

Data Analysis, Statistics, Probability
Comparison of NCTM to CCSS

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The Comparisons

Grade Levels	Comparisons	
K-8	PSSM	Focal Points
	PSSM	CCSS
9-12	PSSM	CCSS

PSSM – pre-K through 12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

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PSSM ↔ FOCAL POINTS

Color Coding Legend

Identified in the Grade Band	Not Identified in the Grade Band
<ul style="list-style-type: none"> ● yellow indicates prekindergarten; ● green indicates kindergarten; ● red indicates grade 1; ● blue indicates grade 2. 	<ul style="list-style-type: none"> ● Purple indicates content that appears in the focal points or connections of a grade level that is outside the grade band shown in a table. ○ White indicates content that is not identified as a focal point or connection at any grade level, pre-K–grade 5.

Comparison of PSSM to Focal Points pre-K through 2

● yellow indicates prekindergarten;
● green indicates kindergarten;
● red indicates grade 1;
● blue indicates grade 2.

Data Analysis and Probability, Pre-K–Grade 2

- Pose questions and gather data about themselves and their surroundings
- ● Sort and classify objects according to their attributes and organize data about the objects
- Represent data using concrete objects, pictures, and graphs
- ● Describe parts of the data and the set of data as a whole to determine what the data show
- Discuss events related to students' experiences as likely or unlikely [In Grade 7 Curriculum Focal Points]

Comparison of PSSM to Focal Points Grades 3 through 5

● yellow indicates grade 3;
● green indicates grade 4;
● red indicates grade 5.

Data Analysis and Probability, Grades 3–5

- ● Design investigations to address a question and consider how data-collection methods affect the nature of the data set
- ● ● Collect data using observations, surveys, and experiments
- ● ● Represent data using tables and graphs such as line plots, bar graphs, and line graphs
- Recognize the differences in representing categorical and numerical data

Comparison of PSSM to Focal Points Grades 3 through 5 Continued

● yellow indicates grade 3;
● green indicates grade 4;
● red indicates grade 5.

Data Analysis and Probability, Grades 3–5 (Continued)

- ● ● Describe the shape and important features of a set of data and compare related data sets, with [In Grade 8 Curriculum Focal Points] an emphasis on how the data are distributed
- Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set. [In Grade 8 Curriculum Focal Points]
- ● ● Compare different representations of the same data and evaluate how well each representation shows important aspects of the data. [Also in Grade 8 Curriculum Focal Points]
- ● ● ○ Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions. [Designing such studies is not identified as a focal point or connection.]

Comparison of PSSM to Focal Points Grades 3 through 5 Continued

● yellow indicates grade 3;
● green indicates grade 4;
● red indicates grade 5.

- Describe events as likely or unlikely and discuss the degree of likelihood using such words as *certain*, *equally likely*, and *impossible* [In Grade 7 Curriculum Focal Points]
- Predict the probability of outcomes of simple experiments and test the predictions. [In Grade 7 Curriculum Focal Points]
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1. [In Grade 7 Curriculum Focal Points]

Comparison of PSSM to Focal Points Grades 6 through 8

Data Analysis and Probability, Grades 6-8

- Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population.
- Interpret, compare, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.
- Find, use, and interpret measures of center and spread, including mean and interquartile range.
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem and leaf plots, box plots, and scatterplots.
- Use observations about differences between two or more samples to make inferences about the populations from which the samples were taken.
- Make connections about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit.
- Use computers to formulate new questions and plan new studies to answer them.
- Understand and use appropriate terminology to describe complement and mutually exclusive events.
- Use probability and a basic understanding of probability to make and test conjectures about the results of experiments and simulations.
- Compare probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Legend:
 yellow indicates grade 6C
 green indicates grade 7C
 red indicates grade 8C
 Purple indicates content that appears in the focal points or correlates to a grade level that is not identified as a focal point or correlate at any grade level pre-K-grade 8.
 White indicates content that is not identified as a focal point or correlate at any grade level pre-K-grade 8.

Conclusions

Data Analysis, Statistics, Probability
 Comparison of NCTM to CCSS

PSSM ↔ CCSS

PSSM – pre-K through 12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Understand and apply basic concepts of probability

Pre-K through Grade 2

PSSM	CCSS
<ul style="list-style-type: none"> pose questions and gather data about themselves and their surroundings; sort and classify objects according to their attributes and organize data about the objects; represent data using concrete objects, pictures, and graphs. describe parts of the data and the set of data as a whole to determine what the data show. discuss events related to students' experiences as likely or unlikely. 	K 1 Organize, represent, and interpret data with up to three categories 2 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.

