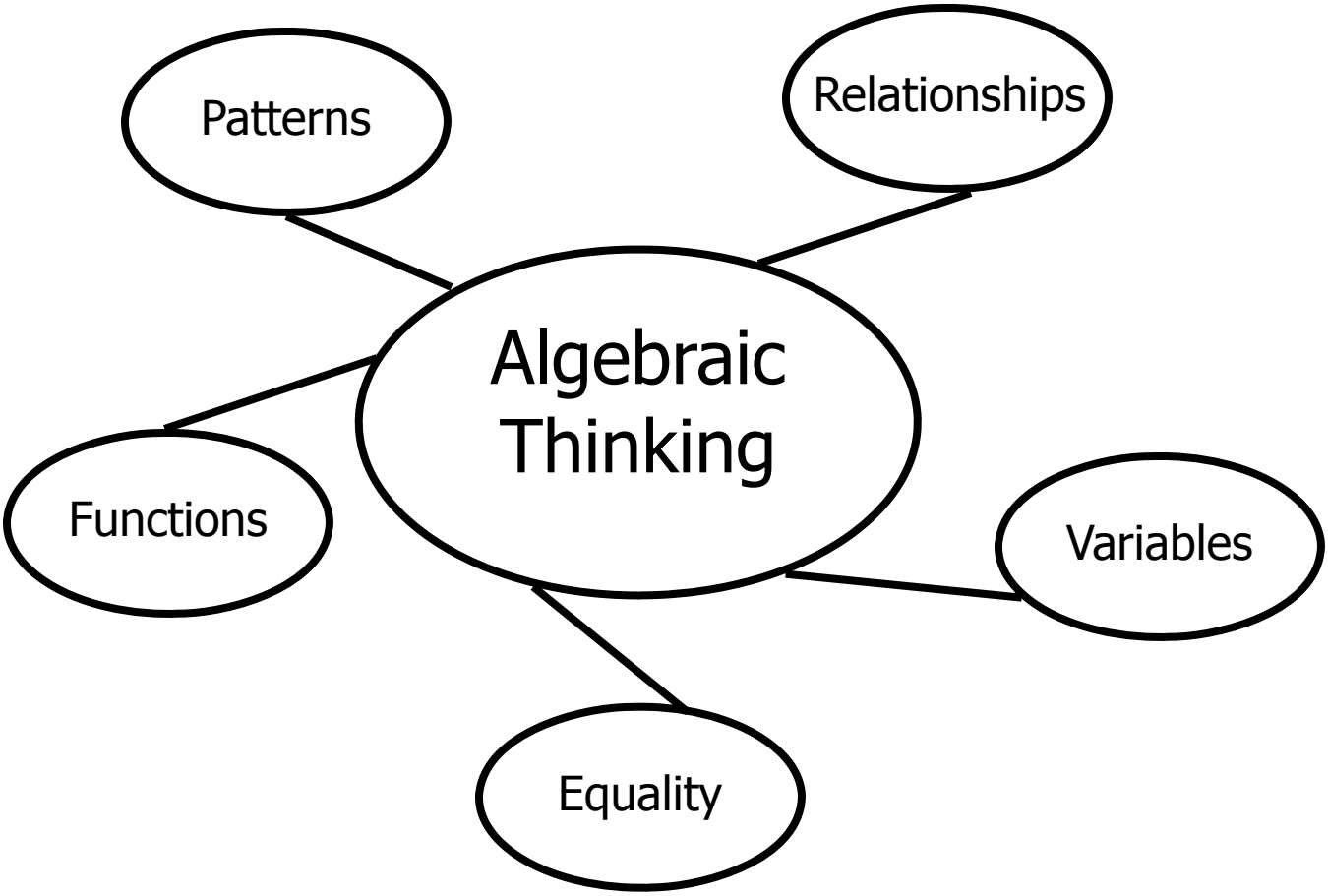
Algebra:
The Missing Variable in Elementary School Mathematics



Kimberly Bender
Kimberlyj_Bender@ccpsnet.net
Victoria Bohidar
Victoria_Bohidar@ccpsnet.net
Kathryn Munson
Kathryn_Munson@ccpsnet.net
Chesterfield County Public Schools

Part-Part-Whole Relationships

It's in the Bag!

Materials: Bags and counters (all one color)

Directions:

Partner One: Fills the bag with counters.

Partner Two: Takes some counters out of the bag and tells the amount taken. For example, "There were 10 counters in the bag and I took out 3 counters. How many counters are in the bag?"

Both partners state how many counters they think are in the bag and then they check and see if they are correct. Partners can record the equation 10-3=7. Partners take turns and repeat process.

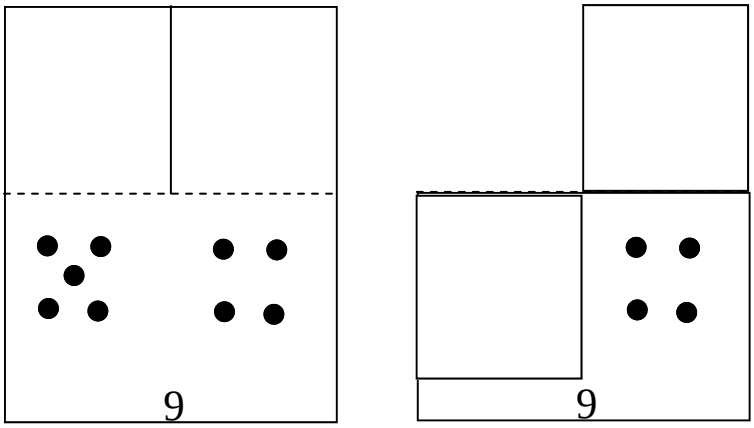
Variation: Use clear bags for students who need extra support.

Interactive Flash Cards

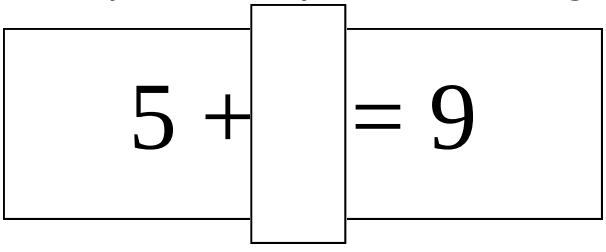
Materials: construction paper, bingo markers, flash cards, sentence strips

Directions:

Create flash cards as shown below so that students can visually work on part-part whole relationships.



Variation: Use store bought flash cards and a paper loop to create missing addend flash cards. Create your own flash cards using sentence strips and use a post-it note or paper loop to create your own missing addend flashcards.



Part-Part-Whole Mats

Materials: Part-Part-Whole Mats, manipulatives

Directions:

Students can move manipulatives on a part-part-whole mat to build an understanding of number combinations. While part-part-whole mats can be used to model any type of problem, they are especially helpful in allowing students access problems where the start number is unknown or the change is unknown.

- Sandra had 12 pennies. She gave some to George. Now she has 8 pennies. How many did she give to George?
- Sandra had some pennies. George gave her 4 more. Now Sandra has 12 pennies. How many pennies did Sandra have to begin with?

