Dynamic Fractions in the Elementary School Classroom

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This session focuses on curriculum materials that are being developed as part of the National Science Foundation project *Introducing Dynamic Number as a Transformative Technology for Number and Early Algebra.*

Dynamic Number is a research and development project at KCP Technologies to extend and operationalize research in number, operation, and early algebra. The project builds on the paradigm of Dynamic Geometry (the interactive and continuous manipulation of geometric shapes and constructions) with a new technological paradigm, Dynamic Number, centered on the direct manipulation of number representations and numerical constructions. Using The Geometer's Sketchpad as a starting point, KCP Technologies is developing new software tools to deepen students' conceptions of integers, fractions, decimals, real numbers, and early algebra in grades 2–8 mathematics.

Project website with free, downloadable Sketchpad activities, teacher notes, worksheets, and instructional movies:

http://www.kcptech.com/dynamicnumber

VIDEO EXCERPT 1: ORDERING FRACTIONS

The context: Three students were ordering the fractions $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$, and $\frac{1}{9}$. When debating between $\frac{1}{7}$ and $\frac{1}{9}$, one of the students said that $\frac{1}{9}$ was bigger because "we are cutting it up into more pieces." The other two students tried to prove their thinking by creating two new fractions, $\frac{1}{2}$ and $\frac{1}{10}$, with Sketchpad. They explained their understanding using a context of sharing a chocolate bar among different numbers of children.

Sara: And the difference between $\frac{1}{2}$ [and $\frac{1}{10}$]…which piece is bigger?

Martin: This one. No…

Sara: Since in $\frac{1}{2}$ … it is two pieces, and in $\frac{1}{10}$, there is 10 pieces. And that's a lot. And that's like…and see how…

Martin: So is it this one? (points to $\frac{1}{10}$)

Sara: But…

Paul: Look at the difference between $\frac{1}{2}$ and $\frac{1}{10}$. Which is bigger?

Martin: This one has two and this one has ten.

Sara: Yeah, but it doesn't matter how much it has; it matters how big of a piece it has or small the piece.

Paul: How is it different? How is it the smallest and the largest?

Sara: See in $\frac{1}{2}$, there is two pieces, but there's…there is two large pieces. But in the $\frac{1}{10}$, they are smaller because there are ten pieces.

Martin studies the screen.

Teacher: What questions do you have for us?

Martin: These are two…and these are ten. All together is ten.

Sara: Fractions, we don't care… we don't care how much do we have. It's two, ten something like that. We only care like how much of a big piece because if you want to share with some kids…For example, if it's a chocolate bar you will want a bigger piece,

right? And if you cut it in half, you get a bigger piece. And that's how you do it in fractions. In ten, if you cut it into ten pieces, you will want more chocolate, but you will only get one little piece.

Martin: So, we cut it like this, like that, like that…

Sara: Yeah. There will only be one piece for you because you are going to share it with ten people. And then $\frac{1}{2}$, you will get a bigger piece instead of $\frac{1}{10}$.

Teacher: So which piece would you rather have…

Paul: $\frac{1}{2}$ or $\frac{1}{10}$?

Martin: I would rather have $\frac{1}{2}$ … because… I will get…two pieces.

Sara: No, you will only get one piece, but a bigger piece. If you get $\frac{1}{10}$, then you will get one small piece, but just for yourself. Which isn't a lot, if you like candy.

VIDEO EXCERPT 2: MEASURING FRACTIONS

Andrea: Hmm … (moves the 1/3 measuring strip to the number line) well, not really actually.

Sonal: Oh—before you move them, make sure you make a prediction.

Andrea: OK.

Emily: $\frac{2}{3}$.

Andrea: Yeah, that's what I was thinking. $\frac{2}{3}$ (*Writes it on the paper and then starts to move the sixths*). And then it will take us a looong time to do this.

Sonal: Before you guys put the sixths there, can I ask you to predict how many sixths you'd need?

