Mathematical Modeling What Does It Mean?

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Introduction to Mathematical Modeling

In a world that is getting more and more complicated all the time, everyone agrees that mathematics is more important than ever. Part of the power of mathematics is how useful it is for so many areas -- banking, business, government, technology, engineering, science and practically every other career area. Major decisions are made every day, like whether to merge companies, give money to research, build a highway, or predict next week's weather. These decisions involve large amounts of data, and affect great numbers of people. The data must be analyzed to find patterns, and the patterns used to project future behaviors. All of that is mathematics, using processes and skills you learn in school, and applying them in new ways to solve real problems.

Have you ever wondered how some people seem to know so much about the world? For example, how do doctors know whether medicine should be taken two or three times a day? How does a wildlife biologist know when an animal is an endangered species? How do animators draw moving pictures that appear natural and "real"? How do engineers design and build things that do complicated tasks like putting together a car? How can these people be sure that things work the way that they do? They are not necessarily super-intelligent or given special abilities. They *have* learned to apply a specific skill to the real issues they encounter in their jobs and lives. They have learned how to use mathematical modeling.

Basically, this means being able to understand a real-world situation well enough to explain it in some kind of mathematical way. The "model", which is the mathematical description of the real-world situation, can come in many forms: words, diagrams, tables, graphs, equations, and even computer simulations. It can *even* be a smaller version of the "real thing" -- the Army Corps of Engineers operates a working model of the San Francisco Bay that is about three football fields long. These descriptions are called models because they have enough features of the situation to resemble it, especially how the various parts work together. Once the model is "built", mathematics can solve the problem that they found in that situation. In doing mathematical modeling, it helps to have some experience, but *anyone* can do it. The only special abilities that one needs to have are an understanding of the basic "rules", and a willingness to keep trying. The focus will be more on understanding what's going on, than in trying to find *an* answer to a *particular* problem.

The Modeling Process

The process of beginning with a situation and gaining understanding about that situation is generally referred to as "modeling". If the understanding comes about through the use of mathematics, the process is known as **mathematical modeling**. This will be *much* clearer after having done it once, but here is a general summary of the main steps in modeling.

Step 1. Identify a situation.
Notice something that you wish to understand, and pose a well-defined question indicating exactly what you wish to know.

Step 2. Simplify the situation.
List the key features (and relationships among those features) that you wish to include for consideration. These are the assumptions on which your model will rest. Also note features and relationships you choose to ignore for now.

Step 3. Build the model. Solve the problem.
Interpret in mathematical terms the features and relationships you have chosen. (Define variables, write equations, draw shapes, measure objects, calculate probabilities, gather data and organize into tables, make graphs, etc.). *That* is the model. Then, apply the model and solve the problem. (Solve the equation, draw inferences from patterns in the data, compare results to a standard result, etc.)

Step 4. Evaluate and revise the model.
Go back to the original situation and see if results of mathematical work make sense. If so, use the model until new information becomes available or assumptions change. If not, reconsider the assumptions you made in step 2 and revise them to be more realistic.

Another way of visualizing the process of mathematical modeling is shown in Figure 1:

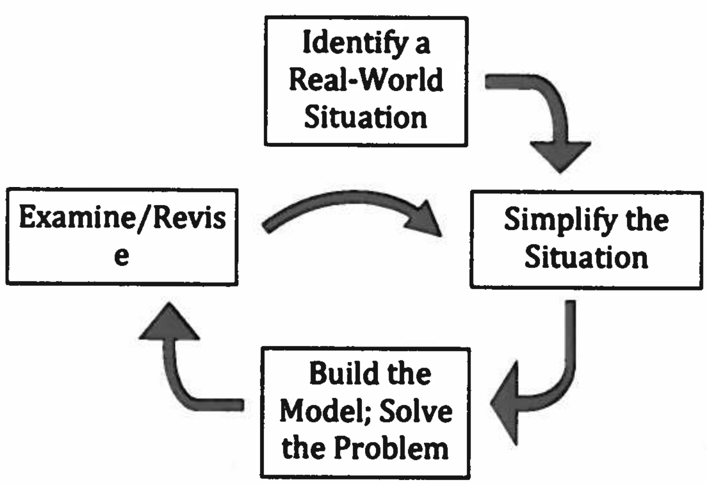


Figure 1.

It's called a process because it's like dance steps: you might go through a revision process several times before you are done making the model. There will be times when it seems as though you are solving what looks like the same problem as before; that is typical in mathematically modeling a situation. If you change any assumptions, you will change the problem, and *that* will require a new solution.
In this first chapter, you will find out much more about what modeling is by *doing* it, but there are several things worth mentioning specifically here.

1. These steps are really just a set of broad guidelines. Each step can involve several smaller steps that depend on the particular situation you are investigating. For example, you might "wrestle" with defining the problem well enough to apply mathematics. In this first chapter, the problem will be clearly stated, so you can focus on how the assumptions govern the model building and affect the solutions.
2. One very helpful principle that guides all modelers is hidden in the second step: *keep it simple.* In general, all models ignore something, and first-draft models usually ignore several things. Good models are simple enough to be understood by someone else, but are still reasonably close to reality. And as long as the assumptions are clearly stated as part of the model, the only criticism can be that it's too simple!
3. Except for the third step, this process can be applied to many tasks; it's helpful for writing essays, preparing a new recipe, or developing a new play in your favorite sport. Rarely does the first attempt at doing something unfamiliar result in the best you can do.

| High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process.
Like every such process, this depends on acquired expertise as well as creativity.

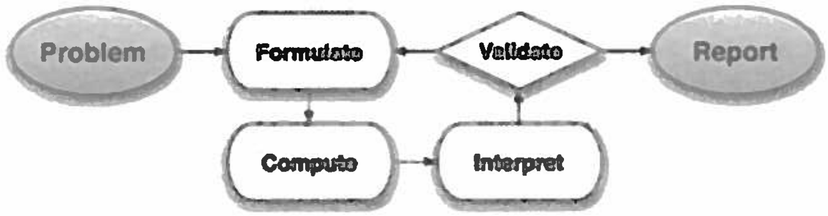
Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.

- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model— for example, graphs of global temperature and atmospheric CO2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol *.*

CCSS-M Mathematical Modeling Standards

Number and Quantity

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Algebra

Seeing Structure in Expressions A-SSE

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.□

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈ 1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

d. Prove simple laws of logarithms. (CA Standard Algebra II – 11.0)

e. Use the definition of logarithms to translate between logarithms in any base. (CA Standard Algebra II – 13.0)

f. Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. (CA Standard Algebra II – 14.0)

4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

Creating Equations A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable including ones with absolute value and use them to solve problems in and out of context, including equations arising from linear functions.

1.1 Judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step. (CA Standard Algebra II 11.2)

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Functions

Interpreting Functions F-IF

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include

reading these from a table).

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

6. Apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. (CA Standard Algebra I – 23.0)

Trigonometric Functions F-TF

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Geometry

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Expressing Geometric Properties with Equations G-GPE

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Geometric Measurement and Dimension G-GMD

Explain volume formulas and use them to solve problems

3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Modeling with Geometry G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder.

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Mathematical Modeling Preparatory Reading

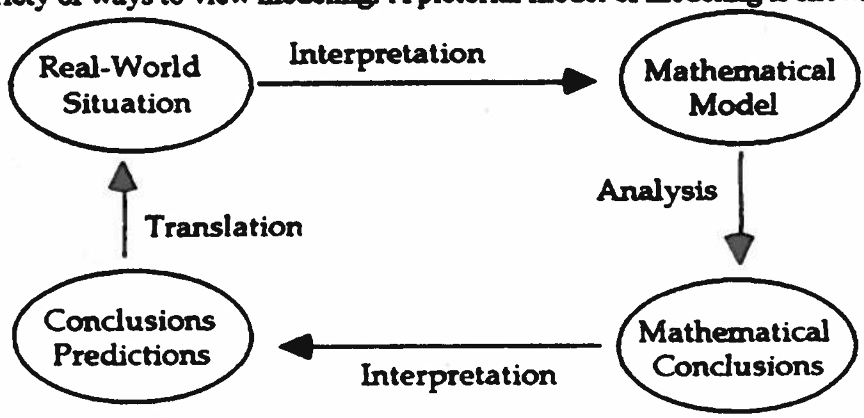
What is mathematical modeling? A model may be physical (as in a scaled version of a ship or airplane) so that characteristics of the original object may be understood while working with a more convenient and similar object. If a model is theoretical, then there is a set of rules or laws that represents an object or phenomenon. A mathematical model uses some mathematical structure (such as tables, graphs, equations, algorithms, etc.) to approximate the features of a phenomenon.

Mathematical modeling is a form of problem solving, but it is much more than computational exercises which are found in many textbooks. When doing mathematical modeling we must consider two overriding criteria: the model should be mathematically accurate and it must accurately portray a situation in the real world. In most mathematics courses, we are accustomed to working towards a correct mathematical solution, but modeling can be more complex in that it considers these solutions in terms of the situation that spawned the mathematical problem.