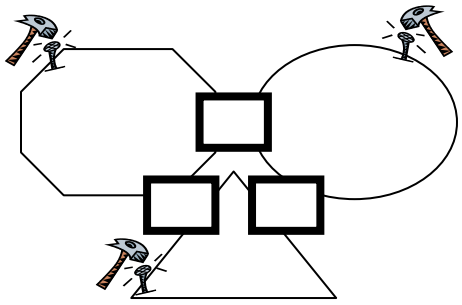
Number Construction Zone

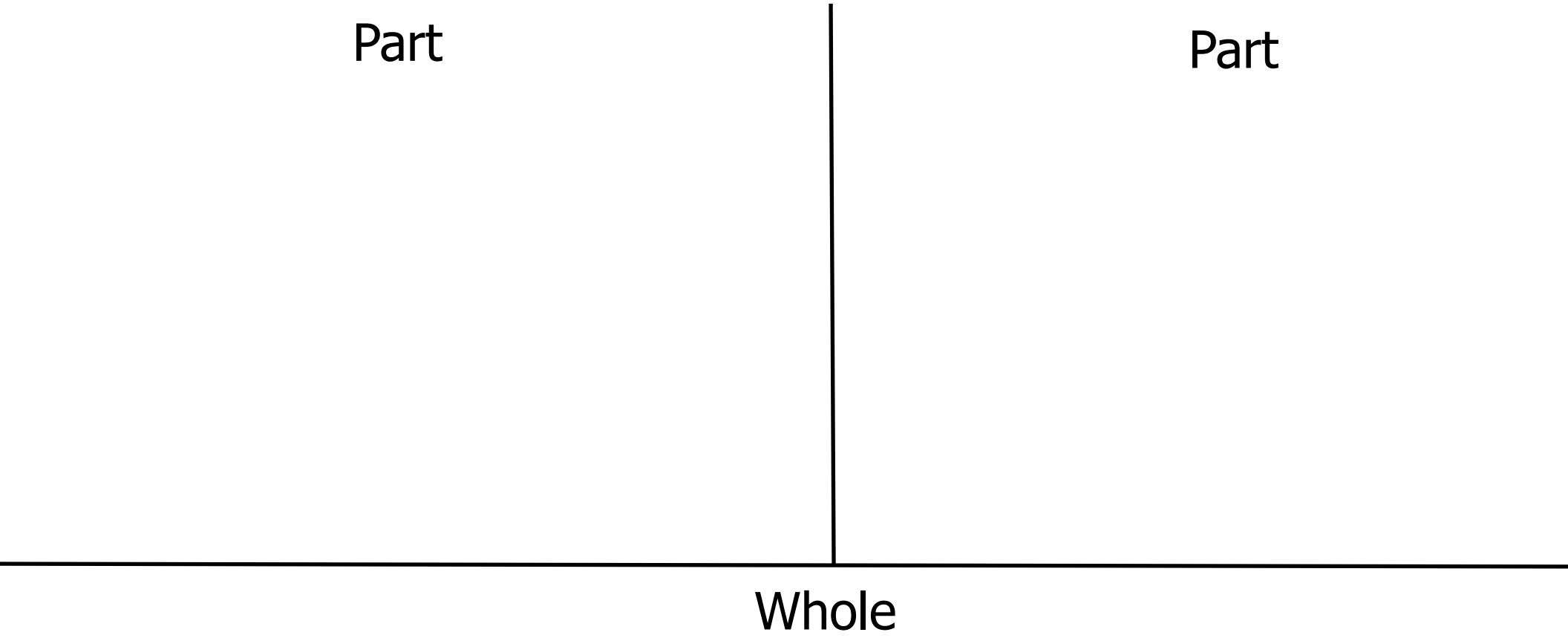
Materials: Number Construction Zone mat, counters

Directions:

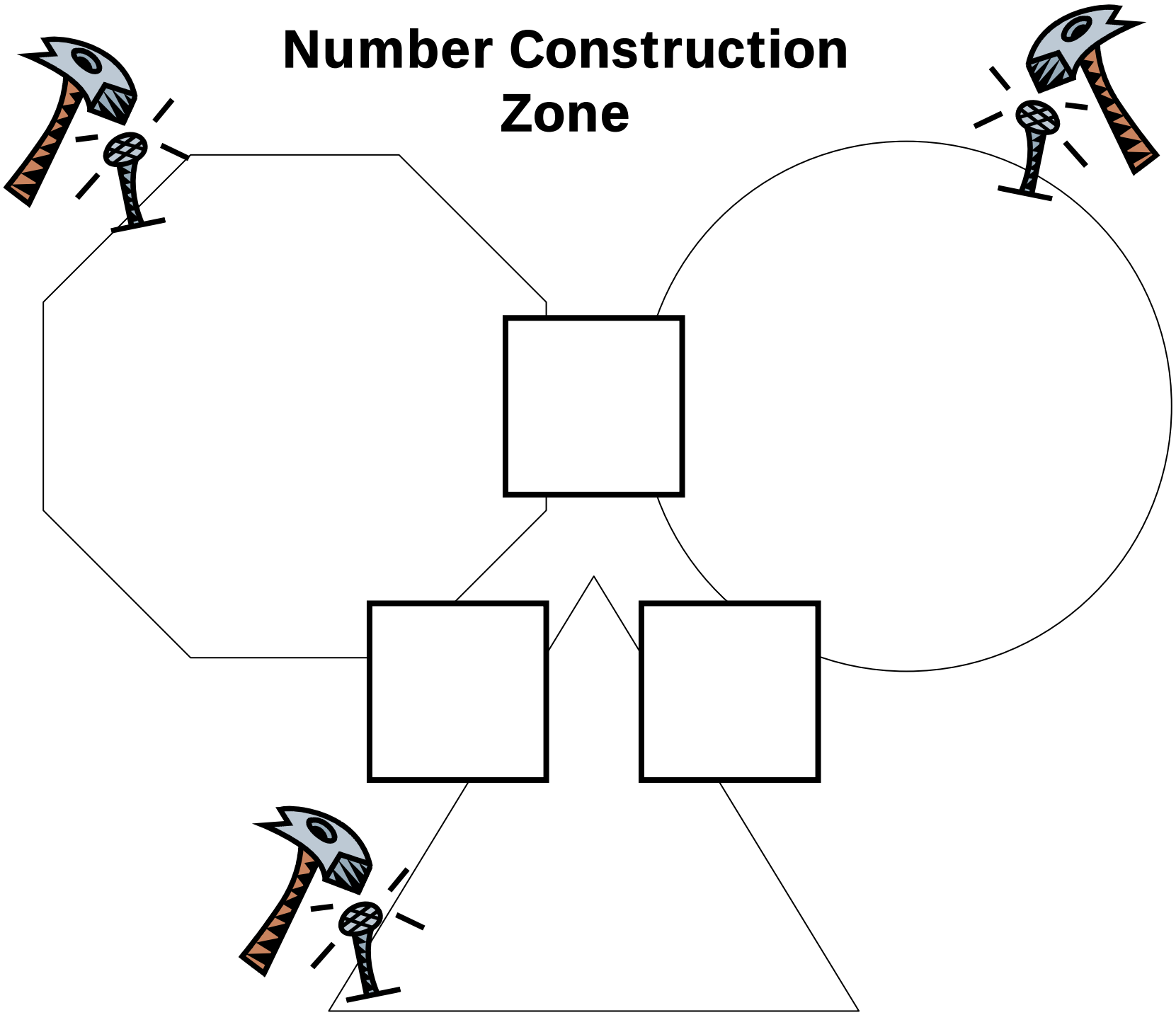
Give students a Number Construction Zone mat and some counters. Have them place a different number of counters in each large shape: the octagon, the triangle, and the circle. Have students add the number of counters in both the hexagon and the circle and write the total in the square that joins them. Do the same for the octagon / triangle and the circle / triangle. Then have students remove the counters and exchange mats with another student. Each student then tries to solve the problem created by someone else by finding the parts of each total. Continue the process until students have had the opportunity to try to solve several Number Construction Zone problems.

Variation: Older students (or students who are becoming more proficient with the activity) can be challenged to solve the problems without the aid of counters or use larger numbers.




Part
Part
Whole

Number Construction Zone



Undercover
Materials: Bowl, counters (all one color)
Directions:
Partner One: Puts some counters under the bowl and states the amount. "I put six counters under the bowl."
Partner One: Puts some counters on top of the bowl and states the amount. "I put 3 counters on top of the bowl. How many counters are there in total?"
Partner Two: "I think that 6+3=9. Let's check and see."
Both partners lift the bowl and check to see if the equation written is correct. Partners take turns and repeat the process.
Variation: Use clear bowls for students who need extra support.

Hide the Beads
Materials: Pipe cleaners, beads
Directions:
Students are given a pipe cleaner with beads on it. If the students are working on combinations to 10 there would be 10 beads on their pipe cleaner.
Partner One: Slides some beads to the side and covers them up. For example, if there are 10 beads on the pipe cleaner partner one could slide 5 beads over and cover them with his hand and say, "There are 10 beads on the pipe cleaner. How many are under my hand?"
Partner Two: "I think there are 5 under your hand because I see 5 and 5 +5=10. Students record equation.
Partner One: Reveals the amount under his hand and they check to see if the equation written is correct.
Partners take turns and repeat process.

Floor Plans

Materials: Unifix or snap cubes (all one color), "Floor Plans" – Note: students should only work with the set of plans for one number at a time

Directions:

Students, using a "floor plan", fill in the floor plan completely with unifix or snap cubes and describe the number by parts. (Two and three make five, or three and one and one make five.) Be aware of how the student describes numbers in parts. Can students begin to make predictions? (If I fill this floor plan for 5 with three cubes, I need two more to finish it.)

Variation: Students can build the floor plans using cubes in two different colors to think about parts in terms of color instead of position.