Andrea: Um, maybe we might need 4, or maybe we might need all of them. But I really think it's around 4 or 5.

Sonal: And what do you think, Sarah?

Emily: I agree, because since the question mark went farther, we had to use more one thirds, and now we have to use one more sixth.

Sonal: How many sixths… are you moving them?

Andrea: Well, I got to $\frac{3}{6}$, so I am really sure that it will be 4.

Sonal: OK.

Andrea places the last sixth on the number line.

Sonal: Can you click back on number two for a second? Because I wonder—how many sixths did it take us to make $\frac{1}{3}$?

Andrea: Two sixths.

Sonal: Two sixths. So than how many sixths would it take us to make *two* thirds? If you click back to number three?

Andrea: Um, 4. Four sixths.

Sonal: Do you guys see that? When you add another third, how many sixths are you adding?

Andrea: Two.

Sonal: Two more sixths.

Emily: I see something,

Andrea: What?

Sonal: Oh, tell her what you see!

Emily: That since last time, on the second one, there were two sixths and one third. And I see that the one third and the one sixths are doubling.

Andrea: Yeah, that's what I was thinking too, because like over here, it was kind of over here somewhere, and then they moved it all the way over here, like they doubled these two, so like it was over here because this was right here so it was over here, so they doubled this and make it come over here. And so now it's over here, it would be double these and double these. Because it was already two of these before, and now it's plus one, no two.

Sonal: I'm getting confused because you're saying "these," and "those," and "this," and "that." Can you tell me what you're talking about? You're pointing, but what doubled, exactly? Be specific.

Andrea: The $\frac{1}{3}$ doubled, and the $\frac{2}{6}$, which is the $\frac{1}{6} + \frac{1}{6}$… the two, it equals $\frac{2}{6}$ so $\frac{2}{6}$ doubled until another $\frac{2}{6}$.

Sonal: So when you double $\frac{1}{3}$, how many thirds do you get?

Andrea: Two.

Sonal: Two what?

Andrea: Thirds.

Sonal: And when you double two sixths, how many sixths do you get?

Andrea: Um, $\frac{4}{6}$.

Sonal: So can I write some of this down, because it's hard for me to keep track of. So when you double (writes 2x) I am going to double $\frac{1}{3}$, you get $\frac{2}{3}$?

Andrea: Mmm hmm.

Sonal: And when you double $\frac{2}{6}$ (writes 2 x $\frac{2}{6}$), you actually get $\frac{4}{6}$? That's really cool! OK.

VIDEO EXCERPT 3: MYSTERY FRACTIONS

Daniel makes the "mystery fraction" 13/24 and then hides its value.

Jason: I think it will be a little bit bigger than one half.

Daniel: Why bigger than a half? How can you tell?

Dylan: Because with the numbers it can't be one half.

Daniel: Why not?

Dylan: Because 24's half is 12, and all the odd numbers don't have half. So 22's half is 11, 20's half is 10, 18's is 9, 16 is 8, and 14 is 7, so there's none. So half can't be it.

Jason: All I know was he's saying like… if the number is equivalent to half, it's probably to be the numerator be…probably…must be half the denominator. So there were no numbers that were half. So it won't be half.

Daniel: That's very interesting.

Sonal: It can't be a half, so then you think it's a tiny bit more than a half.

Daniel: I think both of you caught on to that little trick I did here, wanting you to think it was a half, but neither of you, for a second, thought it was exactly a half. So what might it be if it's close to a half?

Jason: Maybe $\frac{13}{24}$.

Daniel: Is that close to a half?

Dylan: No.

Jason: Yes, because $\frac{12}{24}$ was equivalent to half, just one more… $\frac{1}{24}^{\text{th}}$ bigger.

Dylan: I agree with you because 24's half is 12, and I thought 13 …(inaudible) I got a little confused.

Daniel: Do you want to try it?

Jason: 13 and 24.