Targeted Connections: Making the Drop – Parachutes and Mathematical Modeling

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Standards for Mathematical Practice

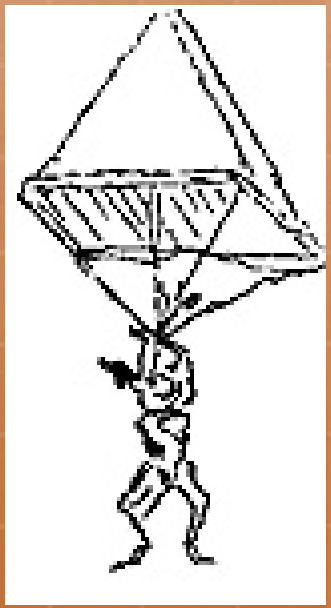
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Scientific and Engineering Practices

- Asking questions and defining problems
- Planning and carrying out investigations
- Analyzing and interpreting data
- Using mathematics and computational thinking
- Constructing explanations / designing solutions
- Engaging in argument from evidence
- Obtaining, evaluating and communicating information

Sketch by Leonardo DaVinci

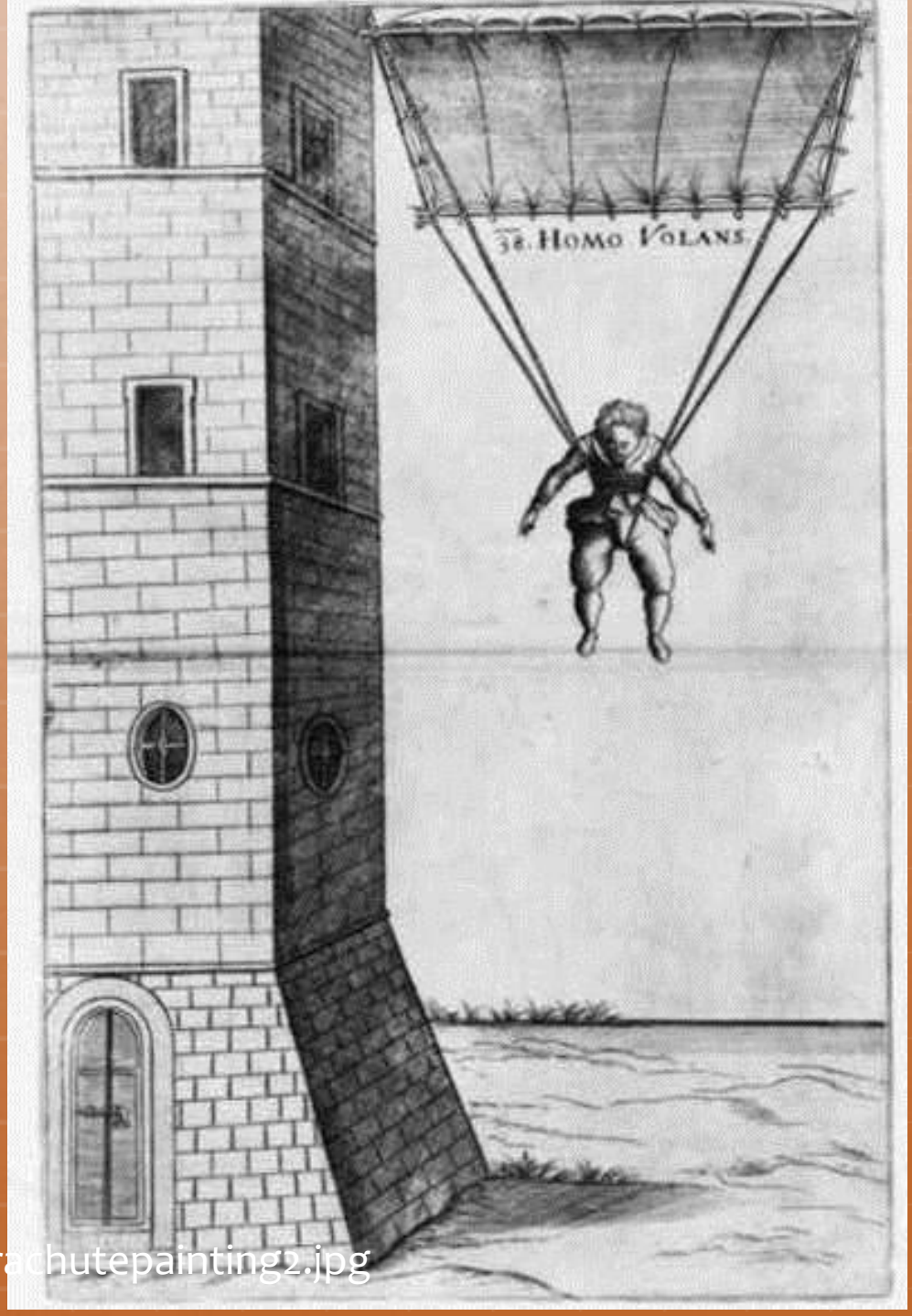
- Imagined parachute
- 1452 - 1519



http://www.parachutehistory.com/eng/drs.html

Faust Vrancic or Veranzio

- Jump in 1617
- Croation
- Rigid design based on DaVinci
- *Machinae Novae*
- The Parachute was called the Homo Volans



http://0.tqn.com/d/inventors/1/5/O/0/1/parachutepainting2.jpg

Veranzio's parachute in Šibenik's Wells Museum

http://upload.wikimedia.org/wikipedia/commons/9/9f/Wells_museum_in_sibenik_2.JPG

Andrew Garnerin

- Jump in 1797
- First recorded jump with a parachute without a rigid frame. He jumped from a hot air balloon.

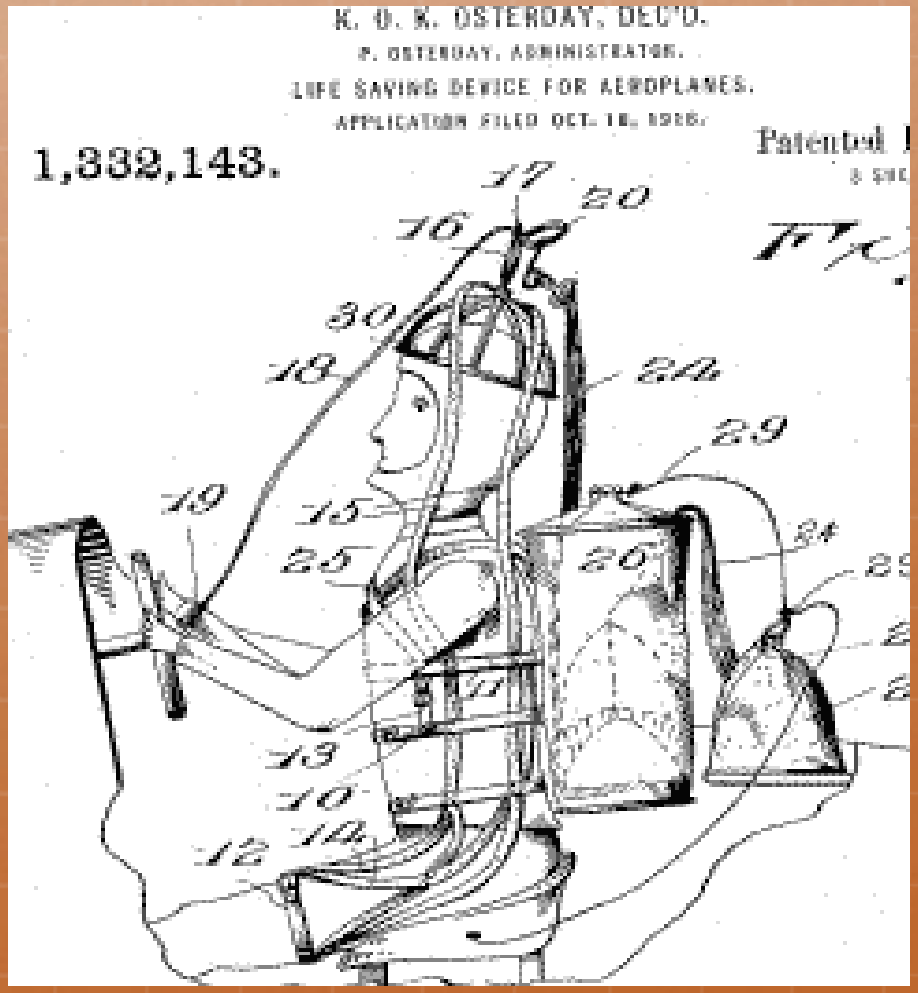
http://0.tqn.com/d/inventors/1/0/M/0/1/parachutepainting.jpg