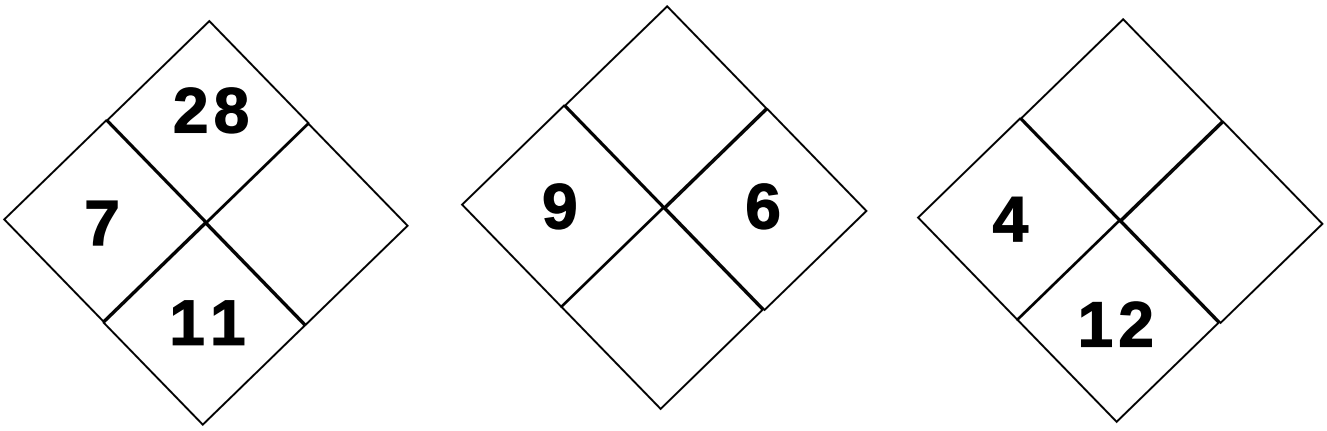
Rhombus Directions

Materials: paper and pencil

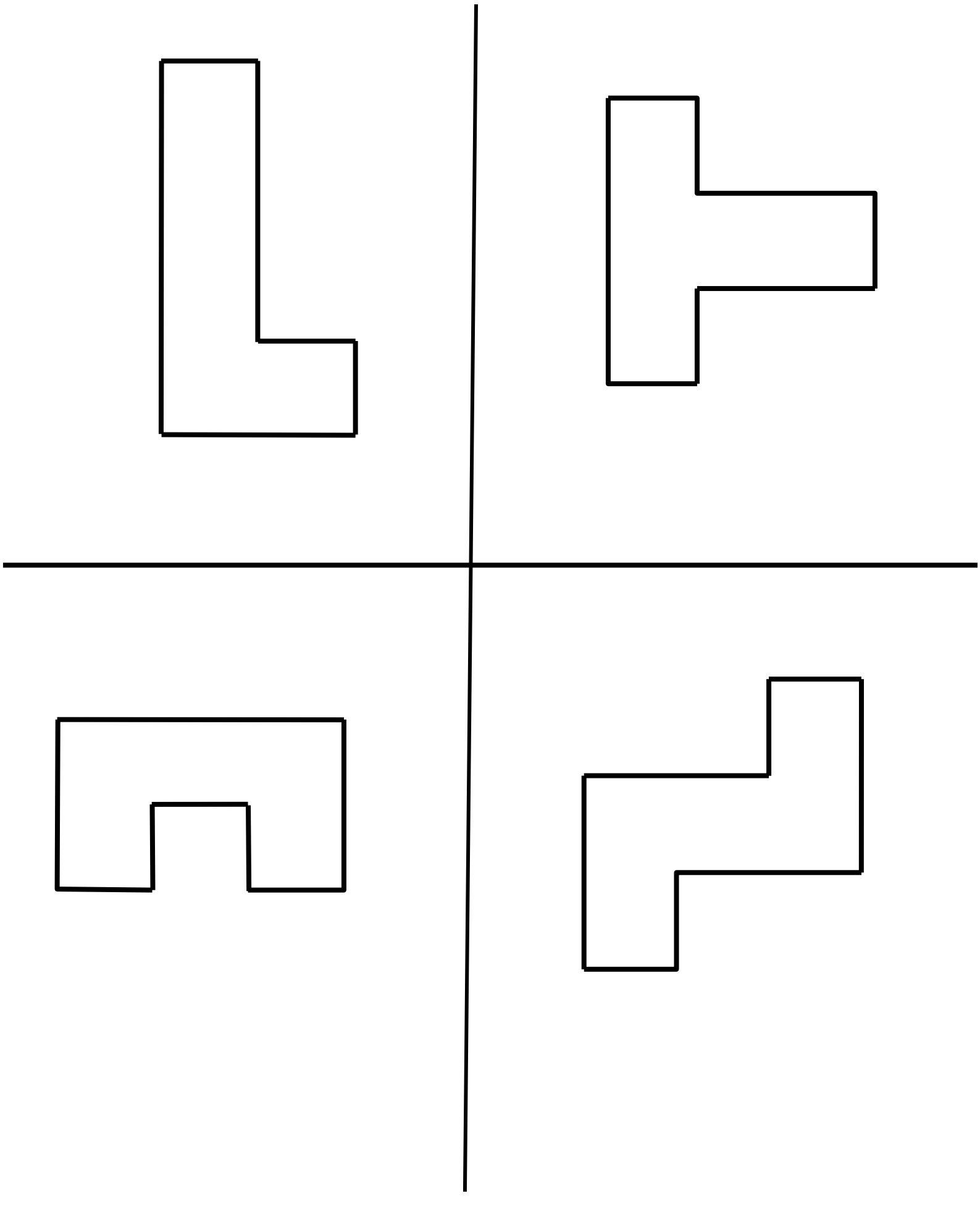
Directions:

Begin by drawing a rhombus on the board and split it into four smaller rhombi. Refer to these as north, south, east and west. Write numbers in the north, south and west rhombi, then ask students to guess what goes in the east. North will be the product of east and west. South will be the sum of east and west. However, you need not tell the students these rules.

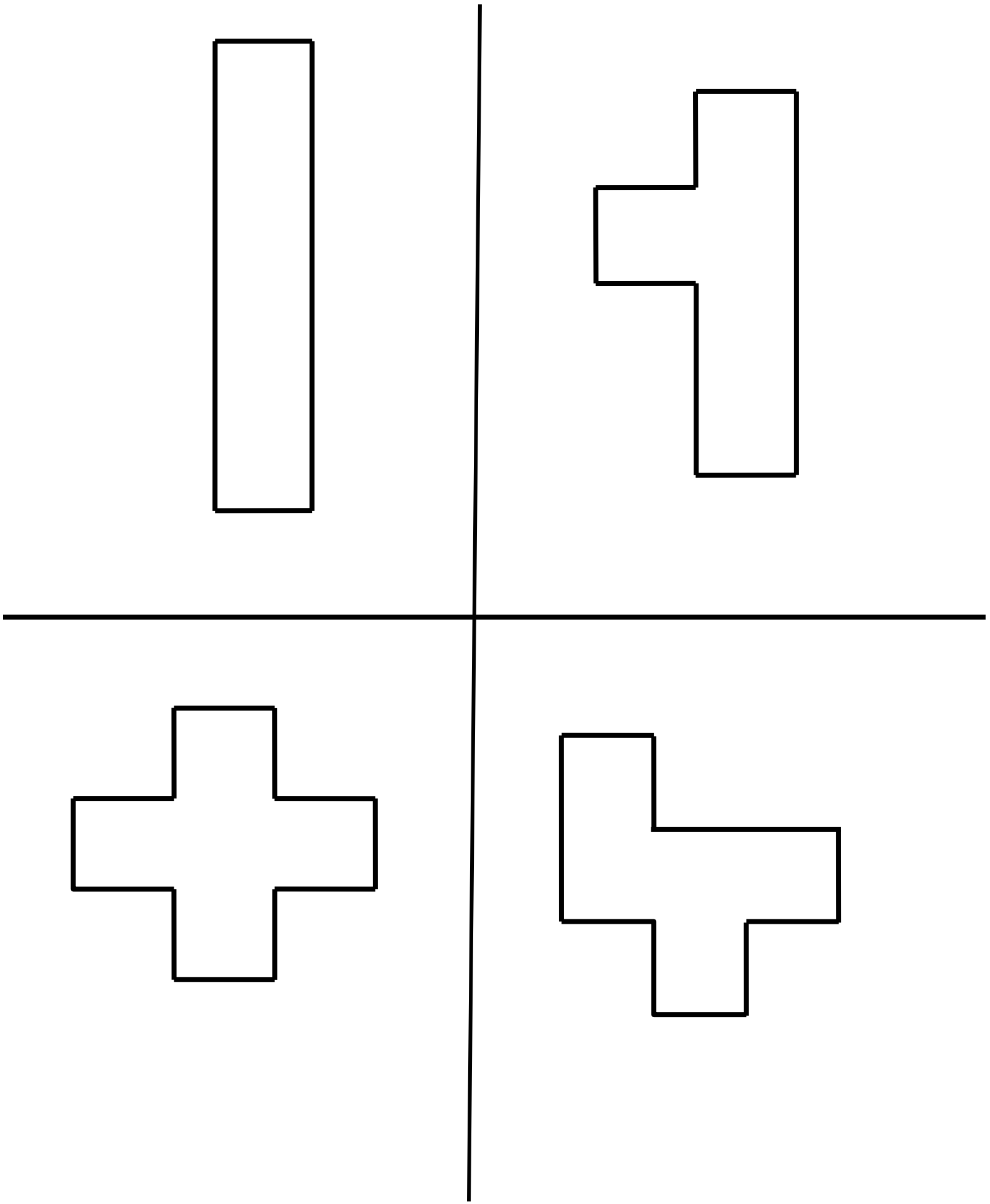
Variation: Once students have caught on to the pattern, vary the information given by filling in only the east and west or east and south rhombi.