Assumptions greatly influence our understanding of the problem and the validity of our solutions. Consider the question of which toothpaste we should buy. What factors are important in determining the solution? If cost is a concern, then we will use the mathematics of computing unit cost. But then we may realize that the cheapest kind does not have a pleasing taste, and narrow our selection based on qualitative considerations. We may then find that the best tasting brand at the lowest cost comes in a large tube, and thus will not be convenient to pack for a trip we are planning. After narrowing the selection further based on a mathematical comparison of size and shape, we make our selection. The question and its solution are considerably different than the textbook question of which toothpaste is the best buy.

Modeling problems are challenging because they consider phenomena that may or may not seem mathematical in nature.

There are a variety of ways to view modeling. A pictorial model of modeling is shown below.



The arrows going from each part to the other indicate a flow of the modeling process.

Another way of looking at the modeling process is portrayed in the chart below.

Mathematical Modeling

ACTION	RESULT
Identify something you want to know, do, or understand	a question in the real world
Select "objects" and the relations among them	identification of key concepts
Decide what you'll keep and what you'll ignore about properties of objects, relations	produce an idealized version of the question
Translate into mathematical terms	develop a mathematical version of the idealized question
Identify field of mathematics you are in	bring out instincts and knowledge of that field
Use mathematical methods and get results	techniques, interesting cases, solutions, theorems, algorithms
Translate back to original field	theory of idealized question
Confront reality . Do you believe what is being said?	Yes-quit - communicate results No - go back to the beginning

The first three actions in the preceding chart are how you view the real world problem and identify the situation you wish to understand. The fourth action is a transition stage between the real world and the mathematical world. The fifth and sixth actions are dealing with a set of specific mathematical processes. The last two actions require you to return to the real world and test the accuracy, sensitivity, and stability of our solution in terms of the problem that you are trying to resolve. Whether you break down the modeling process into many steps or a few, there are some common components.

Yet another way to view the idea of mathematical modeling is looking at the components separately as steps in the process:

1) Identify the problem to be investigated.

2) Determine the important factors (and assumptions about those factors) which exist in a real-world phenomenon and represent those factors as mathematical symbols.

3) Manipulate (analyze) the mathematical relationship.

4) Interpret the mathematical conclusions in the context of the real-world phenomenon. Then evaluate how applicable the conclusions are to the real-world situation. If necessary, reexamine the model's factors and structure.

In this unit, for the sake of clarity, we will use this last modeling model to help in evaluating and guiding you. You should realize that this is just one of many models that could be used. These steps are not always followed in a rigid sequence; you may find yourself interpreting the real-world situation and see the need for other factors to consider as you are working with the mathematical relationship. You may then need to refine the model by adding new assumptions.

In evaluating how applicable your conclusions are, you might also consider how serious are the consequences of the result. For example, if you construct a model of how much food to order for the prom, and you make either an error in the mathematics or in the structure of the model, what would the consequences be (from either ordering too much food or too little food)?

What would the consequences be if you made either a mathematical or contextual error in a model of how many power plants a town needed by the year 2020?

Mathematical models may be quite complicated, partially due to the many assumptions that may influence the validity of a solution. When problems seem quite complex, we sometimes simplify the problem to gather a better understanding of underlying mathematical relationships. Consider the many steps you went through to determine where to put a fire station in Gridville. You were advised to begin by answering the question, "What are important factors to consider in deciding the best location?" You were then encouraged to use charts, diagrams, graphs, equations, and calculations to have formulate a logical conclusion. Some students may have been able to arrive at a good solution to the Gridville problem without having to consider Linear Village and Hermitville. Most mathematical modeling problems do not come with a clear-cut path towards a solution. Rather, it is left to the modeler to determine an appropriate approach towards an answer. In evaluating the approach taken, we ask if the mathematics is valid [For example, do we have an accurate way of computing distance between a house and a fire station? In some problems, does an analysis of residuals indicate that our curve of fit is a good one for our data?]. We also must ask if our plan reflects what we are seeking in the real world [e.g., did we wish to minimize the total distance the fire truck would travel in Gridville or did we wish to minimize the maximum distance the fire truck would travel?].

Sometimes, especially when first attempting a mathematical model, we realize that our original investigation is too complex; it may be beyond the scope of the time we may devote to the problem or our limited mathematical knowledge. For example, we may have been fascinated by morphing (the gradual transformation from one shape to another) but realized that a related concept (animation) is more accessible in a shorter period of time.

In building an understanding of what mathematical modeling is, you will observe how the modeling process has been used by other high school students to solve actual models. In the next section you will look at a mathematical model which has been done by another high school student. Although you may not be able to determine how much work was considered and discarded, you should be able to see how a modeling process was used to form conclusions or predictions which relate to the real world.

Modeling Unit - Overview

Notes for the teacher

This unit is designed as an open ended conclusion for the ARISE curriculum. As such, you may well find yourself feeling uncomfortable with allowing the students the type of freedom that is necessary for a successful modeling unit. Please realize that this is natural. No teacher can know everything and anticipate where the students will lead the discussion and presentation. You are not alone in this feeling.

Mathematics is used in almost every walk of life. It is important for the students to realize this and for them to experience the power that can be theirs by the understanding that comes with this realization. The entire ARISE experience has been a journey to help them come to this realization, along with developing the tools and knowledge that will give them this power. The tools and knowledge are not, in and of themselves, enough however. Students need to learn that they have the ability to transfer these tools and knowledge to help them describe and assist them in answering questions in their life. With this comes true mathematical power.

As the curriculum has progressed, students have been given less direction and more freedom to answer the questions posed in the materials. This is the culmination of that journey. It is with this in mind that the authors give you this advice. Use the least direction as possible to the students when they begin defining their project and proceeding with the development. At the same time, it is necessary to make sure the students understand what is expected.

It is with this in mind that the models in lesson 2 and 3 are offered. It is suggested that you assist students in analyzing, evaluating, and creating these models. At the end of this part of the lesson(s) the students should have a good sense of what can be done mathematically with various approaches, as well as realizing that they are not limited by one rigid format. In other words, there is not one right way to arrive at a final product.

The key question that the students need to answer is "Did I answer the problem I was trying to in a sufficient way?". Your part in this process is to be as adept as possible at leading the students to a mathematically interesting problem and helping them where needed to consider the mathematical accuracy of their solutions and how accurate their findings reflect the real world.

In order to do this, you need to be flexible and willing to admit that you do not have all of the answers. This is difficult for many teachers, but if you have taught ARISE materials in the past, it should not be a new event for you. Knowing others are in the same situation should help the acceptance of this concept. We hope that you will find this to be a rewarding experience.

This unit is divided into four parts. The first part is the overview to the teacher and students. The second part allows the students to get a look at a series of modeling problems, to have them critique the work and to let them know what is possible, as well as give them ideas of mathematical questions that might lead to interesting models. The third section is a brief modeling problem that may be done in a day or two. The fourth lesson allows students to participate in the entire modeling process and is the culmination of the program up to this point. It is reasonable to assume that the first three parts might take a week and the last section might take as long as a month. The time allotted will depend on the situation in the class and the length of the course.

One possible method of presentation would be to do the unit in a single 5-week long exercise. Another way to use this unit is to treat the first two lessons early in the year or term, lesson 3 in another part, and lesson 4 as a day or two per week/month. See the Lesson 4 description for one suggested way to do this.

Lesson 4 will allow the students, along with guidance from the teacher, to work with and analyze problems mathematically that are interesting to them. The materials will provide suggested topics from the topics that ARISE has developed as well as other problems. **Please keep in mind that these are just suggestions**. It is desirable that the students select topics that are of interest to them. The lists are there for ideas and to be used if the student has difficulty deciding upon a topic.

Lesson 4 - Mathematical Modeling

If you have not been able to come up with a problems that interests you and is feasible to develop a model, select from the following problems. Keep in mind as you use the modeling process the steps that were identified in the process. Also keep in mind that you do not have to follow these steps in numerical order. In many cases you may need to cycle through steps 2,3, and 4 many times prior to arriving at a final model and solution. Also it is important to check your results against your assumptions, e.g., if your solution violates your assumptions something needs to be changed.

Use your group to help you with ideas and to read your work. If you need further consultation find someone else to discuss with as well. The goal is to develop the best model and most complete solution given the time and background that you have.

ARISE-specific Topics

(Ed. Will include topics directly suggested as extensions in various ARISE units - The re-write authors should suggest topics as they go through the units)

Project Topics

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Project Topics

1. Analyze the power structure in some political body (US or otherwise).
2. Determine a "fair" way to select the best college (or senior trip site, or ....).
3. How should School Honors (Homecoming Court, Prom, Class officers, etc.) be elected?
4. When will Olympic records level off?
5. What measurements need to be taken, and how should they be used, in order to be able to convert "counter numbers" from one tape player to another, and in to regular times.
6. How can the "roughness" of a shoreline be measured?
7. What is the most efficient shape for a can (or box)?
8. What is the optimal reordering policy for a company which maintains some inventory?
9. What should be my insurance (auto or life) premium?
10. How can the cafeteria avoid running out of food at lunch time?
11. Can false data be used to achieve desired results?
12. How much should companies charge for "pro rata" extended warranties?
13. How much does 40 pounds of bananas weigh?
14. Do a product comparison (experimental design, test of significance, taste testetc.)
15. By how much should airlines overbook flights?
16. Create an index (CPI, leading indicators, trauma score, poverty level, etc.)
17. How do jury size and conviction-vote rules affect conviction/acquittal/hung jury rates?
18. Why is the World Series, Stanley Cup, or NBA finals a "best of seven" tournament? How "discriminating" is this method of playoff?

