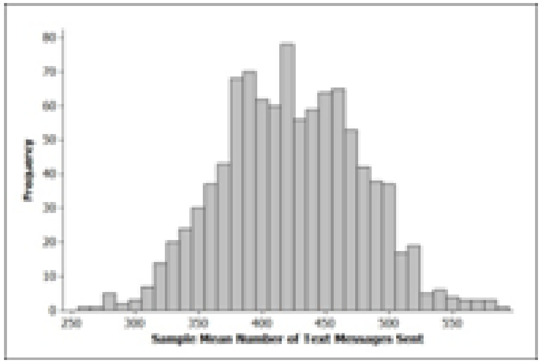


A Population Distribution

Sample Distribution

Sample Distribution The distribution of the values of some variable for a sample selected from the population. It describes individual-to-individual variability in the sample. This is the distribution we get to see and work with.	 Sample Distribution for One Sample of Size 40

Sampling Distribution

Sampling Distribution	
The distribution of the values of a sample statistic (like the sample mean) for all possible samples of a given size that might be selected from the population. It describes how the value of the statistic varies from sample to sample.	 Approximate Sampling Distribution of the Sample Mean for Random Samples of Size 40

Big Idea 4

The way in which data are collected matters.

Big Idea 4

- Statistical methods involve using ***available but usually incomplete*** information to draw conclusions.
- As a result, there is an associated ***risk of error*** that needs to be acknowledged and quantified.

Big Idea 4

- Method of data collection matters because
  - Allows us to **quantify potential errors** in estimation and prediction
  - Determines the **type of inferential conclusions** that can be made

Big Idea 4

- Need to distinguish between two different ways that data are collected
  - *Controlled experiments*
  - *Observational studies*

Big Idea 4

- ***Controlled experiments***
  - Help us learn about the effect of a treatment (drug, therapy, experience)
  - Investigations in which researchers have some level of control over the treatments given to subjects of the study

Big Idea 4

- ***Observational studies***
  - Researchers do not assign subjects to treatments
  - Might be impossible, expensive, or unethical to carry out an experiment
  - Researchers observe characteristics of the groups that subjects are already part of

Big Idea 4

- Well-designed controlled experiments include:
  - A treatment group
  - At least one comparison / control group
  - Random assignment of subjects to treatments

Big Idea 4

- Guiding principle of designed experiments is to make treatment and control groups as much alike as possible ***except*** for the different treatments

Big Idea 4

- **“Is applying duct tape an effective way to eliminate a wart?”**
- The *treatment* is applying duct tape to a wart for a fixed time.
- The *response* is the change in size of the wart in that time.

Big Idea 4

- Recruit subjects
  - No “national registry”
  - Contact dermatologists or physicians
  - No way to randomly select subjects from population of wart sufferers

Big Idea 4

- We cannot give duct tape treatment to everyone
  - If we did, we would never know what would have happened to the warts if we didn't give a treatment
  - We need a control group for comparison

Big Idea 4

- Who gets the duct tape treatment?
- Need random assignment of subjects to treatment and comparison groups

Big Idea 4

- If we didn't use random assignment, we might (out of sympathy) give those with the biggest warts the treatment because they need help the most. This would bias estimates of treatment effect.

Big Idea 4

- We could let wart sufferers choose which treatment to get
  - But we can't be sure there isn't some other difference between the two groups that might account for any differences

Big Idea 4

- *Example*: if we allow subjects to self-select treatment group, maybe younger people would be more willing to try duct tape than older people
- The age of wart sufferers might affect response to the treatment

Big Idea 4

- Random assignment of subjects to treatment / control groups
  - allow us to rule out competing explanations
  - attempts to make each experimental group as much alike as possible, except for the treatment

Big Idea 4

- For observational studies, we desire a sample that is in some way representative of the population
- If the sample is not representative, then we must be suspicious of generalization

Big Idea 4

- **Example**
  - Pew Research Foundation reports that 1/3 of the population sends more than 3000 text messages per month.
  - What?! That’s crazy!

Big Idea 4

- Crazy, at least until you learn that only U.S. teenagers were sampled.
- The “population” Pew refers to is the population of U.S. teenagers, not including adults

Big Idea 4

- How do we get a representative sample?
- One way is through **random selection** from the population of interest

Big Idea 4

- Random samples will not be identical to the population
- But the ways they differ from the population are predictable and quantifiable by considering sampling distributions

Big Idea 4

- **Random** is used in different ways in statistical studies:
  - Random assignment
  - Random selection
- Determine the **type** and **scope** of inference that can be made

Big Idea 4

	Random Selection	No Random Selection
Random Assignment	Infer causality / extend to population	Infer causality / limited to sample at hand
No Random Assignment	Infer to larger population, no causality	No inference, No Causality

Chapter 3
Learning, Teaching and Assessing

Challenge of the Common Core

- All high school mathematics teachers will find themselves in the dual role of being both teachers of mathematics and teachers of statistics.

Recommendations for Teachers

- Developing students' understanding of statistical concepts
  - Look beyond procedural fluency.
    - Ability to "compute" is of relatively little value…
  - Ask Good Questions.
  - Give students the chance to practice "talking statistics".
  - Provide authentic assessments and meaningful feedback.

- Good statistics questions embody these four recommendations.

Not Very Good…

What is the value of the standard deviation of the following 10 numbers?

2009 2015 2002 1979 2032 1991 2016 2030 2001 1990

- Assesses only computational skills.
- Not really a statistics question—lack of context.

Better, But Not Much!

The data given below are the lifetimes (in hours) for 10 light bulbs of a new brand of light bulb that is being considered for use in the football stadium light fixtures at your school. What is the value of the standard deviation of the 10 lifetimes?

- Added context, but focus is still on computation only. Context is irrelevant.

Better…

The data given below are the lifetimes (in hours) for 10 light bulbs of a new brand of light bulb that is being considered for use in the football stadium light fixtures at your school.

a. What is the value of the standard deviation of the lifetimes for the 10 new brand light bulbs?

b. The standard deviation of lifetimes for the brand currently in use is 40 hours. What does the standard deviation you computed for the sample of new brand light bulbs tell you about how the new brand might compare to the old brand?

- Goes beyond computation to include interpretation in context. Context is meaningful.

Even Better.

c. Replacing stadium light bulbs is difficult and requires special equipment. Because of this, rather than replace individual bulbs as they burn out, the school plans to replace *all* of the stadium light bulbs as soon as one burns out. The mean lifetime is 2000 hours for both the current brand and the new brand being considered, and the cost of the two brands is the same. Would you recommend that the school stay with the current brand or change to the new brand? Explain your reasoning.

Requires student to demonstrate conceptual understanding of the standard deviation and what it measures.

More on assessment can also be found in Chapter 3, including rubrics for assessing responses to questions that require interpreting and communicating results and for assessing responses to questions that get at conceptual understanding.

Thanks for coming!

Look for the Essential Understandings in Statistics for Grades 9 – 12 from NCTM this fall.

Also look for Essential Understandings in Statistics for Grades 6 – 8, which should be available around the same time.

Questions?

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