It is better to know some of the questions than all of the answers.
James Thurber

You can observe a lot just by watching.
Yogi Berra

It is possible to store the mind with a million facts and still be entirely uneducated.

Alec Bourne

"We cannot teach people anything; we can only help them discover it within themselves."
-- Galileo Galilei

If you are faced by a difficulty or a controversy in science, an ounce of algebra is worth a ton of verbal argument.
J.B.S. Haldane

The creative principle of science resides in mathematics.
Albert Einstein

The mind is like a parachute: in order to work it has to be open.
author unknown

Making Meaningful Mathematics

using origami

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Assumptions:

mathematics is the search for patterns-
patterns come from problems-
therefore, mathematics is problem solving.

meaningful math is better than meaningless math

teaching is not telling

what does algebra look like?

Find x

$$\frac{x-5}{x} = \frac{2}{3}$$
$$3(x-5) = 2x$$
$$3x - 15 = 2x$$
$$x - 15 = 0$$
$$x = 15$$

$$y - y_1 = m(x - x_1)$$
$$y = mx + b$$
$$Ax + By = C$$

Solve

$$2x^2 - 5x - 10 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\frac{5 \pm \sqrt{25 - (-80)}}{4}$$
$$\frac{5 \pm \sqrt{105}}{4}$$

$$1000 = 500(1.05)^x$$
$$2 = (1.05)^x$$
$$\log 2 = \log (1.05)^x$$
$$\log 2 = x \log(1.05)$$
$$\frac{\log 2}{\log 1.05} = x$$

solve :

$$10 - 2x = x(x-5)$$
$$10 - 2x = x^2 - 5x$$
$$10 = x^2 - 3x$$
$$0 = x^2 - 3x - 10$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$\therefore x = 5 \text{ or } x = -2$$

$$(x+2)(x+5)$$
$$x^2 + 2x + 5x + 10$$
$$x^2 + 7x + 10$$

Find the area of a 10" circle :

$$A = \pi r^2$$
$$A \approx 3.14(10)^2$$
$$A \approx 314 \text{ square inches}$$

the rule of four:

represent a relation in three ways-

as a table of numbers,
as an equation,
as a graph,

and verbally!

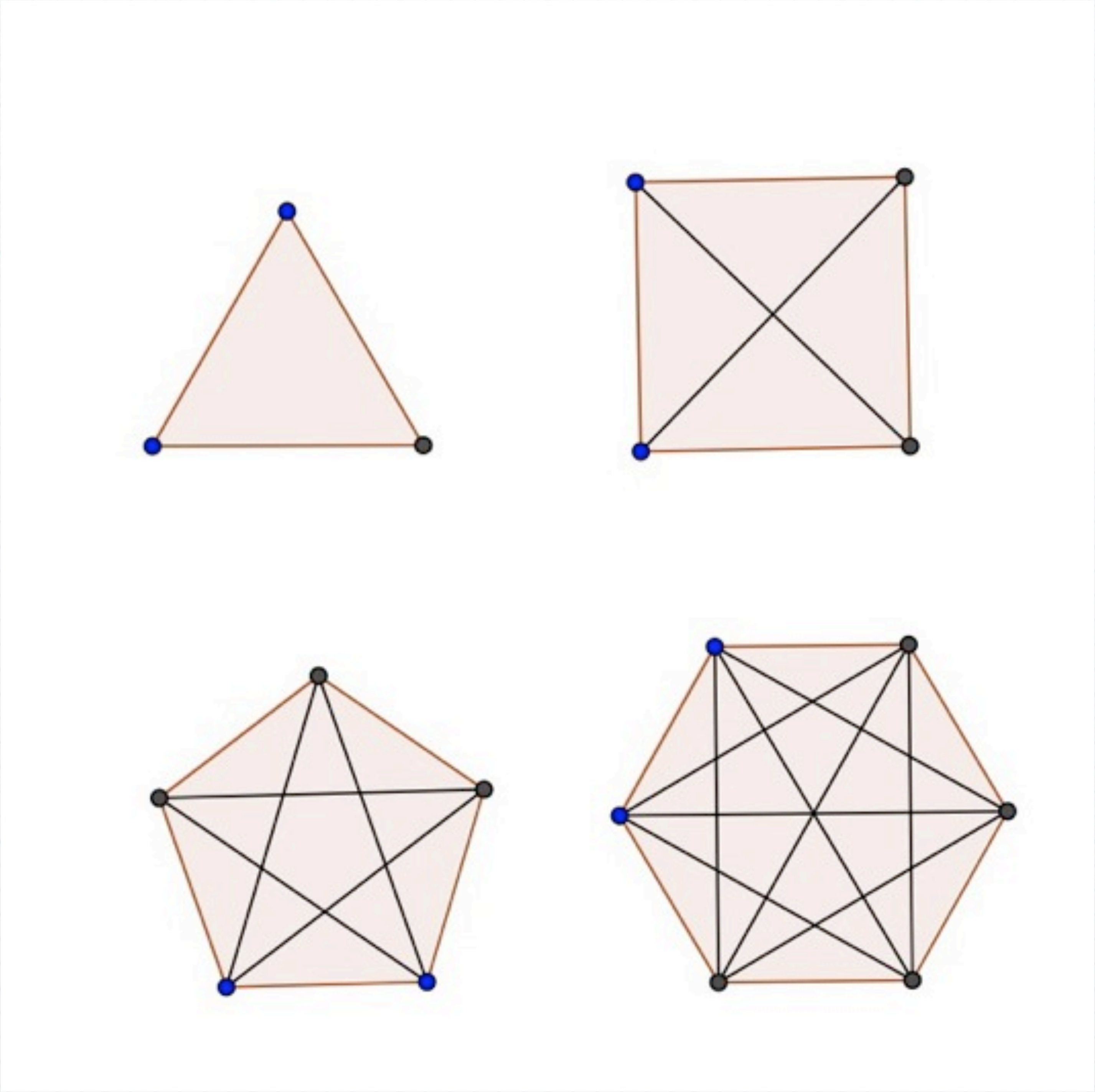
An Example

A triangle has no diagonals.
A square has two diagonals.
A convex pentagon has five diagonals.

The number of diagonals in a convex polygon is a function of the number of sides.

What is the relationship between the number of sides and the number of diagonals in a polygon?

