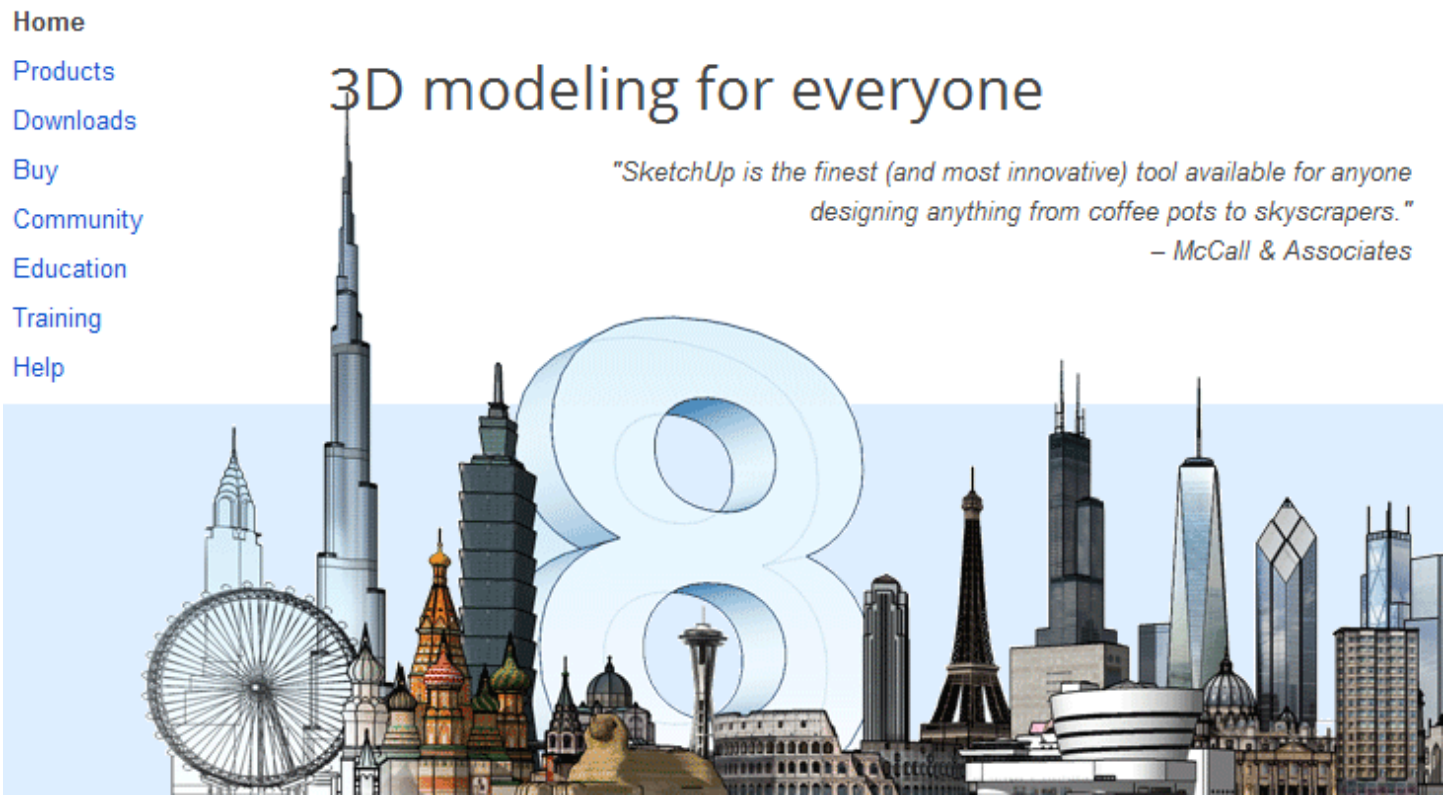
# Exploring Middle Grades Geometry Using Google SketchUp




*National Council of Teachers of Mathematics*
*Annual Conference ~ Philadelphia, PA*
Friday, April 27, 2012