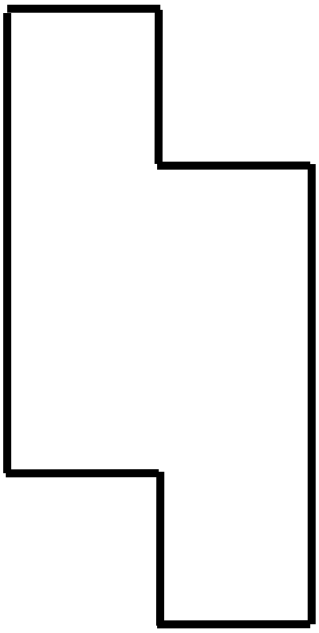
Floor Plans

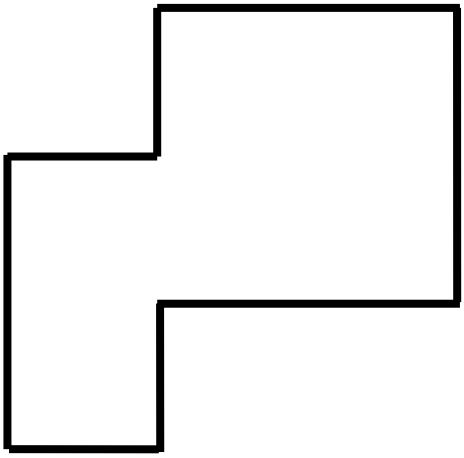


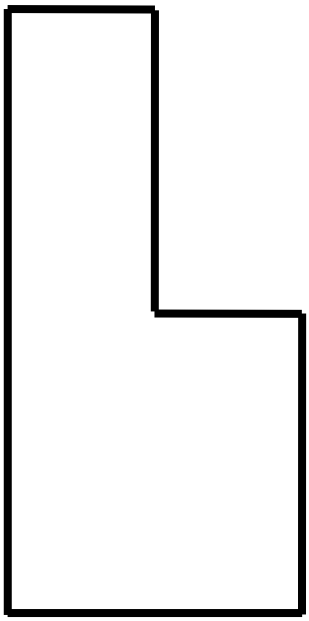
Floor Plans

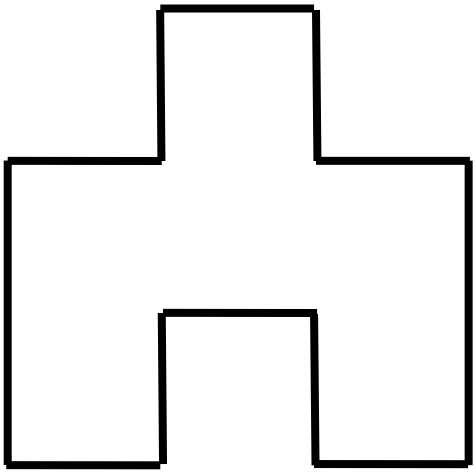


Floor Plans









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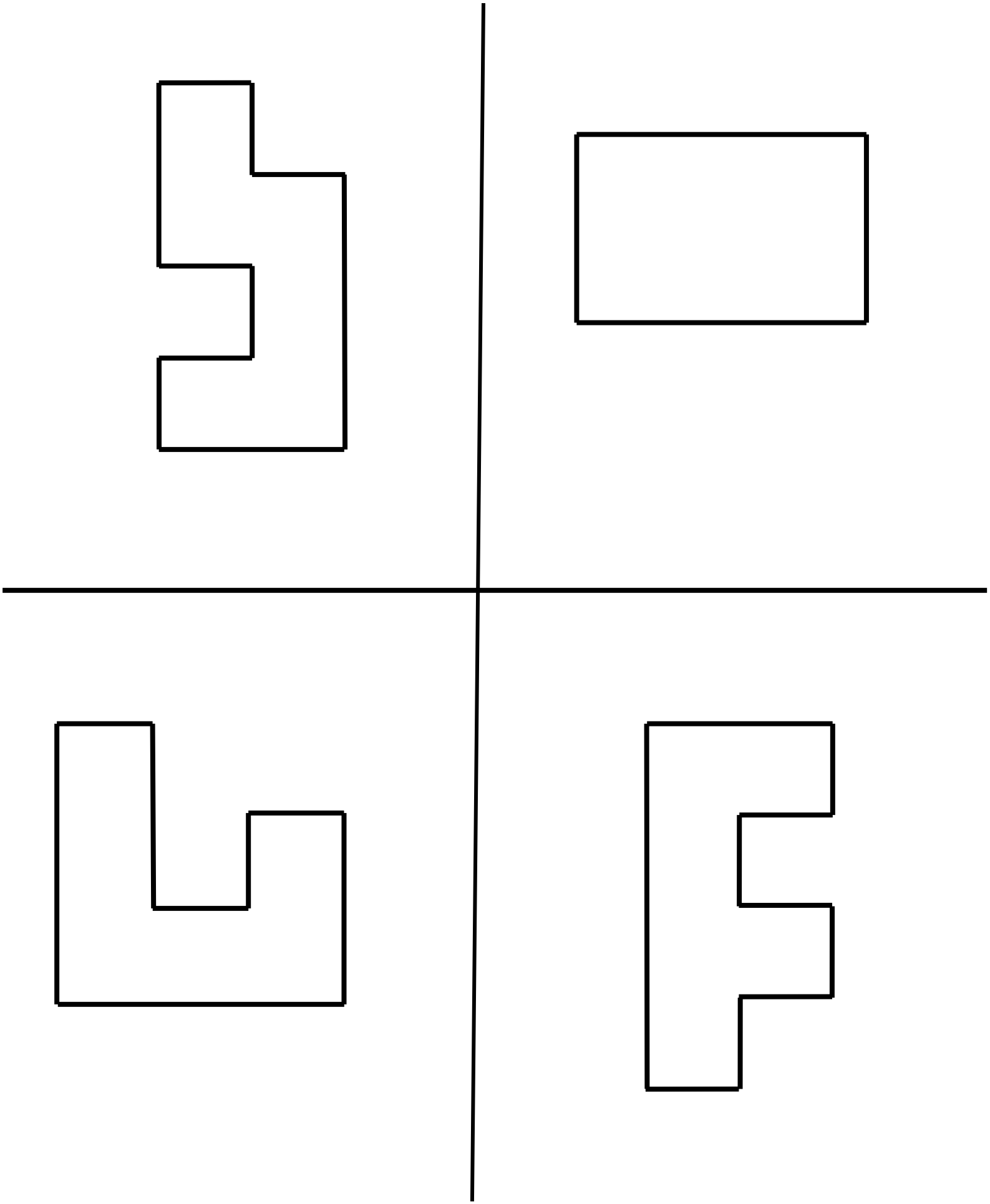
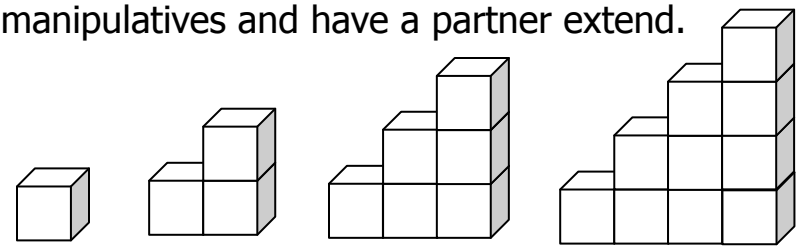
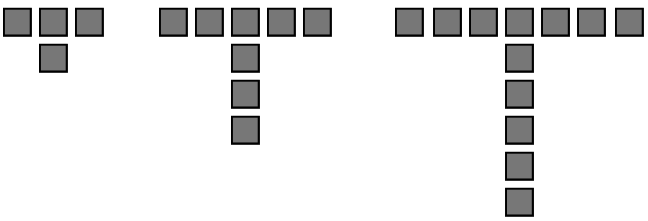


Patterns

Repeating and Growing Patterns
Materials: color tiles, toothpicks, popsicle sticks, cubes, counters
Directions:
Provide pictures of growing and repeating patterns. Students should copy the pattern with an appropriate manipulative, then extend the pattern. Encourage students to describe how the pattern is growing or repeating verbally and in writing.
Variation: Students can create their own growing patterns with manipulatives and have a partner extend.