Grade 3 through Grade 5

PSSM	CCSS
<ul style="list-style-type: none"> design investigations to address a question and consider how data-collection methods affect the nature of the data set; collect data using observations, surveys, and experiments; represent data using tables and graphs such as line plots, bar graphs, and line graphs; recognize the differences in representing categorical and numerical data. describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed; 	<ul style="list-style-type: none"> use measures of center, focusing on the median, and understand what each does and does not indicate about the data set; compare different representations of the same data and evaluate how well each representation shows important aspects of the data.

Grade 3 through Grade 5

PSSM	CCSS
<ul style="list-style-type: none"> Compare different representations of the same data and evaluate how well each representation shows important aspects of the data. Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or prediction. Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible Predict the probability of outcomes of simple experiments and test the predictions. Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1. Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions. Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible; Predict the probability of outcomes of simple experiments and test the predictions; Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1. 	

Grade 3 through Grade 5

Grade	CCSS
3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Generate measurement data by measuring lengths. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs.
4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.
5	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

Conclusions

Grade 6 through Grade 8	
PSSM	
<ul style="list-style-type: none"> Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots. Find, use, and interpret measures of center and spread, including mean and interquartile range; 	<ul style="list-style-type: none"> Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots. Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken; Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit;

Grade 6 through Grade 8 cont.	
PSSM	
<ul style="list-style-type: none"> Use conjectures to formulate new questions and plan new studies to answer them. Understand and use appropriate terminology to describe complementary and mutually exclusive events; 	<ul style="list-style-type: none"> Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Grade 6 through Grade 8	
Grade	CCSS
6	<ul style="list-style-type: none"> Develop understanding of statistical variability. Summarize and describe distributions. Use random sampling to draw inferences about a population. Understand that random sampling tends to produce representative samples and support valid inferences.
7	<ul style="list-style-type: none"> Draw informal comparative inferences about two populations. Investigate chance processes and develop, use, and evaluate probability models. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
8	<ul style="list-style-type: none"> Investigate patterns of association in bivariate data. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects.

Conclusions



High School	
PSSM	
<ul style="list-style-type: none"> Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each; Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments; Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable; Understand histograms, parallel box plots, and scatterplots and use them to display data; Compute basic statistics and understand the distinction between a statistic and a parameter. 	<ul style="list-style-type: none"> For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics; For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools; Display and discuss bivariate data where at least one variable is categorical; recognize how linear transformations of univariate data affect shape, center, and spread; Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled

High School cont.	
PSSM	
<ul style="list-style-type: none"> Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions; Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference; Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions; Understand how basic statistical techniques are used to monitor process characteristics in the workplace. 	<ul style="list-style-type: none"> Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases; Use simulations to construct empirical probability distributions; Compute and interpret the expected value of random variables in simple cases; Understand the concepts of conditional probability and independent events; Understand how to compute the probability of a compound event.

Grade 6 through Grade 8	
CCSS	
Interpreting Categorical and Quantitative Data	
<ul style="list-style-type: none"> Summarize, represent, and interpret data on a single count or measurement variable Summarize, represent, and interpret data on two categorical and quantitative variables Interpret linear models 	
Making Inferences and Justifying Conclusions	
<ul style="list-style-type: none"> Understand and evaluate random processes underlying statistical experiments Make inferences and justify conclusions from sample surveys, experiments and observational studies 	
Conditional Probability and the Rules of Probability	
<ul style="list-style-type: none"> Understand independence and conditional probability and use them to interpret data Use the rules of probability to compute probabilities of compound events in a uniform probability model 	
Using Probability to Make Decisions	
<ul style="list-style-type: none"> Calculate expected values and use them to solve problems Use probability to evaluate outcomes of decisions 	

Conclusions



So Now What?????

How About A Blend?

Academic Standards for Mathematics*

Grades Pre K – High School
February 27, 2012



Pennsylvania Department of Education
*These draft versions of the PA Common Core State Standards pending approval by the State Board.

Organization of the PA Common Core

PA Common Core Standards
Mathematical Content and Mathematical Practice

Standards for Mathematical Content	Standards for Mathematical Practice
2.1 Numbers and Operations A) Counting and Cardinality B) Number and Operations in Base Ten C) Number and Operations—Fractions D) Ratio and Proportional Relationships E) The Number System F) Number and Quantity	Standards for Mathematical Practice <ul style="list-style-type: none"> Make sense of problems and persevere in solving them. Use appropriate tools strategically. Reason abstractly and quantitatively. Attend to precision. Construct viable arguments and critique the reasoning of others. Look for and make use of structure. Model with mathematics. Look for and make sense of regularity in repeated reasoning.
2.2 Algebraic Concepts A) Operations and Algebra: Thinking B) Expressions & Equations C) Functions D) Algebra	
2.3 Geometry A) Geometry	
2.4 Data Analysis and Probability A) Measurement and Data B) Statistics and Probability	

Development and Progression

Mathematical Standards: Development and Progression
Standards for Mathematical Practice

Number of students and progression of standards across grades with emphasis on the modeling of others. Standard for practice: Apply to content standards. Standard for practice: Apply to content standards.