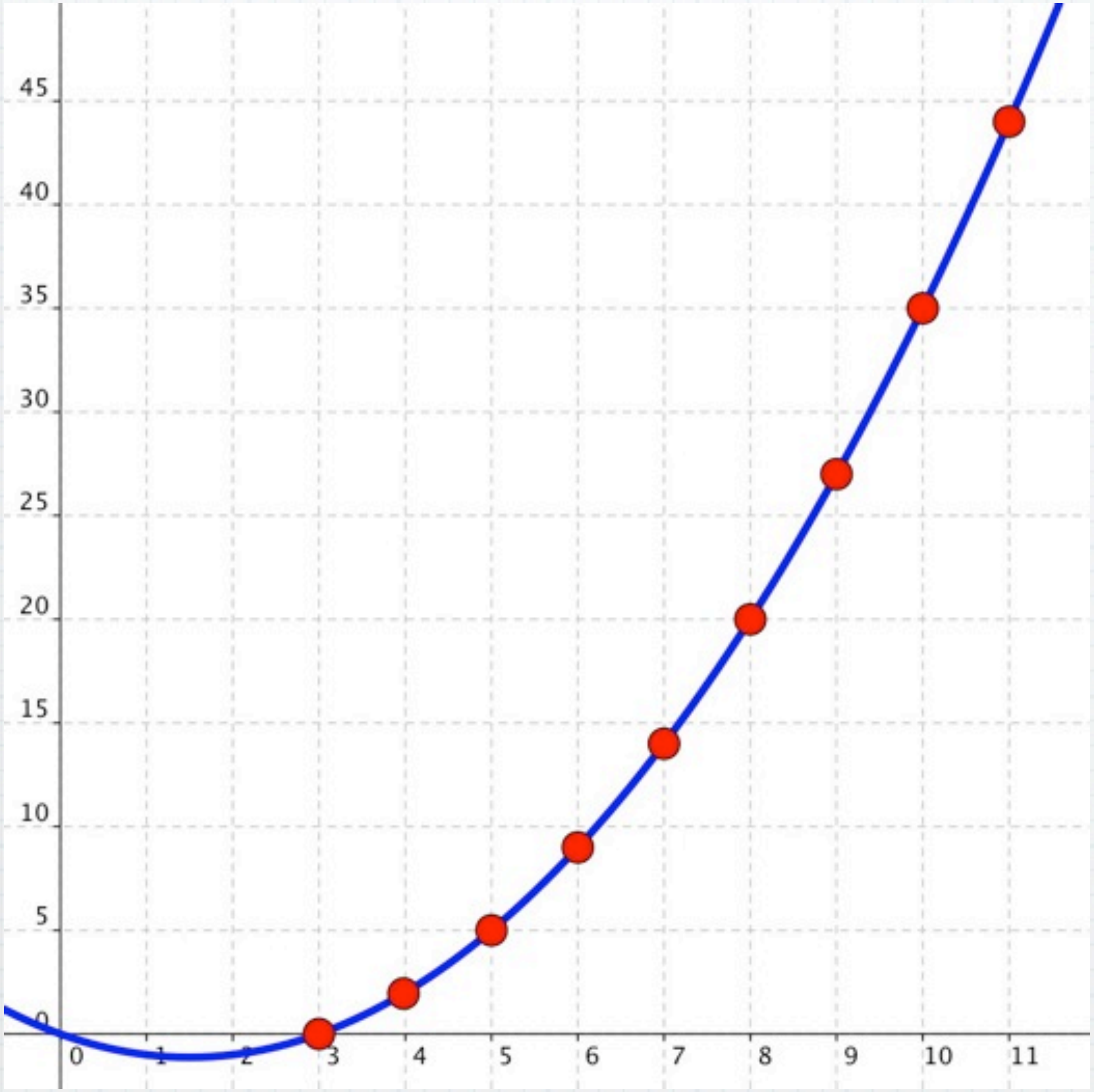
some pictures to help understand the problem:



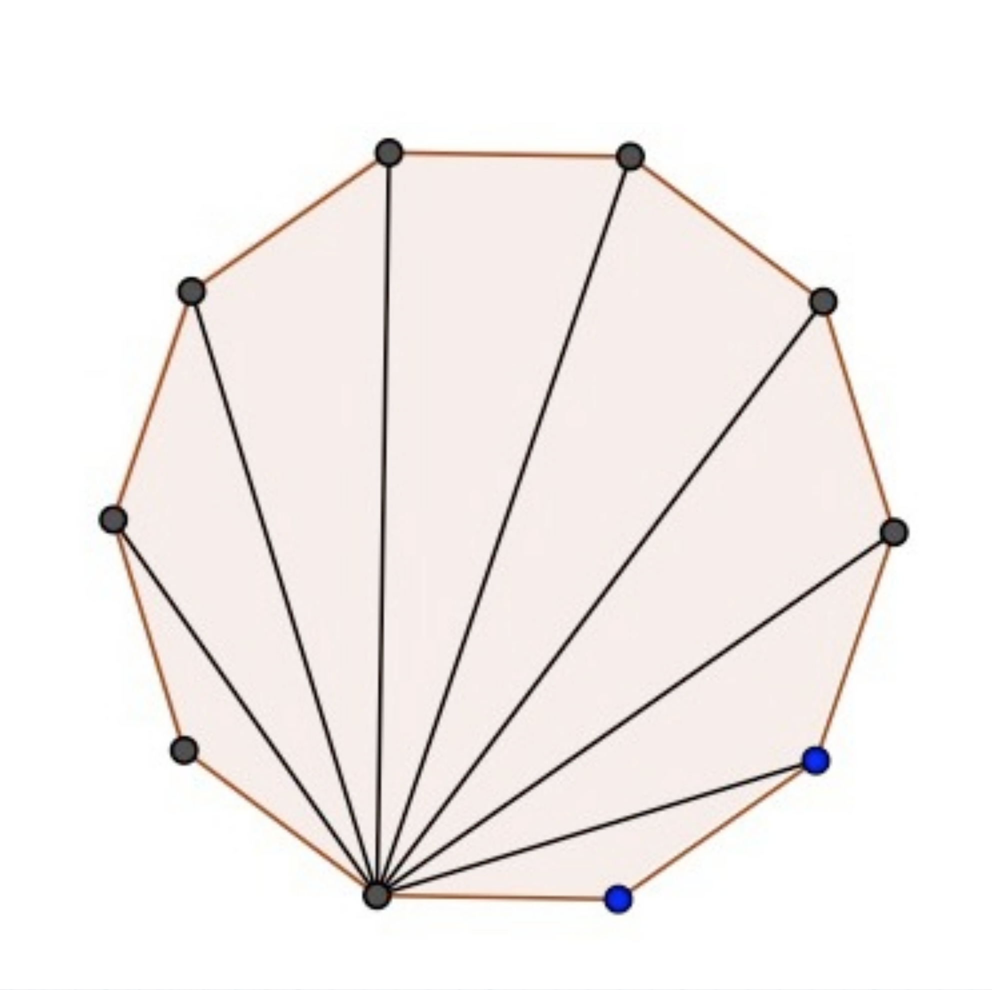
a table showing the relation between sides and diagonals:

sides	diagonals
3	0
4	2
5	5
6	
7	
8	

a graph showing this relation, which is a function:



an verbal explanation of the pattern:

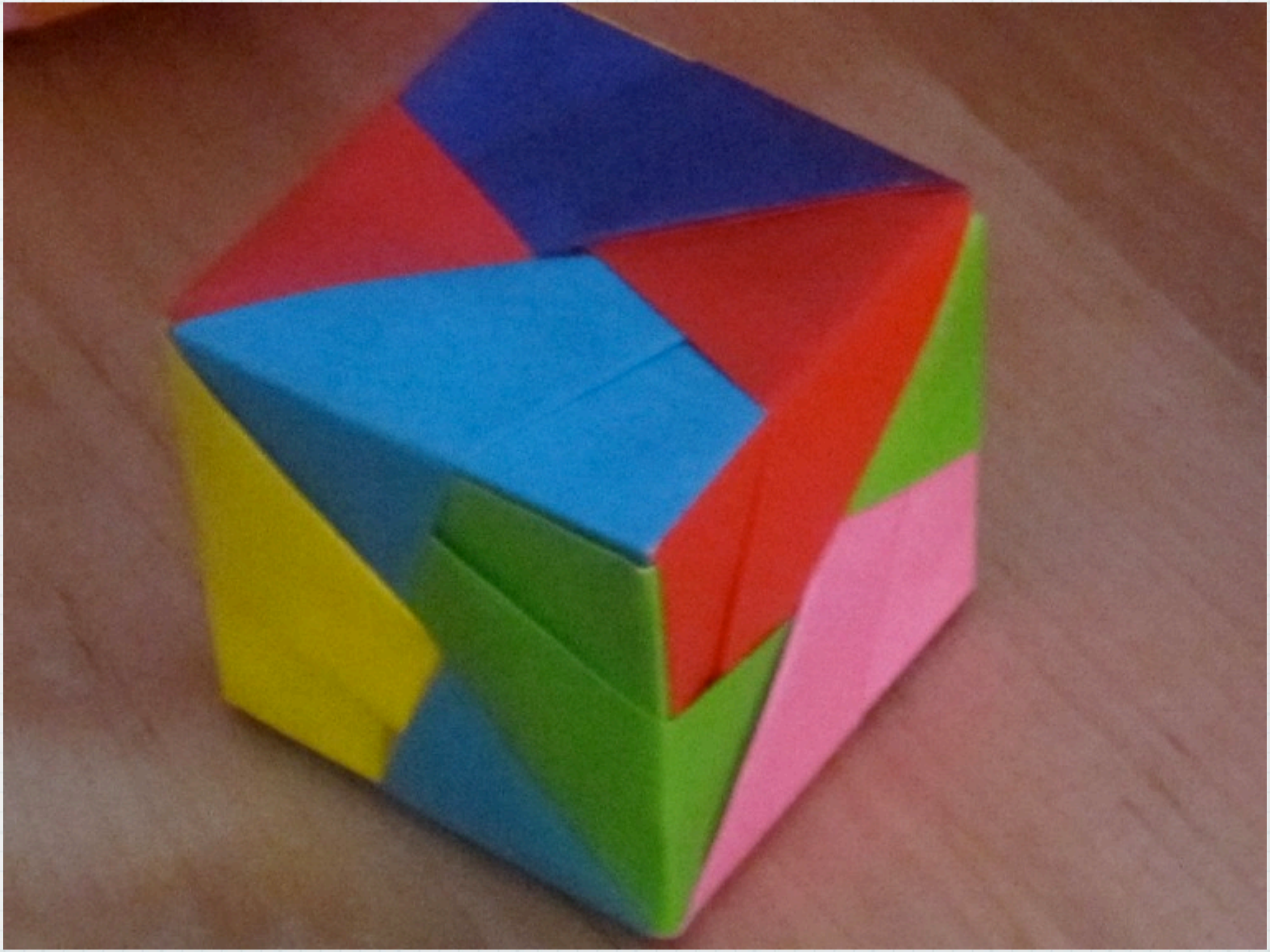


Since every vertex is connected to all other vertices, except three (itself and the two adjacent ones), for 10 sides you would multiply 10 times 7; and since they were all drawn twice, divide by 2.

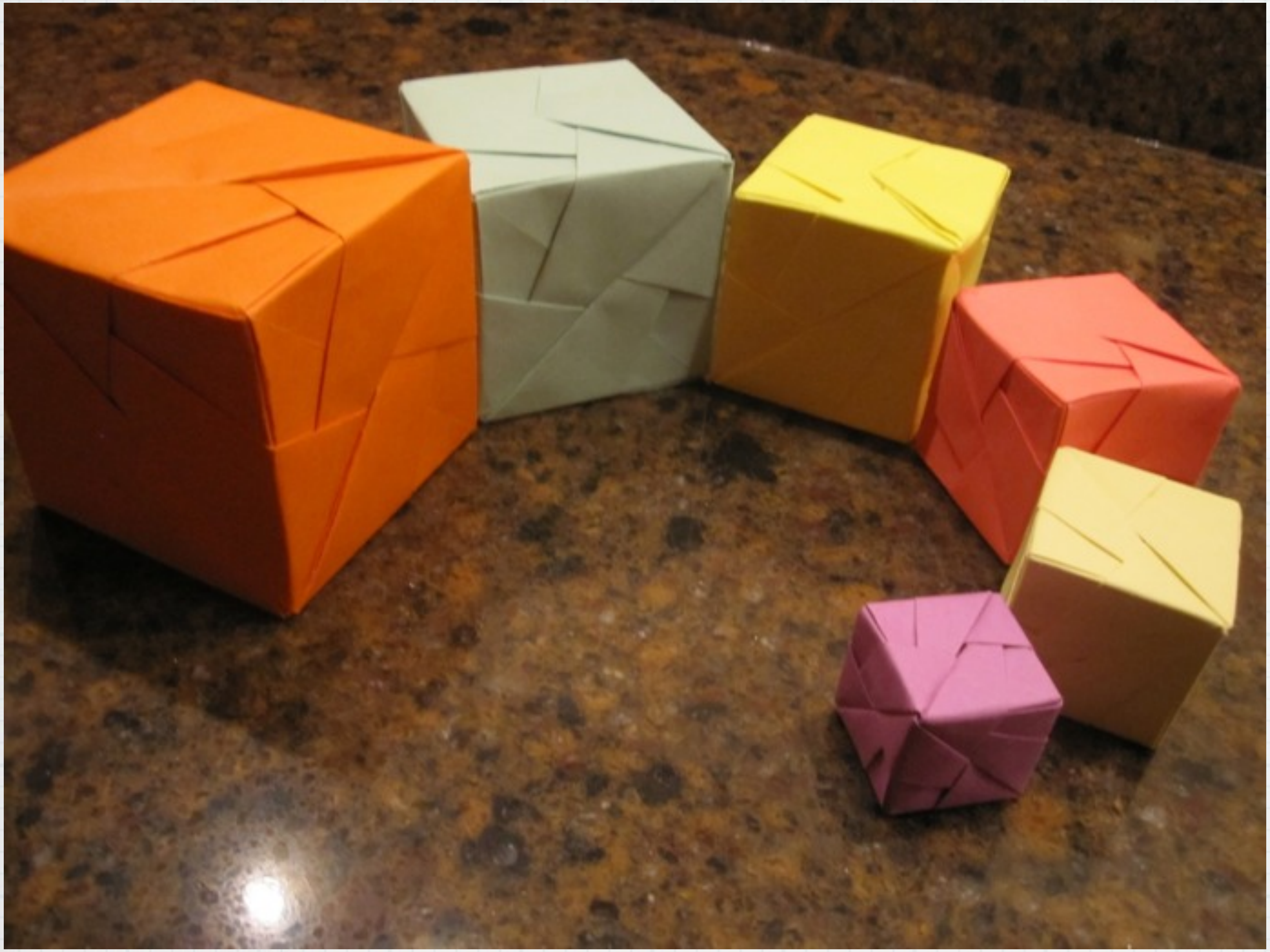
and the equation showing the functional relation:

$$y = \frac{x(x-3)}{2}$$

here is a cube made from square paper.



here are some other cubes, built the same, but starting with square paper of various sizes



As the size of the square that made the cube varies, the volume will vary.