19. How are "streaks" related to probability of success? Develop a method of predicting a team's final win/loss percentage based on its streaks in the first two months of play.

20. Under what conditions is it profitable for a copy center to seek more business?

21. What is the expected length of the World Series, NBA finals, or the Stanley Cup?

22. How many acceptance letters should a college send out during "admissions season?" How does "rolling admissions" affect your conclusions?

23. What is a reasonable description of the pattern of growth in world population?

24. How much should I save for retirement each pay period?

25. Some people invest in mortgages. How should such investors determine the fair price of a "bundle" of mortgages to reflect early payoffs as rates fall?

26. Provide both spatial and temporal analyses of the "beat" heard when two notes are not quite in tune. (What about "phase problems" associated with two sources for the same frequency -- say, from stereo speakers?)

27. Is strength (velocity of release) or form (angle of release) more important for success in the shot put event?

28. Given a map showing the locations of all gas stations and the prices each one charges, how should I determine the best station at which to buy gas?

29. Is it possible for a softball umpire to call balls and strikes accurately by observing only the point at which the catcher catches the ball?

30. Develop the theory behind "magic numbers" in sports standings.

31. How much fuel should be loaded onto an airplane?

32. How should "old" English-unit track and field records be calibrated with "modern" metric-unit records?

33. Is a college education worth the cost?

34. How far behind the car in front of you should you drive? If you observe that spacing, at what speed is traffic flow most efficient?

35. Examine the wagering strategies for Final Jeopardy.

36. What is the most efficient use of parking space at your school?

37. What is the best point from which a soccer player should shoot? Where should the goalkeeper be positioned?

38. Tall tennis players tend to serve more hard serves and fewer spin serves than do short players. Examine the geometry of such serves and servers. What effect does position along the baseline have?

39. Examine the traffic flow control for the Suez (or some other) Canal, and explain how similar logic can be applied to auto traffic in an area undergoing re-paving.

40. How should aircraft be queued to take off?

41. How far apart and how high should lights be hung to provide the most uniform illumination in a room (or along a highway)?

42. How wide should a creek be "channeled" to prevent flooding? How is your answer affected if a dike is to be formed from the dredged bank soil?

43. Should you walk or run when out in the rain?

44. Suppose that when you first see a traffic light ahead of you, it is already red. If there are no other vehicles ahead of you, under what conditions should you begin stopping?

45. Compare the "one line, several servers" of banks and airline baggage check-in to the "many lines, many servers" of McDonald's and K-mart.

46. How can the flow of traffic on the school campus best be controlled? Should "peak periods" be treated in the same manner as "regular hours?"

47. Make recommendations for improving your school's lunch line situation.

48. How should heating or air conditioning be "cycled" in order to reduce costs?

49. When should the air conditioning system for the auditorium (gym) be turned on for an assembly? What about for special nights when the auditorium (gym) is used?

50. Examine the effects of temperature, concentration, etc. on chemical reactions.

51. The most common form of radiation pollution in homes has a very short half-life. How can its concentration be measured accurately if it decays before it can reach the lab to be tested?

52. How can radioactive decay be used to detect art forgeries?

53. What strategy should auto manufacturers adopt in order to meet mileage requirements?

54. What policy should society have toward the consumption of nonrenewable resources such as oil, natural gas, etc.?

55. What should your county or state do about landfills and incinerators?

56. Investigate the greenhouse effect and global warming.

57. How can prisoners' sentences be determined to help relieve overcrowding?

58. Should your state (or the federal government) build more prison space? How much?

59. Profile the demographic changes likely to occur in the United States or your city.

60. Examine the teacher shortage, its future and possible solutions.

61. In what condition should you expect to find the Social Security System by the time you are old enough to retire?

62. Investigate the interaction of two interdependent species in an ecology (predator-prey, competitive hunter, symbiotic, etc.).

63. Examine the wisdom of past human intervention in ecosystems (Sahel, Kabib, etc.).

64. Develop a model of military conflict.

65. Examine the dynamics of blood doping or carbohydrate loading.

66. When should vaccinations for a deadly disease (such as smallpox) be discontinued?

67. How should the government spend its AIDS budget?

68. Explain an economic theory.

69. Model a portion of the economy. (For example, what are the implications of a capital gains tax reduction?)

70. How much should your locale (city, county, etc.) charge for water?

71. Under what conditions does automobile traffic flow smoothly?

72. Explain why cars typically "start off" twice when a traffic light turns green from red. How do the two "waves" of starts propagate through the line of traffic?

73. Plan (schedule) an event (CPM, bin packing, etc.).
74. How long should a traffic signal stay yellow?
75. What is a fair policy for admitting "legacies" to a college?
76. Should (select a road of your choice) be widened?
77. What are fair prices for medical services?
78. How should your state allocate its funds for education?
79. How can the Secretary of State certify a third party candidate's petition for being placed on the ballot?
80. What are the effects of a gas embargo? What economic policy should the government adopt in such a situation?
81. Is it better to lease or buy a car? If you buy, is it better to take out a loan or pay cash? Under what conditions do your answers change?
82. How large and how often should drug doses be in order to maintain both safe and effective results? (Could be modified to consider the effects of alcohol or some other illegal drug.)
83. Examine the dynamics of CFC's and ozone depletion in light of the Montreal Protocol.
84. How should a large set of blood samples be partitioned in order to minimize the expected number of tests which need to be performed?
85. How many chips are there in a Chips Ahoy cookie?
86. What was wrong with the proposed First Amendment to the US Constitution?
87. How does the distance separating "corresponding edges" of a piece of wrapping paper after wrapping a poorly aligned gift depend on the "crookedness" of a package on the (flat) paper?
88. When is the hottest part of the day?

SAMPLE MODEL: MINIMIZING ELEVATOR TIME

THE PROBLEM:

Poor elevator service in an office building. The manager is unwilling to construct additional elevators, being convinced that the problem can be resolved through better scheduling.

FACTS:

- There are five working floors and an unused ground floor.
- 80 people work on each floor.
- There are four elevators in the building.
- Each elevator can hold 10 people.
- Elevators take 22 seconds to load or unload.
- Elevators take 3 seconds to travel between floors.

ASSUMPTIONS:

- Each elevator will carry its capacity (10 people) each trip.
- No elevator will pick up or deliver on its way down.
- All people arrive at the office building at about the same time and will continuously fill the elevators.

PURPOSE:

- To minimize the amount of time it takes the four elevators to deliver the 400 people to their respective floors.

SOLUTION:

- If each elevator stops at each floor, four elevators can deliver 400 people if each elevator carries its capacity 10 times.
- Every stop takes 22 seconds. An elevator must stop 6 times (including initial loading) when it travels to all floors.
- Travel time between successive floors is 3 seconds. Thus, a two-way trip takes 30 seconds of travel time, plus its stopping time. Therefore, 10 round-trips for each elevator would take it

  $((22 \times 6) + (30)) \times 10 = 1620$ sec., or 27 minutes.

Fewer stops would decrease this time. To make the fewest stops, each elevator would make only two stops per trip. Since there are not enough elevators to do this, a different schedule should be implemented.

Require all four elevators to make two trips to the first floor, thus delivering the 80 people in two loads of forty people each. Repeat this process for each floor.

Each trip takes 22 seconds to load and 22 seconds to unload, plus travel time up and down at 3 seconds per floor each way. Therefore the time taken to deliver two round trips to floor X is expressed by the formula $2(44 + 6X)$, or $88 + 12X$. So total time looks like:

Floor 1	88 + 12
Floor 2	88 + 24
Floor 3	88 + 36
Floor 4	88 + 48
Floor 5	88 + 60
Total	440 + 180 = 620 sec. or 10 min 20 sec.

While this more than halves the amount of time, this method is very impractical. A schedule would be needed, and a person missing his elevator would not be able to catch one going to his floor until after morning rush hour. This method would also require an elevator attendant to ensure smooth operation of the elevator scheduling.

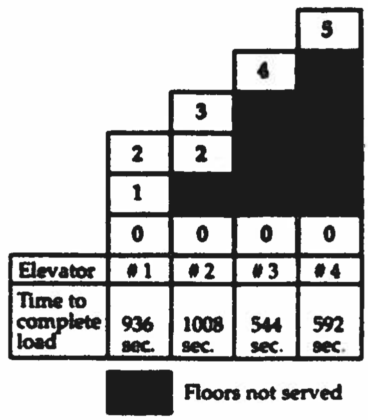
A more practical solution can be realized without confusion or scheduling. Perhaps the fastest way to transport the people to their destinations would be to dedicate elevators for each floor. However, since elevators cannot be added, at least one elevator would have to carry people to two floors. Different solutions were tried.