Suzanne Harper, Miami University, harpersr@muohio.edu
Shannon Driskell, University of Dayton, sdriskell1@udayton.edu

**Description:** Google SketchUp is a 3D modeling program designed for educators, architects, civil engineers, filmmakers, game developers, and related professions. It is designed to be easier to use than other 3D CAD programs. The software (version 8) can be downloaded for free at http://sketchup.google.com/. A great resource for SketchUp is *Google 3D Warehouse* http://sketchup.google.com/intl/en/product/3dwh.html. It's a website that provides a place where modelers can share their models on the internet. It has many models for creating a three dimensional environment. Another great resource for some SketchUp basic skills is available through Dr. Kathryn Shafer's website https://sites.google.com/site/kgshafertpack/su-videos

Complete handout available: http://www.users.muohio.edu/harpersr/conferences/NCTM2012.pdf

## Activity 1: Drawing Special Polyhedra

In this activity, students will construct various polyhedra using Google SketchUp. We will emphasize using appropriate geometric vocabulary such as: face, vertex (vertices), edge, prism, and pyramid. Students will also investigate relationships involving the number of edges, the number of faces, and the number of vertices in different types of polyhedra.

Common Core Standards - Grade 7 Geometry

7.G.2. Draw (freehand, with ruler and protractor, and with technology) geometric (two- and three-dimensional) shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

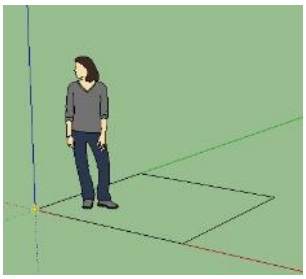
Directions:

1. Using Google SketchUp, draw a 6' x 6' x 6' cube. (Note: For a screen cast of this construction click here.)

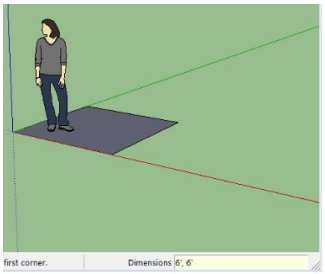
a. Open SketchUp and click the Chose Template Button. Select **Simple Template - Feet and Inches**. Then, click on Start Using SketchUp.

b. After selecting the feet and inches template a Google SketchUp worksheet should open. If this is your first time opening the template, you may want to open the large tool set to view all of the tools. Click View->Toolbars->Large Toolset.

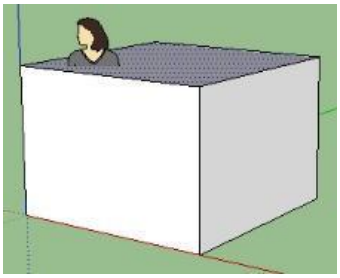
c. Select the rectangle tool and left-click on the origin and begin to drag along the red and green axes.



d. To create a 6' X 6' square, type 6', 6' and press enter. You should see the dimensions you typed appear in the Dimensions box located in the bottom right corner of your screen.

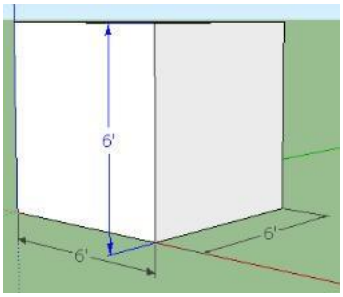



e. Next, select the push/pull tool. Click on the square and simultaneously drag the mouse up to make the square three dimensional.



f. To create a height of 6', type 6' and press enter. The dimension will appear in the box located in the bottom right corner of your screen. Congratulations, you created a 6' x 6' x 6' cube!

g. To verify the dimensions, select the dimensions button. Select two endpoints, drag the dimension to a viewable area, and click to set the dimension.



2. Answer the following questions about the cube. Fill in the blank:

Number of Faces _____Number of Vertices _____ Number of Edges _____

3. Drag your cube to a different location in your window and continue to draw a triangular prism, a non-square rectangular prism, an equilateral triangular pyramid (tetrahedron), and a square pyramid. Draw one more polyhedron not represented in the previous list.