Grade	Pre-K	0	1	2	3	4	5	6	7	8	HS	
2.1 Numbers and Operations												
2.2 Algebraic Concepts												
2.3 Geometry												
2.4 Data Analysis and Probability												

PA CC – pre-K through 5

2.A.PREK Grade PreK	2.A.K Grade K	2.A.1 Grade 1	2.A.2 Grade 2
2.A.PREK.A.4 Classify objects and count the number of objects in each category.	2.A.K.A.4 Classify objects and count the number of objects in each category.	2.A.1.A.4 Represent and interpret data using tables/charts.	2.A.2.A.4 Represent and interpret data using line plots, picture graphs, and bar graphs.
	2.A.3 Grade 3	2.A.4 Grade 4	2.A.5 Grade 5
	2.A.3.A.4 Represent and interpret data using tally charts, tables, pictographs, line plots, and bar graphs.	2.A.4.A.4 Represent and interpret data involving fractions using information provided in a line plot.	2.A.5.A.4 Solve problems involving computation of fractions using information provided in a line plot.

PA CC – Grades 6 through 8

2.A.6 Grade 6	2.A.7 Grade 7	2.A.8 Grade 8
2.A.6.B.1 Use a set of numerical data to develop an understanding of and recognize statistical variability.	2.A.7.B.1 Draw inferences about populations based on random sampling concepts.	2.A.8.B.1 Analyze and/or interpret bivariate data displayed in multiple representations.
2.A.6.B.2 Use numerical data and apply statistical properties to summarize and describe a distribution.	2.A.7.B.2 Draw informal comparative inferences to summarize and describe about two populations.	2.A.8.B.2 Understand that patterns of association can be seen in bivariate data utilizing frequencies.
Intentionally Blank	2.A.7.B.3 Investigate chance processes and develop, use, and evaluate probability models.	Intentionally Blank

PA CC – High School

2.A.HS High School
2.A.HS.B.1 Summarize, represent, and interpret data on a single count or measurement variable.
2.A.HS.B.2 Summarize, represent, and interpret data on two categorical and quantitative variables.
2.A.HS.B.3 Analyze linear models to make interpretations based on the data.
2.A.HS.B.4 Recognize and evaluate random processes underlying statistical experiments.
2.A.HS.B.5 Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
2.A.HS.B.6 Use the concepts of independence and conditional probability to interpret data.
2.A.HS.B.7 Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Use appropriate tools strategically.
- Reason abstractly and quantitatively.
- Attend to precision.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.
- Model with mathematics.
- Look for and make sense of regularity in repeated reasoning.

Grouping the practice standards

1. Make sense of problems and persevere in solving them. 6. Attend to precision.	2. Reason abstractly and quantitatively.	Reasoning and explaining
3. Construct viable arguments and critique the reasoning of others.	4. Model with mathematics.	
5. Use appropriate tools strategically.	7. Look for and make use of structure.	Modeling and using tools
8. Look for and express regularity in repeated reasoning.		Seeing structure and generalizing

Standards for Mathematical Practice in a Classroom



McDonald's Claim

Wikipedia reports that 8% of all Americans eat at McDonalds every day.
310 million Americans and 12,800 McDonalds...
Do you believe the Wikipedia report to be true?
Create a mathematical argument to justify your position.

McDonald's Claim Problem

- Which mathematical *practices* are needed to complete the task?
- What mathematics *content* is needed to complete the task?

Summary

- NCTM and CCSS agree that data analysis, statistics and probability are important
- NCTM and CCSS disagree about timing and ordering to some extent.

But should we blend????

Consider

The Structure is the Standards

Phil Daro, Bill McCallum, Jason Zimba

<http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/#more-422>

What we tend to do?



You have just purchased an expensive Grecian urn and asked the dealer to ship it to your house. He picks up a hammer, shatters it into pieces, and explains that he will send one piece a day in an envelope for the next year.

The Authors' point!

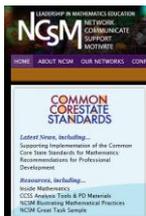
In the Common Core State Standards, individual statements of what students are expected to understand and be able to do are embedded within domain headings and cluster headings designed to convey the structure of the subject. **"The Standards" refers to all elements of the design—the wording of domain headings, cluster headings, and individual statements; the text of the grade level introductions and high school category descriptions; the placement of the standards for mathematical practice at each grade level.**

Their conclusion...

Indeed, it is the nature of mathematics that much new learning is about extending knowledge from prior learning to new situations. For this reason, teachers need to understand the progressions in the standards so they can see where individual students and groups of students are coming from, and where they are heading. But progressions disappear when standards are torn out of context and taught as isolated events.

SO WHAT SHOULD WE DO!

- Attempt to remain true to the vision of the Common Core Standards
- Use available resources from NCSM (www.mathleadership.org) and other sources to inform your implementation



Questions? Comments?