For stops at adjacent floors, where X is the floor of the first (lower) stop, the following times apply:

Initial loading	22 seconds
Trip to floor	$3x$
Unloading at first stop	22
Trip to next floor	3
Unloading at second stop	22
Trip back	$3(x + 1)$
Equation for time	$6x + 72$
Total time for one round trip	$6x + 72$

With floors 5, 4, and 3 having their own elevators, and floors 1 and 2 sharing an elevator, the minimum amount of time necessary to transport all the people to their respective destinations was found to be 1248 sec. or 20.8 minutes. This is the lowest time for the sharing of one elevator by two floors.

FINAL SOLUTION:



A better solution is to have two elevators (instead of only one) take the load of the extra floor. After trying other combinations, it was determined that elevator 1 should serve floors 1 and 2; elevator 2 should serve floors 2 and 3. Floors 4 and 5 would have their own elevators.

Recall that the time taken for one round trip to floor X is given by $44 + 6X$. Therefore, elevator 4 takes 592 seconds to carry the 80 people to floor 5:

$44 + 6(5) = 74$ seconds per round trip.

74×8 trips $= 592$ seconds.

Likewise, elevator 3 takes 544 seconds to carry the 80 people to floor 4:

$44 + 6(4) = 68$ seconds per round trip, for $68 \times 8 = 544$ seconds.

Assume that elevator 2 carries $\frac{1}{2}$ of the people from floor 2 and all of the people from floor 3. Recall that the time taken to deliver on adjacent floors (X and $X + 1$) is given by $6X + 72$.

Thus, elevator 2 takes 1008 seconds to carry the 120 people to floors 2 and 3:

$6(2) + 72 = 84$ per round trip, and $84 \times 12 = 1008$ seconds. (There are 12 trips—80 people to 3, 40 to 2.)

In the same fashion, elevator 1 carries $\frac{1}{2}$ of the people from floor 2 and all of the people to floor 1. Therefore, elevator 1 takes 936 seconds to carry the 120 people to floors 1 and 2:

$6(1) + 72 = 78$; $78 \times 12 = 936$ seconds.

The total time for transporting all the people to their offices using this scheme is 1008 seconds (16.8 minutes), since this is the largest amount of time any given elevator takes to deliver its full load.

IN CONCLUSION:

The final solution shaves nearly 10 minutes off of the original scenario. The first solution was even better but entails too many constraints and could cause major confusion. The second solution is much more practical and fool-proof but assumes that one half of the people going to the second floor use elevator 1 and the other half use elevator 2. This assumption is reasonable since passengers headed for floor 2 will always fill an empty elevator. Even if elevator 2 must carry all 160 people to the second and third floors, this solution is faster than the original scenario by about 5 minutes.

Further Note: If elevator 3 converts, at the end of the first 8 round-trips, to taking the same route as elevator 2 and elevator 4 converts to the same route as elevator 1, it is possible to cut this time down to 826 seconds or 13 minutes 46 seconds. However, this may not be a practical solution, since people may get confused as to where the elevators are going.

SAMPLE MODEL: PASTURELAND

THE PROBLEM

A rancher has a prize bull and some cows on his ranch. He has a large area for pasture that includes a stream running along one edge. He must divide the pasture into two regions, one region large enough for the cows and a smaller region to hold the bull. The cows' grazing pasture must be at least 10,000 sq. feet, and the bull's grazing pasture must be at least 1000 sq. feet. The shape of the pasture is basically a rectangle measuring 120 feet by 150 feet. The stream runs all the way along the 120-foot side. Fencing costs $5/foot and each fence post costs $10. Any straight edge of fence requires a post every 20 feet, and any curved length of fence requires a post every 10 feet. Help this rancher minimize the total cost of fencing.

CONSIDERATIONS AND ASSUMPTIONS

I first looked at the overall area of the pasture to see if there was enough land available for both the bull and cows. Assuming a perfect rectangle, the area of 18,000 square feet in the pasture will hold both.

I next considered the stream. If this stream is not traversible by the cattle, I could use the bank of the stream as part of the fence. However, since the term stream was used instead of the term river, I made the assumption that this was not the case. I therefore assume that I have to fence all sides of the pastures both for the bull and for the cows.

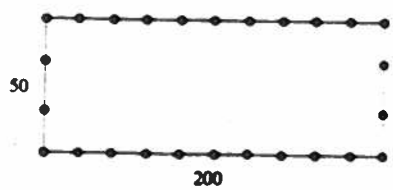
I next considered combining some of the fencing—that is, enclosing one within the other or using a common side. This would save some of the fencing and some of the posts. As I considered this, I realized that cows and bulls need to be separated by more than just a fence. They need some land between them as well. If not, the bull would tear the fence down to get to the cows. If this were not an issue, it would be possible to include all the cattle in a fenced area of 11,000 square feet and not worry about separate pens.

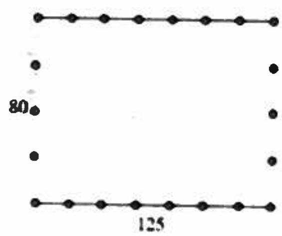
To summarize my assumptions:

1. I am assuming that all sides of each of the pens will need fencing.
2. I am assuming that there will be no common fence. That is, there will be two separate pens.
3. I am assuming that I will enclose the smallest amounts of pasture necessary both for the cows and for the bull (≥1000 for the bull and ≥10000 for the cows) consistent with assumption #4.

4. I will round all side measurements up to the next foot. This is to make sure I have enough area enclosed in the pens and to make measurements simpler.

Given these assumptions, I began to look at different shapes that might be used for the two pens. I wanted to see if there were some shapes that were cheaper than others to build. I decided to limit my examination to 3 different types of figures—a rectangle, a triangle, and a circle. I hoped that by examining these figures one would emerge as a distinct winner in reducing the cost of building the pens.





RECTANGLE

I first looked at a rectangle, assuming an area of 10,000 square feet. I set up a spreadsheet that would examine different dimensions to see if one was cheaper to build. I varied one side in increments of 10 feet to see if a pattern emerged. It was only necessary to go from 10 to 100, since the measurements of the sides would just begin reversing after this. The number of posts needed along one side of length L, including *one* corner, is given by $\lceil L/20 \rceil$. The formula that I used on the spreadsheet to calculate the value in row 5 of column 5 (column 6 was similar) was –Int(–B5/20). The fencing cost is calculated by multiplying $5 by the perimeter, and the post cost is found by multiplying $10 times the number of posts.

Cow pen 10,000							
Rectangle area	Short side	Long side	Perimeter	Posts	Fencing costs	Post cost	Total cost
10,000	10	1000	2020	102	$10,100.00	$1020.00	$11,120.00
10,000	20	500	1040	52	$5200.00	$520.00	$5720.00
10,020	30	334	728	38	$3640.00	$380.00	$4020.00
10,000	40	250	580	30	$2900.00	$300.00	$3200.00
10,000	50	200	500	26	$2500.00	$260.00	$2760.00
10,020	60	167	454	24	$2270.00	$240.00	$2510.00
10,010	70	143	426	24	$2130.00	$240.00	$2370.00
10,000	80	125	410	22	$2050.00	$220.00	$2270.00
10,080	90	112	404	22	$2020.00	$220.00	$2240.00
10,000	100	100	400	20	$2000.00	$200.00	$2200.00

The cost seems lowest when the figure created is a square. I next checked the bull pen for the rectangular configuration.

Bull pen 1000							
Rectangle area	Short side	Long side	Perimeter	Posts	Fencing costs	Post cost	Total cost
1000	5	200	410	12	$2050.00	$120.00	$1220.00
1000	10	100	220	10	$1100.00	$100.00	$920.00
1005	15	67	164	8	$820.00	$80.00	$780.00
1000	20	50	140	8	$700.00	$80.00	$730.00
1000	25	40	130	8	$650.00	$80.00	$720.00
1020	30	34	128	8	$640.00	$80.00	$720.00
1023	31	33	128	8	$640.00	$80.00	$720.00
1024	32	32	128	8	$640.00	$80.00	$720.00
1023	33	31	128	8	$640.00	$80.00	$720.00
1020	34	30	128	8	$640.00	$80.00	$720.00
1015	35	29	128	8	$640.00	$80.00	$720.00
1008	36	28	128	8	$640.00	$80.00	$720.00
1036	37	28	130	8	$650.00	$80.00	$720.00
1026	38	27	130	8	$650.00	$80.00	$730.00

Note that a number of different dimensions lead to exactly the same costs for the bull pen. Therefore, I selected the one that uses the most area for the lowest cost, since I want to have enough pasture to feed the bull.

I needed to check to see if these two figures (a 100 x 100 pen and a 32 x 32 pen) would fit into the same 120 x 150 pastureland and still fit the assumptions that I made. When I looked at this, I was able to place both in the pastureland and have nothing in common if that were desired. It would be possible to reduce the cost by one post by having the pens share a corner. However, since there was no clear reason for doing this, I decided to keep the pens separate.