4. Leonhard Euler was a famous mathematician who noticed a relationship among the vertices, faces, and edges. Using your drawings of various polyhedra, fill in the following table and calculate the last column. Euler's relationship involves the last two columns of the table. Hypothesize a conjecture and verify your conjecture using a polyhedron that is not listed in the table below.

| Polyhedron | Number of Faces (F) | Number of Vertices (V) | Number of Edges (E) | F+ V |
|---|---|---|---|---|
| cube | | | | |
| triangular prism | | | | |
| non-square rectangular prism | | | | |
| tetrahedron | | | | |
| square pyramid | | | | |
| | | | | |
| | | | | |
| | | | | |

Extension:
Consider non-simple polyhedra (polyhedra with holes). Is the relationship you found in #4 still true? Construct 2-3 examples using Google SketchUp to support your claim.

# Activity 2: Cross Sections of a Cube

Activity adapted from: http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html

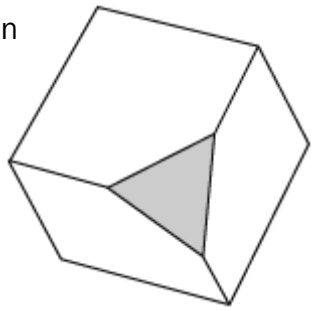
Students will construct a cube using Google SketchUp and use their problem-solving skills to decide how to slice the cube using a single plane to define various shaped cross-sections. Students who have had little experience with spatial-visualization activities will likely have difficulty visualizing these cross sections without a model because they cannot envision the unseen faces of the shape. The use of Google SketchUp will aid in this type of visualization.

Common Core Standards - Grade 7 Geometry

7.G.3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of 2-D objects.

Directions: A cross section is the face you get when you make one slice through an object. To the right is a sample slice through a cube, showing one of the cross sections you can construct.



The polygon formed by the slice is the cross section. The cross section cannot contain any piece of one of the original faces of the cube. In this picture, only the gray piece is a cross section.

1. Using Google SketchUp, construct a cube and slice it using a single plane to create the following cross sections:

| | | |
|---|---|---|
| a) an equilateral triangle | d) a rectangle, that is not a square | g) a hexagon |
| b) a square | e) a triangle, that is not equilateral | h) an octagon |
| c) a pentagon | f) a parallelogram, that is not a rectangle | i) a circle |

Record which of the shapes you were able to create and how you constructed them.

2. A few of the shapes in Problem 1 are impossible to make by slicing a cube. Explain why they are impossible to create.

3. What possible cross sections can you obtain from any right rectangular prism?
Keep a detailed record of your solutions.

4. What possible cross sections can you obtain from any right rectangular pyramid?
Keep a detailed record of your solutions.

5. Compare your solutions to Problems 3 & 4.

6. Find a way to slice a tetrahedron to create a square cross section. Explain your solution to your neighbor.

# Activity 3A: Google SketchUp Scaling Design Project

Activity adapted from: http://www.discoveryeducation.com/teachers/free-lesson-plans/discovering-math-exploring-geometry.cfm

Students will construct a three-dimensional model of a city using Google SketchUp. Working either individually or in a small group, students will prepare a written portfolio on the mathematical construction of a three-dimensional model. The project will consist of a Google SketchUp file, as well as a written mathematical analysis from each group member.

Common Core Standards - Grade 7 Geometry

7.G.1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
7.G.6. Solve real-world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Criteria of the Project

1. Construct a three-dimensional model of a city using Google SketchUp.

   Your city must have at least 8 buildings (clearly labeled A, B, C, etc,) with the following criteria:
   - Building A and Building B are geometrically congruent;
   - Building C and Building D are mathematically similar with a scale factor less than 1;
   - Building E and Building F are mathematically similar with a scale factor greater than 1;
   - Building G is a reflected image of one of the other Buildings; and
   - Building H is a rotated image of one of the other Buildings.