Pattern Tunnel

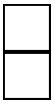
Materials: linking cubes, paper towel roll

Directions:

Students work with partners. Take turns creating a growing or repeating pattern using linking cubes. Slide the train of linking cubes inside a paper towel roll. Push the train of cubes out of the paper towel roll one cube at a time, slowly revealing the pattern. Discuss how the pattern is growing or repeating as it is revealed.

Variation: Create a pattern using linking cubes that is longer than the length of the paper towel tube. Place the paper towel roll over the pattern so that the beginning and end of the pattern is exposed, but the center part is hidden. Have students predict what cubes are hidden by the paper towel roll.

Hundred Chart Patterns

Materials: hundred charts, transparent tiles 

Directions:

Students use a hundred chart and the transparent tile to cover two numbers on the chart. Start by covering 1 and 11. Record the sum of the two numbers on a piece of paper. Slide the tile one square to the right on the hundred chart so that it is covering 2 and 12 then record the sum on the paper. Repeat, sliding the tile one square to the right at a time and recording the sums. Discuss the resulting pattern. Will this pattern continue regardless of the starting location on the hundred chart?

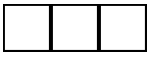
Variation:

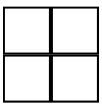
Change the orientation of the tile so that it is covering 1 and 2.

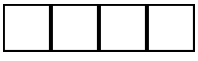
Change the direction that the tile is slid, move down instead of across.

Change the starting location of the tile.

Use a different shaped tile. What patterns result what the tile is a rectangle that covers three or four numbers or a square that covers four numbers.



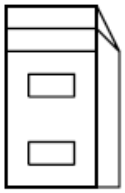




Hundred Chart

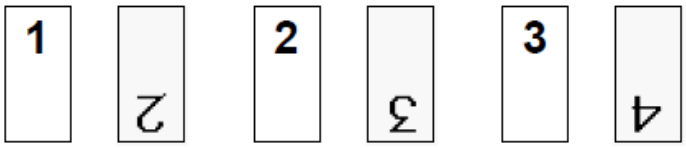
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Functions

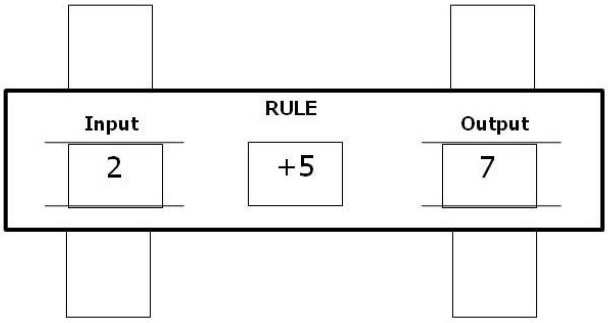


Function Machine Box
Materials: cardboard milk carton
Directions:
Cut a slit across the top of a half-gallon cardboard milk carton about half an inch high and three inches wide. Cut another slit across the bottom. Cut a piece of tag board 3.5 inches wide and 8 inches long. This is the slide the card will travel along. The tab board should go inside the box with the folded flange taped to the outside of the top slit. The slide should curve from the top slit, touch the back of the carton and come out the bottom slit. The cards will be placed into the machine with the input number facing up. It is important that the card "flip" on its way down the slide so that the output number is facing up when the card slides out of the box. Adjust the length of the slide if necessary before taping the bottom folded flange to the bottom slit. Decorate your function machine.

Construction of Cards: Prepare cards as shown. You will want to create about 10 cards per rule. The cards should have a notation about which is the input side. The size of the cards depends upon the size of the opening of the slits in your function machine.



Variation: Create individual student function machines.



In/Out Tables

Materials: manipulatives for growing and repeating patterns

Directions:

Revisit growing patterns with manipulatives. This time focus on the relationship between the figure number and the number of manipulatives needed to create each figure. Have students record this information in an In/Out Table to develop an understanding of the patterns within the table.

Figure 1 Figure 2 Figure 2

Figure Number	Popsicle Sticks Used
1	4
2	7
3	10
X	3X + 1

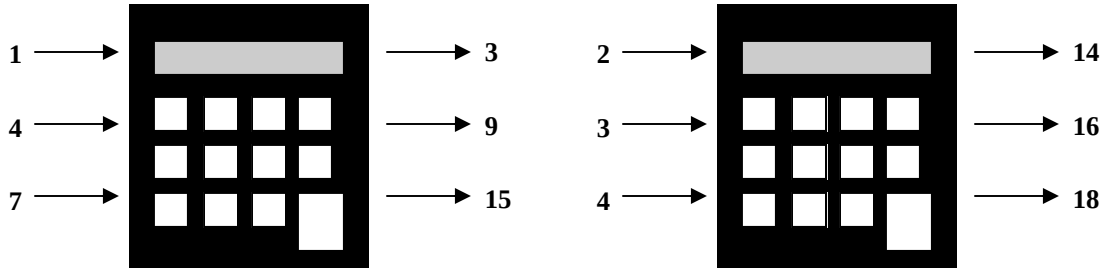
Calculator Functions

Materials: calculators

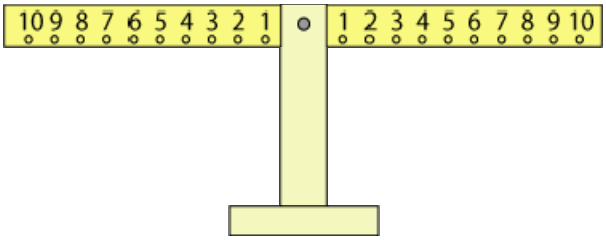
Directions:

For each example, find the sequence that was keyed into the calculator to generate the final displays. Students may use calculators to help with determining the sequence.

Variation: Determine the sequence without using a calculator.

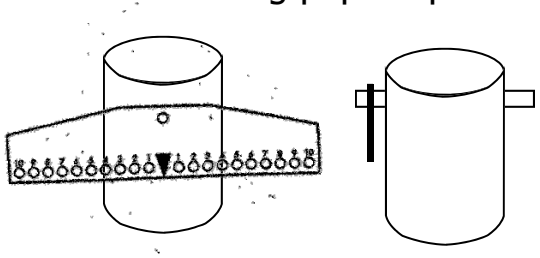


Equality



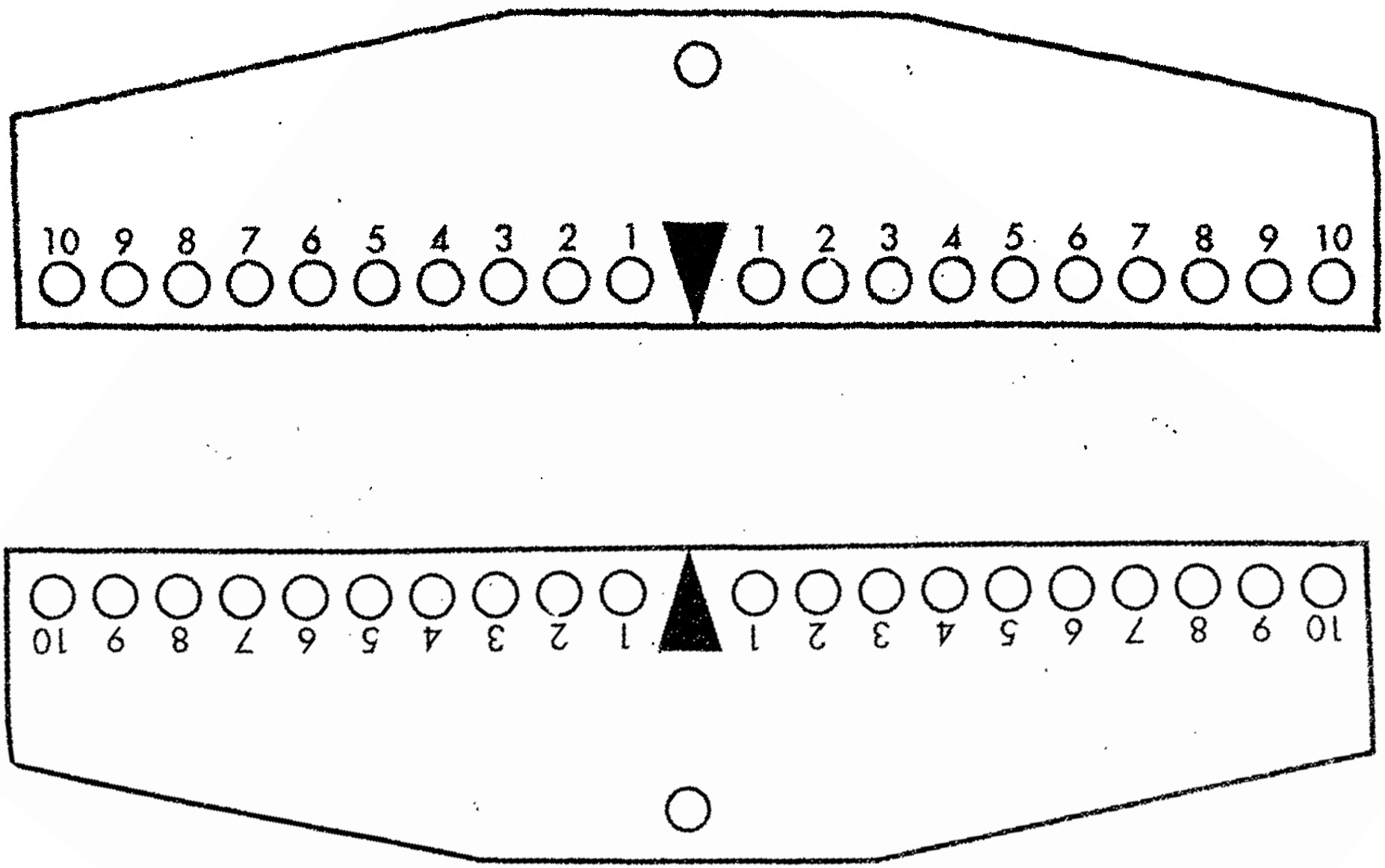
Equabeams
Materials: commercial equabeam or
number balance or
plastic cup, paperclips, pencil, cardstock balance
Directions:
Compare writing a number sentence to using a balance scale. On a balance scale, when the two sides weigh the same and balance the two sides are equal. Students can place counters on a balance scale to show that 4 blue counters and 1 red counter might balance 2 blue counters and 3 red counters. Students can place weights on an Equabeam to show that 4+1=2+3.