Andrew Garnerin

- Three Views of Andrew Garnerin's Parachute -1790s
- *LOC: Tissandier Collection*

- **http://www.spartacus.schoolnet.co.uk/AVgarnerin.htm**

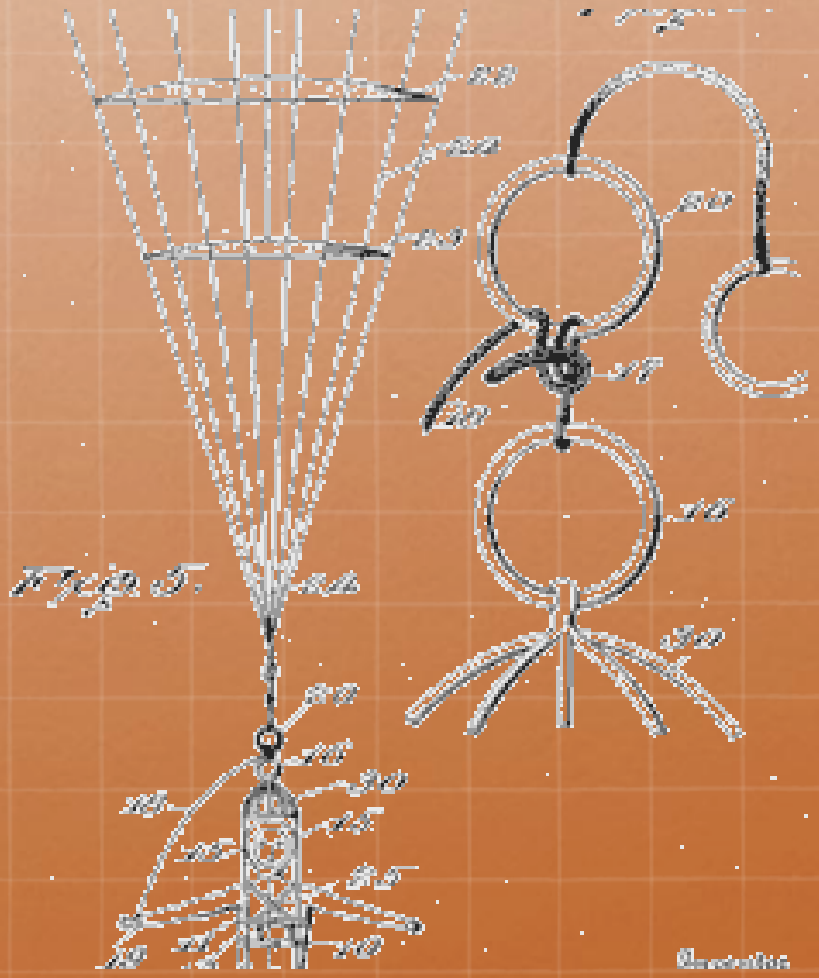
In 1887, Captain Thomas Baldwin invented the first parachute harness.



1920 Parachute Design

Design Change

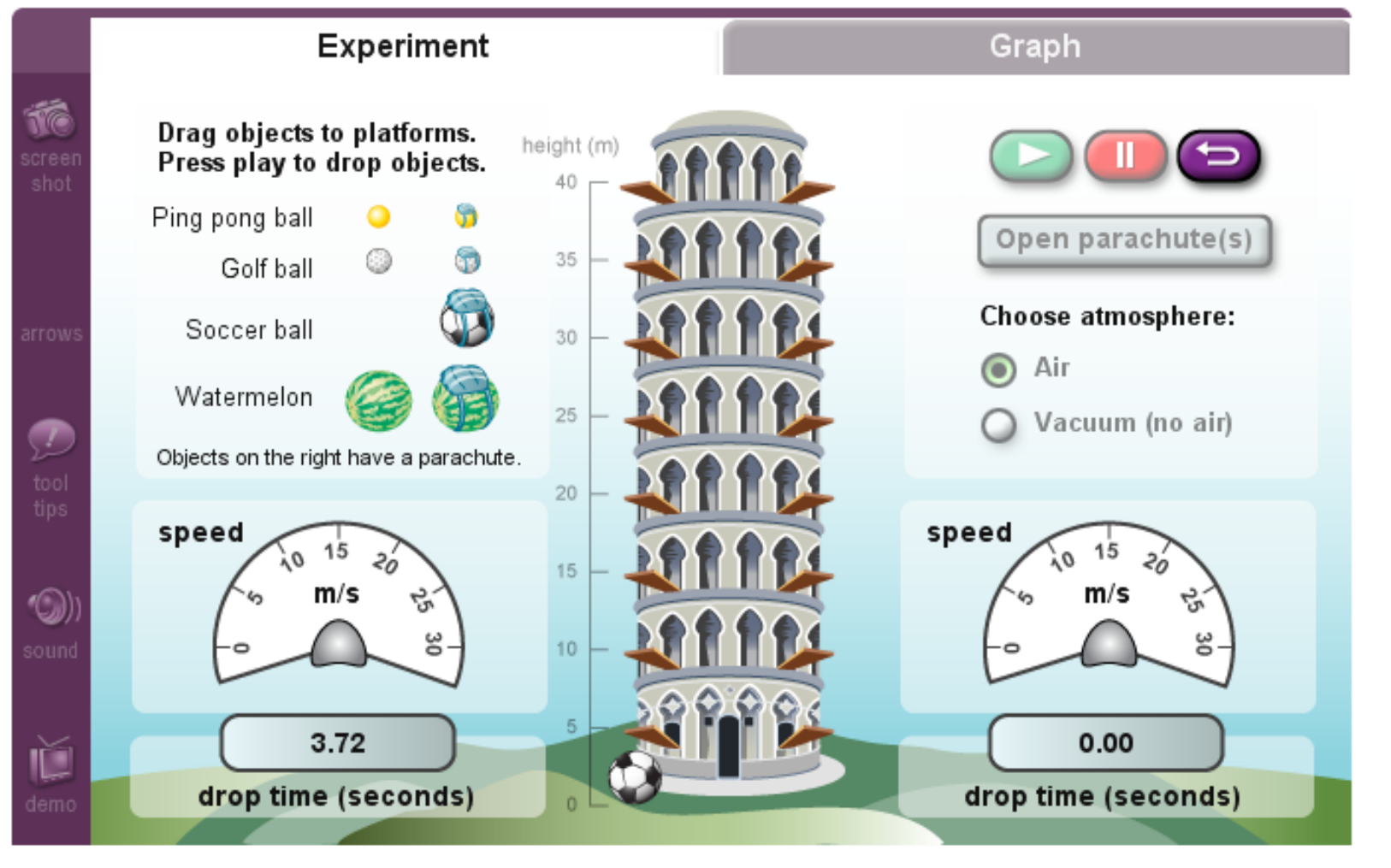
- 1920 Design



A C-17 conducts the first humanitarian aid supply drop during the 2010 Haiti earthquake