But, how?

here is the paper that was used-

the paper ranges in size from a 3" square to an 8" square

How does volume change in relation to changes in the size of the paper used to make the cube?

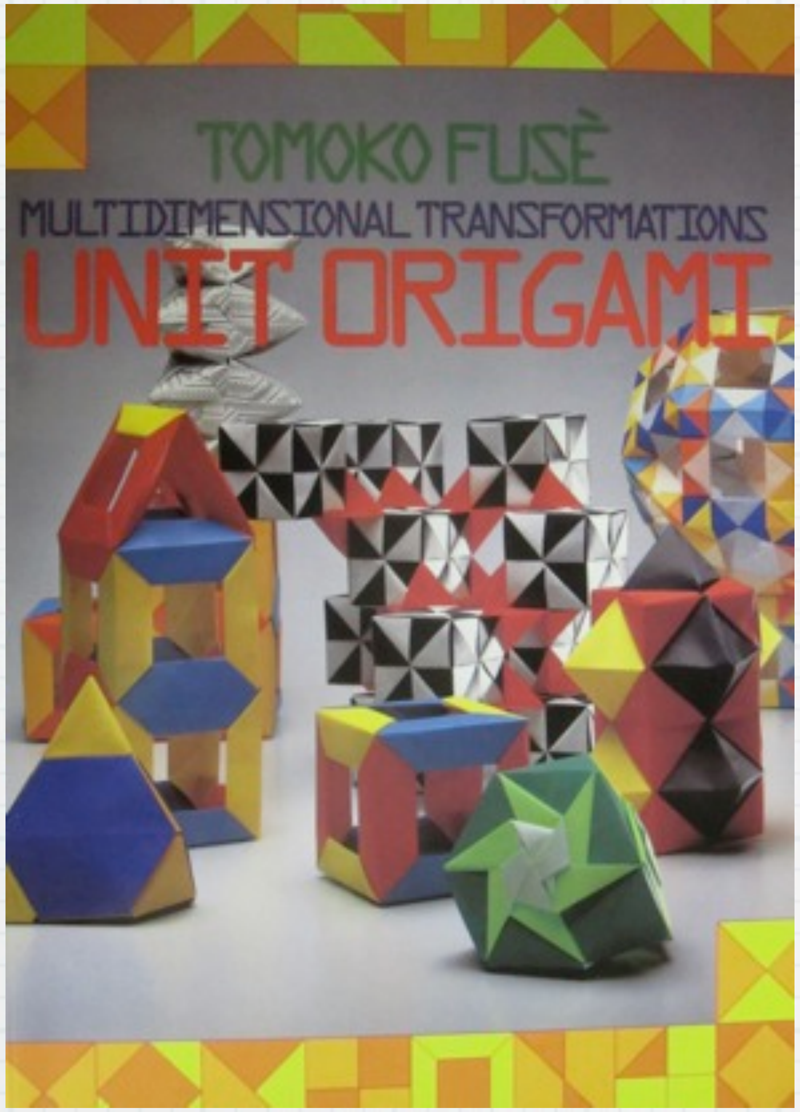
To answer this question we will build cubes, measure their volume, record data in a table, graph the results, and find patterns.

so, let's build a cube-

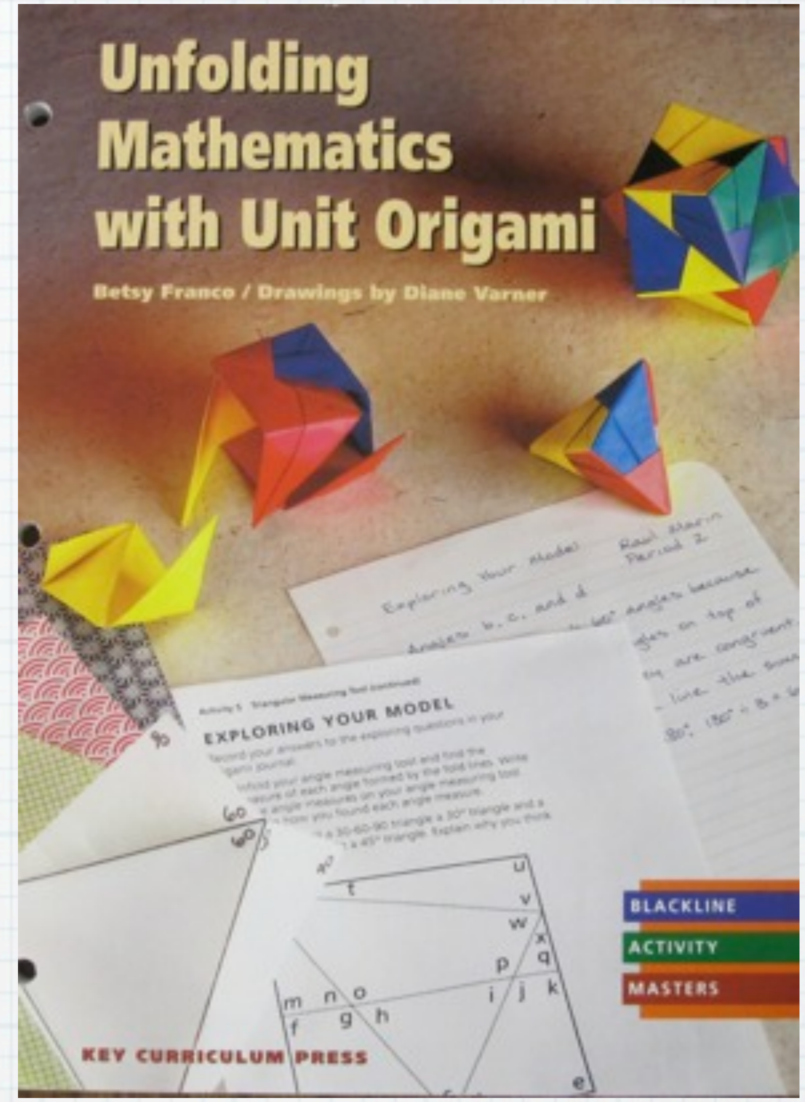
the process is called multidimensional transformations because we transform square paper into a three dimensional cube