Variation: Students can create their own number balances using a plastic cup, a pencil and a cardstock balance cut out. Students can model equations on the balance using paperclips as the weights.



front side

Equality Match
Materials: Equality Match Cards
Directions:
Mix up the cards and lay them face down in a 4 x 6 array.
Each player should write his/her name at the top of one of the columns on the recording sheet. The first player turns over two cards. If they have equivalent expressions, the player takes the two cards and writes an equation linking the two expressions on the recording sheet.
example: 2 + (4 + 10) = (2 + 4) + 10
Once you have recorded the equations, play goes to the next player. If the cards do not have equivalent expressions, turn them back over and your turn is finished. The winner will be the player who writes the most equations on the recording sheet.


10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10

Name: _____ Date: _____

Directions: Cut the expressions shown and find the equivalent expression.

8×3	$18 - 2 - 2$	$(4 \cdot 3) \cdot 5$
5×5	$25 + (15 + 26)$	$100 - 51$
$2 + 4 + 3$	$6 \cdot 1$	3×3
$30 - 6$	$22 + 3$	$(25 + 15) + 26$
$7 + 4 + 3$	$4 \cdot (3 \cdot 5)$	$4 \cdot 1$
7×7	2×2	$3 + 1 + 2$

Name: _____ Date: _____

Directions: Cut the expressions shown and find the equivalent expression.

$10 - 4$	4×4	$2 \cdot 10$
$3 \cdot 5$	$5 + 9$	$60 - 2$
6×1	$72 - 42$	$55 + 45$
$8 + 8$	$16 - 2$	5×6
10×10	$10 + 8 - 3$	$37 + 20 + 1$
$80 - 3$	4×5	11×7

Name: _____ Date: _____

Directions: Cut the expressions shown and find the equivalent expression.

$1 + 7 + 2$	$5 \cdot 7$	9×4
4×4	6×7	$15 - 2 - 2$
8×8	$19 - 8$	$6 \cdot 6$
5×2	21×2	11×3
7×5	$8 \cdot 2$	8×0
$7 - 7$	$32 + 32$	$36 - 3$

Name: _____ Date: _____

Directions: Cut the expressions shown and find the equivalent expression.

=	=
=	=
=	=
=	=
=	=
=	=
=	=
=	=
=	=

Equality Sort

Materials: sorting cards, sorting mat

Directions:

Look at each expression card. Determine if the expressions on the card are equal or not equal. Place the equations on the correct side of the sorting mat.

Variation: After students complete the sort, they can create equality/ inequality statements of their own and record them in the appropriate column.

Number Talk

Materials: Chart paper for recording student solutions

Directions:

Present students with a target number. Students then create expressions that are equivalent to the target number. These expressions can be recorded with the target number or they can be recorded as equivalent to each other.

Example A:

24 = 12 + 12
24 = 3 x 8
24 = 100—76
24 = 57—33

Example B:

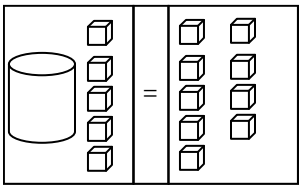
2 x 12 = 24 x 1
20 + 4 = 50—26
17 + 5 + 2 = 45—21
8 x 3 = 25—1

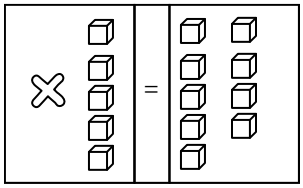
Equal (=)	Not Equal (≠)

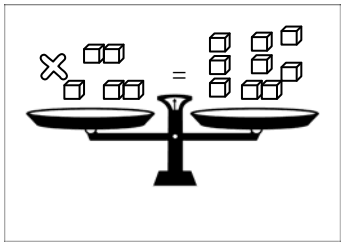
3 + 5 □ 4 + 4	2 + 4 □ 3 + 4
6 - 5 □ 0 + 1	6 + 5 □ 5 + 6
2 + 7 □ 3 + 8	9 - 1 □ 4 + 4
7 – 1 □ 3 + 2	7 + 5 □ 7 + 4
9 + 6 □ 6 + 8	5 + 5 □ 6 + 4
5 + 2 □ 9 - 2	3 + 5 □ 4 + 2

Variables

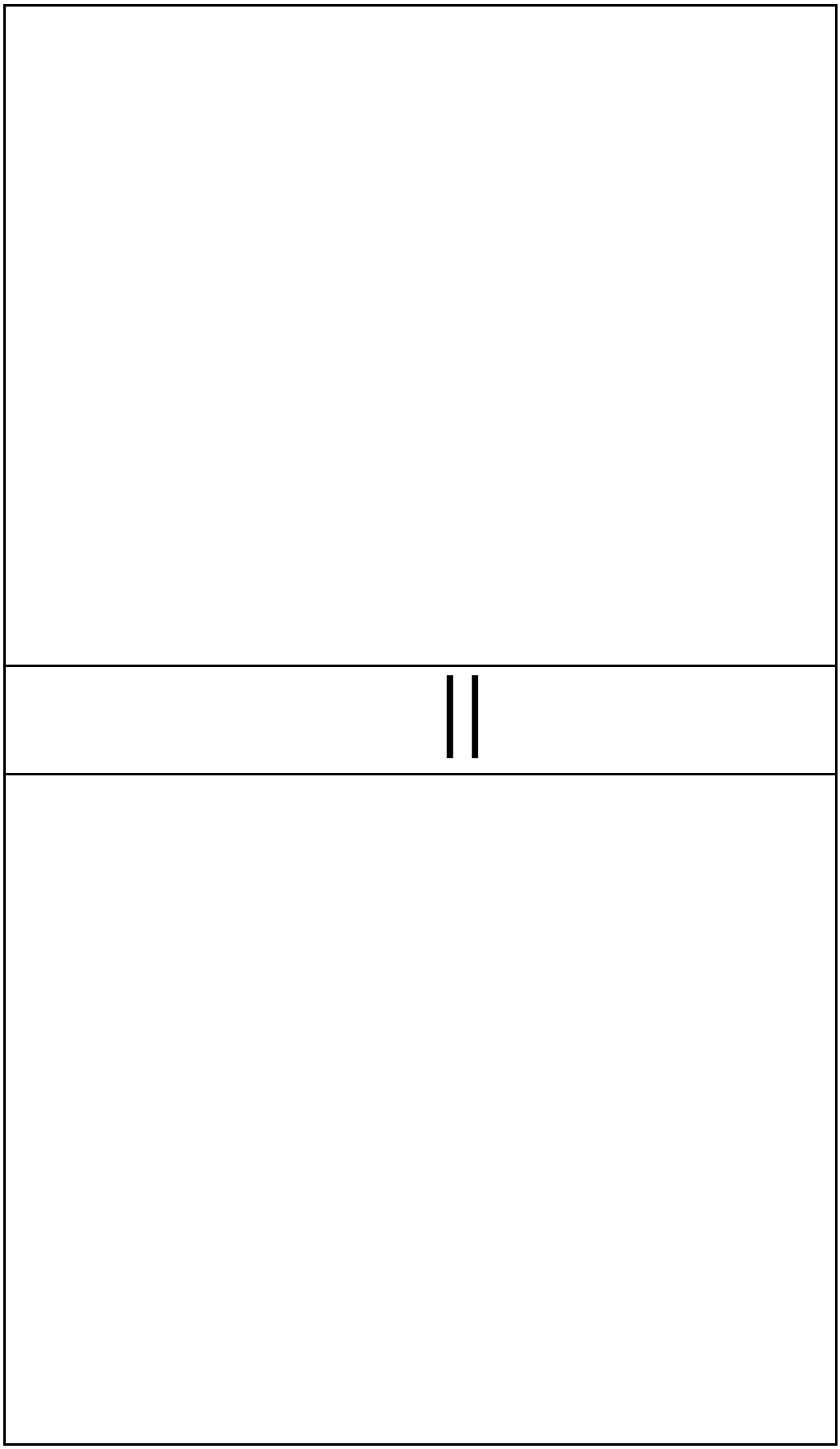
Modeling Mats
Materials: manipulatives, modeling mats, tile spacers, small plastic cups
Directions:
Students can model a variety of equations using cubes and empty plastic cups to represent the variable. To model x + 5 = 9, students would place one empty cup and five cubes on one side of the mat and nine cubes on the other side of the mat.
Variation:
Instead of using small plastic cups to represent the variables, tile spacers could be used. Algebra tiles work can also be used on these mats.