http://en.wikipedia.org/wiki/Airdrop

Free Fall Tower



Explorelearning

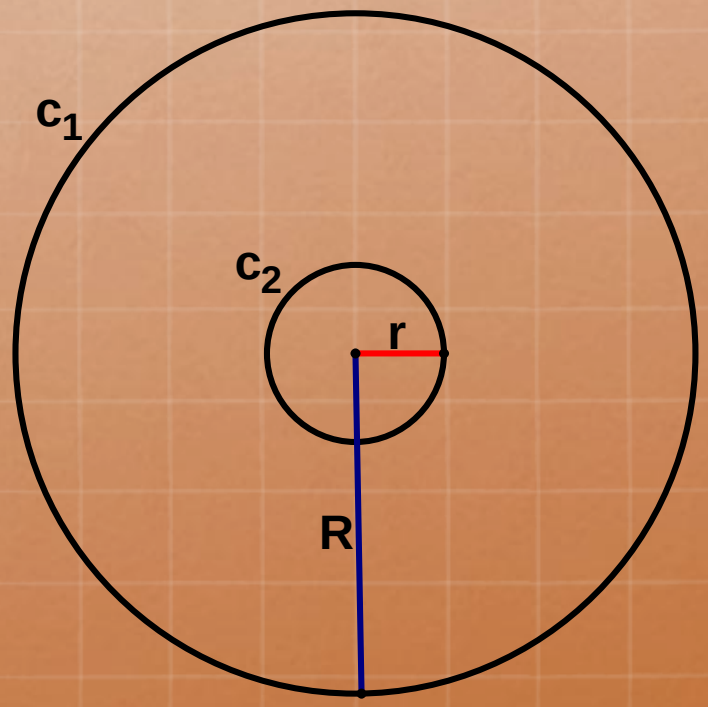
Explorations with Parachutes

- Playing With Parachutes
- Parachute Drop
- Is it surface area or shape?
- Parachutes, Circles and Pi….Oh My