My conclusion, therefore, is that if I use two rectangles, the estimated cost for creating the two pens would be $2200 + $720 = $2920.

CIRCLE

I next looked at producing a pair of circular pens.

Circle	Total area	Radius	Circumference	# of posts	Fencing costs	Post cost	Total cost
Cows	10,000	57	358	36	$1790.00	$360.00	$2150.00
Area used		10,207					
Bull	1000	18	113	12	$565.00	$120.00	$685.00
Area used		1017				**Total of both**	$2835.00

These values are based on rounding the radius to the next highest integer value to guarantee I have at least 10,000 and 1000 square feet in the pens. The next question I asked was: Is it possible to create these two pens in the same pastureland? A circle of radius 57 feet can be enclosed in a square of 114 feet on a side. A circle of radius 18 feet can be enclosed in a square with 36-foot sides. Therefore, both should fit within a rectangle with measurements of 120 by 150. Depending on how the radii are located, there may be one location where the two circles touch. However, since I rounded up for the radii, I should be able to shorten one, or both, slightly so that this is not a problem.

My conclusion is that if I use two circles, the estimated cost for creating the two pens would be $2150 + $685 = $2835.

TRIANGLE

For this part, since the square gave me the best answer on the rectangle part, I will assume that an equilateral triangle will be the best answer when calculating triangle results.

Triangle	Side	Perimeter	Height	Actual area	Posts	Total
Equilateral total area 10,000						
	152	456	132	10,004		$2280.00
Posts	8	24			24	$240.00
					Cow	$2520.00
Equilateral total area 1000						
	49	147	43	1039		$735.00
Posts	3	9			9	$90.00
					Bull	$825.0
					Total	$3345.00

The large triangle can be enclosed in rectangle of 152 x 132 feet. This becomes a problem since the shorter side of the pasture is only 120 feet long. There may be a possibility of fitting other types of triangles into this region. However, if my assumption is correct—that the least expensive of all triangles is equilateral and the equilateral triangle costs $3345—then there is no sense in checking other types since this is already more than the cost of a rectangular or circular configuration.

CONCLUSION

Based on the assumptions made and the calculations performed, the least expensive way to create these two pens is to use two circles, one with a radius of approximately 57 feet and the other with a radius of approximately 18 feet. The cost for this would be around $2835.00. It seems clear that if you are using two separate pens, that the least expensive way to construct them is to use circles, given the three choices I selected.

VARIATIONS

If the cattle cannot cross the stream, it would be possible to use the stream as one of the fences. This would reduce the overall cost and would require recalculating the totals.

If the bull would not break through a shared fence, it would be possible to reduce the costs by using one or more common sides. This would allow a reduction of costs and would allow you to examine a combination of figures.

Although this analysis used separate spreadsheet calculations, it is possible to create functions that describe the different costs based on the area of the overall pasture acreage and side or radius. Graphing these cost equations could help approximate the cheapest cost for producing the two pens. Finding such functions would allow you to use this process for differing initial scenarios, and continuous variations in sides and radii.

SAMPLE MODEL: FACULTY

THE PROBLEM

When confronted with a rise of 142 students in a school of 480, and a capacity for 7 new teachers, in what departments should the new teachers be placed? Placing the new teachers should maintain the ideal student-to-teacher ratio. The current makeup of the (student:teacher) enrollment in each department is: Art (99:1); Biology (319:4); Chemistry (294:3); English (480:5); French (122:1); German (51:1); Spanish (110:1); Mathematics (613:6); Music (95:1); Physics (291:3); and Social Studies (363:4).

INVESTIGATIONS INTO THE SCHEDULING PROBLEM

The initial assumptions that we made for this problem were:

1. No departments will have a larger student:teacher ratio than 125:1.
2. The new students will be distributed along the same ratios that currently exist. That is, for example, approximately the same percentage of the new students will take mathematics as the current percentage.
3. Each new enrollee will take 6 classes.
4. It is clear that this school allows students to take more than one class in a department (math 613) so we will allow the new students this same option.
5. When there is a fractional student increase we will determine whether it makes sense to add an additional student.

The existing population (student and teacher) is listed in the table below.

Subjects	Current enrollment	Number of teachers	Student/teacher ratio	Overall % enrollment/total classes
Art	99	1	99.000	3.49%
Biology	319	4	79.750	11.24%
Chemistry	294	3	98.000	10.36%
English	480	5	96.000	16.92%
French	122	1	122.000	4.30%
German	51	1	51.000	1.80%
Spanish	110	1	110.000	3.88%
Math	613	6	102.167	21.61%
Music	95	1	95.000	3.35%
Physics	291	3	97.000	10.26%
Social Studies	363	4	90.750	12.80%
Totals	2837	30	94.567	

We looked at where these new students will go in relation to the existing population. Using the current overall percentages for each department (3.49% for art, times the 852 new student hours, for example), we arrived at the following set-up for each department. (New hours were rounded to the nearest integer.)

Subjects	Current enrollment	Percent enrollment	New student hours	New student hours (rounded)	Total student hours
Art	99	3.49%	29.735	30	129
Biology	319	11.24%	95.765	96	415
Chemistry	294	10.36%	88.267	88	382
English	480	16.92%	144.158	144	624
French	122	4.30%	36.636	37	159
German	51	1.80%	15.336	15	66
Spanish	110	3.88%	33.058	33	143
Math	613	21.61%	184.117	184	797
Music	95	3.35%	28.542	29	124
Physics	291	10.26%	87.415	87	378
Social Studies	363	12.80%	109.056	109	472
Totals	2837	100%	852.085	852	

We now need to determine in which departments additional teachers are needed, and how many. To do this, we first looked at what would happen if we didn't increase the total number of teachers.

Subjects	Total student hours	Number of teachers	Student hour per teacher
Art	129	1	129
Biology	415	4	103.75
Chemistry	382	3	127.3333333
English	624	5	124.8
French	159	1	159
German	66	1	66
Spanish	143	1	143
Math	797	6	132.8333333
Music	124	1	124
Physics	378	3	126
Social Studies	472	4	118
Totals	3689	30	122.9666667

We then set up a spreadsheet and began to add teachers in various ways. The things that we were looking at were fairness and balance. Balance was a bit easier to observe since we just had to look to see if the ratios were close to each other. Fairness was a bit more difficult since we knew we could never achieve perfect balance and have all departments equally happy. We started by looking at those departments with student hours per teacher higher than the school-wide average (Art, Chemistry, English, French, Spanish, Math, Music, and Physics). Since the alloment of new teachers is too small to permit adding one to each of these departments, we decided to add one teacher to each of these departments that already had more than one teacher.

Subjects	Total student hours	Number of teachers	Student hours per teacher	Add	Student hours per teacher
Art	129	1	129		129
Biology	415	4	103.75		103.75
Chemistry	382	3	127.3333333	1	95.5
English	624	5	124.8	1	104
French	159	1	159		159
German	66	1	66		66
Spanish	143	1	143		143
Math	797	6	132.8333333	1	113.8571429
Music	124	1	124		124
Physics	378	3	126	1	94.5
Social Studies	472	4	118		118
Totals	3689	30		4	

We had 3 more teachers to add. Examining the new Hours per teacher ratios, we decided to begin with the highest and assign the remaining teachers. Therefore French, Spanish, and Art will get the last 3 teachers. The results are as follows:

Subjects	Total student hours	Number of teachers	Student hours per teacher	Add	Student hours per teacher
Art	129	1	129.000	1	64.500
Biology	415	4	103.750		103.750
Chemistry	382	3	127.333	1	95.500
English	624	5	124.800	1	104.000
French	159	1	159.000	1	79.500
German	66	1	66.000		66.000
Spanish	143	1	143.000	1	71.500
Math	797	6	132.833	1	113.857
Music	124	1	124.000		124.000
Physics	378	3	126.000	1	94.500
Social Studies	472	4	118.000		118.000
Totals	3689	30		7	

Obviously, there will be some departments not satisfied with the results. Social Studies, for instance, increases enrollment by over 100 student hours and gets no additional teacher.

CONCLUSIONS

Using the assumptions that we listed at the beginning, the 7 teachers hired should be in Art, Chemistry, English, French, Spanish, Math, and Physics. We did wonder why Physical Education wasn't listed among the departments.

VARIATIONS

If it were possible to hire a teacher who could teach in more than one department, it would open up many more options. It then might be necessary to look at individual courses. If Physical Education or some other department were added, it would also be necessary to staff it.

SAMPLE MODEL: FLU EPIDEMIC

THE PROBLEM

A certain population of 20,000 people is hit by a strain of flu. One percent of the susceptible population is stricken each day. The flu effects last 5 days, after which the person is then immune to this virus. Ten percent of the population is naturally immune to the virus.

a) If the epidemic were allowed to run its course, how long would it take for the entire population to become immune?

b) The Health Service would like to institute an immunization program in an attempt to knock out the epidemic quickly—within a month, if possible. However, they would also like to inoculate as few people as possible because the serum does occasionally make people feel sick. Devise an immunization procedure that will meet these guidelines.