Another more common name is
UNIT ORIGAMI

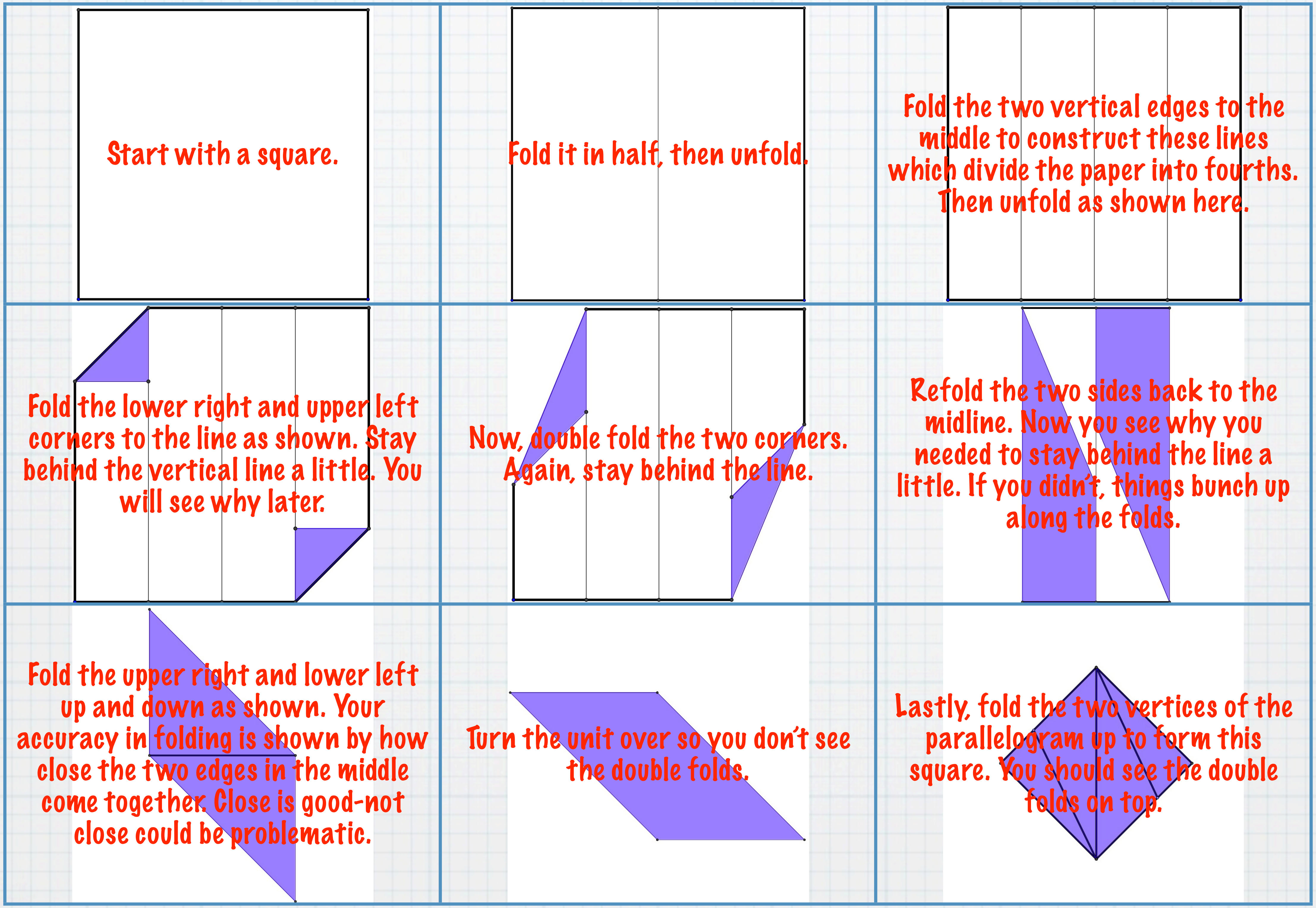
two very useful books-highly recommended.



Unit Origami, Tomoko Fuse



Unfolding Mathematics with Unit Origami, Key Curriculum Press


Start with a square.
Fold it in half, then unfold.
Fold the two vertical edges to the middle to construct these lines which divide the paper into fourths. Then unfold as shown here.
Fold the lower right and upper left corners to the line as shown. Stay behind the vertical line a little. You will see why later.
Now, double fold the two corners. Again, stay behind the line.
Refold the two sides back to the midline. Now you see why you needed to stay behind the line a little. If you didn't, things bunch up along the folds.
Fold the upper right and lower left up and down as shown. Your accuracy in folding is shown by how close the two edges in the middle come together. Close is good-not close could be problematic.
Turn the unit over so you don't see the double folds.
Lastly, fold the two vertices of the parallelogram up to form this square. You should see the double folds on top.

This is one UNIT.

We need 5 more UNITS to construct a cube.

Finding the volume.

Student understanding of volume can be connected to a formula with little understanding about what the number represents.

Therefore we won't use a formula, but fill the cube and count.

Then, we will use a formula.

Fill your cube (the one made from 6" squares) with beans.

Count the beans.

Agree as a table on a number.

If you have time you could build another cube from 4" squares or use the small cubes that are on your table made from 3" squares.

We are gathering data.

Here are my results:

plot the points in geogebra

what information can you get from the graph?

what patterns do you notice in the table?

is this relation linear?

is it direct (proportional)?

predict the volume (in beans) if you started with 12" paper squres.

original square	number of beans
1	
2	
3	44
4	116
5	242
5.5	320
6	
7	660
8	1012
9	
10	