Other Parachute Materials

- Parachutes: Boston Museum
- Barbie Bungee

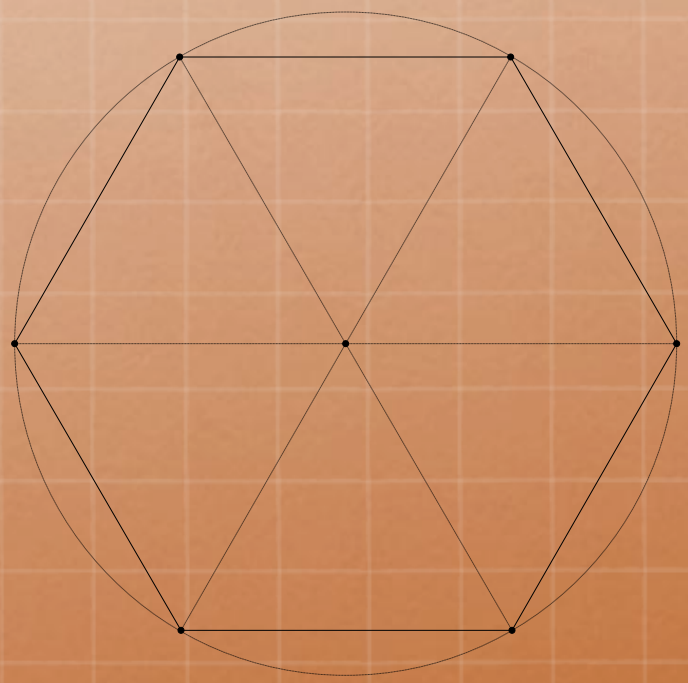
Circular Parachutes

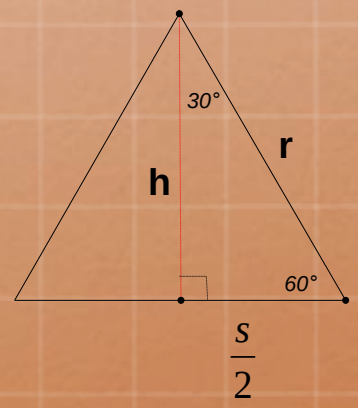


Area of Parachute = Area of c_1 *- Area of* c_2

Chute area = $\pi R^2 - \pi r^2$

Area of a regular Hexagon

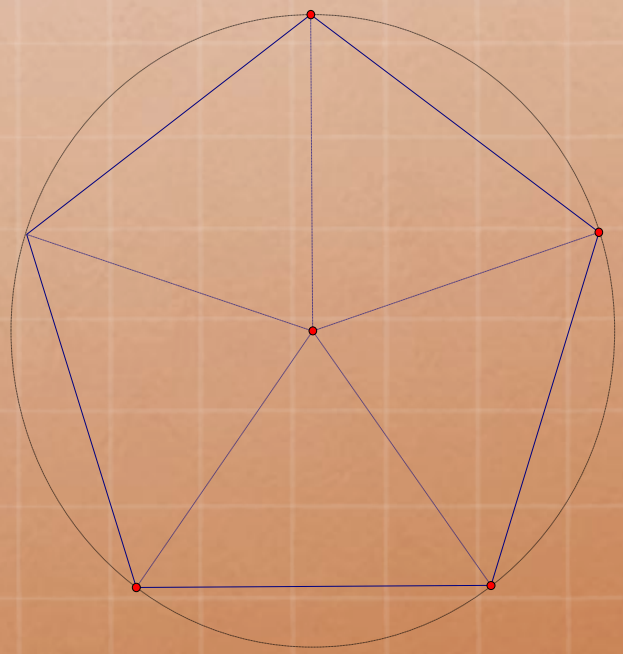


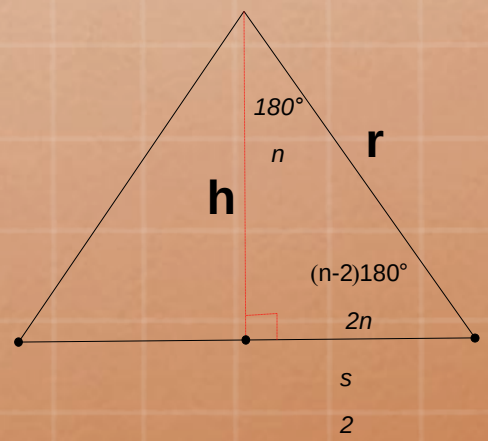


$$h = \frac{\sqrt{3}}{2}$$

$$A = \left(\frac{s}{2}\right)\left(\frac{s\sqrt{3}}{2}\right)$$

Area of a regular polygon





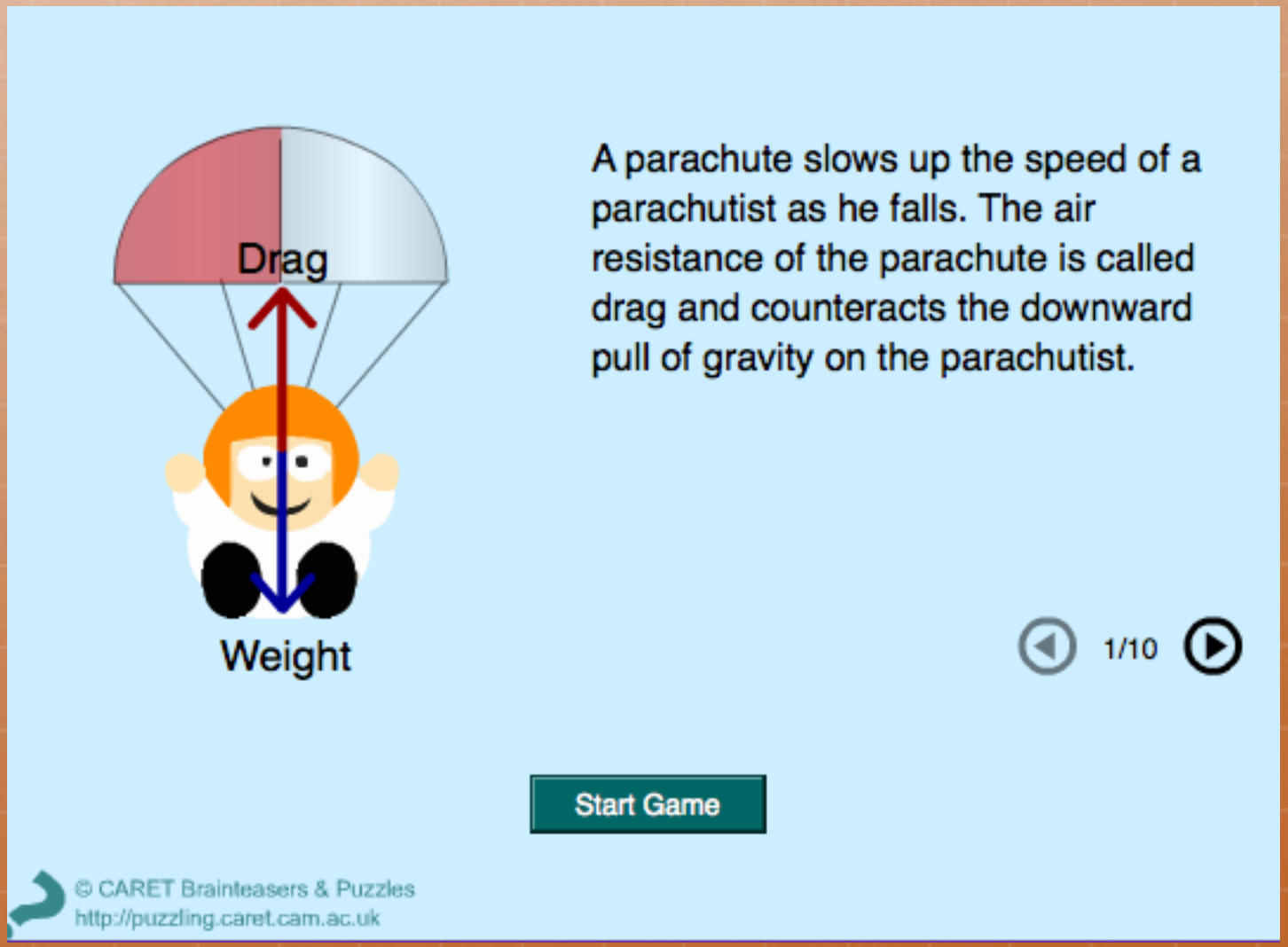
$$A = \frac{1}{2}bh = \frac{s}{2}h$$

$$\sin\left(\frac{180°}{n}\right) = \left(\frac{\frac{s}{2}}{r}\right) \quad OR \quad \frac{s}{2} = r\left(\sin\frac{180°}{n}\right)$$

$$\cos\left(\frac{180°}{n}\right) = \left(\frac{h}{r}\right) \quad OR \quad \frac{h}{2} = r\left(\cos\frac{180°}{n}\right)$$

$$A = r^2 \sin\left(\frac{180°}{n}\right)\cos\left(\frac{180°}{n}\right)$$

Ranger Danger Dan



http://puzzling.caret.cam.ac.uk/game.php?game=parachute

Ranger Danger Dan data

Relationship between area and Terminal Velocity					
Canopy Area	Hole Area	Total Area	Terminal Velocity	Ratio of area	Ratio of velocities
4.91	0.2	4.71	6.51		
9.08	0.71	8.37	4.88	16/9	3/4
11.04	0.44	10.6	4.34	9/4	2/3
19.24	0.4	18.84	3.25	4	1/2
43.59	1.2	42.39	2.17	9	1/3

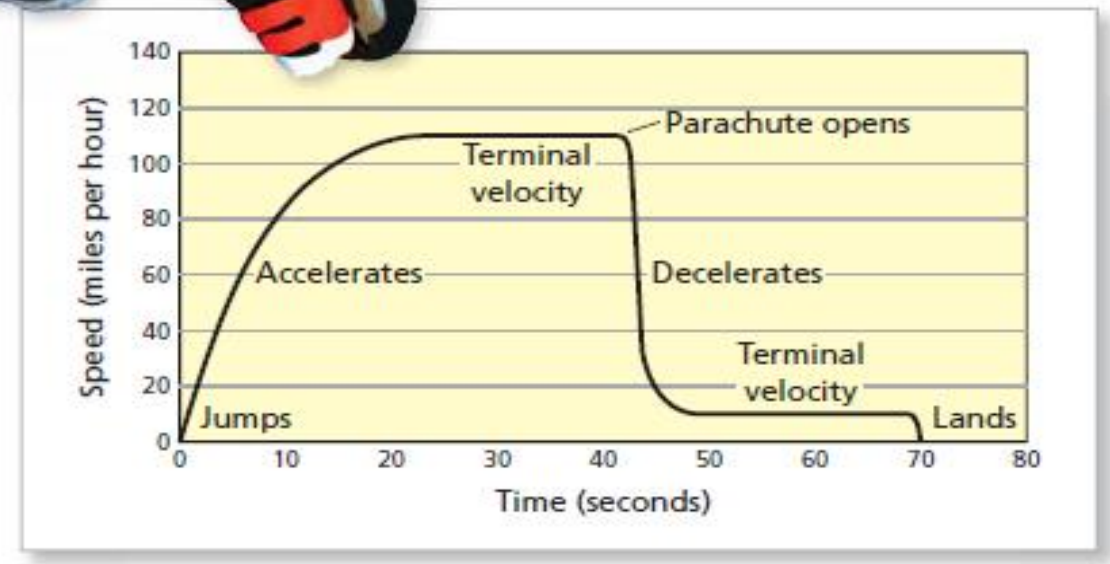
Math and You

When an object falls, it accelerates. As its speed increases, the air resistance increases. Eventually, the air resistance will offset the weight of the object, and the object will reach "terminal velocity."