ASSUMPTIONS

- There are 30 days in a month.
- Once infected, a person is no longer part of the susceptible population.
- Getting sick from the inoculation is no different than getting sick from having the disease.
- Immunity from inoculation is effective immediately.
- No inoculations will be given to people who already have the disease.
- Inoculations occur at the start of each day, before any new infections for that day.
- There is no limit to the supply of vaccine or of staff to administer it.
- Calculations will be done without intermediate rounding.

SOLUTION

a) If the epidemic were allowed to run its course, how long would it take for the entire population to become immune?

10% immune => $0.10 \times 20{,}000 = 2000$ people are immune initially

=> $20{,}000 - 2000 = 18{,}000$ are susceptible

If 1% of the susceptible population is infected each day, then 99% of the susceptible population is still susceptible at the end of the day.

$18{,}000(0.99)^n$ people are still susceptible after n days (assuming no intermediate rounding).

We decided to find the day on which $18{,}000\,(0.99)^n < 1$. Using a graphing calculator, we entered $18000\,(0.99)^x$ as one function and 1 as another function. We used the window *xmin* = 500, *xmax* = 2000, *ymin* = 0, *ymax* = 118 (since $18{,}000\,(0.99)^{500}$ is around 118) and calculated the intersection of these two functions. We found it to be around $x = 974.9$ and $y = 1$. Logs confirmed that result. This means that after around 975 days, the entire population has either gotten the flu or is immune. Five days after that 980 days, the flu should have run its course.

[Using $18{,}000\,(0.99)^n < 0.5$, instead, gives a duration of about 1049 days. In light of this wide range of reasonable values, together with the uncertainty of how rounding and randomness affect real results, perhaps the best answer here is "about 1000 days." That's almost 3 years!]

b) The Health Service would like to institute an immunization program in an attempt to knock out the epidemic quickly, within a month, if possible. However, they would also like to inoculate as few people as possible because the serum does occasionally make people feel sick.

Initially, 2000 people are immune, 18,000 people are susceptible.

$18{,}000(0.99)^n$ people are still susceptible after n days => in 30 days, the number of susceptible people will be $18{,}000(0.99)^{30} = 13{,}315$.

If the flu runs unchallenged, there will be 13,315 people to inoculate on the 30th day.

If the inoculations were evenly spaced over the month, what number of inoculations would result in 0 people being susceptible at the end of day 30?

Using a spreadsheet and trial and error, we found that by inoculating 511 people on days 1 through 29 and by inoculating 530 people on day 30, all would have either contracted the flu or be immune to it. The spreadsheet calculates the values in the following way:

- Immune—Sum of the previous immune, the number inoculated, and the number that contracted the flu.
- Number Inoculated—this was the explanatory variable. We changed this value to try to get to 0 contracting after 30 days.
- Number Contracting—1% of the previous day's Number Susceptible.
- Number Susceptible—18,000 minus the sum of the Immune, Inoculated, and Contracting.

Day	Number of immune	Number of inoculated	Number contracting	Number susceptible
0	2000	0	0	18,000
1	2000	511	180	17,309
2	2691	511	173	16,625
3	3375	511	166	15,948
4	4052	511	159	15,277
5	4723	511	153	14,613
6	5387	511	146	13,956
7	6044	511	140	13,306
8	6694	511	133	12,662
9	7338	511	127	12,024
10	7976	511	120	11,393
11	8607	511	114	10,768
12	9232	511	108	10,149
13	9851	511	101	9537
14	10,463	511	95	8930
15	11,070	511	89	8330
16	11,670	511	83	7736
17	12,264	511	77	7147
18	12,853	511	71	6565
19	13,435	511	66	5988
20	14,012	511	60	5417
21	14,583	511	54	4852
22	15,148	511	49	4293
23	15,707	511	43	3739
24	16,261	511	37	3190
25	16,810	511	32	2647
26	17,353	511	26	2110
27	17,890	511	21	1578
28	18,422	511	16	1051
29	18,949	511	11	530
30	19,470	530	0	0
		15,349	2651	

CONCLUSIONS

The model that we created indicates that we need to inoculate a total of 15,349 over 30 days, and that 2651 people will contract the flu during that time.

VARIATIONS

Since we know that our model predicts 2651 people contracting the flu and 15,349 getting the vaccine, we could try to inoculate a larger group to start with and then reduce this number further into the month. This would be interesting for further investigation.

Assessment

Because of the subjective nature of a mathematical model, it may be difficult for some teachers to evaluate the work which their students have done. As Henry Pollak said, "The criteria by which success in real-world mathematical problem solving must be judged will be in part non-mathematical. This fact will inevitably be difficult for some mathematicians and mathematics educators to accept." Not only may that be hard to accept, but it also may be hard to assess. As has been stated previously, a mathematical model is as good as its justification. We especially appreciate creative and sophisticated models, but ultimately we are most interested in whether the model is mathematically correct and has a reasonable interpretation in the real world based on the assumptions initially stated.

Some teachers break down the modeling assignments into components such as a written and oral report. A teacher arrived at a grade which was weighted so that 50% of the grade came from the quality of the model itself, 25% from the paper, and 25% from the oral presentation. One teacher has found that students were especially responsive to doing and explaining "posters" rather than formal reports. Some teachers assign grades at intermediate steps before a final product. [Due dates for a topic, bibliography, outline, draft, and final project are helpful to students and usually yield better results.]

Some teachers may be comfortable with assigning letter grades. You may find that most students receive grades of A or B; this is not unusual. The exception might be those students who obviously did not exert sufficient effort, ignored some important factors or assumptions, incorrectly did mathematics, or did a poor job of interpreting the results in a real-world context.

Some teachers may be more comfortable using a quantifiable scale rather than letter grades. Each teacher will wish to develop his or her own method of grading. For those teachers who wish to consult specific methods used by others, the appendix includes samples of grade sheets and criteria. Science teachers (with lab reports) and history teachers (with research papers) are also good resources to consult about grading final papers.

T7.3

Sample Assessment Form

Title: ________________________ Modeling Assessment

Topic	Points Possible	Points Awarded	Comments
Analysis/Understanding of Problem: Does the modeler indicate a clear and thorough understanding of the contextual situation? Is the problem well formulated?	2		
Reasonability of Assumptions/Factors: Does the modeler make reasonable assumptions? Are the factors that are included those that are most significant in the problem?	2		
Mathematical Accuracy: Is all the mathematics included in the problem accurate? Is it consistent with the assumptions made in step 2? Does it follow from the assumptions?	4		
Completeness: How complete is the result? Are appropriate visual representations (graphs, tables, diagrams) used? Did the modeler return to check the results with the contextual problem?	2		
Presentation (Written and Oral): Was the presentation clear, coherent, and complete?	2		
Total	12		

Appendix A

One teacher asked each student to fill out the following grade sheet for each group which did a presentation:

Title:		
Team Members:	Points	Scale
clarity		(4-10)
teamwork		(4-10)
visual		(10-25)
accuracy		(4-10)
style		(1-5)
Comments:		

This accounted for 60% (60 points) of the modeling grade.

Each student was also asked to evaluate his or her team members (4-10 points or 10% of the grade), for each member including self. Contribution to the group, cooperation, and cohesion of the group were to be considered.

All of the above was averaged. The teacher felt more confident about assigning a grade which included peer assessment.

The teacher alone evaluated the written report based on the following scale:

Title:		
Team Members:	Points	Scale
~clarity		(4-10)
style (grammar, etc.)		(1-5)
completeness		(1-5)
accuracy		(4-10)
Comments:		

Appendix B

One teacher asks students to evaluate each other's oral presentations by filling out the following grade sheet. Each criterion is graded on a scale of 0 to 4, and an average is achieved in each category.

Mathematical Modeling
Review of Oral Project Presentation

Presenter(s):________________________________ Date: ________

Topic: ________________________________

[0,4] Average TEAMWORK: ______

COMPLETENESS / CORRECTNESS:
Background: ______
Objectives: ______
Sample Selection or Assumptions and Extensions: ______
Method of Analysis: ______
Conclusions: ______
______

DELIVERY:
No read: ______
Organization: ______
Duration: ______
Formality: ______
Clarity: ______
______

VISUAL AIDS:
Neatness: ______
Propriety: ______
______

QUESTION / ANSWER: ______
Average: ______
Grade:(0=55, 3.5=90, 4=100) ______
Comments: ________________________________

Here is one teacher's directions to the schedule of deadlines for various parts of the modeling and project process.

Term Project Timeline

Proposal	Friday
Progress Report	2 weeks from Friday
Final Written Report	4 weeks from Friday
Oral Report (Optional)...............	TBA

Proposals

By no later than Friday select one of the topics listed under the Second Semester heading of the Topics List on which you would like to work. You may have some ideas about things which sound interesting to you; that's the most important quality of a good topic. (Some topics listed as First Semester topics may also be appropriate, but these must be cleared in advance.) You are encouraged to discuss with me any aspect of any potential project you wish, whether it's the mathematics, the setting, or some other feature.