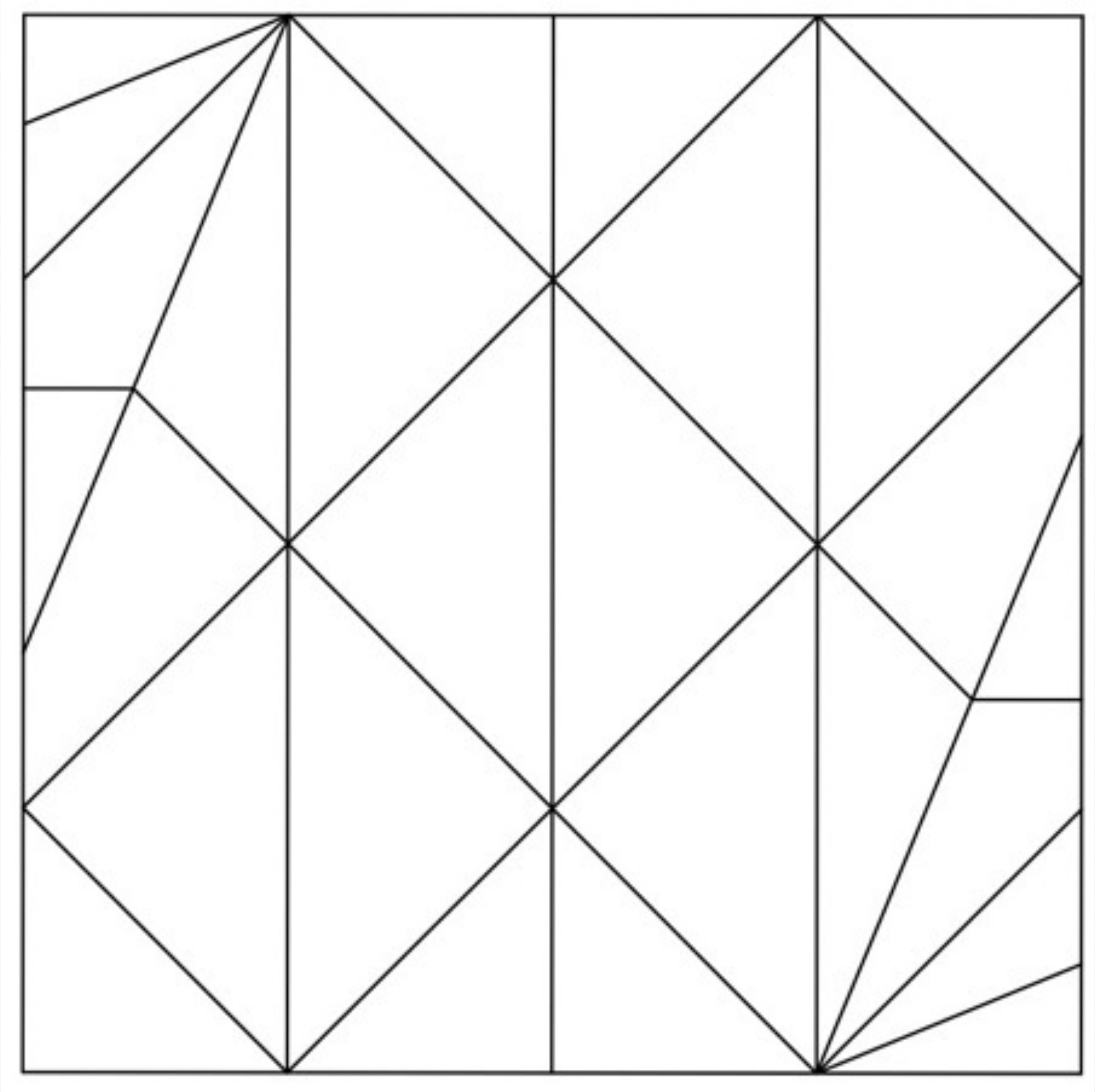
“the goal is not to ‘cover’ mathematics, but to ‘uncover’ it.”

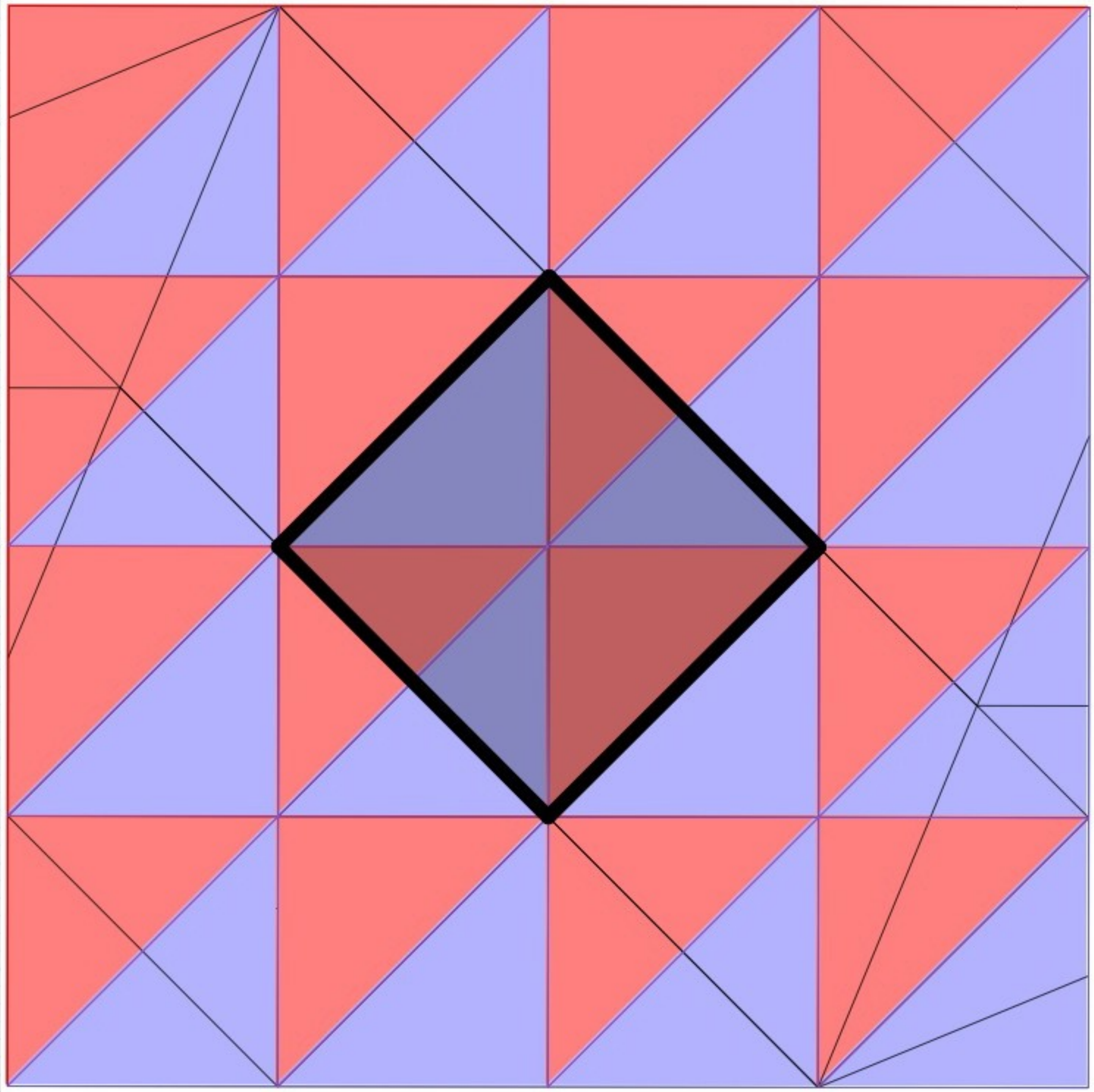
Marion Walters (paraphrased)

So, we will unfold a unit to uncover some of the mathematics.

where is the face of the cube and what is its area?
where is the length of the cube, and what is that length?