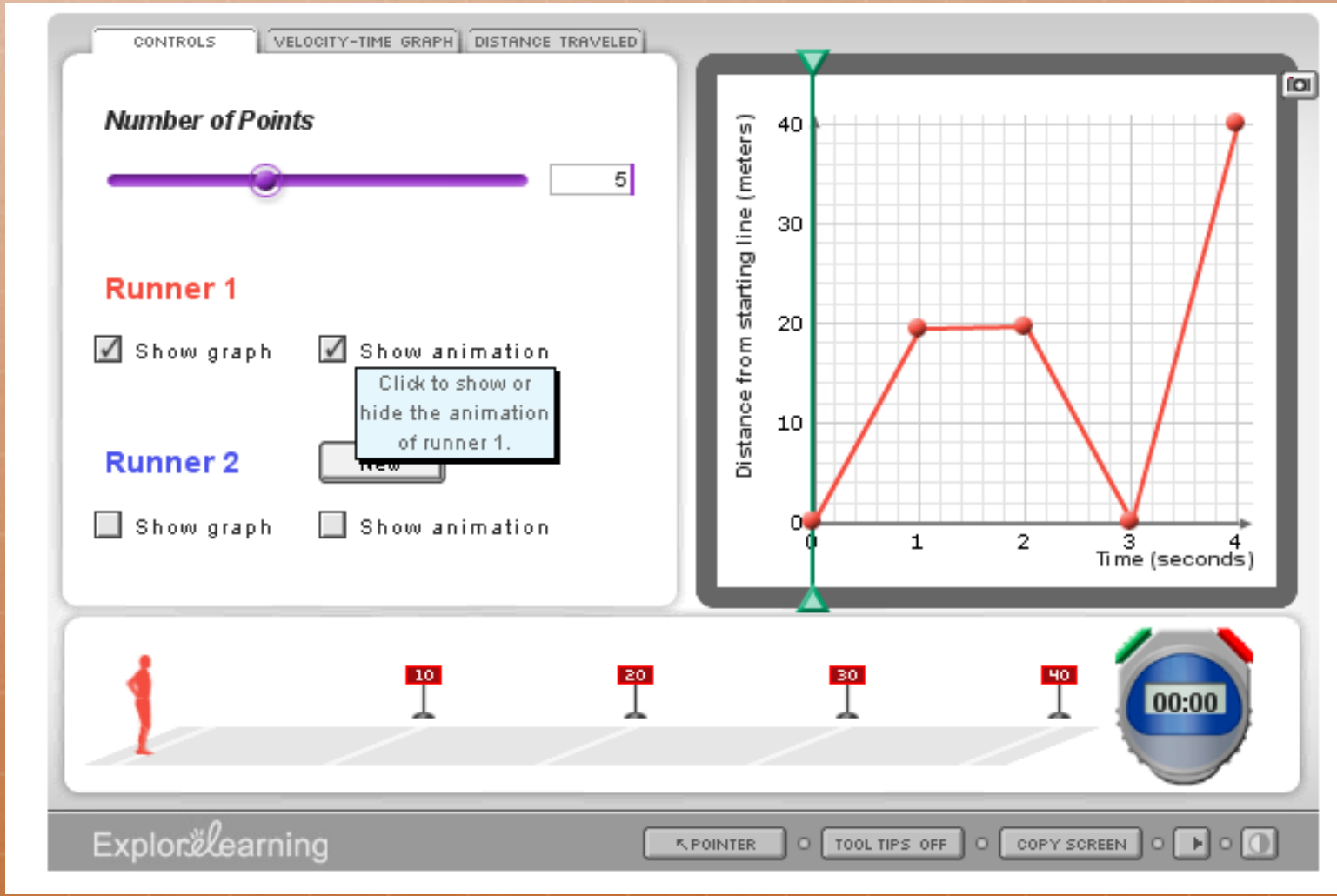
Use the graph to find each velocity in *feet per second*.

a. Terminal velocity *before* the parachute opens—during free fall

b. Terminal velocity *after* the parachute opens



Interpret Graphs



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Parachute Drag

Parachute Drag

Introduction	Directions	Questions	Activities	Enable Comparative Graphs	?
RUN!	Restart Model	Back to Main	Links	Enable Tabular Display	?

Inputs:

? Initial Displacement (500-1500 m):
1000

? Mass (50-100 kg):
75

? Cross-Sectional Area (0.5-10 m^2):
7

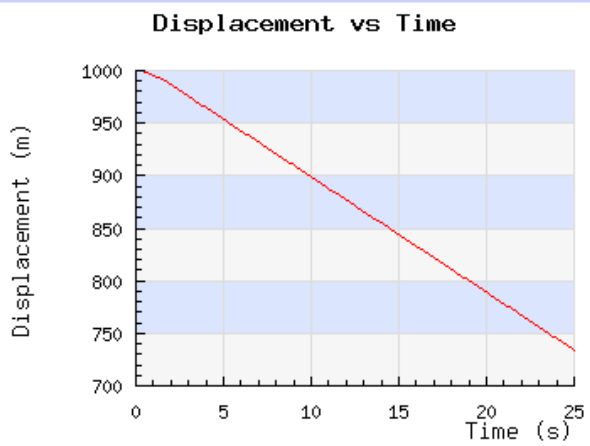
? Drag Coefficient (1-2):
1.5

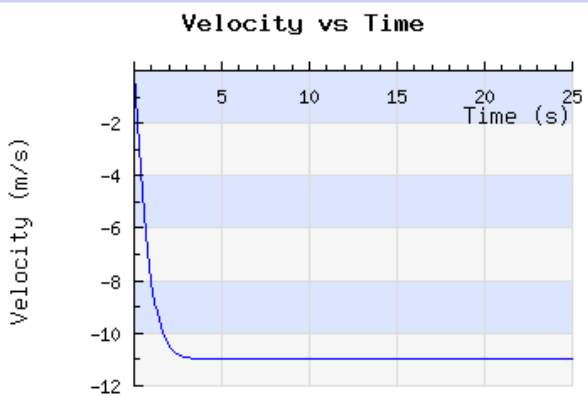
? Time til Chute Opens (0-25 s):
0

Messages:

Results are displayed below.

Results:



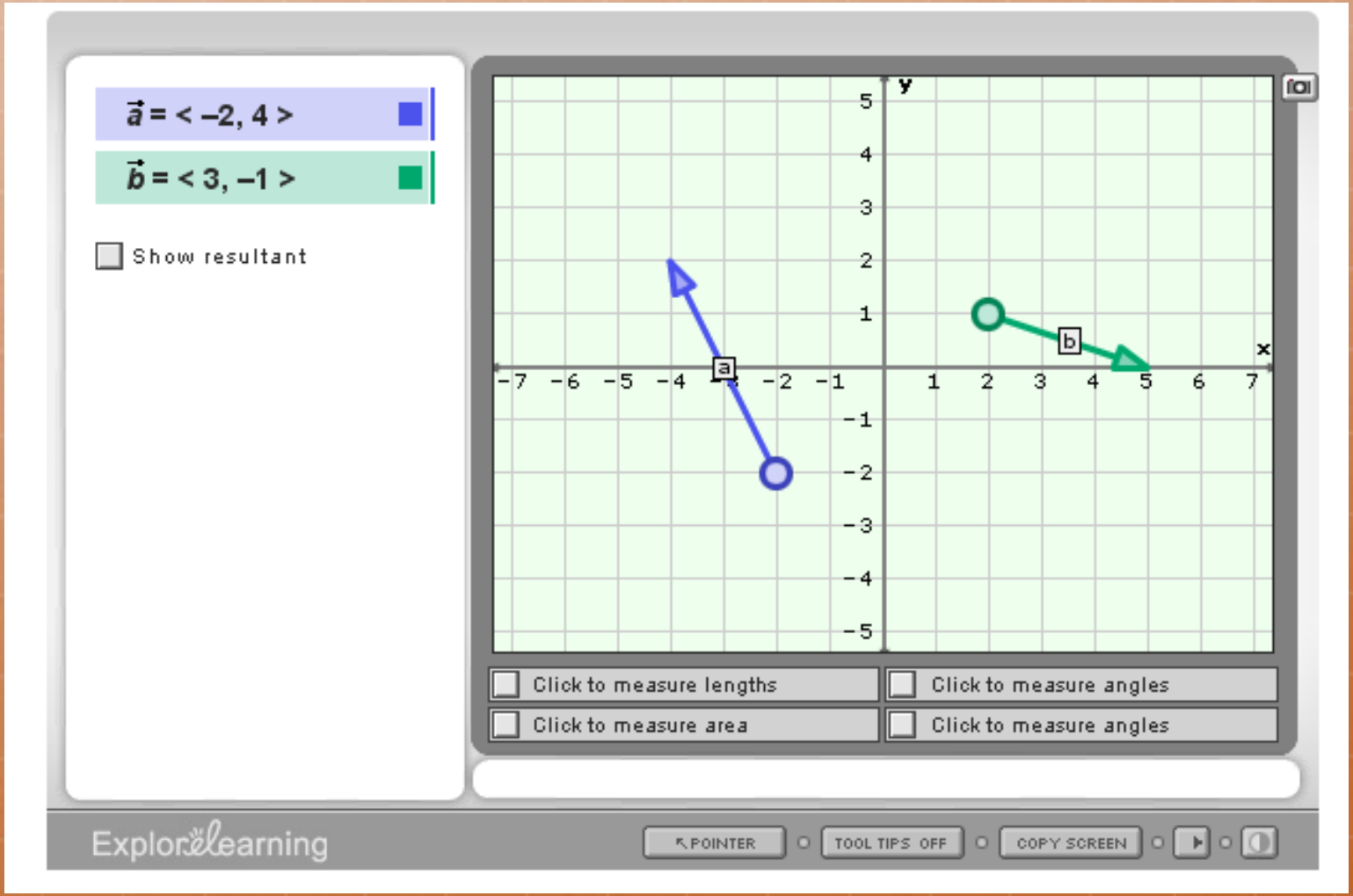


Drag Force vs Time

Drag to Weight Ratio vs Time

http://mvhs.shodor.org/mvhsproj/drag/drag.html

Adding Vectors



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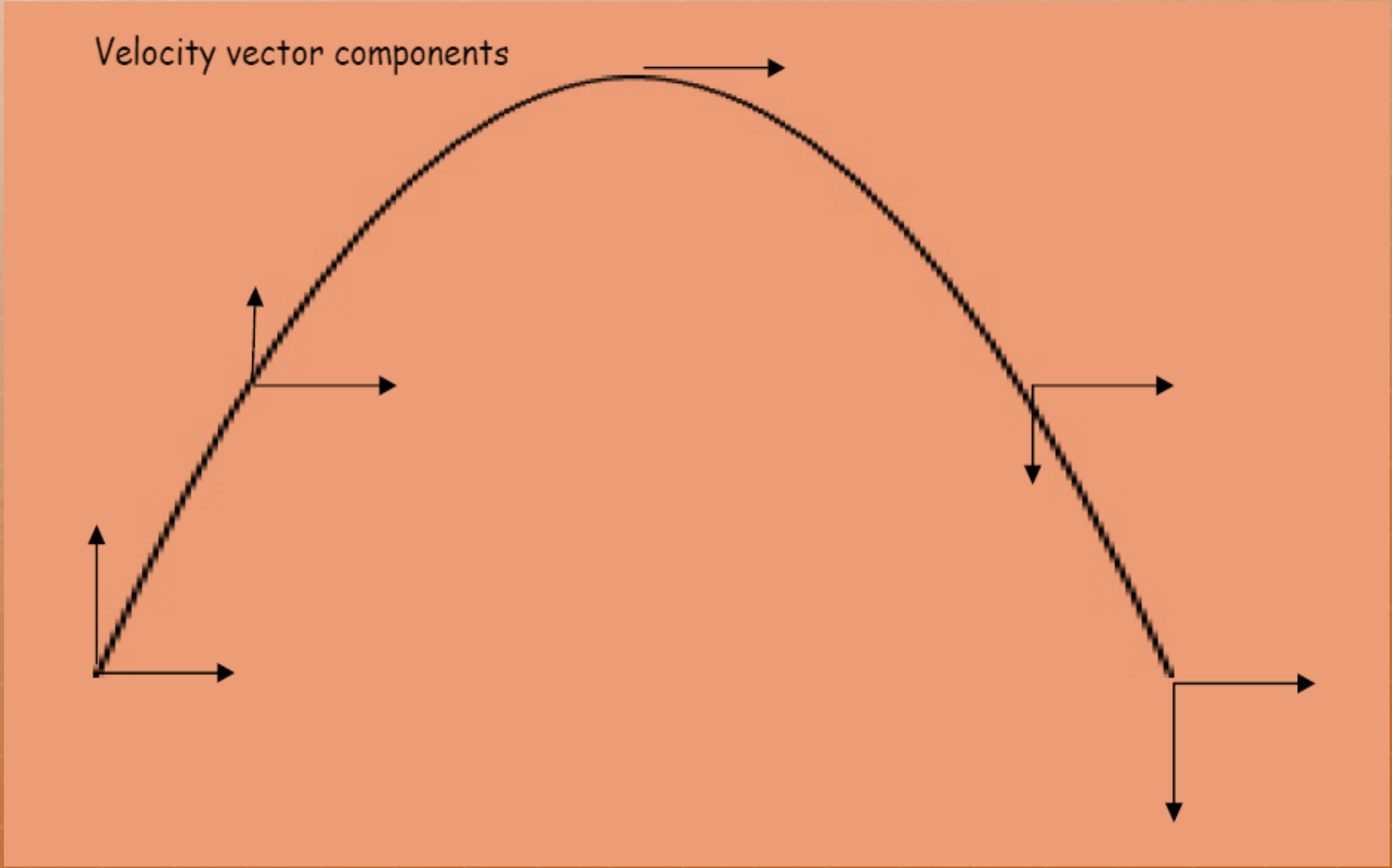
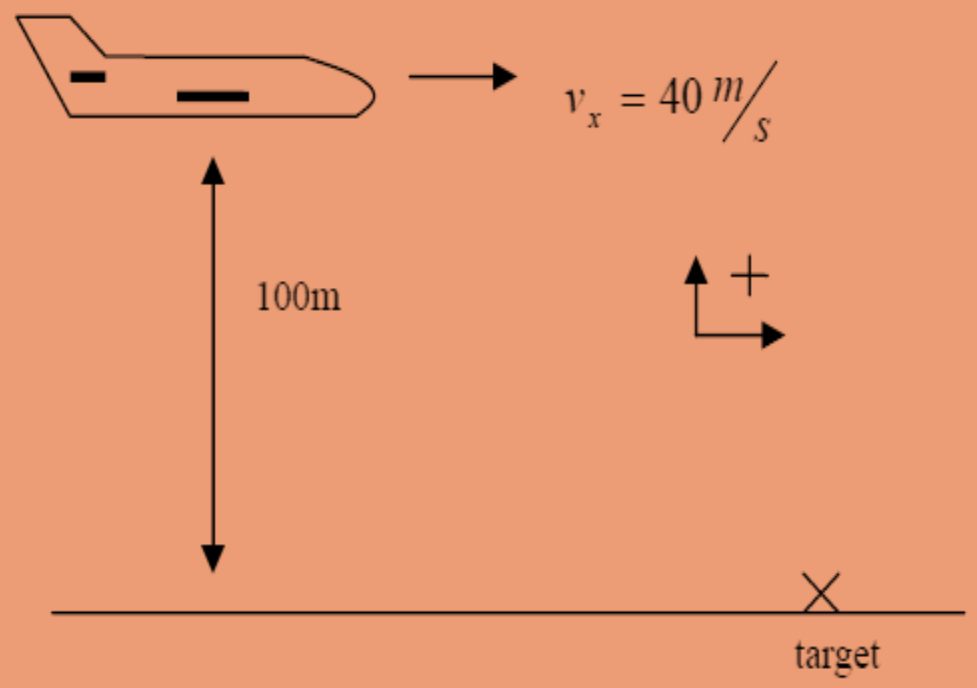
Motion in a Plane/Parabolic Motion



Motion in a Plane/Parabolic Motion

Consider a supply airplane attempting to airdrop a box onto a target marked on the ground as shown below.