   Also, add streets to the city using line segments; and name your streets using the Text tool.

2. Write a paragraph about your Building construction process using Google SketchUp. Include a description of the streets using the terms such as parallel, perpendicular, and intersecting. Your description should be written in complete sentences using appropriate mathematical vocabulary.

3. Compute the surface area of Buildings C and E. Using the scale factor between Buildings C and D, describe how to compute the surface area of Building D. Then, do the same for Buildings E and F. Your documentation must be clearly written in complete sentences along with mathematical equations using correct measurement units.

4. The City Council would like you to create an actual three-dimensional model of your city out of clay. Since you cannot build the model to its true scale, you must create a plan to scale down your model. Also, before you get started on the model's construction, you must figure out how much clay you will need to purchase to construct all of your buildings. Describe and compute the volume of your buildings in terms of the scale factor you selected. Write about the scaling process and clearly document your volume calculations using correct measurement units.

# Activity 3B: Google SketchUp Scaling Design Project

Adapted from: Shafer, K.G., Severt, G., & Olson, Z.A. (2011). Sketching up the digital duck. *Mathematics Teacher, 105*(4), 262-268.

Students will construct a model of a three-dimensional object using Google SketchUp. Work with your small group (3 people per group) to prepare a written portfolio on the mathematical construction of a three-dimensional object. Your project will consist of both technology and print copies of your construction, as well as a written reflection from each group member.

| Project Component | Description | Guidelines |
|---|---|---|
| **Section 1:** Design of Solid | The base of your shape must include at least one arc and one angle that is not 90°. The height must be greater than 1cm (use metric measurements throughout the project). | Submit a sketch of your base design (hand drawn on graph paper). |
| **Section 2:** Construction of the Base in GeoGebra | Create a net of the base in GeoGebra with the segments and angle measurements shown on the page. | Print out a copy of the GeoGebra file. |
| Construction of Solid | Cut out matching bases from cardboard, and use one strip of construction paper for the lateral edges. Tape the three pieces together. | Submit your solid, clearly labeled with group member names. |
| **Section 3:** Construction of Solid in Google SketchUp | Use Google SketchUp to create a three-dimensional image of your shape with the linear measurements shown. | Print out two different views from the Google SketchUp file. |
| **Section 4:** Surface Area and Volume | Clearly document the dimensions of your shape and the computation used to find the lateral surface area, total surface area, and volume. | Your documentation must be written in complete sentences, along with mathematical equations using correct measurement units. It must be easy to follow. |
| **Section 5:** Scaling of Solid | Using the scale factor of either **0.75** or **1.25** (your choice), clearly document the dimensions of your shape and the computations used to find the lateral surface area, total surface area, and volume. | Your documentation must include a Google SketchUp image with linear measurements shown; and a description written in complete sentences along with mathematical equations using correct measurement units. It must be easy to follow. |
| **Section 6:** Personal Reflection | Answer the following prompts:<br>• Document the contributions made by you and your group members.<br>• Reflect on this project as a future middle-school geometry teacher. | Type and submit your reflection as the last pages of your portfolio. |

Adapted from: Shafer, K.G., Severt, G., & Olson, Z.A. (2011). Sketching up the digital duck. *Mathematics Teacher, 105*(4), 262-8.

# SketchUp 8 Quick Reference Card

Windows

## Large Tool Set


## Solid Tools


## Dynamic Components


## Sandbox (Terrain)


## Standard Views


## Style


## Google


Display additional toolbars by choosing View > Toolbars from the menu bar.


| | | |
|---|---|---|
| Middle Button (Wheel) | Scroll | Zoom |
| | Click-Drag | Orbit |
| | Shift+Click-Drag | Pan |
| | Double-Click | re-center view |
| Right Mouse Button | Click | show context menu |