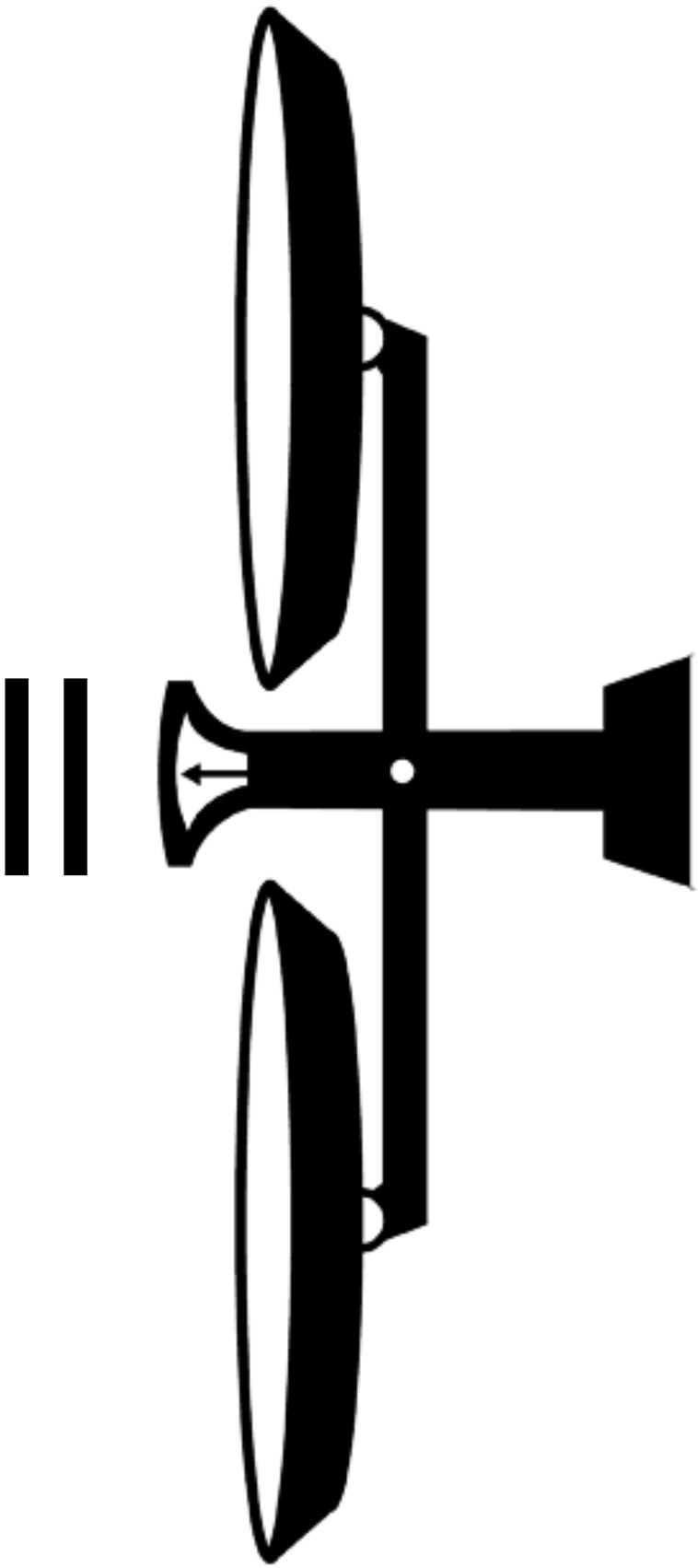






Expression Match
Materials: Equivalent Expressions Cards
Directions:
Students work together to match verbal expressions with algebraic expressions. Cut apart the verbal expression cards. Then cut apart the algebraic expression cards. Keep the two sets separate. Students should choose a verbal expression card and then work together to think of an algebraic expression that matches the verbal expression. Then they look for the card containing that algebraic expression.





five more than twice something	Last summer, Max earned 5 stickers for every book he read. He earned 40 stickers in all for reading books. Which number sentence could be used to find *B*, the number of books Max read?
Eight times something	<table><tr><th>Input (*A*)</th><th>Output (*B*)</th></tr><tr><td>7</td><td>14</td></tr><tr><td>12</td><td>19</td></tr><tr><td>20</td><td>27</td></tr></table> Which could be the rule for this input-output table?
Three more than something	Paco had 32 trading cards. He gave *B* trading cards to his friend. Which expression tells how many cards Paco has now?
Fifteen less than a number	*B* stands for the number of hours of sleep that Ken gets each night. Which expression represents the number of hours of sleep that Ken gets in 1 week?

Fifteen times something	The length of a rectangle is 3 more than its width. If *B* is the length, which expression represents the width of the rectangle?
Carla set up tables and chairs for a party. Carla set up *B* tables. Each table has 4 chairs. If *B* represents any number of tables, which expression represents the total number of chairs that Carla set up?	Which expression represents the product of *B* and 32?
B stands for the number of stamps that John had. His sister gave him 32 more stamps. Which expression represents the number of stamps that John has now?	Andre has 6 pears. Martha has some pears. Together they have 10 pears. Which number sentence would you use to show how many pears they have together?
Allen caught two less fish than Antonio. Antonio caught 48. Choose the correct number sentence to figure how many fish Allen has.	Mike had 12 cans. Together, Mike and Lisa had 21 cans. How many cans did Lisa have?

Lisa jogged *B* miles yesterday and 8 more miles today. Which expression represents the total number of miles that Lila jogged?	Emma checked out 4 library books. Amanda and Emma checked out 11 books altogether. How many library books did Amanda check out? Which number sentence represents this problem?
Sophie caught twice as many fish as her dad. Which expression represents the number of fish that Sophie caught?	Mrs. Cardarella is baking 85 cookies for her 5th grade students who are going on a picnic. She packs 5 cookies in each bag. Which number sentence can be used to find the number of bags, *B*, that she will need?
B represents the number of cookies in a bag that Mrs. Hill wants to divide evenly among 8 classes. Write an expression that shows the number of cookies each class will receive.	Mrs. Beckner has 32 students. She breaks them into *B* equal groups. Which expression represents the number of students in each group?
Mrs. Deaver has *B* pencils and 32 students. If she divides the pencils evenly, which expression represents the amount of pencils each student will get?	Which number sentence is true for all pairs of values in the table shown below? Input / Output *A* / *B* 15 / 3 25 / 5 10 / 2 30 / 6

$32B$	$6 + B = 10$
$85 \div 5 = B$	$B \div 8$
$4 + B = 11$	$48 - 2 = B$
$12 + B = 21$	$B - 3$

$2B + 5$	$A + 7 = B$
$5B = 40$	$32 - B$
$32 \div B$	$B \times 2$
$7B$	$8B$

$15B$	$B + 3$
$B + 32$	$B \div 32$
$A \div 5 = B$	$B - 15$
$B + 8$	$4B$

Solving Equations Through Movement

Materials: Picture cards

Directions:

Hand out a card to each student.

Ask for 3 "moon and stars" and a sail boat to stand on the right side of the equal sign.

Ask for 5 "moon and stars" to stand on the left side of the equal sign.

Ask how to write the modeled problem algebraically.

Ask the question, "How can we find out how many "moon and stars" equal one sail boat?"

Have students act out the operation (take 3 moon and stars away from both sides).

Have an audience member explain the process and write the process algebraically.

Have the participants sit down.