$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2$$

$$-100m = 0 - (4.9m \cdot s^{-2})t^2$$

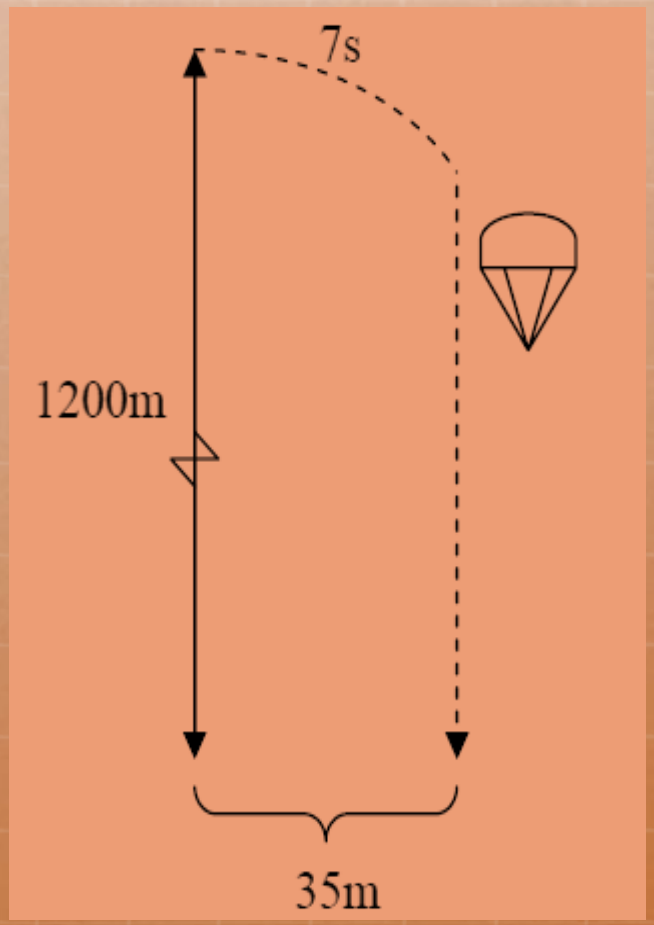
$$t = 4.5s$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x - x_0 = (+40m \cdot s^{-1})(4.5s) + 0$$

$$x - x_0 = 180m$$

Motion in a Plane/Parabolic Motion



Given: $y - y_0 = -1200m$

$a = -9.8 m/s^2$

$t = 7s$

$$y - y_0 = v_{0y}t + \frac{1}{2}at^2$$

$$y - y_0 = (-4.9m/s^2)(7s)^2$$

$$y - y_0 = -240m$$

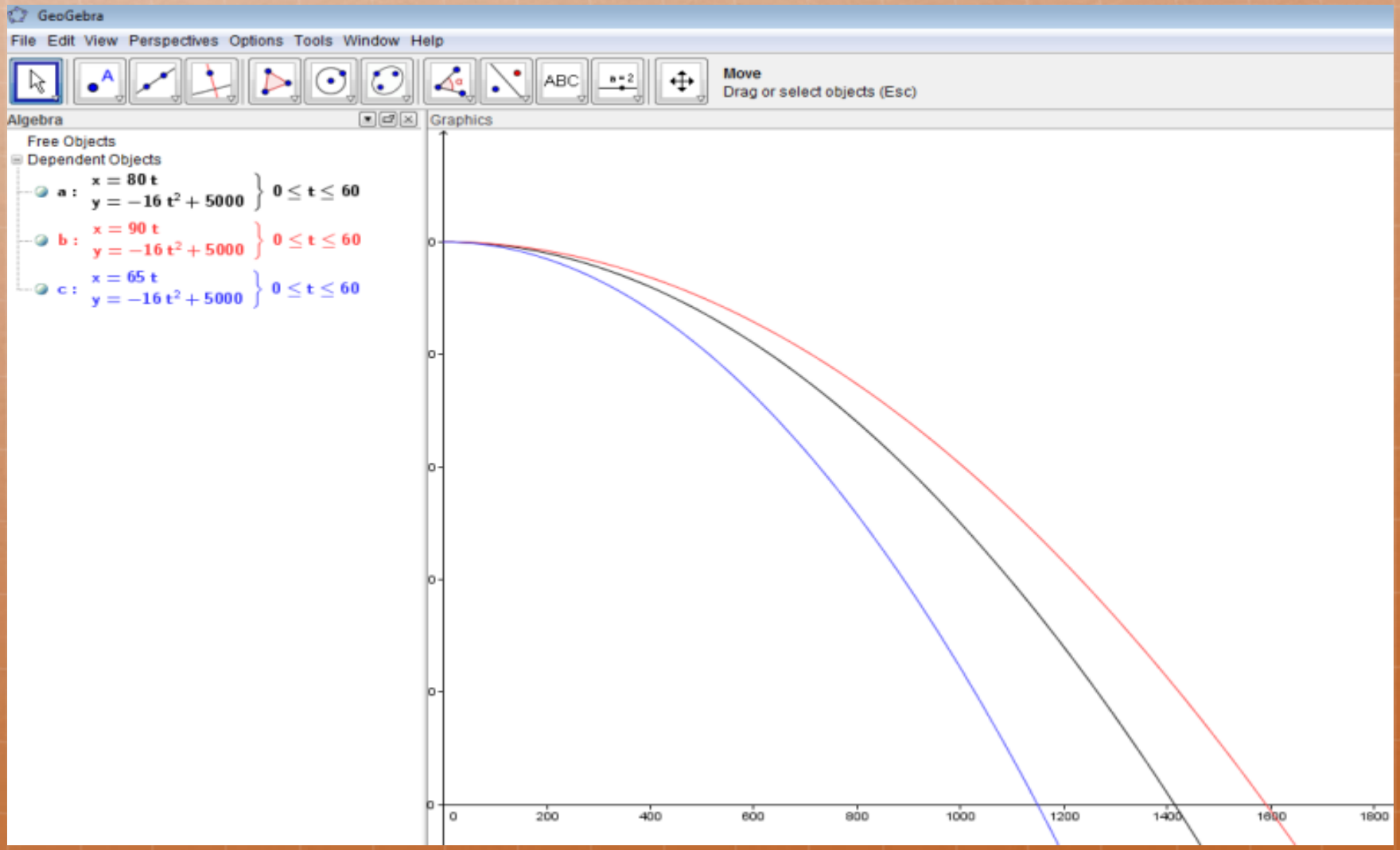
Since $v_x = v_{0x}$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