Google SketchUp

| Tool | Operation | Instructions |
|---|---|---|
| Arc (A) | Bulge | specify bulge amount by typing a number and Enter |
| | Radius | specify radius by typing a number, the R key, and Enter |
| | Segments | specify number of segments by typing a number, the S key, and Enter |
| Circle (C) | Shift | lock in current plane |
| | Radius | specify radius by typing a number and Enter |
| | Segments | specify number of segments by typing a number, the S key, and Enter |
| Eraser (E) | Ctrl | soften/smooth (use on edges to make adjacent faces appear curved) |
| | Shift | hide |
| | Ctrl+Shift | unsoften/unsmooth |
| Follow Me | Alt | use face perimeter as extrusion path |
| | Better Way | first Select path, then choose the Follow Me tool, then click on the face to extrude |
| Line (L) | Shift | lock in current inference direction |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Length | specify length by typing a number and Enter |
| Look Around | Eye Height | specify eye height by typing a number and Enter |
| Move (M) | Ctrl | move a copy |
| | Shift | hold down to lock in current inference direction |
| | Alt | auto-fold (allow move even if it means adding extra edges and faces) |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Distance | specify move distance by typing a number and Enter |
| | External Array | n copies in a row: move first copy, type a number, the X key, and Enter |
| | Internal Array | n copies in between: move first copy, type a number, the / key, and Enter |
| Offset (F) | Double-Click | apply last offset amount to this face |
| | Distance | specify an offset distance by typing a number and Enter |
| Orbit (O) | Ctrl | hold down to disable "gravity-weighted" orbiting |
| | Shift | hold down to activate Pan tool |
| Paint Bucket (B) | Ctrl | paint all matching adjacent faces |
| | Shift | paint all matching faces in the model |
| | Ctrl+Shift | paint all matching faces on the same object |
| | Alt | hold down to sample material |
| Push/Pull (P) | Ctrl | push/pull a copy of the face (leaving the original face in place) |
| | Double-Click | apply last push/pull amount to this face |
| | Distance | specify a push/pull amount by typing a number and Enter |
| Rectangle (R) | Dimensions | specify dimensions by typing length, width and Enter ie. 20,40 |
| Rotate (Q) | Ctrl | rotate a copy |
| | Angle | specify an angle by typing a number and Enter |
| | Slope | specify an angle as a slope by typing a rise, a colon (:), a run, and Enter ie. 3:12 |
| Scale (S) | Ctrl | hold down to scale about center |
| | Shift | hold down to scale uniformly (don't distort) |
| | Amount | specify a scale factor by typing a number and Enter ie. 1.5 = 150% |
| | Length | specify a scale length by typing a number, a unit type, and Enter ie. 10m |
| Select (Spacebar) | Ctrl | add to selection |
| | Shift | add/subtract from selection |
| | Ctrl+Shift | subtract from selection |
| Tape Measure (T) | Ctrl | create a new Guide |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Resize | resize model: measure a distance, type intended size, and Enter |
| Zoom (Z) | Shift | hold down and click-drag mouse to change Field of View |



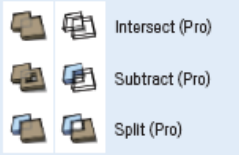
Harper, S.R. & Driskell, S.O. (April 2012). Exploring middle grades geometry using Google SketchUp. Session presented at the National Council of Teachers of Mathematics, Philadelphia, PA. Complete handout available at: http://www.users.muohio.edu/harpersr/conferences/NCTM2012.pdf

# SketchUp 8 Quick Reference Card

Mac OS X

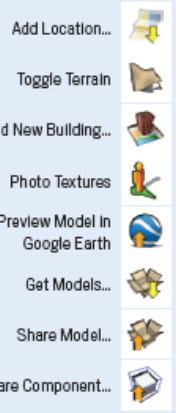
### Large Tool Set


### Solid Tools

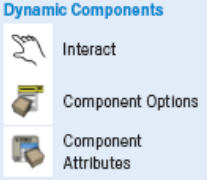



### Sandbox (Terrain)


### Google

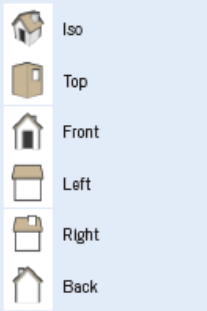



### Dynamic Components




### Style


### Standard Views




To add other tools, right-click the toolbar (at the top of your document window) and choose "Customize Toobar..."