assume the original square to be 8"





length of original square	resulting length of cube	resulting area of one face of cube	resulting volume of cube
1		$\frac{1}{8}(1)^2$	
2	$\sqrt{\frac{1}{8}(2)^2}$	$\frac{1}{8}(2)^2$	$\left(\sqrt{\frac{1}{8}(2)^2}\right)^3$
3		$\frac{1}{8}(3)^2$	
4	$\sqrt{\frac{1}{8}(4)^2}$	$\frac{1}{8}(4)^2$	$\left(\sqrt{\frac{1}{8}(4)^2}\right)^3$
5		$\frac{1}{8}(5)^2$	
6	$\sqrt{\frac{1}{8}(6)^2}$	$\frac{1}{8}(6)^2$	$\left(\sqrt{\frac{1}{8}(6)^2}\right)^3$
7		$\frac{1}{8}(7)^2$	
8	$\sqrt{\frac{1}{8}(8)^2}$	$\frac{1}{8}(8)^2$	$\left(\sqrt{\frac{1}{8}(8)^2}\right)^3$
9		$\frac{1}{8}(9)^2$	
10	$\sqrt{\frac{1}{8}(10)^2}$	$\frac{1}{8}(10)^2$	$\left(\sqrt{\frac{1}{8}(10)^2}\right)^3$
x	$\sqrt{\frac{1}{8}(x)^2}$	$\frac{1}{8}(x)^2$	$\left(\sqrt{\frac{1}{8}(x)^2}\right)^3$

length of original square	resulting length of cube	resulting area of one face of cube	resulting volume of cube
1	0.354	0.125	0.044
2	0.707	0.5	0.354
3	1.061	1.125	1.193
4	1.414	2	2.828
5	1.768	3.125	5.524
6	2.121	4.5	9.546
7	2.475	6.125	15.159
8	2.828	8	22.627
9	3.182	10.125	32.218
10	3.535	12.5	44.194
x			

length of original square	volume using beans	volume in cubic inches	ratio
1			
2			
3	44	1.193	36.8818106
4	116	2.828	41.01838755
5	242	5.524	43.8088342
6	guess = 420	9.546	about 44
7	660	15.159	43.538492
8	1012	22.627	44.7253281
9			
10			
x			

other uses for this unit:

models of volume, surface area, and length

Sierpinski's Carpet in 3 dimensions

models for the Painted Cube problem

construct stellated icosahedron with 30 units, stellated octahedron with 12 units or ....