Ask for a snow flake and 2 sail boats to stand on the right side of the equal sign.

Ask for 3 sail boats to stand on the left side of the equal sign.

Ask the question, "How can we find out how many sail boats equals one snow flake?"

Have students act out the operation then have an audience member explain the process.

Write the equation on the board and show the steps they just performed.

Have the participants sit down.

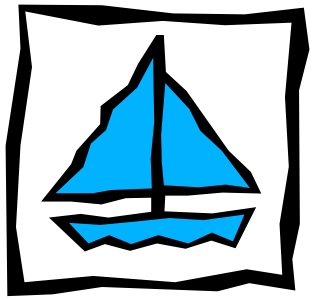
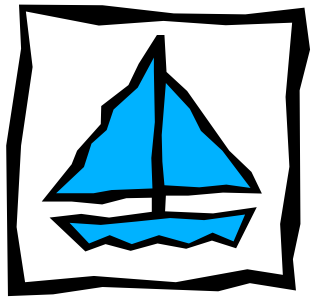
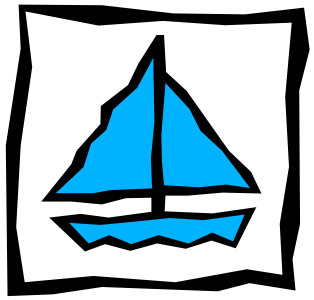
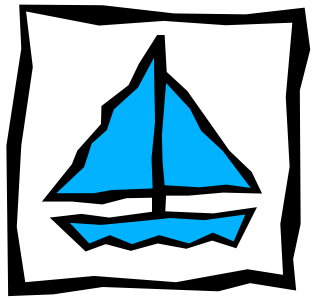
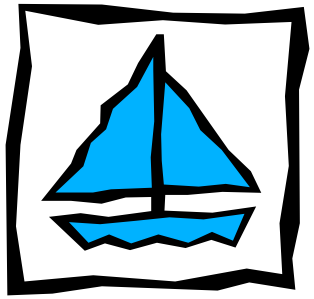
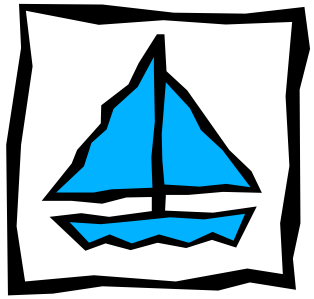
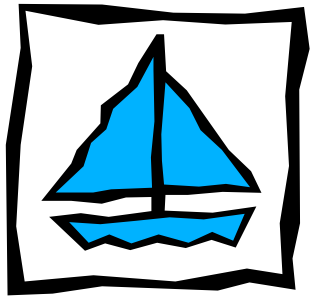
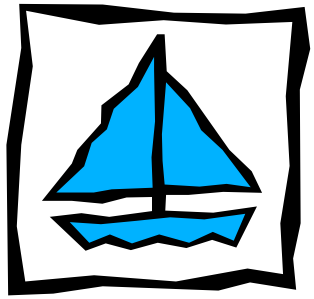
Ask 3 "butterflies" to stand on the right side of the equal sign and 6 "moon and stars" to stand on the left side of the equal sign.

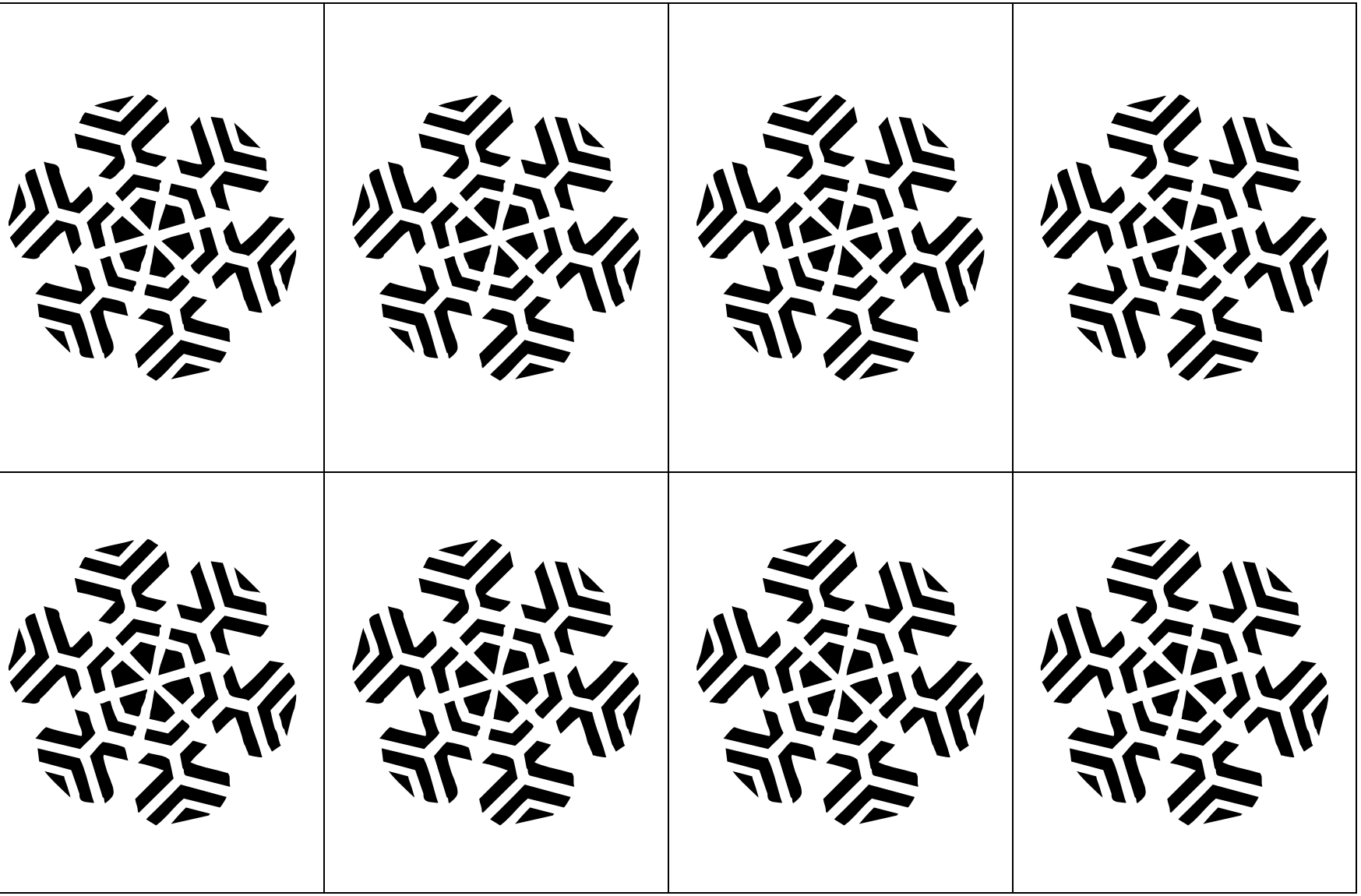
Ask the students how to write the modeled problem algebraically. $3b = 6$

Ask the question, "How can we find out how many "moon and stars" equals one butterfly?"

After they act it out, write the steps on the board.

Continue having students model the process until you feel most of the students understand.

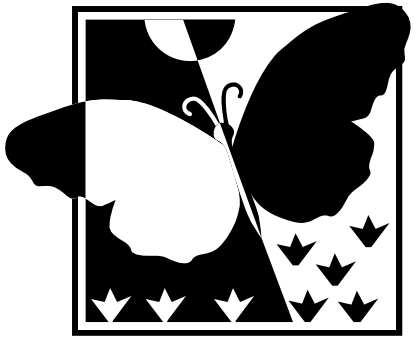
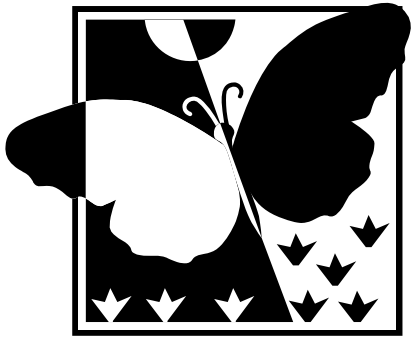
			
			









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popTomatos
Aquation
Plus Block
Mathris
Split-it
Animals with Math Balance
Number Blocks

Websites

Hundreds Board
http://www.pixelpump.co.nz/hundreds3.html

Double Function Machine
http://www.crickweb.co.uk/ks1numeracy.html#fmach
Function Machine
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Fruit Balance
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Calculation Balance
http://www.wmnet.org.uk/wmnet/custom/files_uploaded/uploaded_resources/850/calcbalancev3.swf

Super Sequence Number Patterns
http://www.amblesideprimary.com/ambleweb/mentalmaths/supersequencer.html