| | | |
|---|---|---|
| Middle Button (Wheel) | Scroll | Zoom |
| | Click-Drag | Orbit |
| | Shift+Click-Drag | Pan |
| | Double-Click | re-center view |
| Right Mouse Button | Click | show context menu |

Google SketchUp

| Tool | Operation | Instructions |
|---|---|---|
| Arc (A) | Bulge | specify bulge amount by typing a number and Enter |
| | Radius | specify radius by typing a number, the R key, and Enter |
| | Segments | specify number of segments by typing a number, the S key, and Enter |
| Circle (C) | Shift | lock in current plane |
| | Radius | specify radius by typing a number and Enter |
| | Segments | specify number of segments by typing a number, the S key, and Enter |
| Eraser (E) | Option | soften/smooth (use on edges to make adjacent faces appear curved) |
| | Shift | hide |
| | Option+Shift | unsoften/unsmooth |
| Follow Me | Command | use face perimeter as extrusion path |
| | Better Way | first Select path, then choose the Follow Me tool, then click on the face to extrude |
| Line (L) | Shift | lock in current inference direction |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Length | specify length by typing a number and Enter |
| Look Around | Eye Height | specify eye height by typing a number and Enter |
| Move (M) | Option | move a copy |
| | Shift | hold down to lock in current inference direction |
| | Command | auto-fold (allow move even if it means adding extra edges and faces) |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Distance | specify move distance by typing a number and Enter |
| | External Array | n copies in a row: move first copy, type a number, the X key, and Enter |
| | Internal Array | n copies in between: move first copy, type a number, the / key, and Enter |
| Offset (F) | Double-Click | apply last offset amount to this face |
| | Distance | specify an offset distance by typing a number and Enter |
| Orbit (O) | Option | hold down to disable "gravity-weighted" orbiting |
| | Shift | hold down to activate Pan tool |
| Paint Bucket (B) | Option | paint all matching adjacent faces |
| | Shift | paint all matching faces in the model |
| | Option+Shift | paint all matching faces on the same object |
| | Command | hold down to sample material |
| Push/Pull (P) | Option | push/pull a copy of the face (leaving the original face in place) |
| | Double-Click | apply last push/pull amount to this face |
| | Distance | specify a push/pull amount by typing a number and Enter |
| Rectangle (R) | Dimensions | specify dimensions by typing length, width and Enter ie. 20,40 |
| Rotate (Q) | Option | rotate a copy |
| | Angle | specify an angle by typing a number and Enter |
| | Slope | specify an angle as a slope by typing a rise, a colon (:), a run, and Enter ie. 3:12 |
| Scale (S) | Option | hold down to scale about center |
| | Shift | hold down to scale uniformly (don't distort) |
| | Amount | specify a scale factor by typing a number and Enter ie. 1.5 = 150% |
| | Length | specify a scale length by typing a number, a unit type, and Enter ie. 10m |
| Select (Spacebar) | Option | add to selection |
| | Shift | add/subtract from selection |
| | Option+Shift | subtract from selection |
| Tape Measure (T) | Option | create a new Guide |
| | Arrows | up or down arrow to lock in blue direction; right to lock in red; left to lock in green |
| | Resize | resize model: measure a distance, type intended size, and Enter |
| Zoom (Z) | Shift | hold down and click-drag mouse to change Field of View |



Harper, S.R. & Driskell, S.O. (April 2012). Exploring middle grades geometry using Google SketchUp. Session presented at the National Council of Teachers of Mathematics, Philadelphia, PA. Complete handout available at: http://www.users.muohio.edu/harpersr/conferences/NCTM2012.pdf