here is a stellated icosahedron-

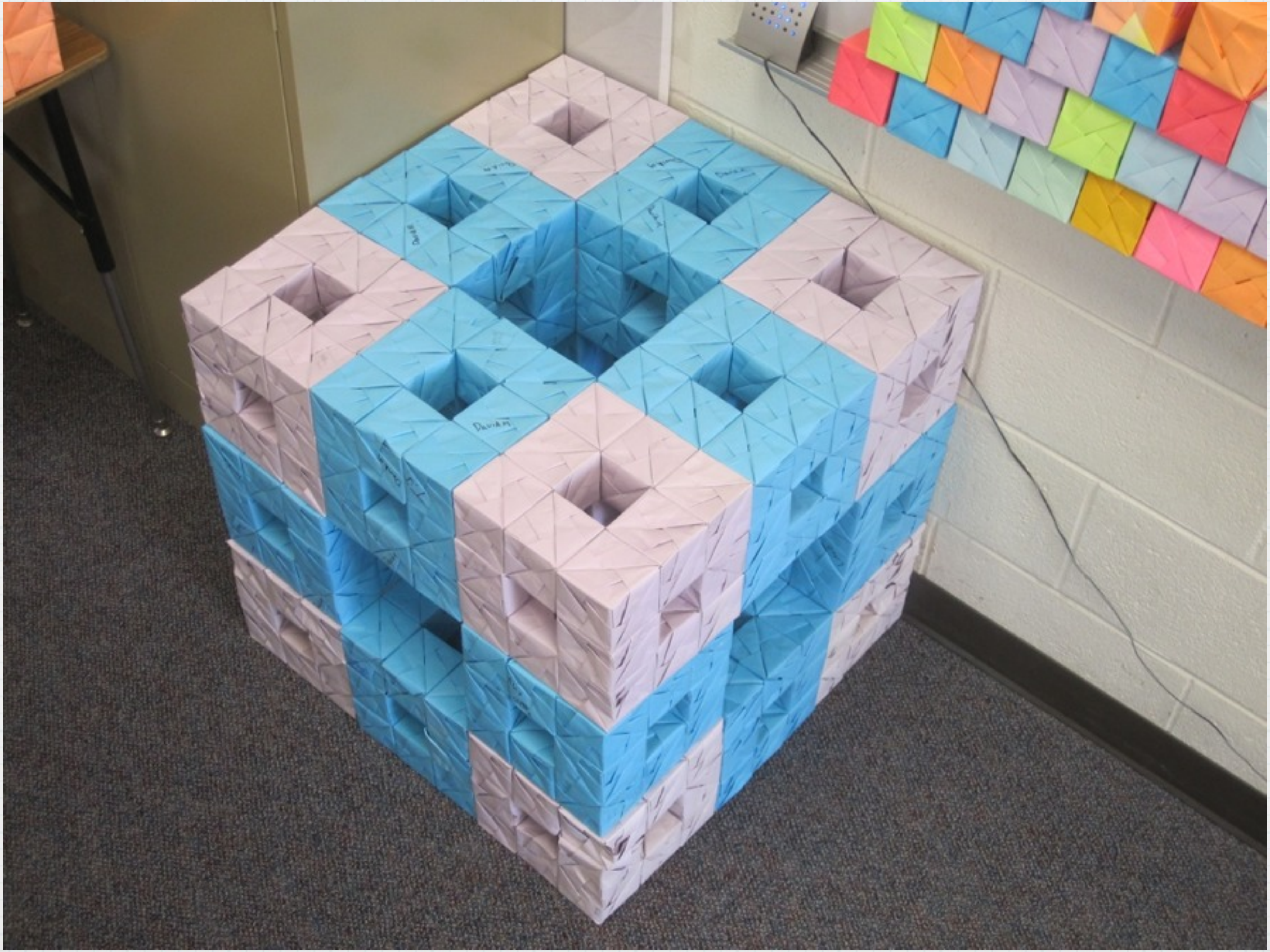
30 units are required

this is a
Buckyball,

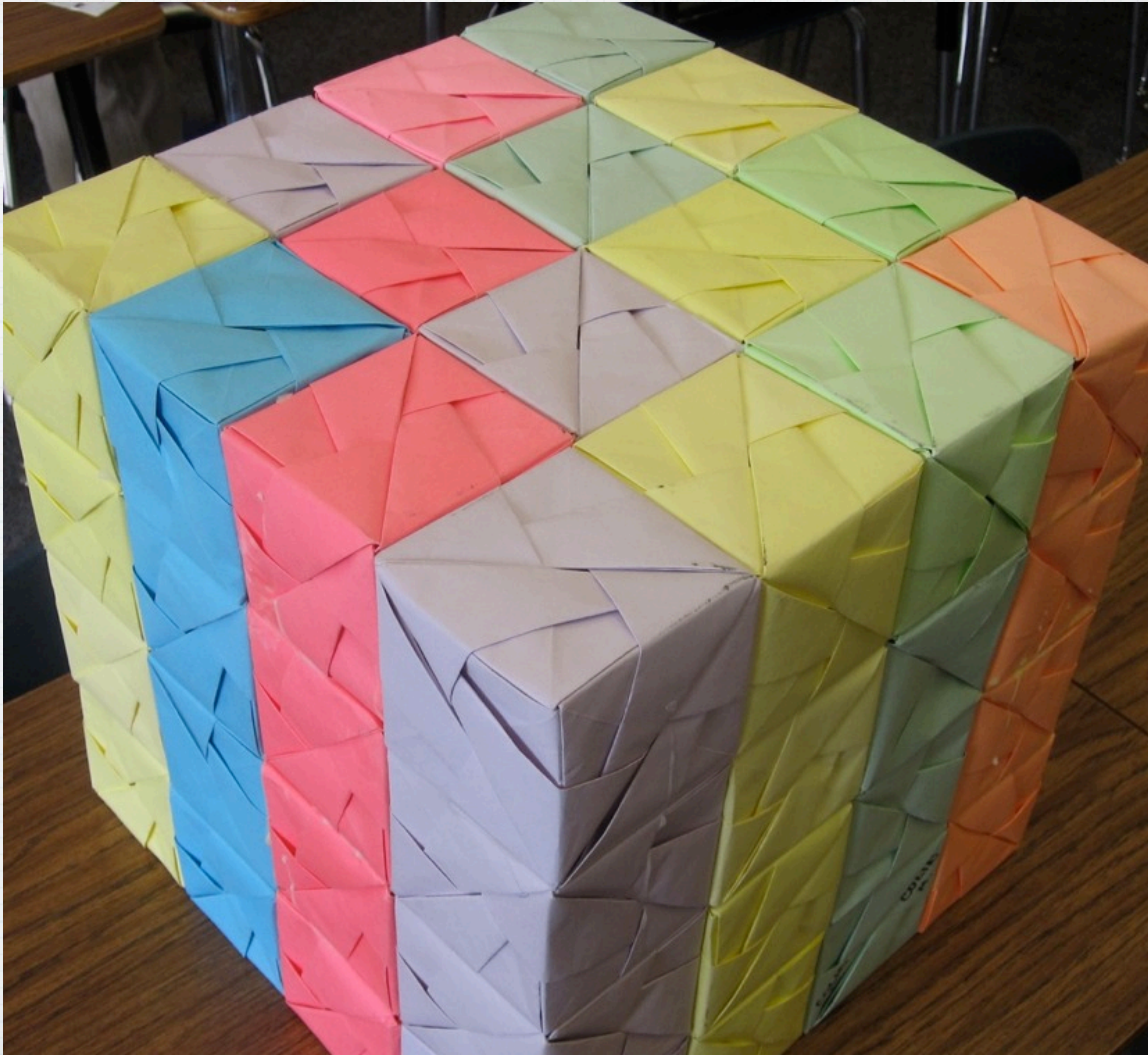
270 units

a science fair project- determining how many structures the unit can make

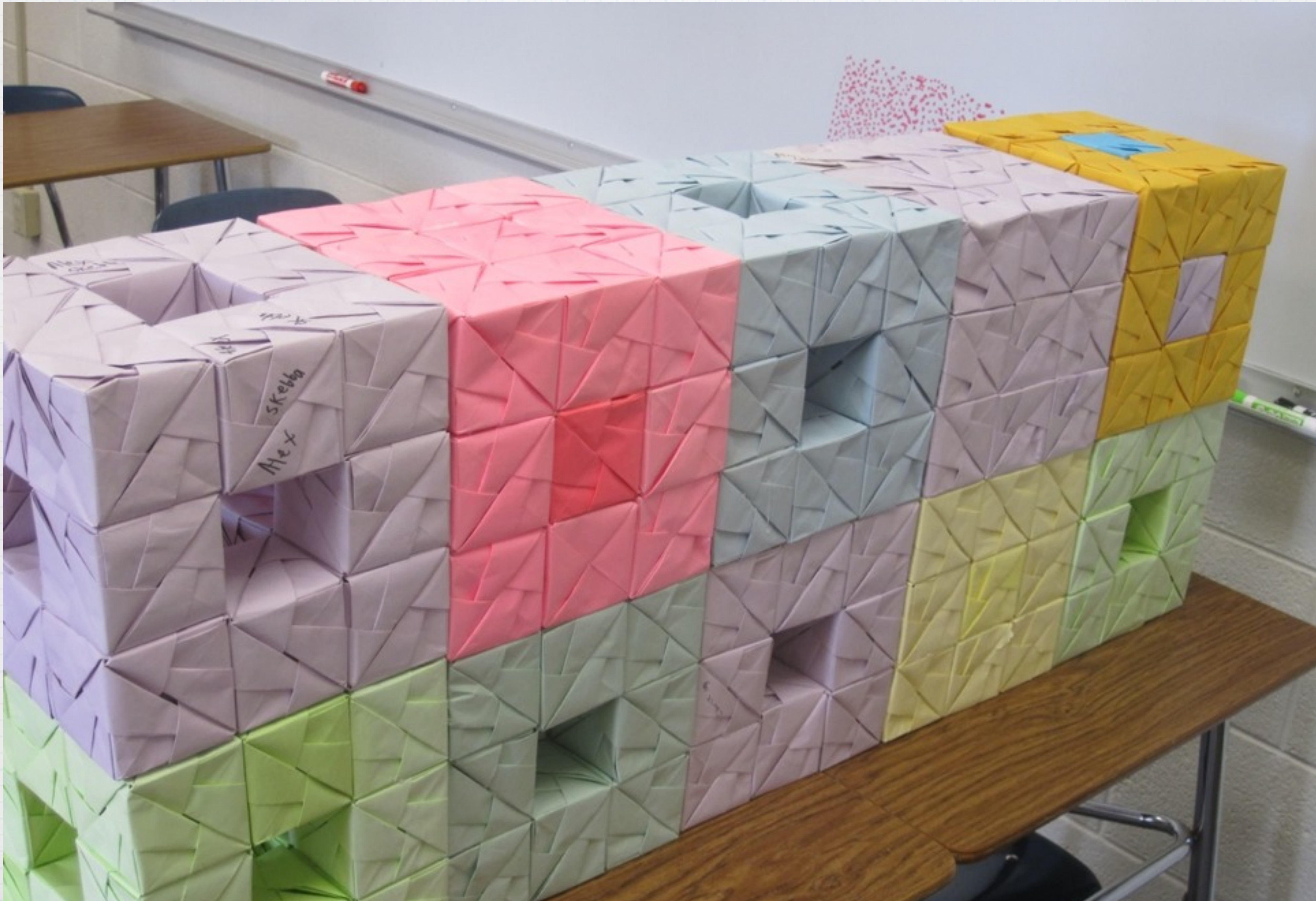
Sierpinski's carpet in 3 dimensions-



a model
for
volume



a wall of cubes!



sources that would be helpful:

handout: this keynote is available in pdf form at

http://piman1.wikispaces.com

the other resources that would be very helpful are the two books

Unit Origami, Tomoko Fuse

Unfolding Mathematics using Unit Origami, Key Curriculum Press

geogebra.org