Specifically, the Proposal should contain an introduction to the problem, laying out the background information needed to understand and appreciate the problem, a clear and precise statement of one or more questions for which you will seek answers, and an indication of your anticipated main lines of attack (the kind of mathematics you expect to be doing). In short, let me know where you are going, and why, and how.

Include in your background information a clear statement of why the problem is of interest to someone else, including to whom it is of interest.

NOTES:

The deadline of Friday is the last day you may submit your proposal. This does not mean it is the only day you may turn it in!! I will be happy to help you revise ideas at any time prior to the deadline, but please put your ideas into written form as clearly as possible first.

The proposal will be graded and will be entered into the gradebook as two (2) quizzes. Only the most recent version, as of Friday, will count. I do not grade by the pound; longer is not necessarily better. What matters is clarity and completeness. If a friend of yours can not understand what you have in mind -- just from what you have written -- then it needs more work.

You may elect to work with a partner on your project, but I must approve of the partnership in advance. If you elect this option, each partner must understand the complete dependence of each on the other -- no freeloading! You must submit, as part of your final written report, a statement detailing the division of labor throughout the term. These "job descriptions" will affect your grade.

Progress Report

No later than two weeks from Friday, you must submit a first draft of your project report. While I do not expect it to be entirely complete at that time, it should be substantially under way and not just a beginning.

The same information that was required in the proposal should again be included. By this point, the problem statement should be refined to a very precise form. Significant progress should have been made in the actual solution, though solutions need not be entirely complete. However, if the solution is not finished, clearly indicate what remains to be done and how it will be done.

To repeat, this should be a draft of your project, not just a list of things you have done and things you plan to do.

In addition to the written progress report, all groups will be required to meet with me at regularly scheduled times so that I may observe the group in action. Of course, individuals may also come by to discuss ideas and questions, but I will not schedule such meetings.

Keep a regular log in your journal of all project work. Include attempts, progress, questions, and feelings. This is not only to help you be able to look back on the process and learn from it but also to allow you to ask me questions and get answers in writing when you don't have time to come talk. Both functions are important.

NOTE:

The progress report will be graded and will count as one major test. Again, the standard to be applied is clarity and completeness, not length.

Written Report

The final written report is due 4 weeks from Friday. It must be typed and conform to the project guidelines which you received on the first day of class. The written report will comprise 50% of your final exam grade.

Oral Report

Oral reports will be optional and will be scheduled during the last two weeks of regular class time. Each report will be permitted 20 minutes and should be organized in the same manner as the written report. Visual aids are recommended but not required. Explanation, of both the problem and the solution, is the goal. The oral report will not be graded.

Another Model for Assessment

Select the group that you wish to work with and the topic you wish to work on. I am expecting some great work on these projects! I will allow you a maximum of two days per week in-class time to work on these. The rest of the time needs to be spent outside of the class.

Assessment of your work will be made up of the following components:

40%	Written paper	Evaluated by me
30%	Oral presentation	Evaluated by classmates and visiting instruc
30%	Group Participation	Evaluated by yourself and others of your gr
10%	Audience Behavior/Participation	Evaluated by me.

In the written paper, I will be looking for accuracy, logic, and presentation. In the oral presentation, you will be evaluated, by the class and visitors, on participation, clarity of purpose, and presence. The group assessment will be an evaluation by all your group of your participation within the group structure, relative to the other members of your group. **Please note**: I will be evaluating the audience during presentations. You may get an additional 10% for positive participation during the presentations.

Model Oral Participation Assessment
(Use a Scale of 1-10, 10 being the best)

Names	Presentation	Clarity	Solution	Questions
Total				
Overall Average				

Names	Presentation	Clarity	Solution	Questions
Total				
Overall Average				

Names	Presentation	Clarity	Solution	Questions
Total				
Overall Average				

Names	Presentation	Clarity	Solution	Questions
Total				
Overall Average				

Names	Presentation	Clarity	Solution	Questions
Total				
Overall Average				

Audience Participation Assessment Form
(All students begin with 10 points)

Name	Score

Modelling – Lesson 2 – Evalutions of Group Members

			Participation
Group Members :	**1.**	______________________	**/10**
	2.	______________________	**/10**
	3.	______________________	**/10**
	4.	______________________	**/10**
	5.	______________________	**/10**

In doing this evaluation, you are to consider the extent to which each member of the group participated their ideas, their time, their efforts. You should evaluate each member of the group as well as yourself. Remember to be Honorbound and to grade each member honestly.

Modelling – Lesson 2 – Evalutions of Group Members

			Participation
Group Members :	**1.**	______________________	**/10**
	2.	______________________	**/10**
	3.	______________________	**/10**
	4.	______________________	**/10**
	5.	______________________	**/10**

In doing this evaluation, you are to consider the extent to which each member of the group participated their ideas, their time, their efforts. You should evaluate each member of the group as well as yourself. Remember to be Honorbound and to grade each member honestly.

Modelling- Lesson 2 - Oral Presentation

Group Members :		Participation	Total
	1. ______________________	/10	/40
	2. ______________________	/10	/40
	3. ______________________	/10	/40
	4. ______________________	/10	/40
	5. ______________________	/10	/40

In determining the individual participation grade of each student you are to consider the extent to which that person was a valid participant of the group presentation. They spoke to the audience in a clear voice and it was evident that they were prepared and knew what they were talking about.

Group Evaluation

Clarity – [___ / 10] *The presentation was clear. You were able to understand the problem and were clear on the different aspects of the model. The group did a good job in explaining the problem, the model and the Mathematics used to solve the problem.*

Accuracy – [___ / 10] *The description was accurate. The students used terminology that was correct and Mathematics that were valid in describing and analyzing the model. The points made were valid and accurate.*

Completeness – [___ / 10] *The group did a good job in analyzing the model. They clearly touched on all the points including what the problem was, the mathematics used, the assumptions, the strengths, the weeknesses, etc..*

Modeling- Lesson 3 - Oral Presentation

		Participation	Total
Group Members :	1. ____________________	/10	/40
	2. ____________________	/10	/40
	3. ____________________	/10	/40
	4. ____________________	/10	/40

In determining the individual participation grade of each student you are to consider the extent to which that person was a valid participant of the group presentation. They spoke to the audience in a clear voice and it was evident that they were prepared and knew what they were talking about.

Group Evaluation

Clarity – [___ / 10] *The presentation was clear. You were able to understand the problem and were clear on the different aspects of the model. The group did a good job in explaining the problem, the model and the Mathematics used to solve the problem.*

Accuracy – [___ / 10] *The description was accurate. The students used terminology that was correct and Mathematics that were valid in describing and analyzing the model. The points made were valid and accurate.*

Completeness – [___ / 10] *The group did a good job in analyzing the model. They clearly touched on all the points including what the problem was, the mathematics used, the assumptions, the strengths, the weaknesses, etc..*

Modeling- Lesson 3 - Oral Presentation

		Participation	Total
Group Members :	1. ____________________	/10	/40
	2. ____________________	/10	/40
	3. ____________________	/10	/40
	4. ____________________	/10	/40

In determining the individual participation grade of each student you are to consider the extent to which that person was a valid participant of the group presentation. They spoke to the audience in a clear voice and it was evident that they were prepared and knew what they were talking about.

Group Evaluation

Clarity – [___ / 10] *The presentation was clear. You were able to understand the problem and were clear on the different aspects of the model. The group did a good job in explaining the problem, the model and the Mathematics used to solve the problem.*

Accuracy – [___ / 10] *The description was accurate. The students used terminology that was correct and Mathematics that were valid in describing and analyzing the model. The points made were valid and accurate.*

Completeness – [___ / 10] *The group did a good job in analyzing the model. They clearly touched on all the points including what the problem was, the mathematics used, the assumptions, the strengths, the weaknesses, etc.*

Algebra II – Analysis of a Student Model

Your task involves analyzing a model which has been written by a high school student. You should first describe the model being sure to fully explain what the student author did so that your pears will understand the model. Secondly, you need to provide a critique of the model using the questions in Lesson 1, Activity 2 as a guide.

Assessment of your work will be made up of the following components:

1. Oral Presentation .... 40 points
   - *evaluated by classmates and by me*
2. Group Participation ..... 10 points
   - *evaluated by yourself and others of your group*

For the Oral Presentation, each student will be evaluated on their individual participation in the presentation. The group as a whole will be evaluated on the clarity of their work, on the accuracy of their work and on the completeness of the analysis. Each of these four aspects will be graded on a scale of 1-10 with 10 being the highest grade.

For the Group Participation, each student will evaluate the other members of the group on their participation within the group structure relative to the other members of the group.

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Assessment of your work will be made up of the following components:

1. Oral Presentation .... 40 points
   - *evaluated by classmates and by me*
2. Group Participation ..... 10 points
   - *evaluated by yourself and others of your group*

For the Oral Presentation. each student will be evaluated on their individual participation in the presentation. The group as a whole will be evaluated on the clarity of their work. on the accuracy of their work and on the completeness of the analysis. Each of these four aspects will be graded on a scale of 1-10 with 10 being the highest grade.