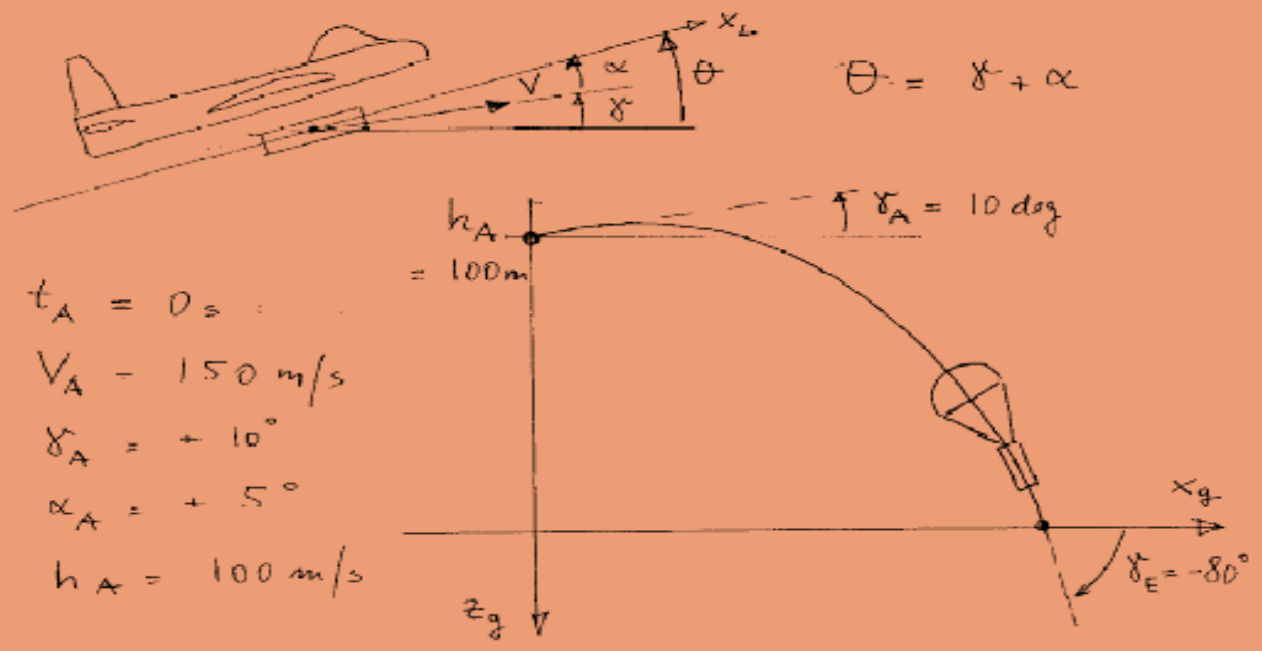
$$= (5m/s)(7s)$$

$$= 35m$$

Explore effect of wind Parametric Equations

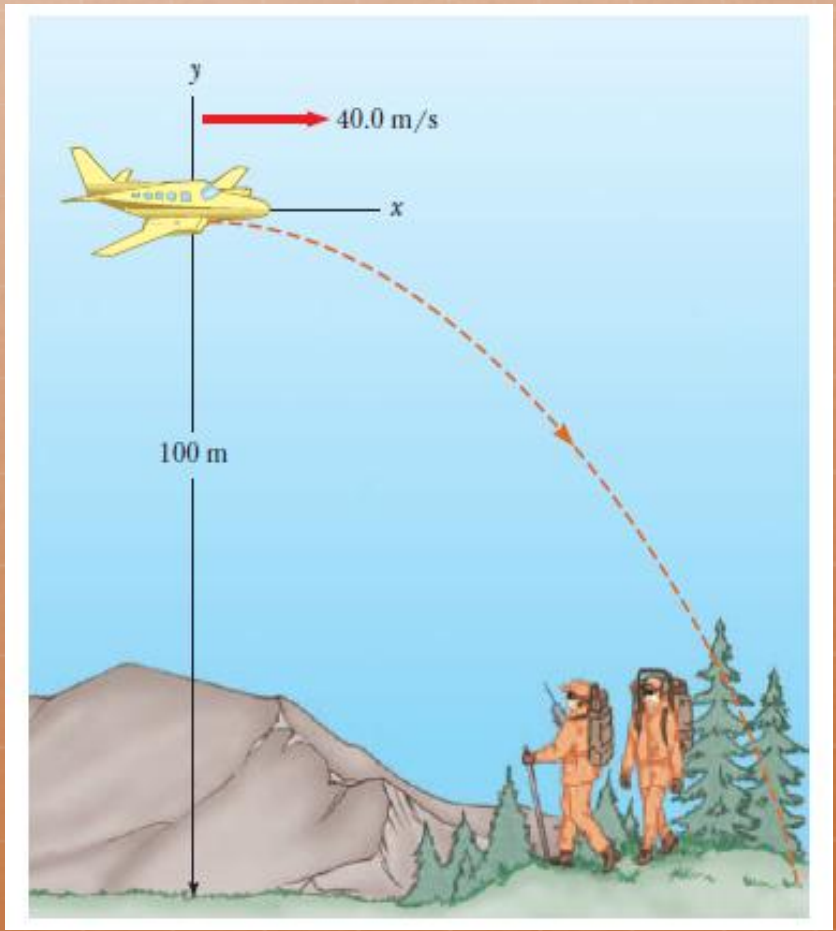


Parachute Flight Dynamics and Trajectory Simulation



Study Case : Example

Vectors and Two-Dimensional Motion



$$\Delta y = y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$y = -(4.90\ \text{m/s}^2)t^2 = -1.00 \times 10^2\ \text{m}$$

$$t = 4.52\ \text{s}$$

$$\Delta x = x - x_0 = v_{0x}t$$

$$x = (40.0\ \text{m/s})(4.52\ \text{s}) = \boxed{181\ \text{m}}$$

$$v_x = v_0 \cos\theta = (40.0\ \text{m/s}) \cos 0° = \boxed{40.0\ \text{m/s}}$$

$$v_y = v_0 \sin\theta - gt = 0 - (9.80\ \text{m/s}^2)(4.52\ \text{s})$$
$$= \boxed{-44.3\ \text{m/s}}$$

Web Links:

- Free Fall Tower: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=650
- Playing With Parachutes: http://www.tryengineering.org/lessons/playingwithparachutes.pdf
- ParachuteDrop: http://www.nationalmuseum.af.mil/shared/media/document/AFD-090709-082.pdf
- Is it Surface Area or Shape? http://www.nationalmuseum.af.mil/shared/media/document/AFD-090710-044.pdf
- Parachutes, Circles or Pi….Oh My: http://www.nationalmuseum.af.mil/shared/media/document/AFD-090710-041.pdf

Web Links:

- Parachutes, Boston Museum: http://www.mos.org/eie/parachutes.php
- Barbie Bungee: http://illuminations.nctm.org/LessonDetail.aspx?id=L646
- Ranger Danger Dan: http://puzzling.caret.cam.ac.uk/game.php?game=parachute
- Math and You: http://www.math.andyou.com/pdf/37.pdf
- Interpret Graphs: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=626&ClassID=0
- Parachute Drag: http://mvhs.shodor.org/coresims/drag/index.php?disptab=&save=0&disp=1&displacement=1000&mass=75&tto=0&area=7&cd=1.5&msg=Results%20are%20displayed%20below.

Web Links:

- Adding Vectors:http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=171
- Motion in a Plance/Parabolic Motion: http://www.physics.isu.edu/~hackmart/motinplane.pdf
- Parachute Flight Dynamics and Trajectory Simulation: http://www.mrc.uidaho.edu/~atkinson/IPPW-3/Parachute%20Course%20Material/10%20Parachute%20Flight%20Dynamics%20and%20Trajectory%20Simulation%20Doher.pdf
- Vectors and Two-Dinensional Motion: http://www.ph.utexas.edu/~turner/classes/302K/SerwayFaughnText/SF_Chapters/SF_Chapter3.pdf

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