For the Group Participation, each student will evaluate the other members of the group on their participation within the group structure relative to the other members of the group.

Group Members:

1. ______________________________ ____/40
2. ______________________________ ____/40

Oral and Visual Presentation Critique

Oral	Visual
___/8 Clarity, speaks to audience, refers to visual	____/2 Well designed , good use of color, texture, shading, pictures & diagrams
___/8 Accuracy, model was correct	____/2 Eye Catching, creative & original
___/8 Completeness, presentation was planned and well presented	____/2 Thoroughly executed, could understand the model by just viewing the visual
___/8 Real-world relativity, thought out	____/2 Complete bibliography
___/32 Total	____/8 Total

Group Members:

1. ______________________________ ____/40
2. ______________________________ ____/40

Oral and Visual Presentation Critique

Oral	Visual
___/8 Clarity, speaks to audience, refers to visual	____/2 Well designed , good use of color, texture, shading, pictures & diagrams
___/8 Accuracy, model was correct	____/2 Eye Catching, creative & original
___/8 Completeness, presentation was planned and well presented	____/2 Thoroughly executed, could understand the model by just viewing the visual
___/8 Real-world relativity, thought out	____/2 Complete bibliography
___/32 Total	____/8 Total

Model Critique

1. **Identify the problem** *(3 points)*

2. **Factors and Assumptions** *(5 points)*

3. **The Mathematics** *(7 points)*

4. **Conclusion** *(5 points)*

In determining the scores of the model you are to evaluate, use the criteria established in Lesson 1, Activity 2 of the Modeling Unit.

TOTAL /20

Model Critique

1. **Identify the problem** *(3 points)*

2. **Factors and Assumptions** *(5 points)*

3. **The Mathematics** *(7 points)*

4. **Conclusion** *(5 points)*

In determining the scores of the model you are to evaluate, use the criteria established in Lesson 1, Activity 2 of the Modeling Unit.

TOTAL /20

Evaluation of Modeling Project

The project should:

1. Identify the situation

- Clearly state the central question /5 points
- Grab the interest of the reader by including facts, quotes and/or pictures to illustrate and give flavor to the investigation /5 points
- States what others have already written on the topic (shows evidence of research) /5 points

2. Simplifies the situation

- Clearly list facts and key features of the model /5 points
- Clearly states the assumptions to be considered in the model /5 points

3. The Model

- Identifies the variables /5 points
- Translates the assumptions into mathematical terms /5 points
- Organizes and analyzes the data /5 points
- Uses tables, scatterplots /5 points
- Identifies an equation or rule with residuals or states the lack of a rule or equation /5 points
- Revises or refines the model in mathematical terms /5 points

4. Evaluates and applies the model

- Illustrates the results of the equation or calculations /5 points
- Uses the results to predict or explain /5 points
- States clearly how the model answers the key question /5 points
- Applies and checks the model results /5 points

5. Bibliography

- States clearly all resources used /5 points

/ 80 total

Mathematical Modeling
Ideas and Resources

MATHEMATICAL MODELING TASK FORCE

1) Math Modeling Task Force Final Documents
https://sites.google.com/a/cmpso.org/caccss-resources/high-school-modeling-task-force/high-school-modeling-resources
Documents available for people who plan to deliver workshops and presentations on mathematical modeling. Please attribute the Task Force, in general, and the publisher of each item specifically.

ADDITIONAL SOURCES FOR MODELING PROBLEMS

2) MAP/MARS Problems from the Shell Centre
http://map.mathshell.org.uk/materials/tasks.php

3) LEMA Project
http://www.lema-project.org/web.lemaproject/web/dvd_2009/english/teacher.html
Several individual problems along with video and teacher training materials.

4) Modeling Problems at COMAP
http://www.mathmodels.org/problems/

5) Handbook on Modeling (COMAP) Sampler
http://www.comap.com/Philly/CCSSModelingHB.pdf
Complete treatment of four problems you can use for math modeling.

6) Dana Center at the University of Texas
www.utdanacenter.org
Mathematical Models Clarifying Activities
http://www.utdanacenter.org/mathtoolkit/instruction/activities/models.php

7) NCTM Illuminations
http://illuminations.nctm.org/LessonDetail.aspx?ID=U142
http://www.figurethis.org/index.html
Figure This! Problems can sometimes be used as lesson openers.

8) Model Math (Greece)
http://www.modelmath.eu/
The identification of real problems that professional engineers face in their daily work and the development of a database of these problems. Based on these problems, a set of modified engineering problems, suitable for elementary and secondary school students will be developed.

9) Teaching Math Grades 5-8
http://www.learner.org/resources/series33.html
Dozens of video clips on mathematical topics within real-life contexts. The videos are funded by Annenberg and can be streamed without cost.

10) Futures Channel
http://www.thefutureschannel.com/index.php

MODELING CONTESTS

11) COMAP Mathematical Contest in Modeling
http://www.comap.com/undergraduate/contests/mcm/previous-contests.php

12) Moody Modeling Problems
http://m3challenge.siam.org/problem/sampleProblems.php
Society for Applied and Industrial Mathematics. Contest problems available.

13) Cornell Mathematical Contest in Modeling
http://www.math.cornell.edu/~mcm/2011/
Problems from the local university and the international contest sponsored by COMAP.

ARTICLES AND MISCELLANEOUS

14) California Mathematics Council
www.cmc-math.org
Links to latest information on Common Core and on newest assessments.

15) History and Issues With Modeling
http://cimm.ucr.ac.cr/ciaem/articulos/universitario/conocimiento/Modelling%20in%20Mathematics%20Classrooms:%20reflections%20on%20past%20developments%20and%20the%20future*Burkhardt,%20Hugh,*Hugh%20Burkhardt%20(USA),%20with%20contributions%20by%20Henry%20Pollak%20(US.pdf
Interview with Hugh Burkhardt and Henry O. Pollak

16) Meaningful Mathematics
http://www.meaningfulmath.org/modeling
Links to a variety of mathematical problems, including Wolfram math content videos.

17) Comprehensive Introduction to Math Modeling
http://www.math.colostate.edu/~pauld/M331/ch1%5B1%5D.pdf

18) Singapore Children Math Modeling
http://www.recsam.edu.my/R&D_Journals/YEAR2009/june2009vol1/mathmodelling(36-61).pdf

19) STAR Analytical Services
http://www.staranalyticalservices.com/index.html
Company that uses math modeling to solve real-life problems. Examples shared.

20) UMAP Modules
http://www.cengage.com/math/book_content/0495011592_giordano/student_cd/START_HERE.html
Higher-level modeling problems from COMAP.

21) Optimization Problems
http://www.shoreline.edu/SarahLeyden/Classes/Math141_old/Optimization.pdf

22) Mathematics in Life Posters
http://www.mathaware.org/index.html
These are the posters in the math tutoring room at CSUSB.

Models

1. **Elevator Problem**

Suppose a building has 5 floors (1 – 5) which are occupied by offices. The ground floor (0) is not used for business purposes. Each floor has 80 people working on it, and there are 4 elevators available. Each elevator can hold 10 people at one time. The elevators take 3 seconds to travel between floors and average 22 seconds on each floor when someone enters or exits. If all of the people arrive at work at about the same time and enter the elevators on the ground floor, how should the elevators be used to get the people to their offices as quickly as possible?

2. **Pasture Land**

A rancher has a prize bull and some cows on his ranch. He has a large area for pasture that includes a stream running along one edge. He must divide the pasture into two regions, one region large enough for the cows and the other, smaller region to hold the bull. The bull's pasture must be at least 1,000 sq. meters (grazing) and the cow pasture must be at least 10,000 sq. meters to provide grazing for the cows. The shape of the pasture is basically a rectangle 120 meters by 150 meters. The river runs all the way along the 120 meter side. Fencing costs $5/meter and each fence post costs $10.00. Any straight edge of fence requires a post every 20 meters and any curved length of fence requires a post every 10 meters. Help the rancher minimize his total cost of fencing.

3. **Faculty**

When confronted with a rise of 142 students in a school of 480, and a capacity for 7 new teachers, in what departments should the new teachers be placed? Placing the new teachers should maintain the ideal student-teacher ratio. The current makeup of the faculty and student enrollment in each department is:
Art 1, 99 Biology 4, 319 Chemistry 3, 294 English 5, 480 French 1, 122 German 1, 51
Spanish 1, 110 Mathematics 6, 613 Music 1, 95 Physics 3, 291 Social Studies 4, 363

4. **Flu epidemic**

A certain population of 20,000 people is hit by a strain of flu. 1% of the susceptible population per day is stricken, the flu effects last 5 days, after which the person is then immune to this virus. 10% of the population is naturally immune to the virus.

1. If the epidemic were allowed to run its course, how long would it take for the entire population to be immune?

2. The Health Service would like to institute an immunization program in an attempt to knock out the epidemic quickly, within a month if possible. However, they would like to inoculate as few people as possible because the serum does occasionally make people feel sick.